Measurements of the electromagnetic form factor of the Proton at JLab

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Abstract

The ratio of the proton’s elastic electromagnetic form factors $G_{E_p}/G_{M_p}$ was obtained by measuring $P_t$ and $P_\ell$, the transverse and longitudinal recoil proton polarization, respectively. For the elastic reaction $e^-p \rightarrow e^-p$, $G_{E_p}/G_{M_p}$ is proportional to $P_t/P_\ell$. The simultaneous measurement of $P_t$ and $P_\ell$ in a polarimeter reduces systematic uncertainties. The results for the ratio $G_{E_p}/G_{M_p}$ measured in Hall A so far show a systematic decrease with increasing $Q^2$, indicating for the first time a definite difference in the distribution of charge and magnetization in the proton. Together these experiments cover the $Q^2$-range of 0.5 to 5.6 GeV\(^2\). A new experiment is currently being prepared, to extend the $Q^2$-range to 9 GeV\(^2\) in Hall C.

1 Introduction

The first direct evidence that the proton has an internal structure came from a measurement of its anomalous magnetic moment 70 years ago by O. Stern[1]; it is 2.79 times larger than that of a Dirac particle of the same mass. The first measurement of the charge radius of the proton, by Hofstadter[2] et al., yielded a value of 0.8 fm quite close to the modern value. Although the structure of the proton has been taken for well known until recently, the experimental results to be reported here show that it held secrets which are only now being revealed.

The nucleon elastic form factors describe the internal structure of the nucleon; in the non-relativistic limit, for small four-momentum transfer squared, $Q^2$, they are Fourier transforms of the charge and magnetization distributions in the nucleon. In the Breit frame the hadron electromagnetic 4-vector current $J_\mu$ has time- and space components proportional to the Sachs form factors $G_{E_p}$ and $G_{M_p}$, respectively. Hence, in this frame, it is generally true that the electric and magnetic form factors $G_{E_p}$ and $G_{M_p}$ are the Fourier transform of the charge and magnetization distributions, respectively. The difficulties associated with
the calculations of the charge and magnetization distributions in the laboratory have been discussed recently by Kelly [3].

The unpolarized elastic \( e p \) cross section can be written in terms of the Sachs form factors \( G_{Ep} \) and \( G_{Mp} \):

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2 \frac{\theta_e}{2}}{4E_e^3 \sin^4 \frac{\theta_e}{2}} \left[ G_{Ep}^2 + \frac{\tau}{\epsilon} G_{Mp}^2 \right] \left( \frac{1}{1 + \tau} \right),
\]

where \( \tau = Q^2 / 4M_p^2 \), \( \epsilon = \left[ 1 + 2(1 + \tau) \tan^2 \left( \frac{\theta_e}{2} \right) \right]^{-1} \) is the polarization of the virtual photon, with values between 0 and 1, \( E_e \) and \( E'_e \) are the energies of the incident and scattered electron, respectively, and \( \theta_e \) is the electron scattering angle in the laboratory frame. \( G_{Ep} \) and \( G_{Mp} \) can be extracted from cross section measurements made at fixed \( Q^2 \), and over a range of \( \epsilon \) values, with the Rosenbluth separation method.

Fig. 1 shows previous results of \( G_{Ep} \) and \( G_{Mp} \) obtained by Rosenbluth separations, plotted as the ratios \( \frac{G_{Ep}}{G_D} \) and \( \frac{G_{Mp}}{\mu_p G_D} \) versus \( Q^2 \), up to 6 GeV\(^2\). Here \( G_D = (1 + Q^2/m_D^2)^{-2} \) is the dipole form factor, with the constant \( m_D^2 \) empirically determined to be 0.71 GeV\(^2\). For \( Q^2 < 1 \) GeV\(^2\), the uncertainties for both \( G_{Ep} \) and \( G_{Mp} \) are only a few percent, and one finds that \( G_{Mp}/\mu_p G_D \simeq G_{Ep}/G_D \simeq 1 \). For \( G_{Ep} \) above \( Q^2 = 1 \) GeV\(^2\), the large uncertainties and the scatter in results between different experiments, as seen in Fig. 1, illustrate the difficulties in obtaining \( G_{Ep} \) by the Rosenbluth separation method. In contrast, the uncertainties for \( G_{Mp} \) remain small up to \( Q^2 = 31.2 \) GeV\(^2\).[4]

As seen in Eq. (1) the \( G_{Mp} \) part of the cross section, which is about \( \mu_p \) times larger than the \( G_{Ep} \) part, is also multiplied by \( \tau \); therefore, as \( Q^2 \) increases, the cross section becomes dominated by the \( G_{Mp} \) term, making the extraction of \( G_{Ep} \) more difficult by the Rosenbluth separation method.

The JLab results have been obtained by measuring the recoil proton polarization in \( \vec{e}p \rightarrow e\vec{p} \) [11, 12] instead of the cross section. For one-photon exchange, in the \( \vec{e}p \rightarrow e\vec{p} \) reaction, the scattering of longitudinally polarized electrons results in a transfer of polarization to the recoil proton with only two non-zero components, \( P_t \) perpendicular to, and \( P_\perp \) parallel to the proton momentum in the scattering plane. For 100% polarized electrons, the polarizations are [13, 14, 15, 16]:

\[
I_0 P_t = -2\sqrt{\tau (1 + \tau)} G_{Ep} G_{Mp} \tan \frac{\theta_e}{2}
\]

\[
I_0 P_\perp = \frac{1}{m_p} (E + E') \sqrt{\tau (1 + \tau)} G_{Mp}^2 \tan^2 \frac{\theta_e}{2}
\]
where \( I_0 \propto G_{Ep}^2 + \frac{G_{Mp}^2}{\mu_p^2} \). Measuring simultaneously these two components and taking their ratio gives the ratio of the form factors:

\[
\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_\ell} \frac{(E + E')}{2m_p} \tan\left(\theta_e^2\right)
\]  

The form factor ratio \( G_{Ep}/G_{Mp} \) at a given \( Q^2 \) can be obtained without change of beam energy or detector angle, eliminating important sources of systematic uncertainties; the principal source of systematic uncertainty remaining comes from the need to account for the precession of the spin in the spectrometer detecting the recoil proton accurately.

## 2 Experiments

In 1998 \( G_{Ep}/G_{Mp} \) was measured at JLab for \( Q^2 \) from 0.5 to 3.5 GeV^2 [11]. Protons and electrons were detected in coincidence in the two high-resolution spectrometers (HRS) of Hall A. The polarization of the recoiling proton was
obtained from the asymmetry of the azimuthal distribution after rescattering the proton in a focal plane polarimeter (FPP) with graphite analyzer.

In 2000 new measurements were made at $Q^2 = 4.0, 4.8$ and $5.6 \text{ GeV}^2$ with overlap points at $Q^2 = 3.0$ and $3.5 \text{ GeV}^2$ [12]. To extend the measurement to these higher $Q^2$, two changes were made. First, to increase the figure-of-merit of the FPP, a CH$_2$ analyzer was used; the thickness was increased from 50 cm of graphite to 100 cm of CH$_2$ (60 cm for $Q^2 = 3.5 \text{ GeV}^2$). Second, the electrons were detected in a lead-glass calorimeter with 9 columns and 17 rows of $15 \times 15 \times 35 \text{ cm}^3$ blocks placed so as to achieve complete solid angle matching with the HRS detecting the proton. At the largest $Q^2$ the solid angle of the calorimeter was 6 times that of the HRS.

The combined results from both experiments are plotted in Fig. 2 as the ratio $\mu_p G_{Ep}/G_{Mp}$. If the $\mu_p G_{Ep}/G_{Mp}$-ratio continues the observed linear decrease with the same slope, it will cross zero at $Q^2 \approx 7.5 \text{ GeV}^2$.

3 Results and Discussion

In exclusive electron scattering at high $Q^2$, the dominant degrees of freedom of the nucleon are the three valence quarks. This is the regime where the perturbative QCD (pQCD) theory can be applied [17]. At moderate $Q^2$ values, the Vector Meson Dominance (VMD) model [18, 19, 20, 21, 22] has been successful in describing the nucleon form factors and hadronic interactions. At $Q^2$ between 1 and $10 \text{ GeV}^2$, relativistic constituent quark models [23, 24, 25, 26, 27, 28, 29, 30] currently give the best understanding of the nucleon form factors, with the strongest dynamical input. Predicting nucleon form factors in the intermediate $1 < Q^2 < 20 \text{ GeV}^2$ region, where soft scattering processes are still dominant compared to hard scattering, is very difficult.

In Fig. 3, the JLab data are compared with the results of various calculations that include the VMD calculation of Lomon [22], the relativistic constituent quark model (CQM) calculations [26, 28, 30], the soliton model calculation of Holzwarth [31], and a new pQCD fit from Brodsky [32]. In the soliton model Holzwarth[31] applies the relativistic corrections due to recoil and incorporates partial coupling to vector mesons. He uses the skyrmion as an extended object with one vector meson propagator and relativistic boost to the Breit frame. The result is shown in Fig. 3 as the dashed and solid curves, corresponding to two different strength of the $\omega$-meson coupling strength, $g_\omega$. This model describes the ratio very well over the $Q^2$ range in this experiment. In 2002 Lomon updated the original VMD calculation of Gari and Kruempelman [20] and obtained good agreement with the JLab data for reasonable parameters for the vector-meson masses and coupling constants.

In Fig. 4 the JLab data are shown as $Q^2$ times $F_2/F_1$, the ratio of the Pauli and Dirac form factors $F_2$ and $F_1$; these are connected to the Sachs form factor as follows:
Figure 3: Comparison of theoretical model calculations with the data from Ref. [11] (solid circles) and from [12] (empty squares). The curves are, black thin solid [30], green dot-dashed and dashed [28], black dashed [26], red solid [22], yellow solid [32] and magenta dashed and solid [31].

Figure 4: $Q^2 F_{2p}/F_{1p}$ versus $Q^2$. The curves shown are from Brodsky [32] (solid line), and from Belitsky et al. [33] (dashed line). The data from Ref. [11] are shown as solid circles, from Ref. [12] as empty squares and from Ref. [10] as empty diamonds.

$$F_2 = \frac{G_{MP} - G_{EP}}{\kappa_p(1 + \tau)}, \quad F_1 = \frac{\tau G_{MP} + G_{EP}}{1 + \tau}$$  \hspace{1cm} (5)$$

where $\kappa_p$ is the anomalous part of the proton magnetic moment, in units of the nuclear magneton $\mu_N$. The prediction of pQCD is that quenching of the spin flip form factor $F_2$ should occur at large $Q^2$, or equivalently helicity conservation should hold true; higher order contributions should make $Q^2 F_2/F_1$ asymptotically constant. Unlike the SLAC [10] data, the JLab data clearly contradict this prediction over the $Q^2$ region covered.

Recently there have been two revisions of the pQCD prediction for the large $Q^2$ behavior of $F_2$. In the first, Brodsky [32] argues that the pQCD motivated behavior of $F_2$ must contain an extra logarithmic term from higher twist contributions; the 3 free parameters $a$, $b$ and $c$ of the expression $F_{2p}/F_{1p} = \frac{1}{1 + (Q^2/c) \ln^a(1 + Q^2/a)}$ were fitted in Ref. [32] to the data of Ref. [11] and [12]
with the result shown as a solid line in Fig. 4. In the second, Belitsky et al. [33] reiterate the fact that the spin of a massless (or very light) quark cannot be flipped by the virtual photon of the $ep$ reaction. For a quark to undergo spin-flip, it must be in a state of non-zero angular momentum with projection $|L_z|=1$. As a result, the standard pQCD prediction for $F_{2p}$ (namely $\propto Q^{-6}$) becomes modified by a logarithmic term such that \[ \frac{F_{2p}}{F_{1p}} = \frac{a}{\Lambda^2} \ln^2 \left( \frac{Q^2}{\Lambda^2} \right), \] where $a$ is a normalization constant; $\Lambda$ is a cutoff constant required to suppress the infrared singularity generated by the very soft part of the quark wave function. Although the constant $a$ in the expression above is not determined, a fit to the data of this paper (augmented by the data of [12]) gives $\Lambda = 290$ MeV, and $a = 0.175$. The soft physics scale of the nucleon is determined by $\Lambda$; its size is of order of the transverse quark momentum in the nucleon. This fit is shown as the dashed line in Fig. 4.

In Fig. 5 the JLab data plotted as $QF_{2p}/F_{1p}$ show a remarkable flattening of the ratio starting at 1-2 GeV$^2$. Inspired by the results of this experiment, Ralston [34] revisited the calculation of the single-quark spin-flip amplitude responsible for the Pauli form factor in the framework of QCD. According to Ralston et al. [34, 35], if quarks in the proton carry orbital angular momentum, then $F_{2p}/F_{1p}$ should behave like $\frac{1}{\sqrt{Q^2}}$ (Ref. [17]). In a different approach, Miller and Frank [36] have shown that imposing Poincaré invariance leads to violation of the helicity conservation rule, which results in the behavior of $F_{2p}/F_{1p}$ observed in the JLab data.

A third phase of the $G_{Ep}/G_{Mp}$ measurements with the recoil polarization technique in Hall C, this time to 9 GeV$^2$, is planned for 2005 [37]. In Fig. 6 the projected error bars are shown at $Q^2$ of 7.5 and 9 GeV$^2$. Also, a new Rosenbluth separation experiment has been done in Hall A in 2002, up to $Q^2=4.1$ GeV$^2$ [38]; the experiment used a technique which strongly reduces systematic uncertainties compared to the standard Rosenbluth separation; the results are expected in mid-2003. A future experiment in JLab Hall A [39], to measure $G_{En}$ up to 3.4 GeV$^2$, will significantly improve our knowledge of the nucleon form factors.

4 Conclusion

The precise new JLab data on $\mu_p G_{Ep}/G_{Mp}$ show that this ratio continues to drop off linearly with increasing $Q^2$ up to 5.6 GeV$^2$. The ratio $F_2/F_1$ does not follow the $1/Q^2$ behavior predicted by pQCD. Thus, the Jlab data may indicate the continuing dominance of soft physics in the $Q^2$-range explored so far. This behavior must be compared with the scaling of $Q^4 G_{Mp}$ seen in Ref. [4], starting at $Q^2 \sim 5$ GeV$^2$, which has been interpreted as the onset of pQCD. The previous discussion emphasizes the need for more and better data at higher $Q^2$, to challenge theoretical models in this difficult range of momentum transfer.
The data presented here clearly show that the ratio $Q^2 F_{2p}/F_{1p}$ is still increasing.
monotonically up to 5.6 GeV². Recently a careful re-examination of the pQCD prediction has led to the inclusion of a logarithmic factor and good agreement with the behavior of $F_2^p/F_1^p$ reported in this paper.

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