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LONGITUDINAL IMPEDANCE MEASUREMENT IN RHIC*

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Abstract

The very clean Schottky spectra of gold beams in RHIC allow an accurate measurement of potential well distortion. By observing the variation in the small amplitude, incoherent synchrotron tune with intensity and bunch length, the intensity dependent longitudinal force can be measured. Dynamical effects associated with coherent motion are not important though some new dynamical effects appear. Measurements were carried out both at injection energy and store, which allowed the space charge and wall contributions to be individually determined.

1 INTRODUCTION

The individual particles within a bunch respond to the local values of the electric and magnetic fields. Consider longitudinal motion and let τ denote the arrival time of a particle with respect to the synchronous particle. For no coherent oscillations, the beam current depends only on τ and, modeling the machine impedance as a pure inductance, the net voltage per turn is given by

$$V(\tau) = V_{rf}(\tau) - L\frac{dI(\tau)}{d\tau}. \quad (1)$$

Note that the L in equation (1) is the sum of contributions from the inductive wall and space charge. The applied voltage causes the particles to oscillate in the longitudinal phase space. Let $\epsilon = E - E_0$ denote the difference between a particle's energy and the energy of the synchronous particle. Use the turn number n as the time-like variable and consider the smooth focusing limit. The equations of motion for a given particle are

$$\frac{d\epsilon}{dn} = qV(\tau) \quad (2)$$

$$\frac{d\tau}{dn} = T_{rev}\eta\frac{\epsilon}{\beta^2 E_0}, \quad (3)$$

where q is the charge of the particle, T_{rev} is the revolution period, $\eta = 1/\gamma_T^2 - 1/\gamma^2$, and $\beta = v/c$.

Assume the bunch has a smooth, symmetric shape with the peak current $\hat{I}$ occurring at $\tau = 0$. For small τ,

$$I(\tau) = \hat{I}[1 - \tau^2/w^2 + O(\tau^4)], \quad (4)$$

where w is the half width at base for a parabolic fit to the current using data near the peak. For small τ equations (1) and (4) give

$$V(\tau) = \hat{V}_{rf}\omega_{rf}\tau\left(1 - \frac{(\omega_{rf}\tau)^2}{6}\right) + L\hat{I}\frac{2\tau}{w^2}. \quad (5)$$

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Consider particles with relatively small synchrotron amplitudes. Using first order perturbation theory and expanding all results to leading order, the synchrotron frequency of such a particle is given by

$$f_s(\hat{\tau}) = f_{s,0}\left[1 + \frac{L\hat{I}}{V_{rf}\omega_{rf}w^2} - \left(\frac{\omega_{rf}\hat{\tau}}{4}\right)^2\right], \quad (6)$$

where $\hat{\tau}$ is the amplitude of the synchrotron oscillation and $f_{s,0}$ is the small amplitude synchrotron frequency for zero beam current.

Let the longitudinal distribution function corresponding to $I(\tau)$ be $\rho(\hat{\tau})$ with $\rho(\hat{\tau})\hat{\tau}d\hat{\tau}$ being the number of particles with synchrotron amplitudes in the interval $[\hat{\tau}, \hat{\tau} + d\hat{\tau}]$. Let kf_0 be the frequency of the revolution line about which the Schottky spectrum will be measured. The Schottky power for synchrotron side band m is given by [1, 2]

$$P(f) = K\int_0^\infty \hat{\tau}d\hat{\tau}\rho(\hat{\tau})J_m^2(2\pi kf_0\hat{\tau})\delta(kf_0 + mf_s(\hat{\tau}) - f), \quad (7)$$

where $J_m(x)$ is the ordinary Bessel function of the first kind of order m, and K is a constant which is proportional to the beam intensity. Assume the net voltage is such that for a given synchrotron frequency there is at most one value of $\hat{\tau}$. Define $\hat{\tau}_{f,m}$ to be the solution of

$$f - kf_0 = mf_s(\hat{\tau}_{f,m}) \approx mf_s(0)[1 - (\omega_{rf}\hat{\tau}_{f,m}/4)^2], \quad (8)$$

where $f_s(0)$ is the small amplitude synchrotron frequency, including intensity dependent effects. Equation (8) may not have a physical solution, so define $G(f, m) = 1$ for a physical solution and zero otherwise. With expression (8) inserted into (7) notice that $df \propto \hat{\tau}d\hat{\tau}$, making closed form integration possible. The Schottky power as a function of frequency is

$$P(f) = K'\sum_{m\neq 0} G(f,m)J_m^2(2\pi kf_0\hat{\tau}_{f,m})\rho(\hat{\tau}_{f,m})/|m|. \quad (9)$$

Figure 1 shows RHIC Schottky spectra [4] using the gold beam and a fit of equation (9). The data have been smoothed to remove the rapid oscillations in the Bessel functions and in equation (9) we have made the approximation

$$J_m^2(x) = \min\left\{\left(\frac{x^m}{2^m m!}\right)^2, \frac{1}{\pi x}\right\}$$

which smoothes the function. Figure 2 shows the details of the $m = -5$ synchrotron sidebands and a fit of equation (9) where we have set $\rho(\hat{\tau}_{f,m}) =$ constant.

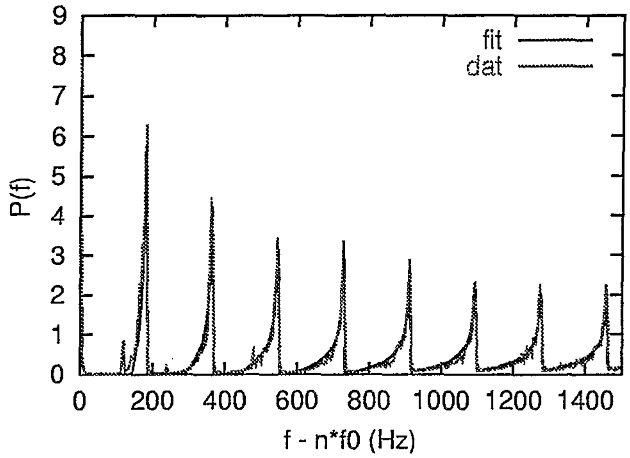

Figure 1: Schottky spectra at injection for Au beams in RHIC and an envelope fit.

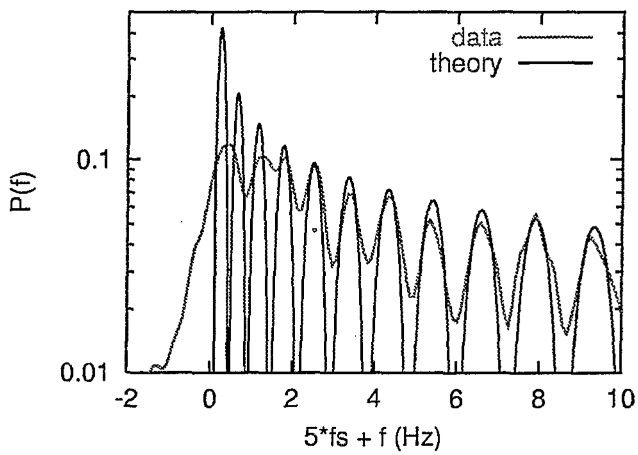

Figure 2: Details of the $m = -5$ synchrotron sidebands for 1×10^8 ions. The blue line is a fit of equation (9) assuming constant $\rho(\tau)$.

Table 1: Machine and beam parameters for gold during the RHIC study at injection and store.

parameter	value
atomic number Z	79
mass number A	197
γ_{inj}	10.5
γ_T	22.8
γ_{store}	107
harmonic no. h	360
intensity range in ions per bunch N_b	$1 \times 10^8 \rightarrow 9 \times 10^8$
emitt. $\epsilon_{N\,x,y\,95\%}$	10μm
range in w_{inj}	$6.7 \rightarrow 7.6$ ns
range in w_{store}	$3.0 \rightarrow 3.9$ ns
$\lvert\hat{V}_{rf}\rvert$	300 kV

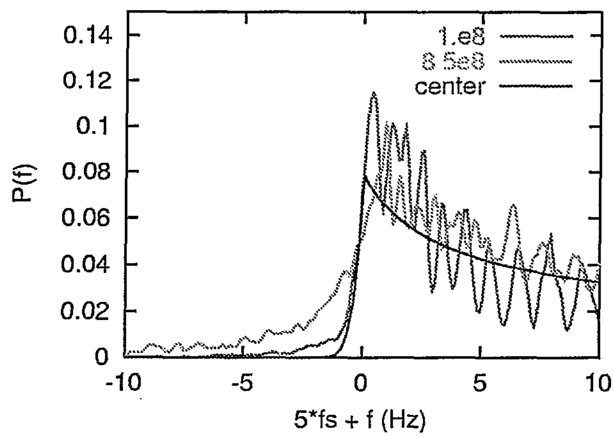

Figure 3: Details of the $m = -5$ synchrotron sidebands for 1×10^8 and 8.5×10^8 ions. Both have been shifted in frequency and normalized to minimize the chi-square between the data and the blue line referred to as "center".

2 APPLICATION TO RHIC

The parameter regime of the experiment is outlined in Table 1. For low intensities the synchrotron sidebands were slightly smoothed versions of equation (9) but for higher intensities an additional smear was present. This effect is illustrated in Figure 3. The green line marked 8.5e8 was for a bunch population of 8.5×10^8 ions and the line marked $1.e8$ was for 1.0×10^8. Both spectra have been shifted and normalized to the curve labeled "center". The total Schottky power for the high intensity trace was 6.4 times the power for the low intensity trace. Given that the low intensity bunch was somewhat longer than the high intensity bunch the data a consistent with real Schottky spectra. The increase in line-width with intensity will be discussed later.

For the data taken at injection only one bunch was present in the machine at a given time. The synchrotron frequency was extracted from the Schottky data by maximizing the cross correlation between between the data and a centering curve, shown in blue in Figure 3. Data for the $m = 5$ and $m = -5$ sidebands were taken and cross correlations were calculated. The difference in frequency was divided by ten yielding an estimate of f_s. For the data taken at store there were six bunches in the ring and the signal from the Schottky cavity was gated to measure the spectrum of one bunch at a time. Only the frequencies of the $m = 5$ and $m = -5$ Schottky peaks were recorded and the difference was divided by ten to estimate f_s.

Values of $\hat{I}$ and w were obtained by digitizing data from the wall current monitor and fitting a parabolic cap to data points near the peak. Instead of $\hat{I}$ we parameterize intensity using the number of ions which yield the fitted curve. This is given by $N = 3\hat{I}w/(4Z)$.

Figures 4 and 5 show the variation in synchrotron tune with N/w^3 for injection and store, respectively. The data at injection are well approximated by a straight line and the slope of the line corresponds to a capacitive impedance with $\omega_0 L = -jZ/n = -5\Omega$. The data at store have more

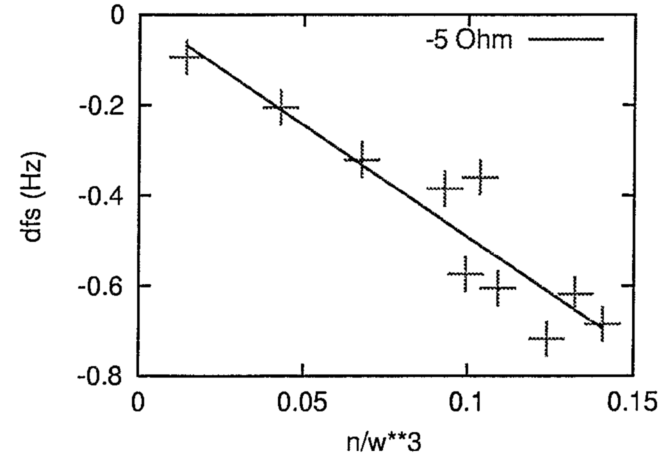

Figure 4: Synchrotron frequency shift versus N/w^3 at injection. N is in units of 10^8 ions and w is in nanoseconds. The crosses mark the data points but don't indicate error bars.

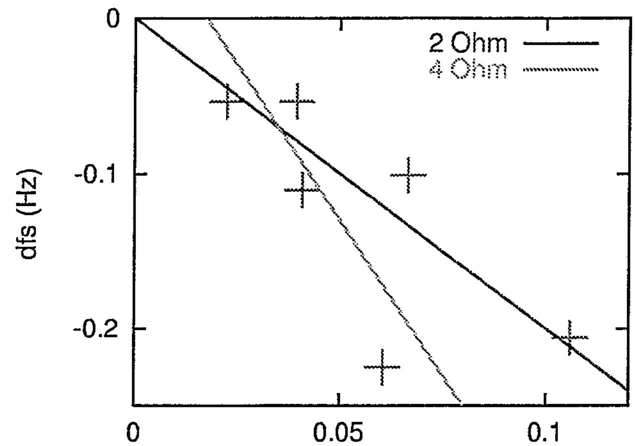

Figure 5: Synchrotron frequency shift versus N/w^3 at store. N is in units of 10^8 ions and w is in nanoseconds. Lines corresponding to $\omega_0 L = 2\Omega$ and $\omega_0 L = 4\Omega$ are shown for reference.

scatter but a value of $\omega_0 L = -jZ/n$ between 2Ω and 4Ω is in reasonable agreement with the data.

For a round Gaussian beam with $\sigma_x = \sigma_y = \sigma$, in a round pipe of radius $b \gg \sigma$, the longitudinal space charge impedance is well approximated by

$$\left.\frac{Z}{n}\right|_{sc} = -j\frac{Z_0}{\beta\gamma^2}\ln\frac{b}{\sigma}. \qquad (10)$$

For gold at injection $\gamma = 10.5$ and $b/\sigma \approx 10$ yielding $Z/n_{sc} = -j7.9\Omega$. At store $\gamma = 107$ so $|Z/n_{sc}| \lesssim 0.1\Omega$. Using $Z/n = -j5\Omega$ at injection, $Z/n = j3\Omega$ at store, and assuming that the inductive wall contribution does not change with energy, gives $Z/n_{sc} = -8\Omega$ at injection. The wall impedance value of $Z/n = j3\Omega$ is larger than previously calculated values which gave $Z/n \approx j\Omega$ [3].

3 DISCUSSION

As is clear from Figure 3 the width of the Schottky spectrum about a single synchrotron sideband increases with intensity. The effect could be due to the fourth order terms neglected in equation (4) but the magnitude and sign of the measured impedance argues against it. To see this consider a Gaussian bunch for which equation (1) gives

$$\begin{aligned} V(\tau) &= \hat{V}_{rf}\sin(\omega_{rf}\tau) + L\hat{I}\frac{\tau}{\sigma_t^2}e^{-\tau^2/2\sigma_t^2} \\ &\approx \hat{V}_{rf}\omega_{rf}\tau + L\hat{I}\frac{\tau}{\sigma_t^2} \\ &- \hat{V}_{rf}\frac{(\omega_{rf}\tau)^3}{6} - L\hat{I}\frac{\tau^3}{2\sigma_t^4} \end{aligned} \qquad (11)$$

Consider the case at injection with $\gamma < \gamma_T$. In this case $\hat{V}_{rf} > 0$ and $L < 0$. The linear terms in equation (11) tend to cancel, leading to a reduction in the synchrotron frequency with intensity. The cubic terms also tend to cancel. As long as the cubic term proportional to L does not change the sign of the cubic term as a whole, the synchrotron frequency will tend to vary more slowly with amplitude than in the low intensity case. This will result in a narrowing of the spectrum, not a broadening. For a numerical estimate take $|Z/n| = 5\Omega$ and $\sigma_t = 4$ ns. The critical intensity for which the cubic term changes sign is 13.7 A. This is 10 times larger than the currents used during the experiment which both removes the possibility of the cubic term changing sign and justifies the neglect of the intensity dependent cubic terms in the data analysis.

Even though the broadening is not well understood its effects are relatively benign. For the data in Figure 3 the peak in the high intensity spectrum is shifted to the right by 0.6 Hz. Since this is the 5th sideband the associated synchrotron frequency shift is 0.12 Hz. The data plotted in Figure 3 correspond to the extreme values of synchrotron frequency shift in Figure 4. Given that the range of the synchrotron frequency shift is 0.6 Hz a 20% error bar will cover the range of possibilities. The measured value of the broadband impedance at $\gamma = 10.7$ is then $Z/n = -j(5 \pm 1)\Omega$, being dominated by space charge. The measured value at $\gamma = 107$ is $Z/n = +j(3 \pm 1)\Omega$, and is dominated by the inductance of the wall.

4 ACKNOWLEDGMENTS

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5 REFERENCES

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