Final report Univ. of Chicago

Progress Report for Grant: DE-FG02-92ER25119

Numerical Methods

Numerical methods for fluid flow are an important part of Dupont's work. The results with Keenan provide an improved method for constructing velocities in the context of so-called mixed-methods for partial differential equations. The work of Blaine Johnston has been directed at simulation of axisymmetric free surface flows. Some of his work is available on the WWW.

Together with a number of collaborators (Fefferman, Majda, Procaccia, Segel), Constantin has extended his previous results concerning geometric-analytic and physical criteria for blow- up in incompressible 3D fluid equations.

Sonoluminescence

The work of Brenner, Lohse, Oxtoby and Dupont on sonoluminescence is focused on understanding the hydrodynamic and diffusion processes involved in stable single bubble sonoluminescence. This remarkable phenomenon concentrates energy by twelve orders of magnitude, and improving our theoretical basis for predictions about it will make it more likely that we will be able to control it in valuable ways.

The patterns that develop in colloidal mixtures were studied by Anette Hosoi and Dupont. A simple reduced-dimensional model of the shocks that can develop in these mixtures sheds new light on the mechanism involved. Several movies of the results of this work are available on the WWW.

Convection and Turbulence

Together with Ch. Doering, Constantin has studied bulk transport (of mass, momentum or heat) in driven dissipative systems. Their most recent success has been a derivation based on the Boussinesq equations of exponents (1/3 and 2/7) in the scaling law for Nusselt number in Rayleigh-Benard convection.

Together with student Wu, Constantin has obtained strong convergence results for the inviscid limit for non-smooth vorticities in 2D. The main point of the research is that it illustrates the non-universal and non-uniform character of the limit.

There has been considerable effort on phenomenological and model studies of turbulence. These included an extensive analyses of experiment. In addition, there was a theoretical analysis of a simple model which attempted to capture some of the most important aspects of turbulence. The model shows some of the multifractal properties seen in real turbulence and has a phase transition in its stability matrix.

Singularities in Droplet Snapoff

Dupont, Kadanoff and Nagel studied singularity formation in partial differential equations. The major point of this work is that one can find singularities in physical situations and that one can predict what the singularities will look like. Then, one can see whether the singularity formation is in fact stable. They found that there is linear stability, but non-linear instability, in certain physical systems such as the breakoff of a liquid drop. Under DOE sponsorship, they studied the process of how a liquid breaks into two or more pieces.

Their studies have focused on a fundamental idea about how liquid drops break, originally introduced by Keller and Miksis, about the dynamics of a droplet close to the point of rupture: Arbitrarily close to breaking there are no external length scales.

DISCLAIMER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

describing the interface, so the dynamics should be *self similar*. Peregrine refined this idea and applied it to their experiments on falling water drops: They point out that for low viscosity fluids there is a range of scales where the thickness of the fluid neck is much larger than the viscous length scale, but much smaller than the length scale where energy is fed into the system. Over this range of scales, the self--similarity hypothesis might be expected to hold.

These considerations suggest that a breaking fluid interface should be described by a similarity solution to the governing hydrodynamic equations. Under the auspices of the DOE Kadanoff et al. performed the first study that succeeded in constructing a similarity solution for a breaking fluid thread in the rupture of a drop in the two dimensional Heleshaw cell. This initial success was followed up with the identification of further similarity solutions for the Heleshaw cell as well as the discovery of a similarity solution for three dimensional droplet fission for fluid threads. In this case the similarity solution is unstable to finite amplitude perturbations, with the critical amplitude for instability approaching zero at the singularity. At the high viscosities where this similarity solution is relevant, the droplet shape is long and slender.

Nagel and collaborators have also studied the question whether similar scaling ideas can explain the interfacial shapes occurring during the breakup of low viscosity fluids, such as a water drop. Based on the above discussion, this translates into the question of whether the self similar singularities in the equations of *inviscid* hydrodynamics describe experiments when the thread thickness is larger than the viscous length scale? To answer this question, we have studied in detail the characteristics of a water drop falling from a nozzle both before and after a fission event. Through a combination of experiments, numerical simulations and theory we argued that the scaling hypothesis fails for this problem. In the absence of viscosity, this instability actually causes a singularity in the curvature of the interface before rupture occurs .

Sedimentation at Contact Lines

When a spilled drop of coffee dries on a solid surface, it leaves a dense, ring-like stain along the perimeter. The coffee---initially dispersed over the entire drop---becomes concentrated into a tiny fraction of it. Such ring deposits are commonplace wherever drops containing dispersed solids evaporate on a surface. Thus ring deposits influence printing, washing and coating processes. They provide a potential means to write or deposit a fine pattern onto a surface. Dupont, Nagel and collaborators have investigated the causes of such deposits. We ascribe the deposition to a previously unexplored form of capillary flow: the contact line of the drying drop is pinned so that liquid evaporating from the edge must be replenished by liquid from the interior. The resulting outward flow can carry virtually all the dispersed material to the edge. This mechanism predicts a distinctive power-law growth of the ring mass with time---a law independent of the particular substrate, carrier fluid or deposited solids. We have verified this law by microscopic observations of colloidal fluids.

Our initial observations show that rings form for a wide variety of substrates, dispersed materials (solutes), and carrier liquids (solvents), as long as (1) the solvent meets the surface at a nonzero contact angle, (2) the contact line is pinned to its initial position, and (3) the solvent evaporates. In addition, we found that mechanisms typically responsible for solute transport---surface tension gradients, solute diffusion, electrostatic, and gravity effects---are negligible in ring formation. Based on these findings, we have identified the minimal ingredients for a quantitative theory. The phenomenon is due to a geometrical constraint: the free surface, constrained by a pinned contact line, squeezes the fluid outward to compensate for evaporative losses.

River Networks

The patterns created by river drainage basins has also been the subject of DOE research at Chicago. A simulation model was proposed by Nagel and Leheny for drainage basin evolution which includes both erosion from precipitation and avalanching on hillslopes. Despite the simplicity of the model, the simulated landscapes evolve in a realistic manner which shares many qualitative features with Glock's model for natural river network evolution. In addition, throughout the evolutionary cycle the model maintains many statistical properties that characterize real river networks.

Granular Dynamics

Constantin, Jaeger, Kadanoff, and Nagel have studied the flow properties of granular media. A detailed understanding of the mechanisms involved in dense granular flows is presently not available despite many years of active research in the engineering and, more recently, condensed matter physics communities . The reasons for this lack of knowledge can be traced to the complexities that derive from the inhomogeneous ("granular") nature of the material and from the nonlinear, dissipative interactions between its constituents, the individual grains.

Using simulations, Kadanoff and his co-workers model granular materials by looking at the dynamics of inelastic collisions. Under the right conditions the partricles can slow down very considerably. This simple system can serve as an 'Ising model' for dynamics. Understanding the way forces are distributed in stationary granular materials is important. for insight into failure modes among other reasons. Jaeger and Nagel collaborated on experiments, simulation, and theory that characterizes the large force inhomogeneities observed in stationary packs of spherical particles.

Jaeger and Nagel studied convection in granular materials culminating in a measurement using MRI to follow the motion of particles being vibrated. They have characterized both the depth and radial dependence of the vertical velocity. This motion was driven by the shaking of the container. They have shown that convective motion in agitated granular materials gives rise to size separation in mixtures of granular particles. Clearly, control over such unmixing is highly desired in all applications that require a uniform, homogeneous shape or size distribution.

As part of the response to that shaking, the material compacts--but very, very slowly. This slow rate of compression is a technological problem in situations in which high compaction is required. They have also quantified this effect experimentally and have constructed several competing theories to model this behavior.

Reprints Supported by DOE Grant

"Infinite Prandtl number convection," P. Constantin with Ch. Doering, J. Stat. Phys., **94**, 159-172 (1999),

"Heat transport in rotating convection," P. Constantin with C. Hallstrom and V. Putkaradze, Physica D **125**, 275-284 (1999).

"Physics at the Breakfast Table – or Waking Up to Physics," [Klopsteg Memorial Lecture (August, 1998)] S. R. Nagel , American Journal of Physics **67**, 17-25 (1999).

"Nonsingular Surface-Quasi-Geostrophic Flow," P. Constantin, Q. Nie, N. Schorghofer, Phys. Letts. A., **241** 168-172 (1998).

"Slow relaxation in granular compaction," E. Ben-Naim, J. B. Knight, E. R. Nowak, H. M. Jaeger, and S. R. Nagel, Physica D **123**, 380-385 (1998).

"Superconvergence and postprocessing of fluxes from lowest order mixed methods on triangles and tetrahedra," T. Dupont and P. T. Keenan, SIAM Jour. Sci. Comput., **19**, 1322-1332 (1998).

"Scaling exponents for Active Scalars," P. Constantin, J. Stat. Phys. **90**, 571-595 (1998).

"Absence of proper non-degenerate generalized self-similar singularities," P. Constantin, J. Stat. Phys. **93**, 777-786 (1998).

"Bounds for heat transport in a porous layer," P. Constantin with Ch. Doering, J. Fluid Mechanics, **376**, 263-296 (1998).

"Breakdown of Scaling in Droplet Fission at High Reynolds Number," M. P. Brenner, J. Eggers, K. Joseph, S. R. Nagel, and X.D. Shi, Phys. Fluids **9**, 1573-1590 (1997)

"Reversibility and Irreversibility in the Packing of Vibrated Granular Material," E. R. Nowak, J. B. Knight, M. Povinelli, H. M. Jaeger, and S. R. Nagel, Powder Technology, **94**, 79-83, (1997).

"Studies of Granular Compaction," E. R. Nowak, M. Povinelli, H. M. Jaeger, S. R. Nagel, J. B. Knight, and E. Ben-Naim, in *Powders and Grains 97* Proceedings of the Third International Conference on Powders and Grains, Durham/North Carolina/18-23 May 1997 edited by R. P. Behringer and J. T. Jenkins (A.A. Balkema, Rotterdam, 1997) pg. 377-380.

"Statistical solutions of the Navier-Stokes equations on the phase space of vorticity and the inviscid limits," P. Constantin with J. Wu, IMA Preprint Series 1442, November 1996; J. Math. Phy., **38**, 3031-3045 (1997).

"Capillary flow as the cause of ring stains from dried liquid drops," R. D. Deegan O. Bakajin, T. F. Dupont, G. Huber, S. R. Nagel, and T. A. Witten, Nature 389, 827-829 (1997).

"The Littlewood-Paley spectrum in 2D turbulence," P. Constantin, Theor. Comp. Fluid Dyn. 9, 183-189 (1997).

"Exponents of bulk heat transfer in convective turbulence," P. Constantin in *Turbulence Modeling and Vortex Dynamics*, Boratav, Eden, Erzan Eds., Springer Lect. Notes in Phys. 491, 17-27 (1997).

"External boundaries and internal shear bands in granular convection," J. B. Knight, Phys. Rev. E 55, 6016-6023 (1997).

J. B. Knight, H. M. Jaeger, and S. R. Nagel, in Advances in Fluidization and Fluid-Particle Systems (eds. King, D., Arastoopour, H., Chen, J. C., Weimer, A. W. & Yang, W.-C.) 109-112 (American Institute of Chemical Engineers, New York, 1997).

"Singular limits in fluid mechanics," P. Constantin in *Current and future directions in applied mathematics*, M. Alber, B. Hu, J. Rosenthal, Edtrs, (1997) Birkhauser, Boston.

"Some Reduced-Dimension Models Based on Numerical Methods," T. Dupont and A.Hosoi, Preprint (1997).

"A note on third order structure functions in turbulence," Q. Nie and S. Tanveer, submitted to Proc. R. Soc. London (1997).

"Breakdown of Scaling in Droplet Fission at High Reynolds Number," M.P. Brenner, J. Eggers, K. Joseph, S.R. Nagel, and X.C. Shi, Phys. Fluids 9 (6) 1573-1590 (1997).

"Sonoluminescence air bubbles rectify argon," D. Lohse, M. Brenner, T. Dupont, S. Hilgenfeldt, and B. Johnston, Phys. Rev. Lett. 78, 1359-1362 (1997).

"Cascade Models of Turbulence and Mixing," L.P. Kadanoff, Tr. J. of Physics 21 1-14 (1997).

"Transitions and Probes in Helium Turbulence," V. Emsellem, L.P. Kadanoff, D, Lohse, P. Tabeling, and J. Wang, Phys. Rev. E., 55 (3) 2672-2681 (1997).

"Scaling and Linear Response in the GOY Turbulence Model," L.P. Kadanoff, D. Lohse, and N. Schorghofer, Physica C 100 165-186 (1997).

"A Hele-Shaw Problem and the Second Painleve Transcendent," A.S. Fokas and S. Tanveer, to appear in Math. Proc. Camb. Phil. Soc. (1996).

"Granular Solids, Liquids and Gases," H. M. Jaeger, S. R. Nagel, and R. P.Behringer, Rev. Mod. Phys. 68, 1259-1273 (1996).

"Sonoluminescence: Air bubbles as chemical reaction chambers," D. Lohse, M. Brenner, and T. Dupont, Submitted to Phys. Rev. Letts. (1996).

"The evolution of an axi-symmetric Stokes bubble with volumetric change," Q. Nie and S. Tanveer, SIAM J. App. Math, Preprint (1996).

"A Theory of Exact Solutions For Annular Viscous Blobs," D. Crowdy and S. Tanveer, submitted to J. Non Lin Sci (1996).

"Dendritic crystal growth for weak undercooling," M. Kunka, M. Foster, and S. Tanveer, submitted to Phys. Rev. E. (1996).

"Inelastic Collapse of Rotating Spheres," N. Schorghofer and T. Zhou, Phys Rev. E 54 (5) 5511-5515 (1996).

"Mechanisms for Stable Single Bubble Sonoluminescence," M. Brenner, D. Lohse, D. Oxtoby, and T. Dupont, Phys. Rev, Letts 76 (7) 1158-1161 (1996).

"Inelastic Collapse of Three Particles," T. Zhou, L.P. Kadanoff, Phys. Rev. E, 54 623 (1996).

"Geometric constraints on potentially singular solutions for the 3-D Euler equations," P. Constantin with Ch. Fefferman and A. Majda, Commun. in PDE 21, 559-571 (1996).

"Anisotropy and scaling corrections in turbulence," D. Lohse and A. Muller-Groeling, Phys. Rev. E, 54 395 (1996).

"Heat transfer in convective turbulence," P. Constantin with Ch. Doering, Nonlinearity 9, 1049-1060 (1996).

"The inviscid limit for non-smooth vorticity," P. Constantin with J. Wu, Indiana U. Math J. 45, 67-81 (1996).

"Variational bounds on energy dissipation in incompressible flows: III. Convection," P. Constantin with Ch. Doering, Phys. Rev E, 53, 5957-5981 (1996).

"Rigorous estimates of small scales in turbulent flows," P. Constantin with Ch. Doering and E. Titi, Journal of Mathematical Physics 37, 6152-6156 (1996).

"A priori estimates for mixed finite element approximations of second order hyperbolic equations with absorbing boundary conditions," T. Dupont, L. Cowsar and M. F. Wheeler, SIAM Jour. Numer. Anal., 33, 492-504 (1996).

"Mechanisms for stable sonoluminescence," T. Dupont, M. P. Brenner, D. Lohse, and D. Oxtoby, Phys. Rev. Lett. 76, 1158 (1996).

"Bubble Shape Oscillations and the Onset of Sonoluminescence," M.P. Brenner, D. Lohse, and T.F. Dupont, Phys. Rev. Letts. 76 (5) 954-957 (1995).

"Inelastic Collisions of Three Particles on a Line as a Two-Dimensional Billiard," P. Constantin, E. Grossman, and M. Mungan, Physica D 83 409-420 (1995).

"Scaling and Dissipation in the GOY Shell Model," L. Kadanoff, D. Lohse, J. Wang, and R. Benzi, Phys. Fluids 7 (3) 617-629 (1995).

"Granular Convection Observed by Magnetic Resonance Imaging," E. E. Ehrichs, H. M. Jaeger, G. S. Karczmar, J. B. Knight, V. Y. Kuperman, and S. R. Nagel, Science 267, 1632-1634 (1995).

"Density Relaxation in a Vibrated Granular Material," J. B. Knight, C. G. Fandrich, C. N. Lau, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E 51, 3957 (1995).

"Breakdown of Hydrodynamics in a One Dimensional System of Inelastic Particles," Y. Du, H. Li, and L.P. Kadanoff, Phys. Rev. Letts. 74 (8) 1268-1271 (1995).

"Bubble shape oscillations and the onset of sonoluminescence," T. Dupont, M. P. Brenner and D. Lohse, Phys. Rev. Let., 75, 954-957 (1995).

"Iterated Instabilities during Droplet Fission," M.B. Brenner, X.D. Shi, and S.R. Nagel, Phys. Rev. Letts., 73 (25) 3391-3394 (1994).

"A Cascade of Structure in a Drop Falling from a Faucet," X. D. Shi, M. P. Brenner and S. R. Nagel, Science 265, 219-222 (1994).

"Drop Formation in a One-Dimensional Approximation of the Navier-Stokes Equation," T. Dupont and J. Eggers, J. Fluid Mech. 262, 205-221 (1994).

"Singularities and Similarities in Interface Flows," A. Bertozzi, M. Brenner, T.F. Dupont, and L.P. Kadanoff, L. Sirovitch Editor, Trends and Perspectives in Applied Mathematics, Springer Verlag Applied Math Series, 100 155-208 (1994).

"What is shaking in the sand box?" H. M. Jaeger, J. B. Knight, C.-h. Liu, and S. R. Nagel, Mat. Res. Soc. Bull. 19, 25 (1994).

"Model for the Evolution of River Networks," R. L. Leheny and S. R. Nagel, Phys. Rev. Lett. 71, 1470-1473 (1993).

"Vibration-Induced Size Separation in Granular Media: The Convection Connection," J. B. Knight, H. M. Jaeger, and S. R. Nagel, Phys. Rev. Lett. 70, 3728-3731 (1993).

"Finite-time singularity formation in Hele-Shaw systems," T. Dupont, R.E. Goldstein, L. P. Kadanoff, and S.-M. Zhou, Phys. Rev. E 47, 4182-4196 (1993).

Students and PostDocs Supported by this DOE Grant

Itai Cohen
Elizabeth Grosman
Anette Hosoi
Greg Huber
Blaine Johnston
Robert Leheny
Lakshminarayanan Mahadevan
Jonathan Miller