Triangular and Y-shaped hadrons in QCD

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Abstract

Gauge invariant extended configurations are considered for 3 fundamental (quarks) or adjoint (gluons) particles. For 3 quarks it is proved that the only possible gauge-invariant configuration is the Y-shaped one, proposed long ago, while the triangular one proposed recently is ruled out. For adjoint sources both configurations are possible. Static potential is computed in all cases. For baryons the Y-shaped configuration leads to a significant depletion of field-strength density at the string-junction position, which decreases the effective potential slope at $R \sim 0.5$ fm by 10-15%, as observed on the lattice. For adjoint sources the Y-shaped and Δ-shaped potentials are very close, leading to almost degenerate masses of 3^{--} 3g glueballs and odderon trajectories.

1. The Y-shaped configuration was suggested long ago [1, 2] and since then was accepted and used in many dynamical calculations [3, 4].

Meanwhile the uniqeness of the Y-shape was questioned in [5], where some arguments have been given that the triangular (Δ-shaped) configuration can be preferred energetically. Moreover, lattice calculations of $3Q$ static potential [6, 7] displayed a potential slope smaller than 3σ, which was attributed by the authors to the Δ-shaped configuration with reference to [5]. (Note however, that in another lattice study [8] this result was not confirmed).

It is one of purposes of this communication to show that the gauge-invariant Δ-shaped configuration is impossible for baryons (or any 3 fundamental sources in SU(3) QCD). At the same time we extend our previous results [9] to show that the static potential of the Y-shaped configuration has a strong depletion at small distances due to a hole in string density at the string-junction position.This fact may explain smaller slope of lattice results in [6, 7].As a second point of this paper we study configuration of 3 adjoint sources (e.g. of 3 valence gluons) and show that it may have two possible shapes: the Y-type and the Δ-type. We compute static potential for both cases. Using that we estimate masses of lowest $3g$ glueballs, lying on the corresponding odderon trajectories, and show that they are close to each other, implying that there are two possible odderon trajectories with two not much different Regge slopes. A short discussion of physical implications of these results concludes the communication.

2. Hadron building in SU(3) starts with listing elementary building blocks: quarks $q^\alpha,\ \alpha = 1, 2, 3$, gluons (or adjoint static sources) $g^a,\ a = 1, ...8$, parallel transporters (PT) in fundamental representation $\Phi^\beta_\alpha(x, y) = (P \exp ig \int A_\mu(z)dz_\mu)^\beta_\alpha$, adjoint parallel transporters $\Phi_{ab}(x, y)$, generators $t^{(a)\beta}_\alpha$ symmetric symbols $\delta^\beta_\alpha,\ \delta_{ab},\ d^{abc}$, and antisymmetric ones, $e_{\alpha\beta\gamma}$ and

f^{abc}. Note that we always use Greek indices for fundamental representation and Latin ones for the adjoint.

To construct a real extended (not point-like) hadron one uses all listed elements, PT included, and forms a white (gauge-invariant) combination. It is convenient to form an extended quark (antiquark) operator

$$q^\alpha(x,Y) \equiv q^\beta(x)\Phi^\alpha_\beta(x,Y);$$

$$\bar{q}_\alpha(x,Y) = \bar{q}_\beta(x)\Phi^\beta_\alpha(x,Y). \tag{1}$$

In this way one has for the Y-shaped baryon:

$$B_Y(x,y,z,Y) = e_{\alpha\beta\gamma}q^\alpha(x,Y)q^\beta(y,Y)q^\gamma(z,Y). \tag{2}$$

One can also define quark operator with two lower indices: $e_{\alpha\beta\gamma}q^\alpha(x) \equiv q_{\beta\gamma}(x)$. However an attempt to create a gauge-invariant combination from 3 operators $q_{\beta\gamma}(x)$ and 3 PT to construct a Δ-type configuration fails: the structure

$$B_\Delta(x,y,z) = q_{\alpha\beta}(x)\Phi^\beta_\gamma(x,y)q_{\gamma\delta}(y)\Phi^\delta_\varepsilon(y,z)q_{\varepsilon\rho}(z)\Phi^\rho_\alpha(z,x) \tag{3}$$

is not gauge invariant, that can be checked directly, substituting in (3) $q^\alpha(x) \to U^\alpha_\beta(x)q^\beta(x)$. One can try all combinations, but it is impossible to form a continuous chain of indices to represent the Δ-type structure using as operators q_α as $q_{\alpha\beta}$. Thus one can conclude that the Y-shaped configuration is the only possible gauge-invariant configuration for baryons.

Consider now the adjoint source $g^a(x)t^{(a)\beta}_\alpha \equiv G^\beta_\alpha(x)$. We do not specify here the Lorentz structure of $g^a(x)$, but only impose condition that it should gauge transform homogeneously, $G^\beta_\alpha \to U^{+\beta}_{\beta'}G^{\beta'}_{\alpha'}U^{\alpha'}_\alpha$. Therefore $g^a(x)$ can be either the field strength $F^a_{\mu\nu}(x)$, or valence gluon field $a^a_\mu(x)$ in the background-field perturbation theory [10]. It is easy to construct a Δ-type configuration for 3 such sources;

$$G_\Delta(x,y,z) = G^\beta_\alpha(x)\Phi^\gamma_\beta(x,y)G^\delta_\gamma(y)\Phi^\varepsilon_\delta(y,z)G^\rho_\varepsilon(z)\Phi^\alpha_\rho(z,x). \tag{4}$$

It is clear that in (4) all repeated indices form gauge-invariant combinations, and $G_\Delta(x,y,z)$ is a gauge-invariant Δ-type configuration, which was used previously for the $3g$ glueball in [11].

But one can persuade oneself that (4) is not the only $3g$ gauge-invariant configuration. Consider adjoint sources and adjoint PT (here distinguishing upper and lower indices is not necessary) and form as in (1) an extended gluon operator:

$$g_a(x,Y) \equiv g^b(x)\Phi_{ab}(x,Y) \tag{5}$$

and an Y-shaped configuration

$$G^{(f)}_Y(x,y,z,Y) = f^{abc}g_a(x,Y)g_b(y,Y)g_c(z,Y). \tag{6}$$

In the same way one constructs $G^{(d)}_Y$ replacing f by d in (6).It is clear that G_Y is gauge-invariant and should be considered on the same grounds as G_Δ.

At this point it is necessary to clarify why (2), (6) can be called Y-shaped while (4) – a Δ-shaped configuration, or in other words how the parallel transporters transform into strings.

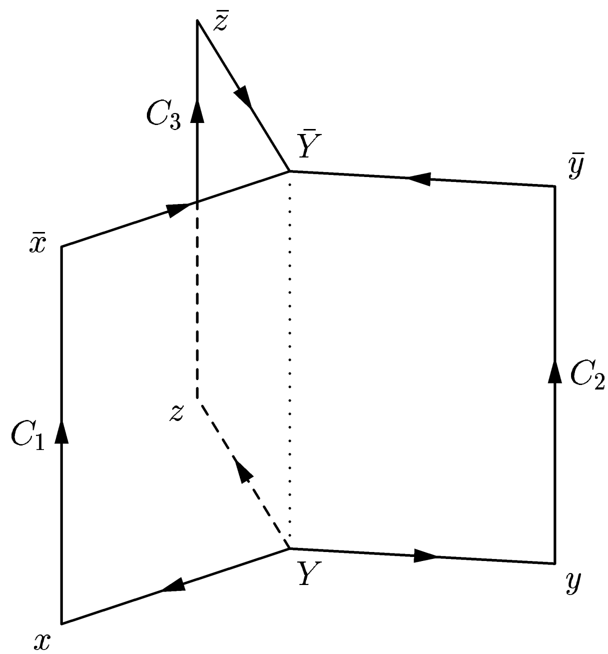

Figure 1: Y-shaped Wilson loop

To this end consider initial and final states made of (2), (4), (6) and for simplicity of arguments take all fundamental and adjoint sources to be static, i.e. propagating only in Euclidean time.

Then the Green's function for the object will be

$$\mathcal{G}_i(\bar{X}, X) = \langle \Psi_i^+(\bar{X})\Psi_i(X)\rangle \tag{7}$$

where $\Psi_i = G_\Delta, G_Y, B_Y$; $X = x, y, z$ for G_Δ and x, y, z, Y otherwise. Now it is important that vacuum average in (7) produces a product of Green's functions for quarks or for valence gluons in the external vacuum gluonic field, which is proportional to the corresponding PT, fundamental – for quarks and adjoint – for gluons. Namely,

$$\langle \bar{q}_\beta(\bar{x}) q^\alpha(x)\rangle \sim \Phi^\alpha_\beta(\bar{x}, x),$$

$$\langle g^a(\bar{x}) g^b(x)\rangle \sim \Phi_{ab}(\bar{x}, x). \tag{8}$$

(This statement is well known for static sources, for relativistic quarks and gluons this follows directly from the exact Fock-Feynman-Schwinger representation (FFSR), see [12, 13] and for a review [14]).

As a result one obtains a gauge-invariant Wilson-loop combination for each Green's function (7). In particular for B_Y (2) one has a familiar 3-lobe Wilson loop W_Y:

$$W_Y(\bar{X}, X) = \mathrm{tr}_Y \prod_{i=1}^{3} W_i(C_i), \tag{9}$$

where $\mathrm{tr}_Y = \frac{1}{6} e_{\alpha\beta\gamma} e_{\alpha'\beta'\gamma'}$, and the contour C_i in the open loop W_i passes from Y to $\bar{Y}$ through points $x, \bar{x}\,(i = 1)$, $\quad y, \bar{y}\,(i = 2)$, or $z, \bar{z}\,(i = 3)$, as shown in Fig.1.

This situation is well-known and was exploited in numerous applications. Relatively less known are the Wilson-loop configurations for G_Y and G_Δ. In the first case the structure is the

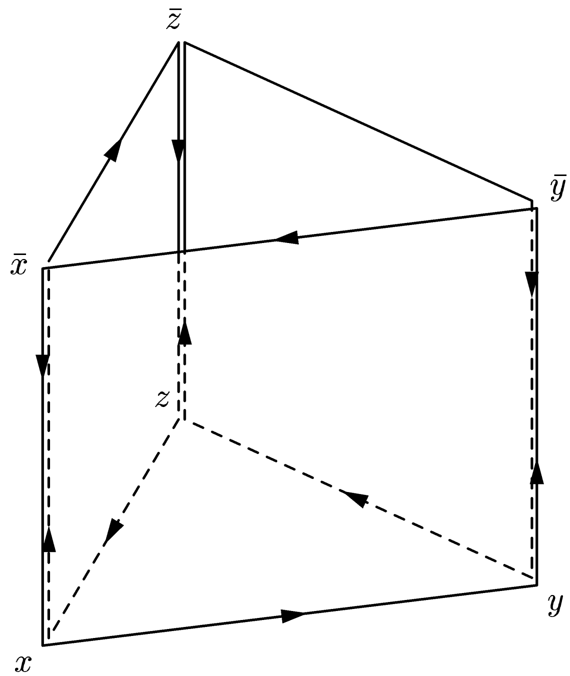

Figure 2: Δ-shaped Wilson loop

same with the replacement of fundamental lines and symbols by the adjoint ones: $e_{\alpha\beta\gamma} \to f^{abc}$ or $e_{\alpha\beta\gamma} \to d^{abc}$, $\Phi_\alpha^\beta \to \Phi_{ab}$, so that the whole structure in (9) is the same with this replacement. Contrary to the baryon case, we can contract adjoint indices in two ways, using antisymmetric symbol f^{abc} or symmetric one d^{abc}. The proper choice is related to the Bose-statistics of the gluon system which ensures the full coordinate-spin function to be symmetric.

In the case of G_Δ using (4) and (8) one can write the resulting structure symbolically as follows

$$\mathcal{G}_\Delta(\bar{X}, X) = \Delta_{a'b'c'}(\bar{x}, \bar{y}, \bar{z})\Phi_{a'a}(\bar{x}, x)\Phi_{b'b}(\bar{y}, y)\Phi_{c'c}(\bar{z}, z)\Delta_{abc}(x, y, z) \tag{10}$$

where we have denoted

$$\Delta_{abc}(x, y, z) = t_\alpha^{(a)\beta}\Phi_\beta^\gamma(x, y)t_\gamma^{(b)\delta}\Phi_\delta^\varepsilon(y, z)t_\varepsilon^{(c)\rho}\Phi_\rho^\alpha(z, x). \tag{11}$$

To understand better the structure of (10), one can use the large N_c approximation, in which case the combination

$$\Psi_{\alpha\alpha'}^{\beta\beta'} \equiv t_{\alpha'}^{(a')\beta'}\Phi_{a'a}(\bar{x}, x)t_\alpha^{(a)\beta} \approx \frac{1}{2}\Phi_\alpha^{\beta'}(x, \bar{x})\Phi_{\alpha'}^\beta(\bar{x}, x). \tag{12}$$

As a result in this approximation $\mathcal{G}_\Delta$ appears to be a product of 3 fundamental closed loops, properly oriented with respect to each other

$$\mathcal{G}_\Delta(\bar{X}, X) \sim W(\bar{x}, \bar{y}|x, y)W(\bar{y}, \bar{z}|y, z)W(\bar{z}, \bar{x}|z, x) \equiv W_\Delta(\bar{X}, X), \tag{13}$$

it is displayed in Fig. 2.

3. Static potentials for configurations (2), (4), (6) can be computed using Field Correlator Method (FCM) [15], through the equation

$$V = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W\rangle, \tag{14}$$

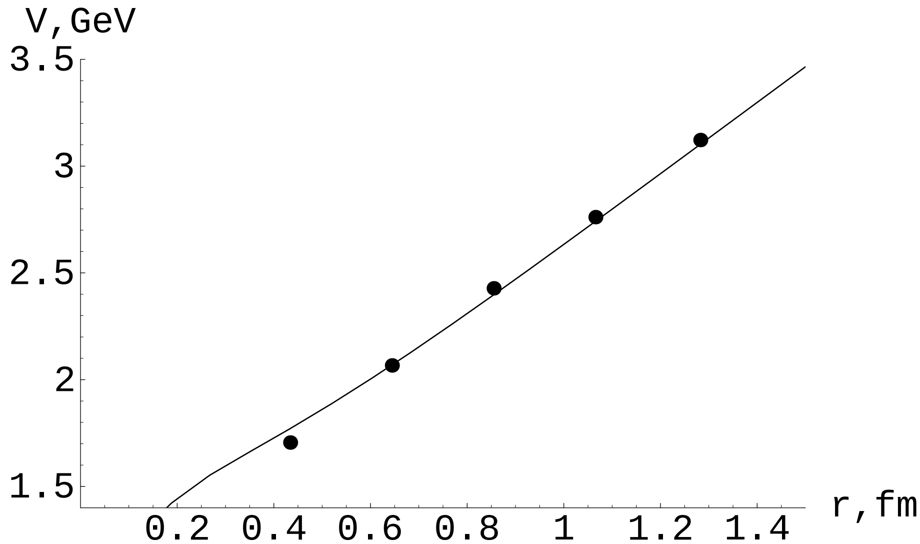

Figure 3: The lattice potential from [7] with $\beta = 5.8$ (points) and the potential $V^{\rm pert}_{\rm (fund)}(r) + V^{(B)}(r)$ (solid curve) with $T_g = 0.12$ fm, $\sigma = 0.188$ GeV^2 and $\alpha_s = 0.4$.

where T is the time extension of the Wilson loop.

For the baryon case this procedure was performed in [9], and here we shall write the result: for the case of 3 quarks at the vertices of an equilateral triangle, at the distance $R_1 = R_2 = R_3 \equiv R$ from the string junction Y, the static baryon potential is

$$V^{(B)}(R) = 3V^{(M)}(R) - V^{(\rm well)}(R), \tag{15}$$

where $V^{(M)}$ is the mesonic confining potential with the asymptotic slope σ, and $V^{(\rm well)}$ appears due to interference of fields on different lobes that causes a strong depletion of the confining fields around the string junction. For the explicit form of the potentials see Appendix. At $R \gtrsim 6T_g$

$$V^{(B)}(R) \approx 3\,\sigma R - 4\left(\frac{3}{\pi} + \frac{1}{\sqrt{3}}\right)\sigma T_g. \tag{16}$$

In the above equations we have considered only nonperturbative confining fields. To obtain the total potential we should add to (15) the perturbative part,

$$V^{\rm pert}_{\rm (fund)}(r) = -\frac{3}{2}\,\frac{C_2(\rm fund)\alpha_s}{r}, \tag{17}$$

where $r = \sqrt{3}R$ is the interquark distance and $C_2(\rm fund) = 4/3$.

In Fig. 3 the lattice data from [7] with $\beta = 5.8$ and the potential $V^{\rm pert}_{\rm (fund)}(r) + V_B(r)$ with $T_g = 0.12$ fm, $\sigma = 0.188$ GeV^2 and $\alpha_s = 0.4$ are shown.

In a similar way one can write for adjoint sources,

$$V^{(G)}_Y(R) = \frac{C_2(\rm adj)}{C_2(\rm fund)} V^{(B)}_Y(R) = \frac{9}{4} V^{(B)}_Y(R) \tag{18}$$

Consider the Δ-configuration in the approximation (12). In this case $V^{(G)}_\Delta(R)$ reduces to the sum of mesonic potentials corresponding to area laws for all three loops minus interference term, and one obtains

$$V^{(G)}_\Delta(r) = 3V^{(M)}(r) - V^{(\rm well)}(r). \tag{19}$$

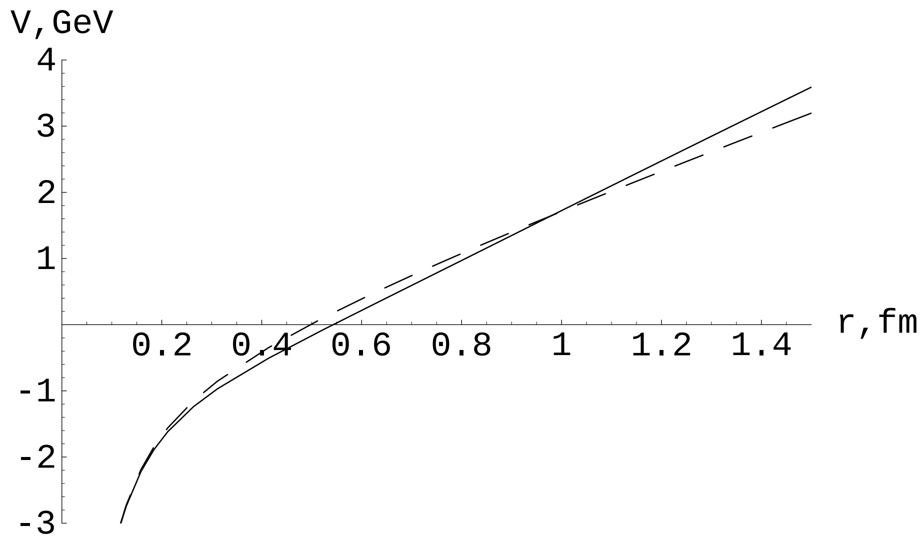

Figure 4: $V_Y^{(G)}(r)+V_{\rm (adj)}^{\rm pert}(r)$ (solid curve) and $V_\Delta^{(G)}(r)+V_{\rm (adj)}^{\rm pert}(r)$ (dashed curve) for $\alpha_s=0.4$, σ=0.18 GeV^2 and $T_g=0.12$ fm.

Along with perturbative potential

$$V_{\rm (adj)}^{\rm pert}(r)=-\frac{3}{2}\frac{C_2({\rm adj})\alpha_s}{r}, \tag{20}$$

where $C_2(\rm adj)=3$, we plot both $V_Y^{(G)}$ and $V_\Delta^{(G)}$ in Fig. 4. We see from the figure that the curves intersect at $r\approx 1$ fm, and at $r<1$ fm their slopes are almost the same and the difference between them is no greater then 100 MeV.

4. To summarize our results, we have found possible gauge-invariant configurations of 3 fundamental or adjoint sources. In particular the Δ-shape configuration debated in literature [5]-[7], is shown to be impossible, and the well-known Y-shaped baryon is the only possibility, while for adjoint sources two possible configurations coexist and yield static potentials differing only a little. This in turn implies that 3-gluon glueballs [11] may be of two distinct types, with no direct transitions between them (quark-containing hadrons must be involved as intermediate states). The mass of the Δ-shaped 3^{--} glueball was found in [11] to be $M_\Delta^{(3g)}=3.51$ GeV ($\sigma_f=0.18$ GeV^2), or 4.03 GeV for $\sigma_f=0.238$ GeV^2 to be compared with lattice one calculated in [17] 4.13±0.29 GeV. The mass of the Y-shaped glueball can easily be computed from the baryon mass calculated in [18], multiplying it by $\sqrt{9/4}=3/2$. In this way one obtains $M_Y^{(3g)}=3.47$ GeV ($\sigma_f=0.18$ GeV^2).

The slope of the corresponding odderon trajectory is almost the same and corresponds to $g-gg$-configuration. Thus one obtains the Δ-odderon (slope)$^{-1}$ to be twice the standard Regge slope, while for Y-odderon it is $\frac{9}{4}$ of the standard slope. In both cases the intercept comes out as in [11] to be rather low (-1.8 for the Y-shape and -2.4 for the Δ-shape) implying very small odderon contribution to reactions under investigation [19] in agreement with measurements.

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A Calculations of the potentials

The baryon Wilson loop for equilateral triangle in bilocal approximation of MFC has the following form [8]:

$$-\ln\langle W_Y(R,T)\rangle = \frac{1}{2}\sum_{a,b=1,2,3} n_i^{(a)} n_j^{(b)} \int_0^R \int_0^R dl\, dl' \int_0^T \int_0^T dx_4\, dx_4' \times$$

$$D_{i4,j4}(l\,\vec{n}^{(a)} - l'\vec{n}^{(b)}, x_4 - x_4'), \tag{A.1}$$

where $\vec{n}^{(a)}$, $a = 1,2,3$, are vectors of unity length directing from the string junction to the a-th quark and $D_{i4,j4}$ are bilocal field correlators, which are expressed through the scalar functions D, D_1 as follows [15],

$$D_{i4,j4}(z) \equiv \frac{g^2}{N_c}\mathrm{tr}\langle E_i(x)\Phi(x,x')E_j(x')\Phi(x',x)\rangle =$$

$$\delta_{ij}D(z) + \frac{1}{2}\left(\frac{\partial}{\partial z_i}z_j + \delta_{ij}\frac{\partial}{\partial z_4}z_4\right)D_1(z); \tag{A.2}$$

$$D(z) = D(0)\exp\left(-\frac{|z|}{T_g}\right), \quad D_1(z) = D_1(0)\exp\left(-\frac{|z|}{T_g}\right), \quad \frac{D_1(0)}{D(0)} \equiv \xi \simeq \frac{1}{3}. \tag{A.3}$$

In the last equation $T_g \approx 0.12$ fm [16] is the so-called vacuum correlation length. One can see from (A.1), (A.2), that the function D_1 does not contribute to the potential, as its second term in (A.2), propotional to z_4, disappears in the $T \to \infty$ limit, and the first term vanishes for symmetry reasons, since

$$\sum_a \vec{n}^{(a)}\vec{z} = 0. \tag{A.4}$$

The baryon potential reads as

$$V^{(B)}(R) = 3V^{(M)}(R) - V^{(\mathrm{well})}(R). \tag{A.5}$$

Here

$$V^{(M)}(R) = \frac{2\sigma}{\pi}\left\{R\int_0^{R/T_g} d\,x\,x\,K_1(x) - T_g\left(2 - \frac{R^2}{T_g^2}K_2\left(\frac{R}{T_g}\right)\right)\right\} \tag{A.6}$$

follows from (A.1) at $a = b$. It is the mesonic confining potential with the asymptotic slope $\sigma \equiv \pi D(0)T_g^2$; K_1 and K_2 are McDonald functions.

$$V^{(\mathrm{well})}(R) = \frac{4}{\sqrt{3}}\sigma T_g - \frac{3\sqrt{3}}{\pi}\frac{\sigma R^2}{T_g}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{d\varphi}{\cos^2\varphi}K_2\left(\frac{\sqrt{3}R}{2T_g\cos\varphi}\right) \tag{A.7}$$

follows from (A.1) at $a \neq b$. As it was shown in [9], $V^{(\mathrm{well})}$ appears due to the strong depletion of the confining field around the string junction.

In the case of Δ-shaped Wilson loop for the equilateral triangle with sides r an integration in (A.1) will go over the sides of the triangle, and we obtain

$$V_\Delta^{(G)}(r) = 3V^{(M)}(r) - V^{(\mathrm{well})}(r). \tag{A.8}$$

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