

# Exclusive annihilation $p\bar{p} \rightarrow \gamma\gamma$ in a generalized parton picture

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Exclusive proton–antiproton annihilation into two photons at large  $s$  ( $\sim 10 \text{ GeV}^2$ ) and  $|t|, |u| \sim s$  can be described by a generalized parton picture analogous to the “soft mechanism” in wide–angle real Compton scattering. The two photons are emitted in the annihilation of a single fast quark and antiquark. The matrix element describing the transition of the  $p\bar{p}$  system to a  $q\bar{q}$  pair can be related to the timelike proton elastic form factors as well as to the quark/antiquark distributions measured in inclusive deep–inelastic scattering. The reaction could be studied with the proposed 1.5–15 GeV high–luminosity antiproton storage ring (HESR) at GSI.

Compton scattering — both real and virtual — is one of the main sources of information on the structure of the nucleon. In particular, inclusive deep–inelastic scattering can be viewed as a measurement of the imaginary part of the forward virtual Compton amplitude, parametrized by the quark and antiquark distributions in the nucleon. More recently, exclusive processes have been considered, which can be described in terms of generalized parton distributions, namely deeply–virtual (DVCS) and wide–angle real (WACS) Compton scattering. It has been argued that at values of  $s \sim \text{few GeV}^2$  and  $|t|, |u| \sim s$  WACS is dominated by the “soft mechanism” [1, 2]. The Compton scattering occurs off a single quark or antiquark in the nucleon. Its emission and absorption by the nucleon is described by double distributions which can be related to the usual quark/antiquark distributions as well as to the elastic form factors of the nucleon [1]. This approach describes well the existing data [3], including the spin asymmetry of the cross section measured recently in the JLAB Hall A experiment [4]. The hard scattering mechanism, in which the struck quark rescatters via gluon exchanges of virtuality  $\sim t$ , is relevant only at asymptotically large  $s$  and  $t$  and cannot account for the measured cross section at JLAB energies [1].

The proposed high–luminosity 1.5–15 GeV antiproton storage ring (HESR) at GSI [5] would offer an opportunity to study the Compton process in the crossed channel, namely exclusive proton–antiproton annihilation into two photons,  $p\bar{p} \rightarrow \gamma\gamma$ . In this letter we argue that this process at large  $s$  and  $|t|, |u| \sim s$  can also be described in a generalized parton picture. The two photons are predominantly emitted in the annihilation of a single “fast” quark and antiquark originating from the proton and antiproton. The new double distributions, describing the transition of the  $p\bar{p}$  system to a  $q\bar{q}$  pair, can be related to the timelike nucleon form factors; by crossing symmetry they are also connected with the usual quark/antiquark distributions in the nucleon. A similar approach based on light–cone wave functions has been used for the processes  $\gamma\gamma \rightarrow \pi\pi$  [6] and  $\gamma\gamma \rightarrow B\bar{B}$  [7]. Exclusive  $p\bar{p}$  annihilation was also studied in the diquark model [8].

The annihilation process  $p(p_1) + \bar{p}(p_2) \rightarrow \gamma(q_1) + \gamma(q_2)$  is characterized by the invariants  $s = 2(p_1 p_2) > 0$  (we neglect the proton mass),  $t = -s \sin^2(\theta/2) < 0$ , and  $u = -s \cos^2(\theta/2) < 0$ , where  $\theta$  is the scattering angle in the center–of–mass frame. We consider the region where  $s$  is much larger than typical light hadron masses,  $s \sim 10 \text{ GeV}^2$ , and  $|t|, |u| \sim s$ , which requires that  $\theta$  be sufficiently far from 0 and  $\pi$  (wide–angle scattering).

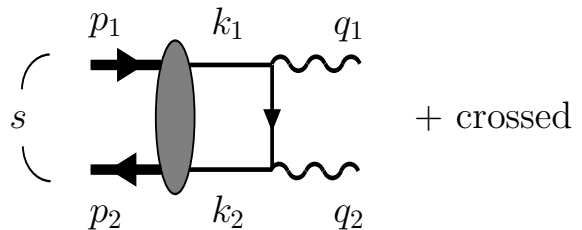


FIG. 1: The “handbag” contribution to  $p\bar{p} \rightarrow \gamma\gamma$  annihilation.

In QED, the annihilation process  $e^+e^- \rightarrow \gamma\gamma$  involves the  $t$  (or  $u$ ) channel exchange of a virtual electron/positron with spacelike four–momentum. In  $p\bar{p} \rightarrow \gamma\gamma$  annihilation in QCD, the exchanged system contains at least three quarks. At large momentum transfer such an exchange is strongly suppressed by the nucleon wave functions. In this situation, the most efficient way of accomplishing a large momentum transfer is the “handbag” diagram, Fig. 1. The amplitude consists of two parts. In the first part, the proton–antiproton system converts to a quark–antiquark pair by exchanging a virtual  $q\bar{q}$  (“diquark”) system. The nucleon wave functions restrict the spacelike virtuality of this system to values of the order of a hadronic scale,  $\lambda^2$ , related to the size of the nucleon. In the second part, the quark–antiquark pair annihilates via exchange of a highly virtual quark/antiquark, much like in QED  $e^+e^- \rightarrow \gamma\gamma$  annihilation. Neglecting transverse momenta, we expand the momenta of the quark and antiquark as

$$k_1 = (1 - x_1)p_1 + x_2 p_2, \quad (1)$$

$$k_2 = x_1 p_1 + (1 - x_2)p_2. \quad (2)$$

The variables  $x_1$  and  $x_2$  obey the “partonic” limits  $0 <$

$x_{1,2} < 1$ . The spacelike virtuality of the exchanged “diquark” system is  $-x_1x_2s$ . The restriction  $|x_1x_2s| < \lambda^2$  implies that for  $s \gg \lambda^2$  the value of either  $x_1$  or  $x_2$  must be small, of the order  $\lambda^2/s$ . This means that either the annihilating quark or the antiquark are “fast”, carrying the major part ( $\sim 1$ ) of the proton or antiproton momentum. This in turn implies that the spacelike virtualities of the quark propagators connecting the photon vertices in the graphs of Fig. 1,  $[-x_1x_2s + (1 - x_1 - x_2)t]$  for the direct and  $[-x_1x_2s + (1 - x_1 - x_2)u]$  for the crossed graph, are large in the kinematics of wide-angle scattering,  $|t|, |u| \sim s$ . Thus, the parton picture of  $p\bar{p} \rightarrow \gamma\gamma$  exclusive annihilation emerging from the “handbag” graph is self-consistent. The inclusion of transverse momenta would change this picture only quantitatively.

The so-called hard scattering mechanism would require the rescattering of the active partons through gluon exchanges of virtuality  $t$ . In the case of WACS such contributions, which are theoretically dominant in the asymptotic region  $|t| \rightarrow \infty$ , are negligible compared to the subasymptotic “handbag” contribution at all experimentally relevant values ( $t \sim \text{few GeV}^2$ ), the main reason being that the hard contributions are numerically suppressed by a factor of  $(\alpha_s/\pi)^2 \approx 1/100$ , see Ref. [1]. The same can be expected for the annihilation channel.

The crucial ingredient in the calculation of the hadronic amplitude is the matrix element describing the amplitude for the conversion of the  $p\bar{p}$  system to a  $q\bar{q}$  pair. Following the treatment of WACS, we parametrize this matrix element by “double distributions” [1, 9, 10], in which the quark and antiquark momenta are measured in terms of the proton and antiproton momenta in a symmetric fashion, using the average and difference,  $p = (p_1 + p_2)/2$  and  $r = p_1 - p_2$ :

$$k_1 = (1 + \alpha)p + xr/2, \quad (3)$$

$$k_2 = (1 - \alpha)p - xr/2. \quad (4)$$

The new variables  $x$  and  $\alpha$  are related to the old ones by  $x = 1 - x_1 - x_2$ ,  $\alpha = x_2 - x_1$ . We parametrize the annihilation-type matrix elements of the quark bilinear vector operators as ( $a$  denotes the quark flavor)

$$\begin{aligned} & \langle 0 | \bar{\psi}_a(-z/2)\gamma_\sigma\psi_a(z/2) | p_1, \lambda_1; p_2, \lambda_2 \rangle \\ &= \bar{v}\gamma_\sigma u \int \int_{|x|+|\alpha|<1} dx d\alpha F_a(x, \alpha; s) e^{-i\alpha(pz) - ix(rz)/2} \\ &+ \dots \end{aligned} \quad (5)$$

The matrix element of the axial vector operator ( $\gamma_\sigma\gamma_5$ ) is parametrized similarly with a double distribution  $G_a(x, \alpha; s)$ . Here  $z$  is a light-like distance, and  $u \equiv u(p_1, \lambda_1)$  and  $\bar{v} \equiv \bar{v}(p_2, \lambda_2)$  are the proton and antiproton spinors. The double distribution  $F_a$  depends on the spectral parameters  $x$  and  $\alpha$ , as well as on  $s$ . Not shown in Eq.(5) for brevity are components of the matrix element of the type  $\epsilon_{\sigma\alpha\beta\gamma}\bar{v}\gamma_\gamma\gamma_5 u$ , which play a role in main-

taining electromagnetic gauge invariance of the “handbag” amplitude, similar to the kinematical twist-3 terms in deeply-virtual Compton scattering [11]. For simplicity we neglect components of the matrix element corresponding to the Pauli and pseudoscalar form factors; they should be included in a more complete treatment.

The new double distributions are related to the time-like elastic form factors of the proton by

$$\sum_a e_a \int \int_{|x|+|\alpha|<1} dx d\alpha F_a(x, \alpha; s) = F_1(s), \quad (6)$$

where  $F_1(s)$  is the Dirac vector form factor. A similar relation holds for the function  $G_a(x, \alpha; s)$  and the axial form factor. Furthermore, in the limit  $s \rightarrow 0$  one can use crossing symmetry to relate the annihilation-type matrix element (5) to the corresponding scattering-type matrix element at  $t = 0$ , parametrized by the usual double distributions of WACS. In particular, this implies

$$\int_{-1+|x|}^{1-|x|} d\alpha F_a(x, \alpha; s = 0) = f_a(x), \quad (7)$$

where  $f_a(x)$  is the usual unpolarized parton density in the proton; in conventional notation  $f_a(x) = \theta(x)q_a(x) - \theta(-x)\bar{q}_a(-x)$ . In a similar way the double distribution  $G_a(x, \alpha; s)$  reduces to the polarized parton density.

To construct an explicit model for the double distributions we make the ansatz

$$F_a(x, \alpha; s) = f_a(x) h_a(x, \alpha) S_a(x, \alpha; s). \quad (8)$$

In the parton density,  $f_a(x)$ , we take into account only the valence quark contribution, *i.e.*, we assume it to be of the form  $f_a(x) = \theta(x)q_a(x)$ ; sea quarks could readily be included. The factor  $h_a(x, \alpha)$  is a profile function, whose integral over  $\alpha$  from  $-1 + |x|$  to  $1 - |x|$  is normalized to unity, *cf.* Eq.(7). Finally,  $S_a(x, \alpha; s)$  is a cutoff function accounting for the  $s$ -dependence of the double distribution, defined such that  $S_a(x, \alpha; s = 0) = 1$ . We model it as (assuming  $x > 0$ )

$$S_a(x, \alpha; s) = \exp\left\{-\frac{[(1-x)^2 - \alpha^2]s}{4x(1-x)\lambda_a^2}\right\}, \quad (9)$$

or, in terms of the original variables  $x_1$  and  $x_2$ ,

$$\exp\left[-\frac{x_1x_2s}{(x_1+x_2)(1-x_1-x_2)\lambda_a^2}\right]. \quad (10)$$

Here  $\lambda_a^2$  is a parameter of dimension mass squared. This cutoff suppresses contributions in which the absolute value of the virtuality of the exchanged “diquark” system,  $|x_1x_2s|$ , is large, and also configurations in which  $x_1 + x_2$  tends to 1 ( $x$  tends to zero). For the double distributions parametrizing scattering-type matrix elements in WACS, this cutoff function can be thought of

as the result of an overlap integral of light-cone wave functions [1, 2]. Such an interpretation is not possible in the annihilation channel. Here, the cutoff function should simply be regarded as an effective manner to restrict the virtuality of the exchanged “spectator” system.

For an estimate of the  $p\bar{p} \rightarrow \gamma\gamma$  amplitude we use the GRV94 LO parametrization of the valence quark densities at a normalization scale of  $1 \text{ GeV}^2$  [12]. We neglect small contributions from strange quarks. The profile function we take to be of the form  $h_a(x, \alpha) = \delta(\alpha)$ ; extended profiles have been suggested in Refs.[9, 10]. For simplicity we choose the cutoff function  $S_a(x, \alpha; s)$  to be the same for both  $u$  and  $d$  flavors. The parameter  $\lambda^2 \equiv \lambda_u^2 = \lambda_d^2$  we determine by fitting the integral over the model double distribution [see Eq.(6)] to the data for the proton vector form factor. Fitting to the form factor data in the spacelike domain [13] we obtain  $\lambda^2 = 0.7 \text{ GeV}^2$ . For large  $s$  the real part of the timelike form factor is approximately two times as large as that of the spacelike form factor for corresponding large  $t$ , see Ref.[14] for a discussion. In order to account for this effect we multiply the timelike form factors entering the double distribution ansatz by a factor of 2. We stress that this should be regarded as a purely phenomenological improvement. The axial vector double distribution  $G_a(x, \alpha; s)$  is modeled analogously, using the GRSV95 LO parametrization for the polarized valence quark densities [15]. For simplicity we use here the same cutoff function as in the vector double distribution  $F_a(x, \alpha; s)$ .

With our model for the double distributions we can compute the  $p\bar{p} \rightarrow \gamma\gamma$  amplitude from the “handbag” graphs of Fig. 1. As the process is dominated by configurations with small  $x_1$  and  $x_2$  (“fast” quark and antiquark), we can simplify the  $q\bar{q} \rightarrow \gamma\gamma$  amplitude by neglecting contributions of order  $x_1 x_2$ ; keeping them would result in  $\lambda^2/s$ -suppressed contributions to the  $p\bar{p} \rightarrow \gamma\gamma$  amplitude which are beyond the accuracy of our approximation. In this way the annihilating quark and antiquark are effectively put on mass shell, which makes it straightforward to maintain transversality of the amplitude (e.m. gauge invariance). The result for the helicity-averaged differential cross section is

$$\frac{d\sigma}{d\cos\theta} = \frac{2\pi\alpha_{\text{em}}^2}{s} \frac{R_V^2(s) \cos^2\theta + R_A^2(s)}{\sin^2\theta}. \quad (11)$$

The information about the structure of the proton is contained in generalized form factors

$$R_V(s) \equiv \sum_a e_a^2 \int \int_{|x|+|\alpha|<1} dx d\alpha \frac{F_a(x, \alpha; s)}{x}, \quad (12)$$

and  $R_A(s)$  defined by the corresponding integral over the double distribution  $G_a(x, \alpha; s)$ . For  $R_V \equiv R_A \equiv 1$  Eq.(11) would reduce to the Klein–Nishina formula for the  $e^+e^- \rightarrow \gamma\gamma$  cross section in QED. Note that our parton picture is applicable only to wide-angle scattering, so

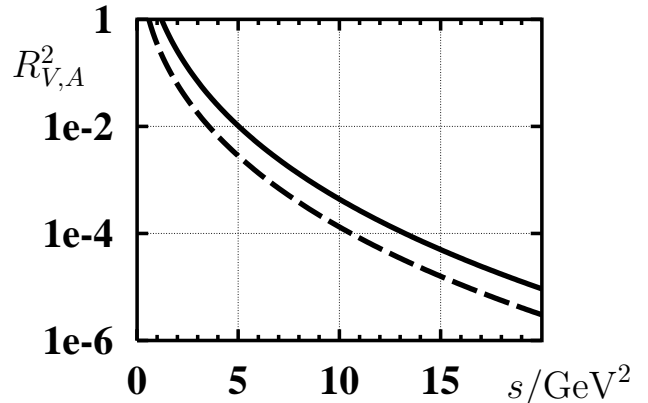


FIG. 2: The squared form factors  $R_V^2(s)$  (solid line) and  $R_A^2(s)$  (dashed line), as calculated from the double distribution model, *cf.* Eq.(12).

the divergence of the expression in Eq.(11) in the limit  $\theta \rightarrow 0$  or  $\pi$  should be regarded as unphysical.

Eq.(11) shows that the contribution of the vector operator to the annihilation cross section,  $R_V$ , is suppressed at large scattering angle relative to that of the axial vector operator,  $R_A$ , by a factor of  $\cos^2\theta$ . This is different from WACS, where the contribution from the axial-vector form factor is suppressed [1].

Compared to the elastic form factor, Eq.(6), the integrand in Eq.(12) contains an additional factor  $1/x$ , which is the “remnant” of the quark propagator in the  $q\bar{q} \rightarrow \gamma\gamma$  amplitude. Note that the integral nevertheless converges at small  $x$ , as the cutoff function  $S_a(x, \alpha; s)$  forces the double distribution to vanish for  $x \rightarrow 0$ , *cf.* Eqs.(8) and (9). More generally, the properties of the double distributions ensure that the integrals are dominated by large values of  $x$ . The numerical results for the squared form factors  $R_V(s)$  and  $R_A(s)$  are shown in Fig. 2.

The results for the form factors  $R_V(s)$  and  $R_A(s)$  turn out to be insensitive to the precise value of the normalization scale of the parton densities in the double distribution model, Eq.(8). This happens due to the correlation of the  $\lambda$  parameter in the cutoff function with the normalization scale of the parton densities [16]. Changing the normalization scale one must change  $\lambda$  such as to refit the form factor data, and the two changes compensate each other in the integral Eq.(12). This fact is important for the consistency of our approach.

In our simple model the timelike double distributions are described by real functions. As a result, the  $p\bar{p} \rightarrow \gamma\gamma$  amplitude is also real (the virtuality of the quark propagators in the “handbag” graphs of Fig. 1 is always spacelike). A more refined treatment should include also the “intrinsic” imaginary part of the double distributions, which is related to the imaginary part of the timelike proton form factor by Eq.(6).

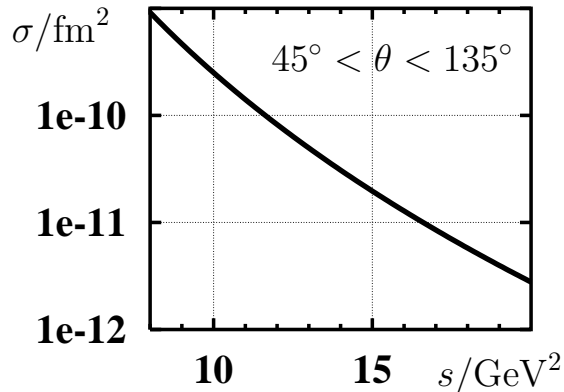


FIG. 3: The  $p\bar{p} \rightarrow \gamma\gamma$  cross section integrated over the range  $45^\circ < \theta < 135^\circ$ , as a function of  $s$ .

It is interesting to estimate the counting rate for  $p\bar{p} \rightarrow \gamma\gamma$  annihilation expected for the proposed 1.5 – 15 GeV antiproton storage ring (HESR) at GSI [5]. With a solid target the luminosity could be as high as  $L = 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ . Since our parton picture applies only in the kinematical region where  $|t|, |u| \sim s$ , we integrate the differential cross section (11) over a range  $\delta < \theta < \pi - \delta$ , with  $\delta > 0$ , excluding small scattering angles. For  $s = 10 \text{ GeV}^2$ , a range of  $45^\circ < \theta < 135^\circ$  corresponds to  $|t|, |u| > 1.5 \text{ GeV}^2$ , which seems to be a reasonable boundary in view of the experience with wide-angle Compton scattering [1]. The cross section integrated over this fixed angular range (divided by 2 to account for the identical particles in the final state) is shown in Fig. 3 as a function of  $s$ . At  $s = 10 \text{ GeV}^2$  our model predicts an integrated cross section of  $0.25 \times 10^{-9} \text{ fm}^2$ , corresponding to a counting rate of  $0.5 \times 10^{-3} \text{ sec}^{-1}$ , that is  $O(10^3)$  events per month. The process should thus be measurable with reasonable statistics.

To summarize, we have outlined a generalized parton picture of exclusive  $p\bar{p} \rightarrow \gamma\gamma$  annihilation, based on the “handbag” graph and certain assumptions about the soft matrix element describing the conversion of the  $p\bar{p}$  system to a  $q\bar{q}$  pair. The approximations made in the present treatment can be refined in many ways, most notably by including other components of the soft matrix element

(Pauli and pseudoscalar form factor), and transverse momenta of the partons. Also, the picture proposed here can be applied to polarization observables, as well as to baryon–antibaryon production in  $\gamma\gamma$  reactions.

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