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Modeling Forces in High-Temperature Superconductors

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Abstract--We have developed a simple model that uses computed shielding currents to determine the forces acting on a high-temperature superconductor (HTS). The model has been applied to measurements of the force between HTS and permanent magnets (PM). Results show the expected hysteretic variation of force as the HTS moves first toward and then away from a permanent magnet, including the reversal of the sign of the force. Optimization of the shielding currents is carried out through a simulated annealing algorithm in a C++ program that repeatedly calls a commercial electromagnetic software code. Agreement with measured forces is encouraging.

Index terms--Electromagnetic analysis, finite element methods, high-temperature superconductors, magnetic forces, modeling, optimization methods, simulated annealing.

I. INTRODUCTION

High-temperature superconductors (HTS) hold both intrinsic interest and commercial potential. Consequently many investigators have tried to model the characteristics of HTS, particularly the shielding currents, magnetic forces, and magnetic stiffness [1]. Many of these attempts [2,3] have used the Bean model. Recent ones [4] have used the Preisach hysteresis model as well, because the shielding currents and forces exhibit hysteretic behavior. Most often this modeling has required the development of entirely new software or, at best, the modification [5] of commercial codes. The simple model described here permits the use of a commercial electromagnetic computational code, in this case OPERA-2D [6], without any modification of the code.

II. THE MODEL

Our model represents the HTS by regions of conductor that carry (a) no current, (b) the (specified, constant) critical current density J_c , or (c) the negative of the critical current density. This way of treating the shielding currents was suggested by a paper of Sanchez and Navau [7]. The boundaries of these current-carrying regions within the HTS are varied to minimize the cost function (also called the objective function), which in this case is the integral of the square of flux density within a specified interior portion of the HTS that carries no current. The locations of these boundaries are constrained by the condition that any change of sign in current always originates at the exterior surface of the HTS. Minimization of the cost function is carried out through a simulated annealing procedure.

III. OPTIMIZATION THROUGH SIMULATED ANNEALING

The global optimization technique known as simulated annealing (SA) [8] operates by analogy with the thermodynamic process known as annealing, through which a system achieves a stable state of minimum energy by being cooled slowly to thermal equilibrium.

In the Metropolis algorithm [9] for SA, small random changes are introduced into the configuration of a system, and the change in the system's cost function ΔE is calculated. If ΔE is negative, the changed configuration is accepted; if ΔE is positive, it is accepted with a probability $\exp(-\Delta E/T)$, where T is analogous to the thermodynamic temperature. We have implemented the Metropolis algorithm using C++ and the commercial electromagnetics modeling software OPERA-2D. Our code consists of a set of C++ class libraries and a program that calls OPERA-2D to calculate the magnetic flux integral that serves as our cost function. These SA libraries do not depend on the use of the OPERA-2D software and could operate just as well with a different electromagnetics modeling package.

Simkin and Trowbridge [10] optimized an electromagnetics problem with six continuous variables by using SA to begin the process and then switching over to a direct gradient-descent method.

Fig. 1. Schematic of the force measurements. An HTS hexagonal prism (modeled as a cylinder) is located above a cylindrical PM; the force measured as a function of the separation d between them.

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IV. EXAMPLE: HTS CYLINDER ABOVE A PERMANENT MAGNET DISK

Argonne National Laboratory, along with several other laboratories worldwide, is developing flywheel energy storage (FES) for electric utilities [11, 12], an endeavor made practical by the existence of nearly frictionless HTS bearings. In the process of building several experimental flywheel units, it was necessary to measure the force between the HTS and a neodymium-iron-boron permanent magnet (PM). The yttrium-barium-copper-oxide (YBCO) HTS was in the form of hexagonal prisms, but in the computation the HTS was treated as a cylinder of the same volume. The geometry is shown schematically in Fig. 1.

The computations are performed with OPERA-2D in the axisymmetrical mode, using the magnetostatic solver. An HTS cylinder with height 14.22 mm and radius 10.97 mm, is located above and coaxial with a cylindrical PM with height 12.7 mm and radius 6.35 mm. As in the experimental measurements, the HTS cylinder is moved in axially to a position 14 mm above the PM, and then moved further in as close as 2 mm. It then is moved away from the disk, out to 14 mm. At each position, coordinates of the shielding current region are specified in the HTS, and the cost function described above is evaluated over the current-free region indicated by the square labeled S in Fig. 2. The six coordinates that represent the four corners (N_1 – N_4) of the boundary between positive current and no current (incoming) or the boundary between negative current and positive current (outgoing) are then varied to minimize the cost function ΔE .

Fig. 2. Cross-section of the cylindrical HTS. The nodes N_1 – N_4 define the boundary between regions with and without current. Coordinates of the nodes are varied to minimize the integral of flux density squared over the volume S .

Fig. 3. Convergence of fluxes and forces as a function of accepted iteration. (a) Flux-density integral for all iterations; (b) For last 60 iterations; (c) Computed force for all iterations; (d) For last 60 iterations. The case shown is for $J=18$ A/mm², $d=8$ mm moving in.

Fig. 4. Distribution of shielding current in HTS for three separations between HTS and PM. Top, 14 mm, moving in; middle, 2 mm, moving in; bottom, 14 mm, moving out. The cost function is evaluated over the small white square. Diagonal lines: $J = 18 \text{ A/mm}^2$. Cross hatch: $J = -18 \text{ A/mm}^2$.

With each successful iteration t , the temperature T is decremented according to $T(t) = \alpha^t T(0)$, where α is typically 0.9. We choose $T(0)$ to be somewhat higher than the maximum change in ΔE found in a test run with a very large value of $T(0)$. The final value of T will be determined by whichever of the following three criteria first occurs: (a) maximum number of successful iterations no greater than 300, (b) total number of successful and unsuccessful iterations no greater than 500 to 1500, (c) no more than a 5% change in the force in the previous 50 to 150 successful iterations.

To achieve convergence in a reasonable time, the range over which any coordinate can move must, on average, decrease with time [10], but large changes still must be possible to avoid being trapped in a local minimum. We achieve this by accepting a randomly generated coordinate change Δr with a probability p determined by a Gaussian distribution,

$$p = \exp\left(\frac{-\Delta r^2}{k\gamma^t R_0}\right)$$

where k is a constant analogous to Boltzmann's constant, γ is the range step multiplier (typically 0.9), t is the number of successful iterations, and R_0 is the geometrical limit on the range. If a randomly generated number is greater than the value p , the move is rejected. Because the range obeys a Gaussian distribution, large changes in coordinate can be accepted, but with low probability.

Regardless of the values of the coordinates found from this Gaussian distribution, the boundaries of a current region must not intersect the region S over which the cost function is evaluated, nor may boundaries of positive and negative current regions intersect.

For each successful iteration, the force is computed as an integral of the Maxwell stress tensor around the HTS. Force computations with the Lorentz force over the volume of the HTS agree with those using the Maxwell stress tensor.

V. RESULTS

Figure 3 shows the cost function (integral of the square of the flux density) and the computed force as a function of iteration number for a typical run. Fluxes and forces were computed for current densities of 18 and 36 A/mm^2 , although the actual value is probably closer to 400 A/mm^2 . For higher J values the small thickness of the regions carrying currents can cause meshing problems. At 18 and 36 A/mm^2 , the minimization typically reduces the value of the integral from a very high value to somewhere between 0.2 and 500. After the value falls below 2000 or so, the force varies little.

Figure 4 shows the distribution of shielding current in the HTS for three separations of HTS and PM as the HTS first approaches, then recedes from, the PM.

Figure 5 shows how the computed force varies with separation for two values of J . For comparison, Fig. 6 shows the measured forces, averaged over 50 HTS samples. Note that the force reverses sign for both the measurements and the

computations. The shapes of the curves in Fig. 5 and Fig. 6 are similar, but the magnitudes are smaller in Fig. 5, largely because of the smaller J values. Computation at larger values of J yielded a force at 2 mm of 5.2 N for $J=72 \text{ A/mm}^2$ (lower than the measured 9.9 N), and 13.6 N for $J=400 \text{ A/mm}^2$ (higher than measured).

When multiple runs are made with the same separation, the computed forces are combined, and each result is weighted inversely by the corresponding value of the cost function. In practice, the combined value is not sensitive to how the individual values are weighted.

VII. DISCUSSION

Computation times ranged from 3.4 to 24.5 hours on a 133-MHz HP9000/770 computer. In general, higher current densities required more iterations and more time for the force to converge. A more efficient optimization might result if an initial SA process, as described here, were followed by a direct search, in the manner described in [10].

The C++ package of optimization subroutines will be made available through the Energy Science and Technology Software Center in Oak Ridge, TN.

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REFERENCES

- [1] F. C. Moon, *Superconducting Levitation*, John Wiley, New York, 1994, pp. 54-56.
- [2] M. Uesaka, Y. Yoshida, N. Takeda, and K. Miya, "Experimental and Numerical Analysis of Three-Dimensional High- T_c Superconducting Levitation Systems," *Int. J. of Appl. Electromag. Mater.*, vol. 4, pp. 13-25, 1993.
- [3] Alain Bossavit, "Numerical Modelling of Superconductors in Three Dimensions: A Model and a Finite Element Method," *IEEE Trans. Magn.*, vol. 30, pp. 3363-3366, Sept. 1994.
- [4] I. D. Mayergoyz, "Nonlinear Diffusion and Superconducting Hysteresis," *IEEE Trans. Magn.*, vol. 32, pp. 4192-4197, 1996.
- [5] A. O. Hauser, "Calculation of Superconducting Magnetic Bearings using a Commercial FE-Program (ANSYS), *IEEE Trans. Magn.*, vol. 33, pp. 1572-1575, 1997.
- [6] The code OPERA-2D is available from Vector Fields, Ltd., Oxford, UK, or from Vector Fields, Inc., Aurora, Illinois, USA
- [7] A. Sanchez and C. Navau, "Vertical Force, Magnetic Stiffness and Damping for Levitating Type-II Superconductors," *Physica C*, vol. 268, pp. 46-52, 1996.
- [8] S. Kirkpatrick, C. Gelatt, and M. Vecchi, "Optimization by Simulated Annealing," *Science*, vol. 220, pp. 671-680, 1983.
- [9] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller, "Equation of State Calculations by Fast Computing Machines," *J. Chem. Phys.*, vol. 21, pp. 1087-1092, 1953.
- [10] J. Simkin and C. W. Trowbridge, "Optimizing Electromagnetic Devices Combining Direct Search Methods with Simulated Annealing," *IEEE Trans. Magn.*, vol. 28, pp. 1541-1544, 1992.
- [11] J. R. Hull, T. M. Mulcahy, K. L. Uherka, R. A. Erck, and R. G. Abboud, "Flywheel Energy Storage Using Superconducting Magnetic Bearings," *Appl. Supercond.*, Vol. 2, pp. 449-455, 1994.
- [12] T. M. Mulcahy, J. R. Hull, K. L. Uherka, R. G. Abboud, J. H. Wise, D. W. Carnegie, C. E. Bakis, and C. W. Gabrys, "A Permanent-Magnet Rotor for a High-Temperature Superconducting Bearing," *IEEE Trans. Magn.*, vol. 32, pp. 2609-2612, 1996.

Fig. 5. Computed force acting on HTS vs. separation d for $J=18$ and 36 A/mm^2 . Above: moving in. Below: moving out. Where multiple runs were made for the same condition, the results were weighted by the inverse of the flux-density integral and averaged.

Fig. 6. Measured force acting on HTS vs. separation d , averaged over 50 HTS samples. Error bars show three times the standard deviation of the mean.