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STRUCTURAL ANALYSIS OF CLOSURE CAP BARRIERS
A Pre-test Study for the Bentonite Mat Demonstration Project (U)

By

CHUNG GONG

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Structural Analysis of Closure Cap Barriers, A Pre-test Study for the Bentonite Mat Demonstration Project (U)

by

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ISSUED: OCTOBER 1993

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SRTC SAVANNAH RIVER TECHNOLOGY CENTER, AIKEN, SC, 29808
WESTINGHOUSE SAVANNAH RIVER COMPANY

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Structural Analysis of Closure Cap Barriers

A Pre-test Study for the Bentonite Mat Demonstration Project (U)

Chung Gong

0.0 EXECUTIVE ABSTRACT

According to the EPA-recommended closure cap design a waste site can either be covered with a single layer cap made of 36 inches of compacted soil (clay) or with a multilayer cap consisting of an upper vegetative layer underlain by a drainage layer over a low permeability layer.

The Bentonite Mat Demonstration Project (BMDP) is a field demonstration study to determine the construction / installation requirements, permeability, and subsidence performance characteristics of a composite barrier. The composite barrier will consist of on-site sandy-clay blanketed by a bentonite mat and a flexible High Density Polyethylene (HDPE) liner (also called flexible membrane liner). Construction of one control test pad and three bentonite test pads are planned. The control test pad will be used to establish baseline data. Underneath the composite clay cap is a four-foot loose sand layer in which cavities will be created by evacuation of sand.

The present work provides a mathematical model for the BMDP. The mathematical model will be used to simulate the mechanical and structural responses of the composite clay cap during the testing processes. Based upon engineering experience and technical references, a set of nominal soil parameters have been selected. Currently, detailed soil test data and cavity configuration data are not available to validate the mathematical model. Since the configuration of the cavities created in the testing process is irregular and unpredictable, two extreme configurations are considered in this mathematical model, viz., the circular cavity and the infinitely long trench in the sand underneath the cap. This approach will provide bounds for the testing results.

In the literature, soil structures particularly clay caps are usually analyzed with linear elasticity theory. The composite clay caps constructed in the BMDP with various compositions of soils and loose sands are more plastic than elastic in mechanical responses. Large irrecoverable deformation is expected in the cap system with the soils experiencing tremendous distortion. The different responses of soil in tension and compression contribute to the deformation pattern of the cap system. Therefore incremental plasticity theory as well as pressure and tension-compression dependent yielding criteria must be applied to this analysis.

This problem possesses both geometrical and constitutional nonlinearities. The finite element analysis technique is applied to solve this type of problem. A commercial code, ABAQUS[®] that is formulated with the incremental plasticity theory and the Drucker-Prager / *Cap plasticity* model, is used to perform the numerical computations.

An axially symmetric model is constructed for the cap covering circular cavities, whereas a plane strain model is used for the cap bridging the trenches. The two dimensional finite element meshes for both models are identical. Each solid element is about six-inch square for the soils. The flexible membrane

liner is meshed with shell elements. The total number of elements is about 1000. The responses of the cap are calculated for a wide range of cavity sizes of both models. The diameter of circular cavities ranges from 3-feet to 12-feet and the width of the trenches from 3-feet to 10-feet.

The loads upon the cap system are gravity and a 2 psi pressure on the top surface of the cap.

In both models the maximum deflection of the cap is at the center of the cavity and the deflection increases with the size of the cavity. The regions of maximum stresses and large deformation are in the vicinity of the cavity edges and the center of the cavity. The maximum deflection for the 12-foot circular cavity model is 2.13 inches, while the maximum deflection for the cap over the 10-foot trench is 6.08 inches. The composite clay cap that spans over the 10-foot trench reaches its load carrying capacity.

Although the results from the mathematical analysis demonstrate the credible structural behavior of the clay cap, realistic in-situ mechanical properties of materials are needed for a good mathematical model which can accommodate prediction of the actual cap behavior.

Post-test investigation and analysis of the BMDP will provide feedback for the mathematical model. The finite element analysis model can be further improved with the input of test data. Further studies of the composite clay cap barrier are recommended.

The structural analysis of a closure cap is to estimate the settlement and subsidence of the closure cap under specified conditions and to evaluate the integrity of the cap system and methods to mitigate damage due to the differential settlement and subsidence.

1.0 INTRODUCTION

This analysis has been performed by the Engineering Modeling & Simulation Group, Applied Technology Section (ATS), Safety Technology Department (STD) of Savannah River Technology Center of the Westinghouse Savannah River Company (WSRC) in response to a work request from the Environment Restoration Department.

1.1 Background

The WSRC Environmental Protection Department has identified over 100 sites suspected of containing hazardous waste materials at SRS. The majority of these sites require development of closure plans. The EPA-recommended closure cap [US EPA, 1985a] is designed to control percolation of rainfall through the waste by deflection of water to the sides of the cap and thereby preventing infiltration of water through the waste into the ground water. A final cover must meet the following requirements:

- Provide long-term minimization of the migration of liquids through the closed waste site;
- Function with minimum maintenance;
- Promote drainage and minimize erosion and abrasion of the final cover;
- Accommodate settling and subsidence so that the cover integrity is maintained;
- Have a permeability less than or equal to the permeability of any bottom liner system or natural subsoils present.

The EPA-recommended multilayer closure cap system contains the following major components (layers):

1. Two-component top layer;
2. Drainage layer;
3. Low permeability layer.

At the Savannah River Site (SRS), five major waste sites have been closed with a Resource Conservation Recovery Act (RCRA) approved closure cap. The low permeability barrier material used in each closure was kaolin clay. While this material meets the recommended regulatory requirements for a permeability of 1.0×10^{-7} cm/sec, in-situ characteristics vary and proper placement is difficult and requires extensive on-site quality control. Consequently, the use of geosynthetic materials as part of a composite barrier is presently being evaluated for future closures [Frobel, 1991].

2.0 BENTONITE MAT DEMONSTRATION PROJECT (BMDP)

The Bentonite Mat Demonstration Project (BMDP) is a field demonstration study to determine the construction / installation requirements, permeability, and subsidence performance characteristics of a composite barrier [Serrato, 1991]. The composite barrier will consist of on-site sandy-clay blanketed by a bentonite mat and a flexible High Density Polyethylene (HDPE) liner (also called flexible membrane liner). Construction of one control test pad and three bentonite test pads are planned. The control test pad will be used to establish baseline data [Warner, 1982].

The stratigraphy of the bentonite mat pads includes from bottom to top: (1) a four-foot thick loose sand layer, (2) a one-foot thick compacted top soil layer, (3) a two to two-and-one-half-foot thick compacted on-site sandy clay layer, (4) one of three commercially available bentonite mats covered with a 40-mil HDPE liner, and (5) a one-foot thick compacted top soil vegetative layer. In addition, earthen berms will abut the basal sand layer at each test pad. The control test pad will be constructed with this same layering. However, the HDPE liner and bentonite mat will be excluded and the sandy clay will serve as the only barrier to infiltration. Each of the test pads extends 80 feet long and 30 feet wide horizontally with an apron of slope 4 by 1 extending 28.25 feet on each of the four edges (see Figures 1, 2) [Serrato, 1991, EBASCO, 1993]. A brief discussion of each layer is provided below.

Sand Layer: The sand layer is four feet thick with the first foot placed below existing grade. The remaining three feet will be contained within a constructed earthen berm. As presently planned it will be constructed from coarse, poorly graded quarry sand with less than 1% fines. During the BMDP voids or cavities will be created in this layer. This layer will be placed in 12-inch lifts. No compaction shall be done on this layer.

Lower Top Soil Layer: A one foot top soil layer of compacted silty loam soil will be placed on top of the sand layer. The top soil will be free of any vegetation, rocks, roots, and any other material. Each lift will be compacted to 80% of the maximum density. Moisture content shall be within $\pm 4\%$ of the optimum moisture content. This layer primarily serves as a marker unit between the underlying sand layer and overlying sandy clay.

On-Site Sandy Clay Soil Layer: Overlying the topsoil will be a two-foot thick layer of the on-site sandy clay soil. The sandy clay soil will have a minimum hydraulic conductivity of 1×10^{-6} cm/sec and each lift will be compacted to 90% of maximum dry density. The sandy clay layer will be placed with a crown along the longitudinal centerline at a slope of 4%. This layer will provide support of the overlying units as voids develop lower in the section and will act as a secondary barrier to surface water infiltration.

Bentonite Mat / HDPE Liner: At each of the three bentonite test pads, a different commercially available bentonite mat will be placed on the top of the sandy clay layer. An HDPE liner will overlay the bentonite mat. Together, the bentonite mat and HDPE will act as a composite barrier to surface water infiltration. The seam placement of the HDPE will coincide with that in the bentonite mat. That will approximate a worst case scenario for subsidence and strain testing. Each HDPE seam will be heat welded together according to manufacture's specifications. As noted above, this layer will not be present in the control test pad.

Topsoil / Vegetative Layer: The uppermost layer will be a one-foot thick top soil cover. It will be the same material and only the pad side slopes will be compacted to the same specification as the topsoil layer underlying the sandy clay layer. It will provide physical protection of the HDPE and minimize the adverse impact of climatic factors.

Earthen Berms: The earthen berms will be constructed from on-site soils. The soil will be placed in lifts and compacted to a minimum of 90% of the maximum density. Moisture content shall be $\pm 4\%$ of the optimum moisture content. Their primary function is to provide lateral support to the test pad embankments.

2.1 Bentonite Mats

A bentonite mat is bentonite clay combined with a geosynthetic fabric or a flexible membrane liner. The clay has a low permeability coefficient that meets recommended regulatory requirements. It also has a high swelling capacity and when hydrated is self-healing. Another benefit is that construction and installation requirements are not as intense as those for placing a kaolin clay cap, since the desired material permeability is achieved primarily during the manufacturing process. While a bentonite mat could be used as a single barrier, combination with a flexible membrane liner provides an even better "composite" barrier system. Further, if this composite barrier is placed over the low permeability site sandy clay a superior cap should result. Three commercially available bentonite mats will be tested: Claymax®, Bentomat® and Gundseal®. Their individual characteristics (especially the width and associated seam placement) impact significantly on the layout of any monitoring instrumentation [Bhutani, et al., 1992].

Structurally, all three bentonite mats have negligible reliable mechanical strength. Under various composite and drainage (and hydration) conditions the maximum cohesion is less than 4.0 psi [Bhutani, et al., 1992].

The primary functions of the flexible membrane liners (FMLs) in the closure cap barrier are to minimize the migration of liquids through the closed land disposal unit and to provide a permeability less than or equal to the bottom liner system or natural subsoil(s) present. A few polymers have achieved widespread acceptance as FML materials for RCRA closure caps. These are: (1) high density polyethylene (HDPE), (2) low density polyethylene (LDPE), (3) linear low density polyethylene (LLDPE), and (4) polyvinyl chloride (PVC).

Since most of the polymers used in the manufacture of FMLs are thermoplastics, the amount or degree of crystallinity varies from as little as 30% in some polyvinyl chloride (PVC) to as high as 75% in HDPEs. Crystallinity is very important in regard to a polymeric geosynthetic behavior. When crystallinity is increased, the following results may be expected:

Table 1. Effect of Crystallinity Increase on HDPE

INCREASED	DECREASED
stiffness and hardness	permeability
heat resistance	elongation or strain at failure
tensile strength	flexibility
modulus	impact strength
chemical resistance	stress crack resistance

Because of its chemical stability, HDPE has become the preferred material for underground applications in a number of industries. The main advantages of polyethylene over other polymers for use as an FML are its extremely low permeability to water vapor, its high chemical resistance and, therefore, resistance to weathering and its good mechanical properties which give superior strength without resorting to reinforcement, and its resistance to penetration (Table 2).

Table 2. Primary Properties of Polyethylene (ASTM D 3350)

PROPERTY	ASTM TEST METHOD	TYPE I	TYPE II	TYPE III
Density, g / cc	D 1505	0.910~0.925	0.926~0.940	0.941~0.955 *
Melt index	D 1238	> 1.0	1.0 to 0.4	< 0.4 to 0.15
Flexural modulus, psi	D 790	< 20,000	20,000 to < 40,000	40,000 to < 80,000
Tensile strength at yield, psi	D 638	< 2,200	2,200 to < 2,600	2,600 to < 3,000
Environmental stress crack resistance	D 1693			
a. Test condition		A	B	C
b. Test duration, hr.		48	24	192
c. Failure, max. %		50	50	20
Hydrostatic design basis, psi, @ 73.4 °F	D 2837	800	1,000	1,250

* The density of Type III Polyethylene, according to D 1248, is 0.941~0.959 (g/cc), while following the "long-standing" practice Type III (High Density) = 0.941~0.965 (g/cc).

Normally FMLs are made from type II or medium density polyethylenes in 40 or 60 mil thickness which gives optimum strength / flexibility characteristics. However with the addition of 2 to 3% of carbon to improve weathering resistance, the material is in the type III density range and is known as HDPE.

Polymeric materials have a very broad modulus or stiffness varying from soft foam-like materials to high modulus structural materials. Those used in waste management facilities have intermediate stiffness. Their uniaxial elongation at rupture can range from 15% to as much as 1,000%. Polymeric compositions are viscoelastic and therefore sensitive to temperature and rate of deformation. When polymeric materials are subjected to deformation they exhibit, in varying degrees, both viscous and elastic behavior. The elastic component has the behavior of a metal spring and is independent of temperature and rate of deformation. The viscous component however, behaves like a dash pot used in damping a shock wave and is highly dependent on the rate of deformation and temperature.

Polymers tend to creep and to relax in stress. Compared to other construction materials such as wood or steel, polymeric materials have a relatively high tendency to creep. As creep progresses, the molecules slip to new positions from which they do not recover. This results in a permanent deformation. Creep is an important characteristic of FMLs due to their long-term exposure. Polymeric materials have thermal coefficients approximately 5 to 10 times greater than those of metal and concrete. Stiffer FMLs, such as polyethylenes, are greatly affected by temperature changes. Temperature changes can cause considerable deformation, buckling, and flexing of a liner.

FMLs encounter a variety of physical stresses when used as components of a cover system. These stresses take place primarily during the construction and during the early service life. Some of the potential physical stresses are as follows:

- Stresses during installation due to the laying out of the FML on the ground.
- Stresses during placement of a soil cover on an FML.
- Stresses due to dropped tools, etc., which could result in puncture.
- Stresses due to traffic.
- Shrinkage stresses at toes of slopes due to heating of the FML and inadequate allowance for shrinkage.
- Low temperature stresses due to inadequate allowance for thermal contraction.
- Stresses over irregularly shaped surfaces due to large aggregates next to the surface of the FML.
- Biaxial stresses which may cause rupture at low elongations, particularly of semicrystalline FMLs.

2.2 Creation of Cavities

In the Bentonite Mat Demonstration Project (BMDP), the loose, dry and friable sand underneath the composite barrier is used to simulate the buried wastes. The cavities in the sand layer will be created by evacuating sand through open-end PVC pipes. There are two available vacuum tools by which the sand may be withdrawn: (1) a heavy-duty shop vacuum, and (2) a heavy-duty mobile vacuum loader truck. The preferred system will be determined during the initial tests conducted on the control pad.

Though the location and volume of a cavity can be approximately measured, the geometrical configuration of the cavity cannot be sculptured as designed. The process of sand evacuation is a relatively slow operation such that at each small increment of sand removal there is time for the cap system to reach mechanical equilibrium. Essentially the deformation of the composite cap system and the creation of cavities are concurrently in progress.

From the point of view of mechanics, the sand evacuation process is sufficiently slow such that the effect of the inertial force in this analysis is negligible and quasi-static analysis is a suitable approach. Soils and the geosynthetic composite are highly inelastic materials. Each incremental process of deformation and cavity development is irreversible. Consequently, the final deformation configuration of the cap system is also a function of the process of the sand evacuation.

The actual design of the overall multilayer cap system and that of its components (layers) depends on the characteristics of the site. Factors for consideration include location and availability of low permeability soil, stockpiling of topsoil, restricting height to provide stable slopes, and site use beyond the postclosure care period.

The structural analysis of the Closure Cap Barriers will provide technical data about:

- The settlement and subsidence of the closure cap under specified conditions:
 - _ Configuration of the cap
 - _ Mechanical properties of the construction materials
 - _ Applied load and degradation of the underneath Waste
- The integrity of the cap system and methods to mitigate damage due to the differential settlement and subsidence.

3.0 MATHEMATICAL MODELING

The analysis of the closure cap involves both geometric and material nonlinearities and structural discontinuities. Finite Element Analysis (FEA) is one of the best numerical techniques for solving complex problems like the multilayer clay caps [US EPA, 1985b].

The mechanical properties of a soil (or any geosynthetic material) are a function of the stress state (hydrostatic pressure, second and third invariants of the deviatoric stresses) as well as moisture content, voids' ratio, temperature, construction process (or manufacture process) and other physical and/or chemical parameters. In the field, the transient variation and inhomogeneity of the parameters in the soil and the uncertainty in measuring these parameters causes more ambiguity in soil data. For preliminary calculations, a range of values of some essential material parameters must be obtained.

The primary effort in this preliminary study is to provide the computational modeling capability for the Bentonite Mat Demonstration Project (BMDP). The major purpose of the computational modeling is twofold: (1) to simulate the mechanical behavior of the composite closure barrier during the testing operation; (2) based upon testing results, to predict the mechanical behavior of the existing as well as prospective new composite closure barriers.

3.1 Geometrical Modeling

During the sand evacuation process in the clay cap, the composite layers deform irreversibly at each increment of the sand movement and the shape of the cavity cannot be predetermined. In fact, the geometrical configuration of the cavity created in the sand bed can be any irregular shape. For a realistic model, a circular cavity and a long trench in the sand bed should cover most of the actual cases. Since the size of the cavity in the test has not been decided, a series of cavities with diameters (or widths) range from 3-feet to 10-feet (12-foot diameter for circular cavity) are included in the calculation. Also the sand base may have relative motion with respect to the soil foundation, a part of the soil foundation should be modeled in this analysis (see Figure 3).

3.2 Material Modeling

This composite closure cap barrier, suggested in the BMDP, consists of at least seven different materials. The mechanical properties of the composite materials vary quite widely.

3.2.1 Soils

In this BMDP the primary construction materials are soils. Mechanically the strength of the soil is a function of the stress state in the soil element. Any mathematical model for soil should include the instantaneous stress state (hydrostatic pressure, second and third invariants of the deviatoric stresses) in the formula. There are several mathematical formulations derived from the classical theories of soil mechanics. The well-known Mohr-Coulomb strength envelope indicates that the shear strength of the soil is a function of the hydrostatic pressure. The Mohr-Coulomb theory is applicable to the biaxial stress state (in the case of triaxial stress state 3-D Mohr circle can be used, but the intermediate principal stress has no determinant effect on the strength). In order to model the geomaterials rationally at least two stress invariants should be considered, i.e., the first invariant of Cauchy stresses and the second invariant of deviatoric stresses. Drucker and Prager [Drucker and Prager, 1952], formulated a model for the behavior of granular materials. Although this model was originally intended for granular materials, some of its extensions make it useful for other pressure dependent materials; it may be used to provide simple approximations to the behavior of some composite materials where differences in tensile versus compressive characteristics are significant.

The inelastic constitutive theories provided in ABAQUS® for modeling cohesionless materials are based on the critical state plasticity theory developed by Roscoe and his colleagues at Cambridge [Schofield et al., 1968 and Parry, 1972]. The specific model implemented is an extension of the "modified Cam-clay" theory. The modified Cam-clay theory is a classical plasticity model. It uses a strain rate decomposition in which the rate of mechanical deformation of the soil is decomposed into an elastic and a plastic part. This model consists of an elasticity theory, a yield surface, a flow rule, and a hardening rule. A key feature of the model is the hardening/softening concept, which is developed around the introduction of a "critical state" surface: the locus of effective stress states where unrestricted, purely deviatoric, plastic flow of the soil skeleton occurs under constant effective stress.

In the BMDP the hydraulic permeability of the soils is considerably low, the effect of fluid flow and soil skeleton deformation interaction is negligible. Therefore, instead of the modified Cam-clay model, the Drucker-Prager / *Cap plasticity* model is chosen. This model is evolved from the original Drucker-Prager model. The modified Drucker-Prager / *Cap plasticity* model is intended for geological materials which exhibit pressure dependent yield. The yield surface includes two main segments: a shear failure surface, providing dominantly shearing flow, and a "cap" that intersects the equivalent pressure stress axis. There is a transition region between these segments, introduced to provide a smooth surface. The cap serves two main purposes: it bounds the yield surface in hydrostatic compression, thus providing an inelastic hardening mechanism to represent plastic compaction, and it helps to control volume dilatancy when the material yields in shear, by providing softening as a function of the inelastic volume increase created as the material yields on the Drucker-Prager shear failure and transition yield surfaces. The model uses the associated flow rule in the cap region and the non-associated flow rule in the shear failure and transition regions.

ABAQUS® version 5.2 has implemented the modified Drucker-Prager / *cap plasticity* model that, in different form [DiMaggio, et al., 1971, Sandler, et al., 1976], has been proved very effective in soil modeling. In order to apply the *cap plasticity* model to a soil structure analysis, a spectrum of mechanical properties of the soil must be obtained through extensive in-situ soil tests. The modified Drucker-Prager / *cap plasticity* model is a member in the general structure of the theory of incremental plasticity which will be briefly discussed in Appendix I. We have obtained a set of parameters of soil elasticity which were derived from laboratory ultrasonic test of *saturated* soil samples [Yu, et al., 1993].

**Table 3. Mechanical Properties of Saturated Soils
(Ultrasonic Test at Core Laboratories)**

SAMPLE	Density (g / cc)	Modulus of Elasticity (psi)	Shear Modulus (psi)	Poisson's Ratio
Top Soil, TS-1	2.04	6.78e+05	2.54e+05	0.335
Top Soil, TS-2	1.99	6.44e+05	2.41e+05	0.337
Gravel, GL-1	2.04	6.64e+05	2.41e+05	0.375
Gravel, GL-2	2.05	8.43e+05	3.12e+05	0.349
Dixie Clay, D-1	1.93	5.99e+05	2.17e+05	0.379
Dixie Clay, D-2	1.95	6.31e+05	2.31e+05	0.367
Grace Clay, GE-1	1.89	2.70e+05	9.90e+04	0.365
Grace Clay, GE-2	1.83	2.60e+05	9.63e+04	0.347
Sand, S-1	1.92	5.00e+05	1.83e+05	0.365
Sand, S-2	1.96	4.88e+05	1.78e+05	0.369
Burma Road Backfill BRS-1	2.06	4.50e+05	1.58e+05	0.424
Burma Road Backfill BRS-2	1.98	5.21e+05	1.85e+05	0.411
Turner Road Backfill TRS-1	2.01	6.31e+05	2.29e+05	0.379
Turner Road Backfill TRS-2	1.99	6.16e+05	2.22e+05	0.386

3.2.2 Bentonite Mat and Flexible Membrane Liner

The bentonite mats to be tested in the BMDP are made of commercially available materials. The three geosynthetic mats are Claymax®, Bentomat® and Gundseal®. The main characteristics of these man-made materials are the low hydraulic permeability. Mechanically, however, they provide little, if any, strength. Compared with sandy clay and the flexible membrane liners, the strength of the three geosynthetic materials is within the range of material testing statistical errors. Therefore, in the stress analysis, the stiffness of the bentonite mat may be ignored.

Among the various polymers, high density polyethylene (HDPE) is chosen as the flexible membrane liner (FML) for the composite clay cap. Mechanically the HDPE is a viscoelastic material that is sensitive to the variation of temperature and rate of deformation. The elastic behavior of the HDPE is a function of deformation gradient [Malvern, 1969]. In general the static isothermal elastic behavior of the HDPE can be modeled with hyperelasticity [see e.g., Truesdell and Toupin, 1960]. In ABAQUS®, two versions of hyperelasticity formulations are available, viz., POLYNOMIAL and OGDEN. The parameters in either formulation must be determined by performing a series of mechanical tests of the material to be modeled. The tests include uniaxial (compression and/or tension), biaxial (compression and/or tension), planar (or pure shear), and volumetric compression tests. For the time being none of the tests is performed. In this BMDP the FML and the geosynthetic mat are directly laying over the sandy clay. The deformation of the

sandy clay may not be too severe before it collapses. Consequently the stretch in the FML may be small enough to fit the elastic-plastic model. The ASTM specifications provide some material parameters suitable for the elastic-plastic formulation. As stated previously, normally FMLs are made from type II or medium density polyethylenes in 40 or 60 mil thickness that gives optimum strength / flexibility characteristics. However with the addition of 2 to 3% of carbon to improve weathering resistance, the material is in the type III density range and is known as HDPE. Accordingly, the FML may have the density of Type III polyethylene but the mechanical properties of Type II.

Table 4. *The Elastic - Plastic Properties of the HDPE FML (ASTM)*

<i>Density (g / cc)</i>	<i>Modulus of Elasticity (psi)</i>	<i>Yield Strength (psi)</i>	<i>Poisson's Ratio</i>
0.955	30,000	2,400	0.40

Theory of hyperelasticity is probably one of the best basic techniques for the modeling of HDPE FMLs. In the situation of inadequate experimental data to determine the parameters in the formulas, the deformation of the FMLs being relatively small, and the temperature fluctuation being low during the loading process, the FMLs can be modeled with incremental plasticity [Hill, 1950, Hibbitt, et al., 1992, Wilson, 1991]. The mathematical theory of incremental plasticity is briefly introduced in Appendix I.

4.0 FINITE ELEMENT MODELING OF THE BMDP

The methodology of mathematical modeling of a physical system is usually determined based upon (1) the geometrical configuration, (2) the mechanical behavior of the materials, (3) the applied loading, (4) the kinematic constraints of the problem, and (5) the availability of the computational facilities. For this BMDP, the horizontal extent of the composite cap system is considerably large compared with the vertical variations of the cap layers. The mechanical properties of the materials in the cap vary from sand, clay to high polymers, if the anisotropy and inhomogeneity of the soils is ignored. The location, configuration, as well as the size of the cavities created in the sand bed are virtually unknown. The interactions between sand and soil layers can best be simulated with sliding surfaces. The finite element code ABAQUS® installed on the CRAY X-MP is the numerical modeling tool available to us. The graphic software, PATRAN® [PATRAN®, 1990], serves as the pre-processor. For this initial study a 2-D model was selected for simplicity.

In order to cover the wide range of cavity variations which may result during testing, two limiting cases are considered. An axially symmetric model with a cylindrical cavity and a plane strain model with a cap covered trench are constructed. In each model, computations are performed for the cap responses to a series of cavities (or trenches) with different sizes. For simplicity, initially vertically straight walled cavities (or trenches) are specified, however, the mechanical property of the sand cannot sustain the vertical walled surface under pressure. For more realistic simulation of the sand behavior, inclined slope surfaces (with 45-degree inclination) are modeled for all the cavities.

In principle, the REMOVE parameter in the * MODEL CHANGE option in ABAQUS® can be invoked to simulate the evacuation of sand in the BMDP. However, in this model, there is a slide line between the covering topsoil and the sand. When the sand elements are removed, the slide line nodes associated with these elements must also be eliminated. In this version of the ABAQUS® the removal of slide line nodes in each analysis step is not provided. An alternate element removal procedure is considered. Instead of gradual removal of sand elements in several steps in a single computation, a series of models created with specific cavity sizes will be analyzed independently. In a nonlinear analysis, the change of procedure in the loading and/or model modification may also perturb the final result. In this BMDP test, the procedure of sand removal is not well defined (in the sense of forming geometrical configuration and location of the cavity). Therefore the effects of procedure change may not be obvious in this analysis.

The fundamental formulation of finite element analysis is based upon the principle of continuum mechanics. Without prescribed gaps, an element in the present finite element method cannot simulate fracturing. In a nonlinear analysis, for the convergence of the solution, the correction of deformation between consecutive iterations within a load increment is limited to the magnitude of only a fraction of the deformation in the previous increment. The evacuation of loose sand in BMDP will certainly induce large deformation in the vicinity of the cavity created. For a close simulation of the sand-evacuation and subsidence interaction, both the space discretization and loading increment must be extremely small. By contemplating the physical parameters in this project and the computational environment provided, the finite element mesh for the 2-D model is developed as follows.

For both the axial symmetric case and the plane strain model, the meshes are composed of six-inch squares, except the sand elements which to accommodate the cavity slope surface are meshed in skewed quadrilateral elements. The thickness of each of the soil layers is closely modeled according to the BMDP design drawings (Figures 1, 2) [EBASCO, 1993]. To minimize the influence of the boundary anomalies, the horizontal length of the model is set to be 20-feet on each side of the line of symmetry. The discretization of the model from top to bottom is listed in Table 5:

Table 5. Type of Elements in the Models

<i>Material in the Layer</i>	<i>Thickness of the Layer inches</i>	<i>Number of Elements in the Thickness</i>	<i>Type of Element in the Axisymmetrical Model</i>	<i>Type of Element in the Plane Strain Model</i>
<i>HDPE FML</i>	<i>0.04</i>	<i>1</i>	<i>SAX1</i>	<i>S4RF</i>
<i>Sandy Clay</i>	<i>30.00</i>	<i>5</i>	<i>CAX4</i>	<i>CPE4</i>
<i>Topsoil</i>	<i>12.00</i>	<i>2</i>	<i>CAX4</i>	<i>CPE4</i>
<i>Sand</i>	<i>48.00</i>	<i>8</i>	<i>CAX4</i>	<i>CPE4</i>
<i>Soil Foundation</i>	<i>24.00</i>	<i>4</i>	<i>CAX4</i>	<i>CPE4</i>

Horizontally there are 40 elements in each of the layers (see Figure 4 for the element mesh)

In these two 2-D models all the element types are two dimensional except the element S4RF that is a 3-D shell element. The trench in the 2-D model has a length considerably longer than other dimensions of the model, thus the plane strain model is a proper approximation. The element CPE4 is a plane strain 4-node bilinear element. There is no comparable plane strain shell element in ABAQUS® for the flexible membrane liner. With constraints in the degrees of freedom.3, 4, and 5, the 3-D shell element S4RF shall behave as a plane strain shell element.

From the limited material data found in the books of soil mechanics and the ASTM specifications, the material parameters for the finite element model can be derived as follows:

Because of insufficient test data for the HDPE FMLs, the material model "hyperelasticity" in ABAQUS® is not suitable in this analysis. Instead, the HDPE FMLs are modeled as an elastic-plastic material with perfect plasticity.

Table 6. The Elastic - Plastic Properties of the HDPE FML

<i>Density (lbm/cubic in.)</i>	<i>Modulus of Elasticity (psi)</i>	<i>Yield Strength (psi)</i>	<i>Poisson's Ratio</i>
<i>0.0345</i>	<i>30,000</i>	<i>2,400</i>	<i>0.40</i>

The soils (the soil foundation, sand, topsoil, and sandy clay) will be simulated with the modified Drucker-Prager / Cap plasticity model. From a field trip observation and the typical range of values of the soil properties provided in a foundation analysis book [Bowles, 1988], a set of model parameters is selected.

Table 7. Modulus of Elasticity of Soils

SOIL	STATIC STRESS-STRAIN MODULUS E_s			
	From (ksf)	To (ksf)	From (psi)	To (psi)
Clay: very soft	50	250	347	1736
Clay: Soft	100	500	694	3472
Clay: medium	300	1000	2083	6944
Clay: hard	1000	2000	6944	13889
Clay: sandy	500	5000	3472	34722
Glacial till: loose	200	3200	1389	22222
Glacial till: dense	3000	15000	20833	104167
Glacial: very dense	10000	30000	69444	208333
Loess	300	1200	2083	8333
Sand: silty	150	450	1042	3125
loose	200	500	1389	3472
dense	1000	1700	6944	11806

Table 8. Angle of Internal Friction of Soils

Soil	Angle of Internal Friction for the type of test		
	Unconsolidated undrained (degrees)	Consolidated undrained (degrees)	Consolidated drained (degrees)
Gravel: medium size	40 ~ 55		40 ~ 55
Gravel: sandy	35 ~ 50		35 ~ 50
Sand: loose dry	28 ~ 34		
Sand: loose saturated	28 ~ 34		
Sand: dense dry	35 ~ 46		43 ~ 50
Sand: dense saturated	1 ~ 2 less than dense dry		43 ~ 50
Silt or silty sand: loose	20 ~ 22		27 ~ 30
Silt or silty sand: dense	25 ~ 30		30 ~ 35
Clay	0 if saturated	3 ~ 20	20 ~ 42

Table 9. Poisson's Ratio of Soils

Soil	Poisson's ratio ν	
	From	To
Clay, saturated	0.4	0.5
Clay, unsaturated	0.1	0.3
Sandy Clay	0.2	0.3
Silt	0.3	0.35
Sand, gravelly sand	-0.1	1.0 *
Sand, commonly used	0.3 **	0.4
Rock	0.1	0.4
loess	0.1	0.3
Ice	0.36	
Concrete	0.15	

* For the uniqueness of the elastostatic solution, Poisson's ratio must satisfy the condition [Love, 1944, Malvern, 1969,]: $-1 < \nu < 0.5$.

** For a very loosely packed sand that can be sucked out with vacuum tube the void ratio will be high and consequently the Poisson's ratio will be low.

The *cap plasticity* hardening parameters should be determined from the consolidation tests of the soil. These parameters are closely related to the compression modulus according to Kögler and Scheidig [Jumikis, 1962, Kögler and Scheidig, 1948]. The approximate range of variation in the average compression modulus, M_v - values, for several soils is shown in the table, after Kögler and Scheidig:

Table 10. Modulus of Compression of Soils

SOIL	COMPRESSION MODULUS M_v			
	From (kg/cm ²)	To (kg/cm ²)	From (psi)	To (psi)
Clay, plastic	5	40	71.1	568.8
Clay, stiff	40	80	568.8	1137.6
Clay, medium hard	80	150	1137.6	2133.0
Sand, loose	100	200	1422.0	2844.0
Sand, dense	500	800	7110.0	11376.0
Gravel, sandy gravel, dense	1000	2000	14220.0	28440.0

Remolded clays usually exhibit thixotropy in load carrying capacity [Terzaghi, et al., 1948, Jumikis, A. R., 1962]. Because of lack of sufficient experimental data, the thixotropy, and other sensitivity factors [Mitchell, 1993] of soil, especially the sandy clay, are not considered. Without in-situ soil test data, the parameters in the *cap plasticity* model will be determined by matching a certain expected deformation of the clay cap system. Current design criterion allows for a maximum deflection of one-foot over a ten-foot trench. One foot displacement at the center of the three and one half feet thick clay cap over a ten-foot wide trench may cause tremendously large deformation in the cap, such that the structural system can no longer sustain the deformed configuration. After a series of parametric studies, the ten-foot trench can only hold a maximum six-inch deflection without collapse. With the requirement of maximum possible deflection in consideration and review of the data in some of the soil mechanics books, the corresponding model parameters can be deciphered as in Table 11:

Table 11. Parameters for the Drucker-Prager / Cap Plasticity Model

SOIL	FOUNDATION	SAND	TOP SOIL	SANDY CLAY
E (psi)	2,000.0	500.0	1,000.0	2778.0
ν	0.30	0.20	0.22	0.25
d (psi)	12.5	4.5	7.5	14.5
β (degrees)	25.0	20.0	25.0	30.0
R	0.3	0.3	0.3	0.3
$\epsilon_{vol}^{pl}(0)$	0.005	0.001	0.003	0.006
α	0.05	0.05	0.05	0.05
K	0.8	0.8	0.8	0.8
$Hp0$ (psi)	20.0	10.0	15.0	20.0
$Vp0$	0.0	0.0	0.0	0.0
$Hp1$ (psi)	40.0	20.0	30.0	40.0
$Vp1$	0.01	0.01	0.01	0.01

In the table:

E = Modulus of elasticity.

ν = Poisson's ratio.

d = Material cohesion in the p-t plane.

β = Material angle of friction in the p-t plane.

R = Cap eccentricity parameter.

$\epsilon_{vol}^{pl}(0)$ = Initial cap yield surface position.

α = Transition surface radius parameter.

K = The ratio of the flow stress in triaxial tension to the flow stress in triaxial compression.

H_{p0} = Initial hydrostatic pressure yield stress.

V_{p0} = Absolute value of the corresponding volumetric plastic strain to H_{p0} .

H_{p1} = First hydrostatic pressure yield stress.

V_{p1} = Absolute value of the corresponding volumetric plastic strain to H_{p1} .

The driving force in this analysis is the gravitational force of all the soil components. The topsoil vegetative layer is not a part of the structural members, however, the weight of this layer is applied on the top of the flexible membrane liner (FML). Since the soil cannot maintain a vertical straight surface under over-pressure, the part of top surface from the free edge to two and one half feet inward is set traction free. The density of the soils is 2.0 grams per cubic centimeter or 0.07225 mass pounds per cubic inch.

5.0 RESULTS

In this analysis the main interest is to find the mechanical responses of the composite clay cap to the gravitational loading and the various configurations of the cavity created in the sand underneath the cap. In order to cover a wide range of cavity configurations without resorting to 3-D computations, two extreme cases are considered. The axially symmetric model and the plane strain model are formulated in similar 2-D geometry. Both models are analyzed with a range of cavity sizes.

A comprehensive mechanical study of each loading and cap configuration case in detail is not warranted for the present investigation. However, the analyses of the overall cap mechanical behavior and the maximum stresses in each case will provide understanding in the correlation between the test data and the mathematical modeling. Among the various displacement components and stress components, the maximum vertical displacement, maximum principal tension, maximum principal compression and the maximum von Mises equivalent stress are considered. The vertical displacement can be directly related to the differential settlement and subsidence of the composite cap measured in the field. The maximum principal tension is the possible largest tensile stress in a soil element. The existence of the tensile stress in a soil element may imply structural crack. The maximum principal compression in a soil element is also a different measurement of strength of the soil at the location of the element. In the Mohr-Coulomb model, the failure surface is a function of the shear stress. While in the three dimensional continuum mechanics, the corresponding shear stress of the *cap plasticity* model is the von Mises equivalent stress (see the Appendix I, for detail). A stress-path (in the p - t plane, i.e., the plane of hydrostatic pressure and the von Mises equivalent stress with the modification of the K parameter) plot at each of the critical locations in the model will provide information for the intrinsic soil behavior. To prepare for this type of plots for even a few elements in each case is very time consuming and it is beyond the scope of the present project.

5.1 Cap covering Circular Cavities

Structurally, under the same loading, the composite clay cap sitting over a circular cavity will deform less than the cap over a trench with the width equal to the diameter of the circular cavity. However the differentiation of deformations between clay cap that covers a circular cavity and that bridges an infinitely long trench will not be elaborated.

There is no in-situ soil data available, also the parameters in the mathematical soil model have no one-to-one correspondence with real physical data. For instance, the cohesion and angle of friction in the model, which though have some physical implication, are not exactly the same quantities measured in the laboratory. After making cross reference to several soil mechanics books and reports, the values of a set of parameters are chosen. The results of a series of test calculations indicate that the stiffness of the composite clay cap with the chosen material parameters was still too strong, such that the deformation of the cap was consistently less than a fraction of an inch over a wide range of cavity sizes. The material parameters are iteratively modified until the composite clay cap that is placed over a ten-foot wide trench under gravity and two pounds per square inch top pressure can barely sustain a maximum deflection of six inches. The set of parameters that preserves the physical values for most of the material constants literally is applied to all the cases in this analysis.

The composite clay cap covering circular cavities range from 3-feet to 12-feet in diameter, deforms only slightly. Except some local stress (or strain) concentrations, the cap remains intact. The following table lists the *maximum* values of vertical displacement, principal tension, principal compression and von Mises equivalent stress at various locations in the cap.

Table 12. *Maximum Displacement and Stresses in Clay Cap over Circular Cavities*

<i>Diameter of Cavity</i>	<i>Vertical Displacement</i>		<i>Principal Tension</i>		<i>Principal Compression</i>		<i>von Mises Equivalent Stress</i>	
<i>feet</i>	<i>inches</i>	<i>Node</i>	<i>psi</i>	<i>Element</i>	<i>psi</i>	<i>Element</i>	<i>psi</i>	<i>Element</i>
3.0	0.8006	8	1.383	17	7.384	29	6.876	29
4.0	0.8672	8	2.345	3	8.805	36	7.742	36
5.0	0.9192	8	2.780	3	9.322	43	8.354	43
6.0	0.9911	7	3.469	3	9.588	50	8.685	50
7.0	1.0880	7	4.355	3	9.749	57	8.884	57
8.0	1.2110	6	5.368	3	9.714	64	8.873	64
9.0	1.3740	6	6.530	3	10.08	7	9.187	71
10.0	1.5750	6	7.829	3	12.11	7	10.33	7
11.0	1.8210	6	9.506	3	14.36	7	12.62	7
12.0	2.1270	5	11.45	3	16.95	7	15.28	7

The maximum displacement, though shifts from node 8 to node 5 as the diameter of the cavity increases, is always along the center axis of the cap. The actual variation of the maximum values along the axis is extremely small. Node 8 is at the top of the cap and node 1 is at the bottom (see Figures 5, 6 and the Node, Element maps of the composite clay cap system in Appendix II). As the diameter of the cavity increases, the radial compression at the top of the cap center also increases due to the reduction of radius of curvature. The Poisson's ratio effect in the top center region of the cap slightly reduces its compressive strain. Consequently the vertical displacement varies slightly along the central axis. The maximum deflection of the cap is plotted against the diameter of the cavity in Figure 7. The curve is fairly smooth and in the shape of a parabola. Of course, without resorting to nonlinear curve fitting process, the actual function of the cavity diameter cannot be determined (i.e., the degree of the polynomial may be beyond 2).

The maximum principal tension in the cap is always at the interface of the sandy clay layer and the topsoil layer along the central axis. Except for the 3-foot diameter case, the maximum stress occurs in element 17 that is the third element from the center line (aligned with the edge of the cavity) and at the same interface. Along the central axis, the bottom element (element 1) has larger tensile strain than the elements above it during downward loading. Comparatively the material stiffness of the topsoil is much weaker than that of the sandy clay on the top. With the same strain the stress in the topsoil will be smaller than that in the sandy clay. According to the *cap plasticity* model, the (hydrostatic) tension cut-off of a material equals to the cohesion times the cotangent of the angle of friction. Since all the angles of friction in this analysis are less than 45 degrees, the tension cut-off value for each of the soils is larger than the corresponding cohesion. The hydrostatic tension is consistently less than or equal to the maximum principal tension. If tensile stress is considered as a failure indication of the sandy clay layer of the cap, the maximum value of 11.45 psi for 12-foot diameter cavity is less than the sandy clay cohesion (14.5 psi). However, the high hydrostatic tension (7.689 psi) in this element reduces the failure limit (in shear, i.e., the von Mises equivalent stress) and this element is under plastic deformation (to be discussed later). The maximum principal tension in the clay cap is plotted in Figure 8.

Each of the maximum stresses value in the tables is the maximum stress at an integration point in the designated element in which there are four integration points. The element stress is the average of the stresses at the four integration points. The calculated element stress is less than or equal to the maximum stress. The maximum stresses shown in the tables are certainly the extreme values.

The location of the maximum principal compression in the cap moves with the edge of the cavity. The maximum principal compression in the cap for each cavity diameter is located about two elements (12 inches) further from the edge of the cavity. At the edge of the cavity the abrupt change of geometry and the ensuing deformation of the cap shift most of the loading and weight of the cap to the rim of the cavity. The loose sand underneath the cap is the softest material in this system. Immediately the pressure at the rim crushes the sharp edge and depresses the sand bed further beyond the edge. About 12-inches from the edge of the cavity the deformation of the sand and the cap develops a wider belt. The pressure in the rim area is redistributed over the belt zone such that the stresses induced in the sand and the cap can still maintain stable equilibrium. If the stresses in the region cannot sustain a stable equilibrium the structure will undergo further deformation. By observation of the deformation in the cap-sand interface region, there are relative slipping and gap closing motions between these two layers in contact. Although the principal compression in the rim area remains high, the maximum values shift to the top element along the center axis as the cavity diameter equals 9-feet or bigger. The maximum principal compression in the clay cap is plotted in Figure 9.

The overall deformation pattern of the composite clay cap is complicated by the layers of different materials and the inelastic properties of the materials. Nevertheless, the cap deforms like a thick plate over a circular cavity. The region at the cap top directly over the rim of the cavity should be under tension. However, the principal tension in this region is slightly less than that along the center axis at the interface of the sandy clay and the topsoil. This pattern of stress distribution shows that the HDPE flexible membrane liner shares the burden of loading in this area.

The von Mises stress is a function of the second invariant of the deviatoric stresses that are the stress components with the hydrostatic pressure part deleted. The von Mises stress in three dimensional mechanics has the characteristics of shear stress. The location of the maximum von Mises stress is usually also the location of maximum shear stress. At the region about 12-inches from the cavity rim the shear stress reaches maximum in the cap, since the rim area carries most of the loading from the cap. The maximum von Mises stress (Figure 10) is in the same region of maximum shear stress component (τ_{rz}), S_{12} . The shear stress S_{12} is steadily increasing as the diameter of the cavity increases. However, the maximum value of the von Mises stress shifts to the top center area of the cap when the diameter of the cavity is above 9 feet. In this top center area the radial compression increases rapidly with the diameter of cavity, whereas the vertical component of stress remains constant. This stress state generates high shear stress in this area. This high shear stress in this element can be demonstrated with Mohr circles. It shows that the maximum shear stress is on a plane with a normal about 45 (or -45) degrees from the radial axis. The maximum von Mises stress is plotted with respect to the cavity diameters in Figure 10.

The failure surface of a *cap plasticity* model is the envelope of the yield limit along the axis of hydrostatic pressure. The yield limit is in terms of the von Mises equivalent stress. From the material data of the soil, such as the cohesion and the angle of friction, and the calculated stresses, the state of stresses (elastic or plastic, hardening or softening) of a clay cap element can be determined. For cavities with diameter less than 11 feet, the maximum von Mises stress in the clay cap is below the cohesion (14.5 psi at $p=0$). For the cap with 12-foot cavity, the von Mises stress, $q=15.28$ psi, is slightly over the cohesion. The hydrostatic pressure in this area is 11.75 psi that provides additional 6.78 ($=11.75 \tan 30$) psi to the failure limit.

The regions of severe deformation in the cap are the regions in the vicinity of the central axis and the circular section above the rim of the cavity. The soil behavior in these areas is manifested by examining the stress and strain status of six elements in the cap (see Figures 5, 6). The elements along the central axis are the bottom element in the topsoil (number 1), the interfacial element in the sandy clay (number 3) and the top element in the sandy clay (number 7). The three elements above the rim of the cavity are in the same order but their numbers vary with the location of the cavity edges.

The behavior of a soil element is a function of at least five stress-strain variables and eight material parameters in the *cap plasticity* model. The magnitude of hydrostatic pressure and von Mises equivalent stress determine the stress path of the element in the *cap plasticity* model. The plastic volumetric strain associated with consolidation parameters fixes the position of the model cap. The Drucker-Prager failure envelope is a function of the angle of friction and cohesion as well as the hydrostatic pressure. With all

these stress bounds in the $p-t$ space the stress state of a specific element can be defined. The following tables (Tables 13, 14) show the stress state of selected elements in the topsoil and the sandy clay layers.

Mechanically, elastic deformation implies that the deformation will be restored with the removal of the applied loads, while the plastic stress state will cause permanent deformation in the element. As long as the structure can stably sustain the (elastic and/or plastic) deformation, the structural system may not collapse imminently. The phenomenon of soil creep is beyond the scope of the present analysis

For the cavity size larger than 9 feet, the elements at the bottom of the cap (in the topsoil layer) are yielded. Whereas, for the elements in the sandy clay layer, only the interfacial element (number 3) along the central axis is in the plastic stress state at cavity diameter equal to 12 feet.

Table 13. Stress State of Topsoil Elements in the Clay Cap Covering Circular Cavities.

Diameter of Cavity feet	Element Number	Hydrostatic Pressure psi	Third stress Invariant psi	von Mises Stress psi	Plastic volumetric Strain	Position of Cap psi	Failure Envelope psi	Modified von Mises Stress psi	Failure Margin psi	State of Stress
3	1	-0.791	-0.985	0.986	0.000000	19.50	7.131	0.986	6.145	Elastic
4	1	-1.350	-1.860	1.860	0.000000	19.50	6.870	1.860	5.010	Elastic
5	1	-1.449	-1.994	1.994	0.000000	19.50	6.824	1.994	4.830	Elastic
6	1	-1.750	-2.461	2.461	0.000000	19.50	6.684	2.461	4.223	Elastic
7	1	-2.165	-3.103	3.103	0.000000	19.50	6.490	3.103	3.387	Elastic
8	1	-2.651	-3.850	3.850	0.000000	19.50	6.264	3.850	2.414	Elastic
9	1	-3.218	-4.728	4.728	0.000000	19.50	5.999	4.728	1.271	Elastic
10	1	-3.856	-5.703	5.704	0.000000	19.50	5.702	5.704	-0.002	Plastic
11	1	-4.058	-5.683	5.683	0.001122	19.50	5.608	5.683	-0.075	Plastic
12	1	-4.061	-5.677	5.678	0.002257	19.50	5.606	5.678	-0.072	Plastic
3	29	3.119	-5.795	6.410	0.000000	19.50	8.954	6.619	2.335	Elastic
4	36	3.645	-7.738	7.742	0.000000	19.50	9.200	7.743	1.456	Elastic
5	43	3.758	-8.334	8.354	0.000000	19.50	9.252	8.361	0.891	Elastic
6	50	3.802	-8.669	8.685	0.000000	19.50	9.273	8.691	0.582	Elastic
7	57	3.827	-8.881	8.884	0.000000	19.50	9.285	8.885	0.399	Elastic
8	64	3.800	-8.869	8.873	0.000000	19.50	9.272	8.874	0.397	Elastic
9	71	3.914	-9.154	9.187	0.000000	19.50	9.325	9.199	0.126	Elastic
10	78	4.092	-9.271	9.371	0.000033	19.50	9.408	9.408	0.000	Plastic
11	85	4.316	-9.250	9.442	0.000096	19.50	9.513	9.513	0.000	Plastic
12	92	4.569	-9.208	9.518	0.000162	19.50	9.631	9.631	0.000	Plastic

Table 14. Stress State of Sandy Clay Elements in the Clay Cap Covering Circular Cavities.

Diameter of Cavity	Element Number	Hydrostatic Pressure	Third stress Invariant	von Mises Stress	Plastic volumetric Strain	Position of Cap	Failure Envelope	Modified von Mises Stress	Failure Margin	State of Stress
feet		psi	psi	psi		psi	psi	psi	psi	
3	3	-0.364	-1.214	1.227	0.000000	32.00	14.290	1.232	13.058	Elastic
4	3	-1.572	-2.193	2.197	0.000000	32.00	13.592	2.198	11.394	Elastic
5	3	-1.809	-2.740	2.746	0.000000	32.00	13.456	2.748	10.707	Elastic
6	3	-2.246	-3.470	3.475	0.000000	32.00	13.203	3.477	9.726	Elastic
7	3	-2.808	-4.405	4.411	0.000000	32.00	12.879	4.413	8.466	Elastic
8	3	-3.446	-5.476	5.484	0.000000	32.00	12.510	5.487	7.023	Elastic
9	3	-4.177	-6.712	6.721	0.000000	32.00	12.088	6.724	5.364	Elastic
10	3	-4.994	-8.085	8.096	0.000000	32.00	11.617	8.100	3.517	Elastic
11	3	-6.114	-9.611	9.627	0.000000	32.00	10.970	9.633	1.337	Elastic
12	3	-7.689	-10.510	10.530	0.000456	32.00	10.061	10.537	-0.477	Plastic
3	7	1.260	-1.356	1.369	0.000000	32.00	15.227	1.374	13.854	Elastic
4	7	2.434	0.897	0.906	0.000000	32.00	15.905	1.129	14.777	Elastic
5	7	3.109	1.896	1.898	0.000000	32.00	16.295	2.372	13.923	Elastic
6	7	3.943	3.177	3.178	0.000000	32.00	16.776	3.972	12.804	Elastic
7	7	4.931	4.697	4.698	0.000000	32.00	17.347	5.872	11.475	Elastic
8	7	6.020	6.373	6.375	0.000000	32.00	17.976	7.968	10.008	Elastic
9	7	7.258	8.276	8.278	0.000000	32.00	18.690	10.347	8.344	Elastic
10	7	8.590	10.330	10.330	0.000000	32.00	19.459	12.913	6.547	Elastic
11	7	10.060	12.620	12.620	0.000000	32.00	20.308	15.775	4.533	Elastic
12	7	11.750	15.280	15.280	0.000000	32.00	21.284	19.100	2.184	Elastic
3	31	1.257	-4.788	4.872	0.000000	32.00	15.226	4.903	10.323	Elastic
4	38	1.118	-5.769	5.872	0.000000	32.00	15.145	5.910	9.236	Elastic
5	45	1.207	-6.065	6.127	0.000000	32.00	15.197	6.150	9.047	Elastic
6	52	1.291	-6.369	6.391	0.000000	32.00	15.245	6.399	8.846	Elastic
7	59	1.370	-6.743	6.746	0.000000	32.00	15.291	6.747	8.544	Elastic
8	66	1.468	-7.078	7.082	0.000000	32.00	15.348	7.083	8.264	Elastic
9	73	1.561	-7.497	7.529	0.000000	32.00	15.401	7.541	7.860	Elastic
10	80	1.716	-7.935	8.037	0.000000	32.00	15.491	8.075	7.416	Elastic
11	87	1.934	-8.314	8.557	0.000000	32.00	15.617	8.646	6.971	Elastic
12	94	2.236	-8.598	9.114	0.000000	32.00	15.791	9.297	6.494	Elastic
3	35	0.689	-2.268	2.392	0.000000	32.00	14.898	2.436	12.462	Elastic
4	42	0.994	-1.077	2.291	0.000000	32.00	15.074	2.548	12.526	Elastic
5	49	1.066	1.723	2.454	0.000000	32.00	15.115	2.867	12.249	Elastic
6	56	1.125	2.500	2.788	0.000000	32.00	15.150	3.388	11.762	Elastic
7	63	1.172	3.212	3.286	0.000000	32.00	15.177	4.080	11.096	Elastic
8	70	1.188	3.937	3.940	0.000000	32.00	15.186	4.924	10.262	Elastic
9	77	1.202	4.740	4.764	0.000000	32.00	15.194	5.946	9.248	Elastic
10	84	1.174	5.662	5.772	0.000000	32.00	15.178	7.175	8.003	Elastic
11	91	1.090	6.737	6.960	0.000000	32.00	15.129	8.619	6.510	Elastic
12	98	0.937	8.039	8.375	0.000000	32.00	15.041	10.348	4.693	Elastic

5.2 Cap covering Trenches

Structurally, to deform a plate into a non-developable surface (such as a spherical surface) will certainly spend more energy than to deform the same plate into a developable surface (such as a cylindrical surface) to the same extent. Therefore, under the same loading conditions, the composite clay cap that spans over a trench will deform much more than the cap covering a circular cavity. The maximum deflection of the cap spanning a trench of 10-feet is about quadruple that of the cap covering a circular cavity of the same size.

The maximum vertical displacement, principal tension, principal compression, and von Mises equivalent stress for the composite clay cap spanning over a range of trench widths are listed in Table 15. Most of the discussions about the cap covering the circular cavities are applicable to the trench cases.

In the case of 10-foot trench, the calculation ends at 99.3% of the total load. Since the gap (0.7%) of loading is so small, the response for the full-loaded case is obtained by linear extrapolation. In the tables, the 99.3% loaded case is marked with an asterisk (*).

Table 15. Maximum Displacement and Stresses in Clay Cap over Trenches

Width of Trench	Vertical Displacement		Principal Tension		Principal Compression		von Mises Equivalent Stress	
feet	inches	Node	psi	Element	psi	Element	psi	Element
3.0	0.9905	8	3.141	17	9.622	29	9.098	29
4.0	1.3300	7	6.654	3	12.33	36	10.01	38
5.0	1.5300	6	7.475	3	13.58	43	11.63	38
6.0	1.7970	6	9.148	3	15.76	7	13.27	7
7.0	2.1750	6	11.30	3	19.70	7	16.97	7
8.0	2.7240	5	12.02	3	23.89	7	20.18	7
9.0	3.7480	6	12.55	3	25.39	7	20.71	7
10.0 *	6.0370	6	13.32	4	25.79	21	20.83	21
10.0	6.0796	6	13.41	4	25.97	21	20.98	21

The maximum displacement (Figure 11) in the trench model progresses much faster with the width of the trench than that in the circular cavity model. As the width of the trench reaches 10 feet, the number of calculation iterations increases for each load increment. In the meantime the size of increment is reduced consecutively until it hits the limit of 1.0E-05. With the load increment reduced to this tiny order of magnitude it still requires more than 7000 increments to complete the calculation. Judging by the amount of time needed to finish the calculation and most of all the truncation error introduced in each operation of the calculation, the calculation should be terminated at this limit. Particularly in this case, the material softening occurs in the topsoil and the sandy clay layers. The strength of the soil in the severely deformed areas progressively deteriorates as the applied load increases. In this situation the solution may not converge. It suggests that further loading may initiate collapse of the structural system.

In actual test pad the situation is different. As the sand masses are gradually removed from underneath the composite clay cap, the cap deforms and settles with each movement of the sand. The cap actually subsides with the sand layer, that is that the cap and the sand layer may keep in contact during the cavitation process. The sand layer may still provide partial support to the cap in the cavity area. In the mathematical model, the cavity is an empty space that provides no support to the cap. In the interface area that extends outward from the trench edge, the cap and sand layer show large relative displacements. The sand elements are severely distorted in this interfacial region. The termination of calculation is mostly due to the slow convergence of solution in this region. In the test pad, the process of gradually smoothing out the edges along the cavities through cap deformation, creates a situation between cavity enlargement and sand subsidence. The finite element model, in theory, can provide close simulation of the real test process, if comprehensive material data are available.

From the 3-foot width trench to the 4-foot trench, the maximum principal tension (Figure 12) in the cap (at the interface of the topsoil and sandy clay layers) jumps 3.14 psi to 6.65 psi. That is the largest increment in the loading process. The maximum principal tension in this interfacial element continuously increases as the trench width increases, but at a more gentle pace. The maximum principal compression and the maximum von Mises equivalent stress in the cap can be found in the vicinity of the trench edge until the trench width is extended to 6 feet. The maxima of compression (Figure 13) and von Mises (Figure 14) stress for trench width equal to or larger than 6 feet are located in the top of the cap center. The rate of increment in both stresses is significantly reduced for the trench size greater than 8-feet. The softening of the cap for wider trenches, indicates that the soils in the region have been stressed into plastic deformation. The material behavior of the clay cap can be explained in the frame work of the *cap plasticity* model as previously discussed in the case of circular cavities.

The mechanical properties of the topsoil layer are comparatively weaker. The areas at the bottom of the cap are heavily stressed, particularly tensile stress at the center and compression in the vicinity of the edge of the trench. The center region starts plastic deformation at a trench width of 6 feet. In the region near the trench edge the topsoil elements deform plastically as soon as the trench widens to 4 feet. In Table 16, the stress state of the topsoil elements in the critical areas is analyzed according to the *cap plasticity* model. The element near the edge of the trench is picked as the element 12 inches from the edge. As the cap deforms and the trench size increases, the location of maximum compression varies. Also the location of the worst damaged element changes with the size of trench. Essentially the elements of maximum stress are deformed plastically and are spread in a zone fanned out from the edge of the trench. The elements selected in the table are those which show consistency in stress pattern.

Table 16. *Stress State of Topsoil Elements in the Clay Cap Covering Trenches.*

Width of Trench	Element Number	Hydrostatic Pressure	Third stress Invariant	von Mises Stress	Plastic volumetric Strain	Position of Cap	Failure Envelope	Modified von Mises Stress	Failure Margin	State of Stress
feet		psi	psi	psi		psi	psi	psi	psi	
3	1	-0.951	2.092	2.287	0.000000	19.50	7.057	2.792	4.265	Elastic
4	1	-1.997	4.272	4.629	0.000000	19.50	6.569	5.662	0.906	Elastic
5	1	-2.215	4.759	5.164	0.000000	19.50	6.467	6.315	0.152	Elastic
6	1	-2.705	4.760	5.264	0.000673	19.02	6.239	6.409	-0.170	Plastic
7	1	-3.014	4.112	5.303	0.001888	17.18	6.095	6.275	-0.180	Plastic
8	1	-3.374	2.361	5.385	0.003802	15.00	5.927	6.115	-0.188	Plastic
9	1	-3.620	-3.729	5.461	0.008569	15.00	5.812	5.926	-0.114	Plastic
10*	1	-3.663	-4.102	5.473	0.020222	15.00	5.792	5.869	-0.077	Plastic
10	1	-3.689	-4.131	5.512	0.020365	14.90	5.780	5.910	-0.131	Plastic
3	29	4.132	-8.501	9.098	0.000000	19.50	9.427	9.308	0.119	Elastic
4	36	5.786	-9.608	9.994	0.000552	19.50	10.198	10.133	0.065	Plastic
5	43	6.778	-10.130	10.420	0.000953	19.36	10.661	10.526	0.135	Plastic
6	50	7.216	-10.230	10.610	0.001080	19.04	10.865	10.747	0.117	Plastic
7	57	7.585	-10.510	10.790	0.001100	19.18	11.037	10.892	0.145	Plastic
8	64	7.693	-10.680	10.890	0.000838	19.46	11.087	10.967	0.120	Plastic
9	71	8.058	-11.010	11.140	0.000502	19.50	11.258	11.188	0.069	Plastic
10*	78	8.234	-11.280	11.290	0.000051	19.50	11.340	11.294	0.046	Plastic
10	78	8.292	-11.360	11.370	0.000051	19.36	11.367	11.373	-0.007	Plastic

Table 17. Stress State of Sandy Clay Elements in the Clay Cap Covering Trenches.

Width of Trench feet	Element Number	Hydrostatic Pressure psi	Third stress Invariant psi	von Mises Stress psi	Plastic volumetric Strain	Position of Cap psi	Failure Envelope psi	Modified von Mises Stress psi	Failure Margin psi	State of Stress
3	3	-0.712	2.078	2.810	0.000000	32.00	14.089	3.303	10.786	Elastic
4	3	-2.514	5.606	6.479	0.000000	32.00	13.049	7.814	5.235	Elastic
5	3	-2.783	6.320	7.356	0.000000	32.00	12.893	8.859	4.035	Elastic
6	3	-3.476	7.577	8.870	0.000000	32.00	12.493	10.670	1.823	Elastic
7	3	-5.211	8.845	10.010	0.000454	32.00	11.491	12.124	-0.633	Plastic
8	3	-6.251	7.796	9.964	0.002036	29.11	10.891	11.806	-0.915	Plastic
9	3	-7.259	-5.916	9.983	0.005922	21.39	10.309	10.971	-0.662	Plastic
10*	3	-7.657	-7.276	10.010	0.016565	20.00	10.079	10.781	-0.701	Plastic
10	3	-7.711	-7.327	10.081	0.016682	20.00	10.048	10.857	-0.809	Plastic
3	7	2.773	-3.176	3.180	0.000000	32.00	16.101	3.181	12.919	Elastic
4	7	4.989	-8.103	8.438	0.000000	32.00	17.380	8.559	8.822	Elastic
5	7	5.898	-9.978	10.500	0.000000	32.00	17.905	10.686	7.219	Elastic
6	7	7.088	-12.470	13.270	0.000000	32.00	18.592	13.552	5.040	Elastic
7	7	8.639	-15.770	16.970	0.000000	32.00	19.488	17.389	2.099	Elastic
8	7	10.760	-18.650	20.180	0.000168	32.00	20.712	20.711	0.001	Plastic
9	7	12.050	-18.490	20.710	0.001065	30.49	21.457	21.456	0.001	Plastic
10*	7	12.230	-17.910	20.660	0.002863	26.91	21.561	21.560	0.001	Plastic
10	7	12.316	-18.036	20.806	0.002884	26.72	21.611	21.712	-0.101	Plastic
3	31	1.631	-5.129	6.872	0.000000	32.00	15.442	7.374	8.068	Elastic
4	38	1.232	-6.044	10.010	0.000000	32.00	15.211	10.986	4.225	Elastic
5	45	1.402	-6.672	10.890	0.000000	32.00	15.309	11.938	3.371	Elastic
6	52	1.936	-7.410	10.890	0.000000	32.00	15.618	11.822	3.795	Elastic
7	59	2.988	-8.250	10.420	0.000000	32.00	16.225	11.076	5.149	Elastic
8	66	4.392	-9.213	10.400	0.000000	32.00	17.036	10.796	6.239	Elastic
9	73	5.995	-10.510	11.170	0.000000	32.00	17.961	11.403	6.558	Elastic
10*	80	7.582	-12.600	13.200	0.000000	32.00	18.877	13.415	5.463	Elastic
10	60	7.635	-12.689	13.293	0.000000	32.00	18.908	13.509	5.399	Elastic
3	35	1.449	-1.843	1.846	0.000000	32.00	15.337	1.847	13.489	Elastic
4	42	1.952	-2.727	2.810	0.000000	32.00	15.627	2.840	12.787	Elastic
5	49	1.813	-2.925	3.041	0.000000	32.00	15.547	3.083	12.464	Elastic
6	56	1.521	-3.020	3.343	0.000000	32.00	15.378	3.453	11.925	Elastic
7	63	0.955	-3.076	4.169	0.000000	32.00	15.051	4.481	10.570	Elastic
8	70	-0.048	1.459	5.964	0.000000	32.00	14.472	6.720	7.752	Elastic
9	77	-1.737	6.616	9.699	0.000000	32.00	13.497	11.296	2.201	Elastic
10*	84	-4.718	6.103	11.310	0.002627	28.45	11.776	12.946	-1.170	Plastic
10	84	-4.751	6.146	11.390	0.002645	28.25	11.757	13.037	-1.280	Plastic

The sandy clay layer with stronger mechanical properties is cushioned with topsoil at the bottom and the HDPE flexible membrane liner on the top. In the circular cavity cap system the sandy clay layer virtually remains elastic for all the cavity sizes until the diameter of the cavity reaches 12 feet. Then only the element at the bottom center interface has yielded. For the clay cap over the trenches (see Table 17), the bottom center interfacial element starts plastic deformation at 7-foot width and the top center element reaches yield at 8-foot width. The interfacial element in the vicinity of the trench edge is not at yield for all sizes of trench studied. The top element above the trench edge deforms plastically for trench width reaching 10 feet. As noted previously the calculation for the 10-foot trench case stopped at 99.3% of the total load. In the table an asterisk (*) is used to mark this case and the results obtained with extrapolation are also recorded.

Since the composite clay cap is not totally confined during the loading process, the soil elements in the cap are not "consolidated", i.e., the incremental plastic volumetric strains are positive for all the elements in plastic deformation. The stress path of the elements in plastic stress state may not reach the cap in the *cap plasticity* model, instead the stress path moves along the Drucker-Prager shear failure surface. Once the stress path touches the Drucker-Prager shear failure surface, the cap will move back and the soil is softening in the mechanical sense. The trend of softening in elements is one of the factors that lead to the deterioration of a soil structure.

More applied surface load is needed in the circular cavity cap system than that in the trench cap system to produce the same magnitude of displacement at the center. A plate being bent along multiple axes is certainly stiffer than the one with only simple bending. The configuration of cavity created in the test is unpredictable. Nevertheless, the bending of the composite clay cap will be multi-axial in the test. Therefore the test results will fall in between the results obtained from the calculations of the two cap systems with extreme cavity configurations, provided that the mechanical properties of the materials in the mathematical model are close to those used in the test.

The HDPE flexible membrane liner (FML) on the top of the composite clay cap provides a certain degree of protection to the cap system. The liner reduces the stresses in the elements on the top of the cap. The liner remains in the elastic state for all sizes of circular cavity or trench considered in this analysis. The maximum stresses in the liner are listed in the table.

Table 18. *Maximum Stresses and Strains in the HDPE FML*

<i>Type of Stress or Strain</i>	<i>12-foot Circular Cavity</i>		<i>10-foot Trench</i>	
	<i>Magnitude</i>	<i>Node</i>	<i>Magnitude</i>	<i>Node</i>
<i>Maximum Tension (psi)</i>	87.44	144	593.5	136
<i>Maximum Compression (psi)</i>	273.8	8	1096.8	8
<i>Maximum von Mises Stress (psi)</i>	268.1	8	955.7	8
<i>Maximum Tensile Strain</i>	0.295 %	136	1.662 %	136
<i>Maximum Compressive Strain</i>	0.508 %	8	3.070 %	8

The maximum stresses in the FML are below the yield strength, 2,400 psi. However, the high compressive stress in the central region of the FML may cause buckling and delaminating from the sandy clay. In order to count on the strength of the FML in the composite clay cap system, sufficient connectivity between the soil and the FML must be provided. Since the thickness of the FML is only 40 mils, the effective force transmitted to the sandy clay is small.

5.3 Stress Contours

The stress contours of the clay cap are plotted over the deformed configuration. The stress contours are plotted for the 12-foot circular cavity cap and the 10-foot trench case. These two sets of plots demonstrate the characteristics of the responses due to different structural configurations. The maximum values of the stresses have been discussed in previous sections. The stress contours and the deformed structures provide an overall understanding of the stress distribution and material behavior of the composite clay cap. In these

plots, the flexible membrane liners are not included for better stress resolution. For the hydrostatic pressure, positive indicates compression, while negative implies tension. The comparison of the responses of the two systems is discussed in Table 19 for clarity.

Table 19. *Structural Responses of the Composite Clay Caps*

Variable	12-foot Circular Cavity	Figure	10-foot Trench	Figure
Deformation	The cap is slightly deformed. The maximum deflection is only 2.13 inches at the center.	15	Six-inch deflection at the center. Large deformation at the Trench edge: The sand elements distorted, the cap sinks into the sand and slides.	16
Hydrostatic Pressure (positive = compression negative = tension)	High compressive pressure at the center top and at the cavity edge. Tension occurs at the interface of topsoil and sandy clay layers along the center axis.	15	High compressive pressure at the center top and at the cavity edge. Tensile stress zones spread from the interface of topsoil and sandy clay layers along the center line to the top of the cap above the trench edge.	16
Principal stress - 1 Principal compression	Maximum compression is in the region of center top and in the vicinity of the cavity edge.	17	Maximum compression is also in the center top and trench edge regions. The zone of compression is much bigger in the trench edge region.	18
Principal stress - 3 Principal tension	Maximum tension is concentrated at the interface of the topsoil and sandy clay layers along the center axis.	19	Though maximum tension is concentrated at the interface of the topsoil and sandy clay layers along the center line, on the top of the cap the principal tension is also high.	20
von Mises equivalent stress	von Mises stress is high in the areas of large shear stresses and the areas of severe distortion. Those areas are at the center top and vicinity of the cavity edge in the cap	21	The clay cap is severely distorted at the center as well as along the trench edge vicinity. The von Mises stress is concentrated in these distorted regions.	22
Third deviatoric stress invariant	This stress in most areas has the magnitude close to that of von Mises stress, but may have negative sign, that depend on the signs of the six deviatoric stress components.	23	The third invariant stress maps a beautiful stress pattern in the clay cap. It also implies uniaxial tension when it is positive and uniaxial compression when it is negative provided its magnitude is equal to that of the von Mises stress.	24
Plastic volumetric strain	This strain includes only the plastic deformation. It clearly indicates that the cap topsoil layer deforms plastically at the center and cavity edge areas. The sandy clay layer is only slightly yield at the interface area along the center.	25	Apparently the clay cap is severely yield, particularly at the bottom and along the center line. Also the region over the clay cap top and in the vicinity of the trench edge	26

6.0 CONCLUSIONS AND RECOMMENDATIONS

The Bentonite Mat Demonstration Project (BMDP) is a field demonstration study to determine the construction / installation requirements, permeability, and subsidence performance characteristics of a composite barrier. The composite barrier will consist of on-site sandy-clay blanketed by a bentonite mat and a flexible High Density Polyethylene (HDPE) liner (also called flexible membrane liner). Construction of one control test pad and three bentonite test pads are planned. The control test pad will be used to

establish baseline data. Underneath the composite clay cap is a four-foot loose sand layer in which cavities will be created by evacuation of sand.

The present work provides a mathematical model for the BMDP. The mathematical model will be used to simulate the mechanical and structural responses of the composite clay cap during the testing processes. Based upon engineering experience and technical references, a set of nominal soil parameters has been selected. Currently, detailed soil test data and cavity configuration data are not available to validate the mathematical model. Since the configuration of the cavities created in the testing process is irregular and unpredictable, two extreme configurations are considered in this mathematical model, viz., the circular cavity and the infinitely long trench in the sand underneath the cap. This approach will provide bounds for the testing results.

In the literature, soil structures particularly clay caps are usually analyzed with linear elasticity theory. The composite clay caps constructed in the BMDP with various compositions of soils and loose sand are more plastic than elastic in mechanical responses. Large irrecoverable deformation is expected in the cap system with the soils experiencing tremendous distortion. The different responses of soil in tension and compression contribute to the deformation pattern of the clay cap system. Therefore incremental plasticity theory as well as pressure and tension-compression dependent yielding criteria must be applied to this analysis.

This problem possesses both geometrical and constitutional nonlinearities. The finite element analysis technique is applied to solve this type of problem. A commercial code, ABAQUS[®] that is formulated with the incremental plasticity theory and the Drucker-Prager / *Cap plasticity* model, is used to perform the numerical computations.

An axially symmetric model is constructed for the cap covering circular cavities, whereas a plane strain model is used for the cap bridging the trenches. The two dimensional finite element meshes for both models are identical. Each solid element is about six-inch square for the soils. The flexible membrane liner is meshed with shell elements. The total number of elements is about 1000 that varies with the size of the cavities in the sand. The responses of the cap are calculated for a wide range of cavity sizes of both models. The diameter of circular cavities ranges from 3-feet to 12-feet and the width of the trenches from 3-feet to 10-feet.

The loads upon the cap system are gravity and a 2 psi pressure on the top surface of the cap.

In both models the maximum deflection of the cap is at the center of the cavity and the deflection increases with the size of the cavity. The regions of maximum stresses and large deformation are in the vicinity of the cavity edges and the center of the cavity. The maximum deflection for the 12-foot circular cavity model is 2.13 inches, while the maximum deflection for the cap over the 10-foot trench is 6.08 inches. The composite clay cap that spans over the 10-foot trench reaches its load carrying capacity.

Although the results from the mathematical analysis demonstrate the credible structural behavior of the clay cap, realistic in-situ mechanical properties of materials are needed for a good mathematical model that can accommodate prediction of the actual cap behavior.

This preliminary study provides a viable methodology for the investigation of structural responses of a soil system. Even though the real physical conditions of the soil system may be different from the conditions set in the mathematical models, the results from the calculation will certainly shed light on the true mechanical behavior of the soil structures. As discussed in the text, sand evacuation (as well as decomposition and deterioration of buried wastes) may create cavities to induce subsidence in the clay cap. However, the deflected cap still rests upon the sand (or the wastes). These phenomena involve crushing of sand elements and reforming of sand masses. Theoretically, all the physically achievable operations can be modeled mathematically. If the finite element analysis technique was used, this kind of process implies extremely small mesh size (as compared with the smallest deformation wave length) in the model. This kind of numerical process is similar to tracing the particle path in a fluid flow.

Under the given loading and boundary conditions a clay cap shows instability over a certain size of cavity. In the field the same cap may collapse over the same size of cavity without obvious indication, because the depressed sand (or waste) provides partial support.

The results obtained in this analysis for both circular cavities and infinitely long trenches will provide bounds for the soil responses in the test project (BMDP), provided that the soil property parameters derived in the mathematical model closely correspond to the in-situ soil data in the test field.

6.1 Recommendations

Mathematical models are the apparatus to simulate the real physical system. In mechanics, the simulation consists three parts, namely the natural conservation laws, the material constitutive relations and the geometry of the physical system. In this analysis, only the conservation laws are precisely modeled. The configuration of the clay cap can be meshed with finite elements as closely as computing devices permit. However, the material properties of soil can only be measured in the field and fit (through fitting process) into model parameters. The common media between the model parameters and the in-situ field test data are the various stress-strain curves. Due to lack of in-situ soil data, the task of this analysis is not finished. Mechanically this model provides sufficient variety for the simulation of the soil structural system. In order to establish the close correlation between the mathematical model and the actual physical process of the system, and to preserve the value of the test project (BMDP), further cooperation among field engineers and mathematical modelers should proceed.

- A comprehensive plan should be started to collect and study the field soil mechanical test properties.
- A series of finite element models should be created accommodating a wide range of field measured soil data, such that a set of best fitted soil parameters can be selected.
- Post-test analysis of the composite clay cap should be carried out to examine the discrepancies between the pre-test and the test results from the BMDP.

Post-test investigation and analysis of the BMDP will provide feedback for the mathematical model. The finite element analysis model can be further improved with the input of the test data. Further studies of the composite clay cap barrier are recommended.

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Figure 2. Bentonite Mat Demonstration Project - Typical Sections.

Bottom of the Foundation Base is fixed in both Directions

Figure 3. Loading and Boundary Conditions of the CLAY CAP

Figure 4. FINITE ELEMENT MESH OF THE COMPOSITE CLAY CAP MODEL

Figure 5.

Element mesh of a part of the Clay Cap with marks of a few cavity locations

Note: The size of each of the elements is 6-inch by 6-inch

Figure 11. ANALYSIS OF COMPOSITE CLAY CAP OVER A TRENCH

Figure 14. ANALYSIS OF COMPOSITE CLAY CAP OVER A TRENCH

ABAQUS

Hydrostatic Pressure 12-foot Circular Cavity

Figure 15. Stress Contours of Hydrostatic Pressure in Clay Cap covering
 a 12-foot Circular Cavity.

ABAQUS

Hydrostatic Pressure 10-foot Trench

Figure 16. Stress Contours of Hydrostatic Pressure in Clay Cap covering a 10-foot Trench.

Principal Stress-1 12-foot Circular Cavity

Figure 17. Stress Contours of Principal Compression in Clay Cap covering a 12-foot Circular Cavity.

ABAQUS

Principal Stress-1 10-foot Trench

Figure 18. Stress Contours of Principal Compression in Clay Cap covering a 10-foot Trench.

ABAQUS

Principal Stress-3 12-foot Circular Cavity

Figure 19. Stress Contours of Principal Tension in Clay Cap covering a 12-foot Circular Cavity.

ABAQUS

Principal Stress-3 10-foot Trench

Figure 20. Stress Contours of Principal Tension in Clay Cap covering a 10-foot Trench.

ABAQUS

von Mises Stress 12-foot Circular Cavity

Figure 21. Stress Contours of von Mises Equivalent Stress in Clay Cap covering a 12-foot Circular Cavity.

ABAQUS

Von Mises Stress 10-foot Trench

Figure 22. Stress Contours of von Mises Equivalent Stress in Clay Cap covering a 10-foot Trench.

ABAQUS

Third Dev-Stress Invariant 12-foot Circular Cavity

Figure 23. Stress Contours of Third Deviatoric Stress Invariant in Clay Cap covering a 12-foot Circular Cavity.

ABAQUS

Third Dev-Stress Invariant 10-foot Trench

Figure 24. Stress Contours of Third Deviatoric Stress Invariant in Clay Cap covering a 10-foot Trench.

ABAQUS

Plastic volumetric Strain 12-foot Circular Cavity

Figure 25. Stress Contours of Plastic Volumetric Strain in Clay Cap covering a 12-foot Circular Cavity.

ABAQUS

Plastic volumetric Strain 10-foot Trench

Figure 26. Stress Contours of Plastic Volumetric Strain in Clay Cap covering a 10-foot Trench.

APPENDIX II

A.1 Incremental plasticity theory

Incremental plasticity theory is based on a few fundamental postulates. A basic assumption of elastic-plastic models is that the deformation can be separated into an elastic part and a plastic part. In its most general form this statement is written as:

$$\mathbf{F} = \mathbf{F}^{el} \cdot \mathbf{F}^{pl}$$

where $\mathbf{F}$ is the total deformation gradient, $\mathbf{F}^{el}$ is the fully recoverable part of the deformation at the point under consideration, and $\mathbf{F}^{pl}$ is defined by $\mathbf{F}^{pl} = [\mathbf{F}^{el}]^{-1} \cdot \mathbf{F}$. The rigid body rotation at the point may be included in the definition of either $\mathbf{F}^{el}$ or $\mathbf{F}^{pl}$ or may be considered separately before or after either part of the decomposition. Conventionally an additive strain rate decomposition:

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^{el} + d\boldsymbol{\varepsilon}^{pl}$$

has been used. Here $d\boldsymbol{\varepsilon}$ is the total mechanical strain rate; $d\boldsymbol{\varepsilon}^{el}$ is the elastic strain rate; and $d\boldsymbol{\varepsilon}^{pl}$ is the plastic strain rate.

In ABAQUS, when the elastic strains are infinitesimal, the rate of deformation definition:

$$d\boldsymbol{\varepsilon} = \text{symm} [\partial \mathbf{v} / \partial \mathbf{x}] dt$$

is applied to the total strain rate in the algorithms developed for all of the plasticity models. Practically the elastic strains in the clay cap soils and the FMLs are considerably small as compared with the inelastic deformations in the composite system. Thus, the use of the additive strain rate decomposition for the models in this analysis is an acceptable approximation.

The elastic part of the response is assumed to be derivable from an elastic strain energy density potential, so that the stress is defined by:

$$\boldsymbol{\sigma} = \partial U / \partial \boldsymbol{\varepsilon}^{el}$$

where U is the strain energy density potential. For the elastic plastic model of von Mises which will be applied to the FMLs, the elasticity is linear, whereas for the soils and foam models the volumetric elastic strain is proportional to the logarithm of the equivalent pressure stress. The yield functions f_i define the limit to the region of pure elastic response, i.e.,

$$f_i(\boldsymbol{\sigma}, \theta, H_\alpha) < 0$$

for pure elastic response. Here θ is the temperature, and H_α are a set of hardening parameters.

When the material is flowing inelastically the inelastic part of the deformation is defined by the flow rule, which can be written as

$$d\boldsymbol{\varepsilon}^{pl} = \sum_i d\lambda_i (\partial g_i / \partial \boldsymbol{\sigma}),$$

where $g_i(\boldsymbol{\sigma}, \theta, H_\alpha)$ is the flow potential for the i th system, and $d\lambda_i$ is a scalar measuring the amount of the plastic flow rate on the i th system.

A.2 *von Mises Equivalent Stress*

The deformation of a solid body can be decomposed into two patterns, a dilatational mode (in which the solid changes volume), and an equivoluminal mode (in which the solid maintains constant volume, but distorts its shape). The corresponding stress fields are the hydrostatic and the deviatoric stresses.

For an isotropic material the criterion of yielding (i.e., in the state of plastic deformation), is a function of the stress state in the material element. In other words, the function is a constitutive law which must be independent of coordinate orientation. Then the yield criterion can be put in the form:

$$q = f(J_1, J_2, J_3)$$

where J_1 , J_2 , and J_3 are the first three invariants of stress tensor σ_{ij} .

Experimental work [Hill, 1950] has shown that for most metals (including steel) the hydrostatic stress has no influence on yielding. In 1913, von Mises suggested that yielding occurred when J'_2 (the second invariant of deviatoric stresses) reached a critical value, and that the yielding function may not involve J'_3 , the third invariant of deviatoric stresses. J'_3 is not considered because its value changes from positive to negative for a triaxial state of stress that changes from tensile to compressive, and the isotropic material yielding does not depend on whether the stresses are tensile or compressive [Malvern, 1969]. In terms of deviatoric stresses, von Mises' yielding criterion can be written as:

$$q = f(J'_2),$$

$$J'_2 = \frac{1}{2} s_{ij} s_{ij},$$

where s_{ij} is the deviatoric stress tensor [Fung, 1965],

$$s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}.$$

Then in terms of Cauchy stresses, σ_{ij}

$$J'_2 = \frac{1}{2} \left(\sigma_{ij} \sigma_{ij} - \frac{1}{3} \sigma_{jj} \sigma_{kk} \right)$$

In terms of principal stresses,

$$J'_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

If σ_y is the yield strength of the material, then the von Mises yielding criterion can be expressed as:

$$q = \sigma_y,$$

where q is the von Mises equivalent stress, in terms of principal stresses:

$$q = \sqrt{3 J'_2} = \left\{ \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \right\}^{\frac{1}{2}}$$

For materials exhibiting strain hardening properties, the equivalent stress or the true stress is a generalized yield criterion for any loading increase that causes additional plastic deformation according to the incremental theory of plasticity.

For a material with appreciable different responses in tension and compression, such as soil, the third invariant of the deviatoric stresses, J'_3 , may be included in the material model.

$$J'_3 = S_{ij} S_{jk} S_{ki} / 3$$

In ABAQUS, the parameter r is used in the formulation of the Cap Model.

$$r = (27 J'_3 / 2)^{1/3} = (9 S_{ij} S_{jk} S_{ki} / 2)^{1/3}$$

The detail of the derivation of the modified Drucker-Prager / cap plasticity model can be found in the ABAQUS theory manual [Hibbitt, et al. 1992].

A.3 Drucker-Prager / Cap Model for Geological Materials

The modified Drucker-Prager / Cap plasticity model is intended for geological materials which exhibit pressure dependent yield. The yield surface include two main segments: a shear failure surface, providing dominantly shearing flow, and a "cap" which intersects the equivalent pressure stress axis (Figure A-1). There is a transition region between these segments, introduced to provide a smooth surface. The cap serves two main purposes: it bounds the yield surface in hydrostatic compression, thus providing an inelastic hardening mechanism to represent plastic compaction, and it helps to control volume dilatancy when the material yields in shear, by providing softening as a function of the inelastic volume increase created as the material yields on the Drucker-Prager shear failure and transition yield surfaces.

The model uses associated flow rule in the cap region and non-associated flow rules in the shear failure and transition yield surfaces. The model is derive from the assumption of additive strain rate decomposition, as discussed in the theory of incremental plasticity. For most geological materials, especially for soils, the range of elastic deformation is considerably small. Therefore, the elastic behavior of the soils may be modeled as linear elastic. The plastic behavior of soils is a function of all the three stress invariants. The yield / failure surfaces used with this model is written in terms of the "standard" three stress invariants:

$$p = - \text{trace} (\sigma) / 3;$$

the von Mises equivalent stress,

$$q = \{ 3 (s : s) / 2 \}^{1/2};$$

and the third invariant of deviatoric stress,

$$r = [9 (s \cdot s : s) / 2]^{1/3}.$$

where s is the stress deviator, defined as

$$s = \sigma + p \mathbf{I},$$

and $\mathbf{I}$ is the unit matrix.

The deviatoric stress measure t is also included in the cap model,

$$t = q [1 + 1/K - (1 - 1/K) (r/q)^3] / 2$$

where $K(\theta, f^\alpha)$ is a material parameter which may depend on temperature, θ , and other predefined field f^α , $\alpha = 1, 2, \dots$. This measure of deviatoric stress is used because it allows matching of different stress values in tension and compression in the deviatoric plane, thereby providing flexibility in fitting experimental results, and a smooth approximation to the Mohr-Coulomb surface. Since $r/q = 1$ in uniaxial tension, $t = q/K$ in this case, and since $r/q = -1$ in uniaxial compression, $t = q$ in that case. When $K = 1$ the dependence on the third deviatoric stress invariant is removed, and the von Mises circle is recovered in the deviatoric plane: $t = q$. Figure A - 2 shows the dependence of t on K . To ensure convexity of the yield surface $0.778 \leq K \leq 1.0$.

With this expression for the deviatoric stress measure, the Drucker-Prager failure surface is written as

$$F_s = t - p \tan \beta - d = 0,$$

where $\beta(\theta, f^\alpha)$ is the material's angle of friction and $d(\theta, f^\alpha)$ is its cohesion.

The cap yield surface has an elliptical shape with constant eccentricity in the meridional (p - t) plane (Figure A-1) and also includes dependence on the third stress invariant in the deviatoric plane (Figure A-2). The cap surface hardens or softens as a function of the volumetric plastic strain: volumetric plastic compaction (when yielding on the cap) causes hardening, while volumetric plastic dilation (when yielding on the shear failure surface) causes softening. The cap yield surface is written as

$$F_c = \{ (p - p_a)^2 + [R t / (1 + \alpha - \alpha / \cos \beta)]^2 \}^{1/2} - R(d + p_a \tan \beta) = 0,$$

where $R(\theta, f^\alpha)$ is a material parameter that controls the shape of the cap, $\alpha(\theta, f^\alpha)$ is a small number which is defined below, and p_a is an evolution parameter that represents the volumetric plastic strain driven hardening / softening. The hardening / softening law is a user defined piecewise linear function relating the hydrostatic compression yield stress, p_b , and the corresponding volumetric plastic strain ϵ_{vol}^{pl} (Figure A-3) $p_b = p_b(\epsilon_{vol}^{pl}, \theta, f^\alpha)$. The evolution parameter p_a is defined as

$$p_a = (p_b - R d) / (1 + R \tan \beta).$$

The parameter α is a small number (typically 0.01 to 0.05) used to define a smooth transition surface between the shear failure surface and the cap:

$$F_t = \{ (p - p_a)^2 + [t + (1 - \alpha / \cos \beta)(d + p_a \tan \beta)]^2 \}^{1/2} - \alpha(d + p_a \tan \beta) = 0.$$

Plastic flow is defined by a flow potential that is associated in the deviatoric plane, associated on the cap in the meridional plane, and non-associated on the failure surface and transition surfaces in the meridional plane. The flow potential surface in the meridional plane is shown in Figure A-4: it is made up of an elliptical portion in the cap region that is identical to the cap yield surface:

$$G_c = \{ (p - p_a)^2 + [R t / (1 + \alpha - \alpha / \cos \beta)]^2 \}^{1/2}$$

and another elliptical portion in the failure and transition regions that provides the non-associated flow component in the model:

$$G_s = \{ [(p - p_a) \tan \beta]^2 + [t / (1 + \alpha - \alpha / \cos \beta)]^2 \}^{1/2}.$$

The two elliptical portions G_c and G_s form a continuous and smooth potential surface.

APPENDIX II

Figure A - 1 Modified Drucker-Prager/Cap model: yield surfaces in the p - t plane.

Figure A - 2 Typical yield/flow surfaces in the deviatoric plane.

Figure A - 3 Typical Cap hardening.

Figure A - 4 Modified Drucker-Prager/Cap model: flow potential in the p - t plane.

Figure A-5 Finite Element Mesh of the Composite Clay Cap Barrier

8	16	24	32	40	48	56	64	72	80	88	96
7	14	21	28	35	42	49	56	63	70	77	
7	15	23	31	39	47	55	63	71	79	87	95
6	13	20	27	34	41	48	55	62	69	76	
6	14	22	30	38	46	54	62	70	78	86	94
5	12	19	26	33	40	47	54	61	68	75	
5	13	21	29	37	45	53	61	69	77	85	93
4	11	18	25	32	39	46	53	60	67	74	
4	12	20	28	36	44	52	60	68	76	84	92
3	10	17	24	31	38	45	52	59	66	73	
3	11	19	27	35	43	51	59	67	75	83	91
2	9	16	23	30	37	44	51	58	65	72	
2	10	18	26	34	42	50	58	66	74	82	90
1	8	15	22	29	36	43	50	57	64	71	
1	9	17	25	33	41	49	57	65	73	81	89

Figure A-6 Element and Nodal Numbers of the Sandy Clay & Topsoil Layers, Part-1

88	96	104	112	120	128	136	144	152	160	168	176
77	84	91	98	105	112	119	126	133	140	147	154
87	95	103	111	119	127	135	143	151	159	167	175
76	83	90	97	104	111	118	125	132	139	146	153
86	94	102	110	118	126	134	142	150	158	166	174
75	82	89	96	103	110	117	124	131	138	145	152
85	93	101	109	117	125	133	141	149	157	165	173
74	81	88	95	102	109	116	123	130	137	144	151
84	92	100	108	116	124	132	140	148	156	164	172
73	80	87	94	101	108	115	122	129	136	143	150
83	91	99	107	115	123	131	139	147	155	163	171
72	79	86	93	100	107	114	121	128	135	142	149
82	90	98	106	114	122	130	138	146	154	162	170
71	78	85	92	99	106	113	120	127	134	141	148
81	89	97	105	113	121	129	137	145	153	161	169

Figure A-7 Element and Nodal Numbers of the Sandy Clay & Topsoil Layers, Part-2

168	176	184	192	200	208	216	224	232	240	248	256
147	154	161	168	175	182	189	196	203	210	217	
167	175	183	191	199	207	215	223	231	239	247	255
146	153	160	167	174	181	188	195	202	209	216	
166	174	182	190	198	206	214	222	230	238	246	254
145	152	159	166	173	180	187	194	201	208	215	
165	173	181	189	197	205	213	221	229	237	245	253
144	151	158	165	172	179	186	193	200	207	214	
164	172	180	188	196	204	212	220	228	236	244	252
143	150	157	164	171	178	185	192	199	206	213	
163	171	179	187	195	203	211	219	227	235	243	251
142	149	156	163	170	177	184	191	198	205	212	
162	170	178	186	194	202	210	218	226	234	242	250
141	148	155	162	169	176	183	190	197	204	211	
161	169	177	185	193	201	209	217	225	233	241	249

Figure A-8 Element and Nodal Numbers of the Sandy Clay & Topsoil Layers, Part-3

248	256	254	272	280	288	296	304	312	320	328
217	224	231	238	245	252	259	266	273	280	
247	255	263	271	279	287	295	303	311	319	327
216	223	230	237	244	251	258	265	272	279	
246	254	262	270	278	286	294	302	310	318	326
215	222	229	236	243	250	257	264	271	278	
245	253	261	269	277	285	293	301	309	317	325
214	221	228	235	242	249	256	263	270	277	
244	252	260	268	276	284	292	300	308	316	324
213	220	227	234	241	248	255	262	269	276	
243	251	259	267	275	283	291	299	307	315	323
212	219	226	233	240	247	254	261	268	275	
242	250	258	266	274	282	290	298	306	314	322
211	218	225	232	239	246	253	260	267	274	
241	249	257	265	273	281	289	297	305	313	321

Figure A-9 Element and Nodal Numbers of the Sandy Clay & Topsoil Layers, Part-4

Figure A-10 Element and Nodal Numbers of the Sand Layer, Part-1

Figure A-12 Element and Nodal Numbers of the Sand Layer, Part-3

Figure A-14 Element and Nodal Numbers of the Sand Layer, Part-5

Figure A-15 Element and Nodal Numbers of the Sand Layer, Part-6

Figure A-16 Element and Nodal Numbers of the Sand Layer, Part-7

Figure A-17 Element and Nodal Numbers of the Sand Layer, Part-8

Figure A-19 Element and Nodal Numbers of the Sand Layer, Part-10

Figure A-21 Element and Nodal Numbers of the Sand Layer, Part-12

Figure A-22 Element and Nodal Numbers of the Sand Layer, Part-13

Figure A-23 Element and Nodal Numbers of the Sand Layer, Part-14

332	347	362	377	386	387	388	390	404	421	437	453	469
824	825	826	827	828	829	830	831	832	833	834	835	836
331	346	361	371	372	373	374	389	406	422	438	454	470
865	866	867	868	869	870	871	872	873	874	875	876	877
330	345	356	357	358	360	375	391	407	423	439	455	471
905	907	908	909	910	911	912	913	914	915	916	917	918
329	341	342	343	344	355	376	392	408	424	440	456	472
947	948	949	950	951	952	953	954	955	956	957	958	959

Figure A-24 Element and Nodal Numbers of the Soil Foundation, Part-1

469	485	501	517	533	549	565	581	597	613	629	645	661
470	486	502	518	534	550	566	582	598	614	630	646	662
471	487	503	519	535	551	567	583	599	615	631	647	663
472	488	504	520	536	552	568	584	600	616	632	648	664

Figure A-25 Element and Nodal Numbers of the Soil Foundation, Part-2

661	677	693	709	725	741	757	773	789	805	821	837	852	865
648	649	650	651	652	653	654	655	656	657	658	659	660	661
662	678	694	710	726	742	758	774	790	806	822	838	853	866
689	690	691	692	693	694	695	696	697	698	699	700	701	702
663	679	695	711	727	743	759	775	791	807	823	839	854	867
930	931	932	933	934	935	936	937	938	939	940	941	942	943
664	680	696	712	728	744	760	776	792	808	824	840	855	868
971	972	973	974	975	976	977	978	979	980	981	982	983	984

Figure A-26 Element and Nodal Numbers of the Soil Foundation, Part-3

741	757	773	789	805	821	837	852	865	876	885	
	854	855	856	857	858	859	860	861	862	863	864
742	758	774	790	806	822	838	853	866	877	886	
	895	896	897	898	899	900	901	902	903	904	905
743	759	775	791	807	823	839	854	867	878	887	
	936	937	938	939	940	941	942	943	944	945	946
744	760	776	792	808	824	840	855	868	879	888	
	977	978	979	980	981	982	983	984	985	986	987

Figure A-27 Element and Nodal Numbers of the Soil Foundation, Part-4