Atmospheric Test Models and Numerical Experiments for the Simulation of the Global Distribution of Weather Data Transponders II. Vertical Transponder Motion Considerations

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Atmospheric Test Models and Numerical Experiments for the Simulation of the Global Distribution of Weather Data Transponders
II. Vertical Transponder Motion Considerations

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ABSTRACT

The vertical motion of constant density atmospheric balloons has been considered via an equation of motion for the vertical displacement of a balloon, due to vertical air motion, which can be numerically solved for balloon positions. Initial calculations are made for a constant density atmosphere. Various vertical wind models with relatively large amplitudes are applied to the model to determine how tightly the balloons are coupled to the reference level and the time scale for the balloons to change to the wind driven reference altitude. A surface launch of a balloon to a 6 km reference altitude is modeled using a detailed atmospheric pressure-density-temperature profile in the equation of motion. The results show the balloons to be relatively tightly coupled (~50 – 100 m) to the reference altitude.
I. INTRODUCTION

In a series of recent reports (Grossman and Molenkamp, 1999, "GM"; Teller et al., 1999) an initial set of calculations were performed to examine the global atmospheric distribution of a constellation of free-floating, neutrally buoyant transponders which provide atmospheric data that can be used to initialize future, high resolution numerical weather prediction models as outlined by Teller et al. (1998). A global, transponder dispersion model consisting of the NCAR CCM3 GCM and the LLNL GRANTOUR parcel transport model was developed to address these issues. The CCM3 GCM supplied the horizontal wind fields at different vertical levels and the GRANTOUR transport model calculated the transponder positions for the given horizontal wind fields at each vertical level. It was assumed for these calculations that the transponders were fixed at their respective pressure levels and any vertical displacements due to air motions were neglected. Also terrain effects were neglected at the higher pressure levels and the transponder motion was unimpeded by collisions with the terrain.

The vertical motion of constant density atmospheric balloons has been considered by Hanna and Hoecker (1971, "HH"). They provide an equation of motion for the vertical displacement of a balloon due to vertical air motion, which can be numerically solved for balloon positions. The main objective of this report is to use this equation of motion to find the limits of the vertical balloon motions and the time for balloons to reach equilibrium altitudes from surface launch points.

II. VERTICAL EQUATION OF MOTION

The vertical equation of motion for a constant density balloon (HH) is:

\[ M_b \frac{\partial \omega_b}{\partial t} = M_a \frac{\partial \omega_a}{\partial t} + \frac{1}{2} M_a \left( \frac{\partial \omega_a}{\partial t} - \frac{\partial \omega_b}{\partial t} \right) - M_b g \left( \frac{\rho_b - \rho_a}{\rho_b} \right) - \rho_a A \frac{C_d}{2} \left( \omega_a - \omega_b \right) \left| \omega_a - \omega_b \right| \]

where;

- \( M_b \) is the mass of the balloon,
- \( M_a \) is the mass of the displaced air,
- \( \omega_b \) is the vertical speed of the balloon,
- \( \omega_a \) is the vertical speed of the air,
- \( g \) is the acceleration of gravity,
- \( A \) is the cross-sectional area of the balloon in the vertical direction,
- \( C_d \) is the balloon drag coefficient,
- \( \rho_b \) is the reference density,
- \( \rho_a \) is the air density.

The term on the left side of Eq. (1) is the net force on the balloon. The terms on the right hand side are the dynamic buoyancy force due to the air acceleration, the acceleration...
Equation (1) can be written in terms of $x$, the displacement from equilibrium altitude, as:

$$\frac{3}{2} \frac{\partial (\omega_x - \omega_{x})}{\partial t} = -g(1 - e^{-x/s}) - \frac{3 C_d}{8 R} (\omega_b - \omega_x)(\omega_b - \omega_x)$$

(2)

where $s [-g(\partial \omega_r)(\rho_\infty)]$ is related to the Brunt Vaisala or “bouncing” frequency. For this problem HH gives a typical drag coefficient of 0.8 and an $s$ value of $\sim 1 \times 10^3$. The balloon radius, as given in the original proposal (Teller et al., 1998), is 5 cm.

Equation (2) can be solved for the equilibrium displacement of a balloon for in a constant air velocity field ($\partial \omega_x/\partial t = 0, \partial \omega_y/\partial t = 0, \omega_z = 0$):

$$1 - e^{-x/s} = \frac{3 C_d}{8 g R} \omega_x^2$$

(3)

For a vertical air velocity of 0.1 m/s, which represents a large value for a 3° x 3° latitude-longitude grid column, Eq.(3) predicts a balloon displacement of $\sim 60$ m. For the range of $s$ values given by HH the displacement range is 55 – 105 m. The equilibrium displacement is inversely proportional to the balloon radius and directly proportional to the square of the air velocity. Larger balloons would be even more tightly coupled to the equilibrium altitude. In terms of pressure differential the displacement would be $\sim 6 \times 10^{-3} \text{Pa}$. These values indicate a tight balloon coupling to the equilibrium pressure levels.

To investigate the time behavior of a constant density balloon, Eq. (2) was solved numerically for several, time varying, vertical air velocities. The first case (Case 1) considers a balloon starting at the equilibrium altitude ($x = 0$) subject to a constant, upward air velocity of 0.1 m/s, with the constraint that $s$ is held constant at $1 \times 10^3$ (constant pressure scale height). The results for Case 1 are shown in Figures 1a and 1b. Figure 1a shows the balloon displacement as a function of time over a time of 90 minutes. The balloon rises exponentially towards its equilibrium altitude with a time constant of 17 – 20 minutes. Figure 1b shows the early time motion of the balloon as it is initially accelerated upward by the air velocity drag force and then decelerated by the buoyant restoring force. The second case (Case 2) is for a balloon starting at the equilibrium position of Case 1 ($x = 60$ m, 0.1 m/s updraft) subject to a constant, downward air velocity of 0.1 m/s, also holding $s$ constant. Figures 2a and 2b show the balloon displacement, velocity, and acceleration for Case 2. The balloon displacement is approximately exponential in time, with a time constant of about 17 – 20 minutes. For both Cases 1 and 2 the equilibrium levels are approximately ± 60 m from the reference level. The third case (Case 3) is for a balloon starting at the zero air velocity level ($x = 0$) and subject to a periodic vertical air velocity $V_a \cos (2\pi t/T)$ where $V_a$ is 0.1 m/s and $T$ is 4 hours. The Case 3 results are shown in Figure 3. The sinusoidal velocity variation causes a sinusoidal displacement variation with an amplitude of approximately 52 m. The fourth case consists of a surface launch subject to the sinusoidal velocity variations of the previous case. For this case, the assumption of constant $s$ over the range of vertical balloon motion is not valid. A representative atmospheric profile giving pressure, temperature, and density is used with Eq. (1) to solve for the balloon motion. The
pressure-temperature-density-altitude profile, representative of a globally and seasonally averaged model atmosphere (Wuebbles et al., 1994), is given in Table 1. Figure 4 shows the vertical balloon displacement as a function of time for a surface launch of a balloon designed to be neutrally buoyant at a reference altitude of 6 km. The time for the balloon to reach 95% of its reference altitude is approximately 1.5 hours. The Case 3 calculation was also repeated with the variable density atmospheric profile for validation of the constant s assumption Case 5.

The Case 5 results are shown in Figure 5. In this calculation the variable density atmospheric profile is used in the calculation of the balloon displacement when subject to a sinusoidal, vertical wind velocity of the form given above for Case 3. The maximum and minimum balloon amplitudes for this case are about ±46 m about the 6 km reference altitude and within 15% of the Case 3 amplitudes.

In summary, we have performed numerical calculations of the vertical displacement about the equilibrium altitude in a constant scale height atmosphere, for a constant density balloon, using different, time varying, vertical velocity profiles. The results show the balloons to be relatively tightly coupled (≈50 – 100 m) to the reference altitude. A surface launch of a balloon to a 6 km reference altitude is modeled using detailed atmospheric pressure-density-temperature profile in the equation of motion. The balloon rise time, in the presence of a sinusoidal variation of the vertical air velocity, to ~95% of its reference altitude is approximately 1.5 hours.

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REFERENCES


FIGURE CAPTIONS

Figure 1  Balloon vertical displacement from zero air velocity reference level (meters, □), velocity (x 100 m/s, ◆), and acceleration (x1000 m/s², O) vs. time (minutes) assuming constant s (Brunt Vaisala or “bouncing” frequency) for a constant upward vertical velocity of 0.1 m/s. Figure 1a. covers the time from 0 – 90 minutes and Figure 1b covers the time from 0 – 3 minutes.

Figure 2  Balloon vertical displacement from zero air velocity reference level (meters, □), velocity (x 100 m/s, ◆), and acceleration (x10⁵ m/s², O) vs. time (minutes) assuming constant s (Brunt Vaisala or “bouncing” frequency), for a constant downward vertical velocity of 0.1 m/s, and a starting position of 60 m above the reference level. Figure 2a. covers the time from 0 – 90 minutes and Figure 2b covers the time from 0 – 12 minutes.

Figure 3  Balloon vertical displacement from zero air velocity reference level (meters, □), velocity (x 100 m/s, ◆), and acceleration (x10⁵ m/s², O) vs. time (minutes) assuming a constant s (Brunt Vaisala or “bouncing” frequency) for a periodic vertical air velocity, Va cos (2πt/T) where Va is 0.1 m/s and T is 4 hours.

Figure 4  Balloon vertical displacement from a surface launch (meters, □), and velocity (x1000 m/s, ◆), vs. time (minutes) for a periodic vertical air velocity, Va cos (2πt/T) where Va is 0.1 m/s and T is 4 hours.

Figure 5  Balloon vertical displacement from zero air velocity reference level (meters, □), velocity (x 100 m/s, ◆), and acceleration (x10⁵ m/s², O) vs. time (minutes), assuming a detailed atmosphere pressure-density-altitude profile, for a periodic vertical air velocity, Va cos (2πt/T) where Va is 0.1 m/s and T is 4 hours.
Balloon Data \((V_a = 0.1 \text{ m/s}, X(0) = 0)\)
Balloon Data ($V_a = -0.1\text{m/s}$, $X(0) = 60\text{m}$)
Balloon Data (V_a = -0.1 m/s, X(0) = 60 m)
Balloon Data [Sin Wave Va (T = 4hr, Va = 0.1m/s), X(0) = 0.]
Balloon Data-Surface Launch [Sin Wave Va(T=4hr, Va=0.1m/s, Xref=6km)]
Balloon Data [Sin Wave $V_a(T=4\text{hr}, V_a=0.1\text{m/s}, X_{\text{ref}}=6\text{km})$]