Bound Nucleon Form Factors, Quark-Hadron Duality, and Nuclear EMC Effect

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We discuss the electromagnetic form factors, axial form factors, and structure functions of a nucleon bound in the quark-meson coupling (QMC) model. Free space nucleon form factors are calculated using the improved cloudy bag model (ICBM). After describing finite nuclei and nuclear matter in the quark-based QMC model, the in-medium modification of the bound nucleon form factors is calculated in the same model. Finally, the bound nucleon structure function, $F_2$, is extracted using the calculated in-medium electromagnetic form factors and Bloom-Gilman (quark-hadron) duality.

I. INTRODUCTION

Partial restoration of chiral symmetry in a nuclear medium, to which the reduction in the mass of a bound nucleon is sometimes ascribed, plays a key role in understanding the medium modification of bound nucleon (hadron) properties. At relatively high energies and/or temperature and/or densities, quark and gluon degrees of freedom are expected to be efficient in describing physical phenomena according to perturbative QCD. However, it is not at all obvious whether such degrees of freedom are indeed necessary and/or efficient in describing low energy nuclear phenomena, such as the static properties of finite nuclei. In this article, we demonstrate that the quark degrees of freedom do indeed seem to be necessary to understand recent polarization transfer measurements in the $^4\text{He}(e,e'p)^3\text{H}$ reaction [1,2], which can not be explained within the best existing treatments of traditional physics (solely based on hadronic degrees of freedom).

Over the past few years there has been of considerable interest in possible changes in bound nucleon properties in a nuclear medium. There is a significant constraint on the possible change in the radius of a bound nucleon based on $y$-scaling — especially in $^3\text{He}$ [3].

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On the other hand, the space (time) component of the effective one-body axial coupling constant is known to be quenched [4] (enhanced [5]) in Gamow-Teller (first-forbidden) nuclear $\beta$ decay, and a change in the charge radius of a bound proton provides a natural suppression of the Coulomb sum rule [6]. One of the most famous nuclear medium effects — the nuclear EMC effect [7], or the change in the inclusive deep-inelastic structure function of a nucleus relative to that of a free nucleon — has stimulated theoretical and experimental efforts for almost two decades now which seek to understand the dynamics responsible for the change in the quark-gluon structure of the nucleon in nuclear medium [8].

Recently the search for evidence for some modification of nucleon properties in medium has been extended to electromagnetic form factors, in polarized $(e,e'p)$ scattering experiments on $^{16}\text{O}$ [9] and $^4\text{He}$ [1,2]. These experiments measured the ratio of transverse to longitudinal polarization of the ejected protons, which for a free nucleon is proportional to the ratio of electric to magnetic elastic form factors [10],

$$\frac{G_E^p}{G_M^p} = \frac{P_x'}{P_x} \frac{E_e + E_e'}{2M_N} \tan(\theta_e/2).$$

Here $P_x'$ and $P_x$ are the transverse and longitudinal polarization transfer observables, $E_e$ and $E_e'$ the incident and recoil electron energies, $\theta_e$ the electron scattering angle, and $M_N$ the nucleon mass. Compared with the traditional cross section measurements, polarization transfer experiments provide more sensitive tests of dynamics, especially of any in-medium changes in the form factor ratios. The feasibility of this technique was first demonstrated in the commissioning experiment at Jefferson Lab on $^{16}\text{O}$ [9] at $Q^2 = 0.8$ GeV$^2$. In the subsequent experiment at MAMI on $^4\text{He}$ [1] at $Q^2 \approx 0.4$ GeV$^2$, and at Jefferson Lab at $Q^2 = 0.5, 1.0, 1.6$ and $2.6$ GeV$^2$, which had much higher statistics, the polarization ratio in $^4\text{He}$ was found to differ by $\approx 10\%$ from that in $^3\text{H}$.

Conventional models using free nucleon form factors and the best phenomenologically determined optical potentials and bound state wave functions, as well as relativistic corrections, meson exchange currents, isobar contributions and final state interactions [11–14], fail to account for the observed effect in $^4\text{He}$ [1,2]. Indeed, full agreement with the data was only obtained when, in addition to these standard nuclear corrections, a small change in the structure of the bound nucleon, which had been estimated within the quark-meson coupling (QMC) model [15–21], was taken into account. The final analysis [2] seems to favor of this scenario even more, although the error bars may still be too large to draw a definite conclusion.

On the other hand, there has recently been considerable interest in the interplay between form factors and structure functions in the context of quark-hadron duality. As observed originally by Bloom and Gilman [22], the $F_2$ structure function measured in inclusive lepton scattering at low $W$ (where $W$ is the mass of the hadronic final state) generally follows a global scaling curve which describes high $W$ data, to which the resonance structure function averages. Furthermore, the equivalence of the averaged resonance and scaling structure functions appears to hold for each resonance region, over restricted intervals of $W$, so that the resonance-scaling duality also exists locally. These findings were dramatically confirmed in recent high-precision measurements of the proton and deuterium $F_2$ structure function at Jefferson Lab [23,24], which demonstrated that local duality works remarkably well for each of the low-lying resonances, including surprisingly the elastic, to rather low values of $Q^2$. 

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In this article we first briefly review how finite nuclei and nuclear matter are treated in the quark-based QMC model [20,21]. We then discuss the modification of the electromagnetic and axial form factors of a bound nucleon in the same model. Finally, using the concept of quark-hadron duality and the calculated bound nucleon electromagnetic form factors, we extract the structure function $F_2$ of the bound nucleon [25]. To the extent that local duality is a good approximation, the relations among the nucleon form factors and structure functions are model independent, and can in fact be used to test the self-consistency of the models. We find that the recent form factor data for a proton bound in $^4$He [1,2] place strong constraints on the medium modification of inclusive structure functions at large Bjorken-$x$. In particular, they appear to disfavor models in which the bulk of the nuclear EM effect is attributed to deformation of the intrinsic nucleon structure off-shell – see e.g. Ref. [26].

This article is organized as follows. In Section II we briefly review the treatment of finite nuclei in the QMC model [20,21]. Then, in Section III, we discuss the in-medium modification of bound nucleon electromagnetic form factors in QMC [15–18] as inferred from the recent polarization transfer experiments, as well as that of axial form factor $G_A(Q^2)$ [27]. In Section IV quark-hadron duality is used to relate the observed form factor modification to that which would be expected in the deep-inelastic structure function $F_2$ [25], and finally, we make a summary in Section V.

II. FINITE NUCLEI AND NUCLEAR MATTER IN THE QMC MODEL

We briefly review the treatment of finite nuclei and symmetric nuclear matter in the QMC model [19–21].

Let us consider static, spherically symmetric nuclei. We adopt the Hartree, mean-field approximation, and ignore the $\rho NN$ tensor coupling as usually adopted in the Hartree treatment of quantum hadrodynamics (QHD) [28]. (See Refs. [20,21] for the discussions about the $\rho NN$ tensor coupling.)

Using the Born-Oppenheimer approximation, mean-field equations of motion are derived for a nucleus in which the quasi-particles moving in single-particle orbits are three-quark clusters with the quantum numbers of a nucleon. Then a relativistic Lagrangian density at the hadronic level can be constructed [20,21], similar to that obtained in QHD [28], which produces the same equations of motion when expanded to the same order in velocity:

$$\mathcal{L}_{QMC} = \bar{\psi}_N(\vec{r}) \left[i\gamma \cdot \partial - M_N^*(\sigma) - (g_\omega \omega(\vec{r}) + g_\rho \tau_3^N b(\vec{r}) + \frac{e}{2}(1 + \tau_3^N) A(\vec{r})) \gamma_0 \right] \psi_N(\vec{r})$$

$$\frac{1}{2}[(\nabla \sigma(\vec{r}))^2 + m_\sigma^2 \sigma(\vec{r})^2] + \frac{1}{2}[(\nabla \omega(\vec{r}))^2 + m_\omega^2 \omega(\vec{r})^2]$$

$$\frac{1}{2}[(\nabla b(\vec{r}))^2 + m_\rho^2 b(\vec{r})^2] + \frac{1}{2}(\nabla A(\vec{r}))^2,$$

where $\psi_N(\vec{r})$ and $b(\vec{r})$ are respectively the nucleon and the $\rho$ meson (the time component in the third direction of isospin) fields, while $m_\sigma$, $m_\omega$ and $m_\rho$ are the masses of the $\sigma$, $\omega$ and $\rho$ meson fields. $g_\omega$ and $g_\rho$ are the $\omega$-$N$ and $\rho$-$N$ coupling constants which are related to the corresponding $(u,d)$-quark-$\omega$, $g_\omega^q$, and $(u,d)$-quark-$\rho$, $g_\rho^q$, coupling constants as $g_\omega = 3g_\omega^q$ and $g_\rho = g_\rho^q$ [20,21]. (Hereafter, we will use notations for the light quark flavors, $q \equiv u,d$.) The
field dependent σ-Ν coupling strength predicted by the QMC model, \( g_\sigma(\sigma) \), related to the Lagrangian density of Eqs. (2) at the hadronic level is defined by:

\[
M_N^* = M_N - g_\sigma(\sigma)\sigma(\vec{r}),
\]

where \( M_N \) is the free nucleon mass. Note that the dependence of these coupling strengths on the applied scalar field must be calculated self-consistently within the quark model [20, 21]. More explicit expression for \( g_\sigma(\sigma) \) will be given later. From the Lagrangian density Eq. (2), a set of equations of motion for the nuclear system is obtained as:

\[
[i\gamma \cdot \partial - M_N^* (\vec{r}) - (g_\omega \omega(\vec{r}) + g_\rho \frac{\tau_3^N}{2} b(\vec{r}) + \frac{e}{2} (1 + \tau_3^N) A(\vec{r}))\gamma_\nu]\psi_N(\vec{r}) = 0,
\]

\[
(-\nabla_\tau^2 + m_\sigma^2)\sigma(\vec{r}) = -\frac{\partial M_N^* (\sigma)}{\partial \sigma}\rho_s(\vec{r}) \equiv g_\sigma C_N(\sigma) \rho_s(\vec{r}),
\]

\[
(-\nabla_\tau^2 + m_\omega^2)\omega(\vec{r}) = g_\omega \rho_B(\vec{r}),
\]

\[
(-\nabla_\tau^2 + m_3^2) b(\vec{r}) = \frac{g_3}{2} \rho_3(\vec{r}),
\]

\[
(-\nabla_\tau^2) A(\vec{r}) = e \rho_p(\vec{r}),
\]

where, \( \rho_s(\vec{r}) \), \( \rho_B(\vec{r}) \), \( \rho_3(\vec{r}) \) and \( \rho_p(\vec{r}) \) are the scalar, baryon, third component of isovector, and proton densities at the position \( \vec{r} \) in the nucleus [20, 21]. On the right hand side of Eq. (5), \( -[\partial M_N^* (\sigma)/\partial \sigma] = g_\sigma C_N(\sigma) \), where \( g_\sigma \equiv g_\sigma(\sigma = 0) \), is a new, and characteristic feature of QMC beyond QHD [28]. The effective mass for the nucleon \( M_N^* \), defined by,

\[
\frac{\partial M_N^* (\sigma)}{\partial \sigma} = -n_q g_\sigma^3 \int_{\text{bag}} d^3 x \bar{\psi}_q(\vec{x}) \psi_q(\vec{x}) \equiv -n_q g_\sigma^3 S_N(\sigma) = -\frac{\partial}{\partial \sigma} [g_\sigma(\sigma)\sigma],
\]

where \( n_q \) is the number of light quarks (u and d), and the MIT bag model quantities and the in-medium bag radius satisfying the mass stability condition are given by [19–21]:

\[
M_N^*(\sigma) = \sum_{q=u,d} n_q \Omega_q^* \frac{z_N}{R_N^*} + \frac{4}{3} \pi (R_N^*)^3 B,
\]

\[
S_N(\sigma) = \left[ \Omega_q^*/2 + m_q^* R_N^*(\Omega_q^* - 1) \right] / \left[ \Omega_q^*(\Omega_q^* - 1) + m_q^* R_N^*/2 \right],
\]

\[
\Omega_q^* = \sqrt{x_q^2 + (R_N^* m_q^*)^2}, \quad m_q^* = m_q - g_q^2 \sigma(\vec{r}),
\]

\[
dM_N^*/dR_N^*|_{R_N^*} = 0.
\]

Here, the MIT bag model quantities are calculated in a local density approximation using the spin and spatial part of the wave functions, \( \psi_q(x) = N_q e^{-ixt}/R_N^* \psi_q(\vec{x}) \) (\( N_q \): the normalization factor), where the wave functions \( \psi_q(x) \) satisfy the Dirac equations for the quarks in the nucleon bag centered at a position \( \vec{r} \) of the nucleus, approximating the constant, mean meson fields within the bag (and neglecting the Coulomb force) (\( |\vec{x} - \vec{r}| \leq R_N^* \ [20, 21] \)):

\[
[i\gamma \cdot \partial_x - (m_q - V^q(\vec{r}) \mp \gamma^0 (V^q(\vec{r}) + \frac{1}{2} V^q(\vec{r}) \pm \frac{1}{2} V^q(\vec{r})) \left( \begin{array}{c} \psi_u(x) \\ \psi_d(x) \end{array} \right) = 0.
\]

The constant, mean-field potentials within the bag centered at the position \( \vec{r} \) of the nucleus, are defined by \( V^q(\vec{r}) = g_q^2 \sigma(\vec{r}) \), \( V^q(\vec{r}) = g_q^2 \omega(\vec{r}) \) and \( V^q(\vec{r}) = g_q^2 b(\vec{r}) \), with \( g_q^2 \), \( g_q^2 \) and \( g_q^2 \).
the corresponding quark-meson coupling constants. The eigenenergies in units of $1/R_N^*$ are given by,

$$\begin{pmatrix} \epsilon_u \\ \epsilon_d \end{pmatrix} = \Omega_q^* \pm R_N^* \left( V^{q}(\hat{r}) \pm \frac{1}{2} V^{\rho}(\hat{r}) \right).$$

(15)

In Eqs. (10) - (13), $z_N$, $z$, $x_q$, and $m_q$ are the parameters for the sum of the c.m. and gluon fluctuation effects, bag pressure, lowest eigenvalues for the quark $q$, respectively, and the corresponding current quark masses. $z_N$ and $B$ are fixed by fitting the nucleon mass in free space. We use the current quark masses $m_{q=u,d} = 5$ MeV, and obtained $z_N = 3.295$ and $B = (170.0 \text{ MeV})^4$ by choosing the bag radius for the nucleon in free space $R_N = 0.8$ fm. The parameters at the hadronic level, which are already fixed by the study of nuclear matter and finite nuclei [21], are as follows: $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $m_\sigma = 418$ MeV, $e^2/4\pi = 1/137.036$, $g_\omega^2/4\pi = 3.12$, $g_\rho^2/4\pi = 5.31$ and $g_\sigma^2/4\pi = 6.93$. Concerning the sign of $m_q^*$ in nucleus in Eq. (12), it reflects nothing but the strength of the attractive, negative scalar potential, and thus naive interpretation of the mass for a physical particle, which is positive, should not be applied.

At the hadronic level, the entire information on the quark dynamics is condensed into the effective coupling $C_N(\sigma)$ of Eq. (5). Furthermore, when $C_N(\sigma) = 1$, which corresponds to a structureless nucleon, the equations of motion given by Eqs. (4)-(8) can be identified with those derived from QHD [28], except for the terms arising from the tensor coupling and the non-linear scalar field interaction introduced beyond naive QHD.

![Graphs](image)

**FIG. 1.** Charge density distributions for $^{40}\text{Ca}$ and $^{208}\text{Pb}$ in QHD [28] and QMC [21].

As examples, we show in FIG. 1 the charge density distributions calculated for $^{40}\text{Ca}$ and $^{208}\text{Pb}$, and also the energy spectra obtained for $^{40}\text{Ca}$ and $^{208}\text{Pb}$ in FIGs. 2 and 3 [21], respectively.

Next, we consider a limit of infinite symmetric nuclear matter [19–21]. In this limit all meson fields become constant, and we denote the mean-values of the $\omega$ and $\sigma$ fields as $\bar{\omega}$ and $\bar{\sigma}$. Then, equations for the $\bar{\omega}$ and self-consistency condition for the $\bar{\sigma}$ are given by [19–21],
\[
\bar{\omega} = \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - k) = \frac{g_\omega}{m_\omega^2} \frac{2k_F^3}{3\pi^2} = \frac{g_\omega}{m_\omega^2} \rho_B, 
\]
\[
\bar{\sigma} = \frac{g_\sigma}{m_\sigma^2} C_N(\bar{\sigma}) \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - k) \frac{M_N^*(\bar{\sigma})}{\sqrt{M_N^{*2}(\bar{\sigma}) + k^2}} = \frac{g_\sigma}{m_\sigma^2} C_N(\bar{\sigma}) \rho_s, 
\]

where \( g_\sigma = 3g_\omega^2 S_N(0) \) (see Eq. (11)), \( k_F \) is the Fermi momentum, \( \rho_B \) and \( \rho_s \) are the baryon and scalar densities, respectively. Note that \( M_N^*(\bar{\sigma}) \) in Eq. (17) must be calculated self-consistently by the MIT bag model, through Eqs. (9) - (14) for a given baryon density. This self-consistency equation for the \( \bar{\sigma} \) is the same as that in QHD, except that in the latter model one has \( C_N(\bar{\sigma}) = 1 \) [28].

**FIG. 2.** Energy spectrum for \(^{40}\text{Ca}\) [21] compared with experiment (Exp.), and that of QHD [28].

**FIG. 3.** Same as FIG. 2 (for \(^{208}\text{Pb}\)).
III. NUCLEAR MEDIUM MODIFICATION OF FORM FACTORS

Let us briefly review the medium modification of the electromagnetic form factors of the nucleon, as suggested in the recent polarization transfer measurements in the $^4\text{He}(\bar{e}, e'p)^3\text{H}$ reaction [1,2].

The first data were analyzed in Ref. [1] using a variety of models, nonrelativistic and relativistic, based on conventional nucleon-nucleon potentials and well-established bound state wave functions, including corrections from meson exchange currents, final state interaction and other effects [11–14]. The observed deviation, which was of order 10%, could only be explained by supplementing the conventional nuclear description with the effects associated with the medium modification of the nucleon internal structure calculated by the QMC model [15–18].

In FIG. 4 we show the “super ratio”, $R/R_{PWIA}$, which was made for the final analysis of the polarization transfer measurements on $^4\text{He}$ [2]. Here, $R_{PWIA}$ stands for the prediction based on the relativistic plane-wave impulse approximation (RPWIA), and $R$ is defined by:

$$R = \frac{(P_\pi'/P_\pi)_{^4\text{He}}}{(P_\pi'/P_\pi)_{^3\text{H}}}.$$  

(18)

![Graph](Image)

**FIG. 4.** Super ratio $R/R_{PWIA}$, as a function of $Q^2$ taken from Ref. [2]. See caption of FIG. 1 in Ref. [2] for detail explanations.

In FIG. 4, the modification of electromagnetic form factors of the bound nucleon in the QMC model (the solid line denoted by “Udias RDWIA + QMC”) has been calculated using the improved cloudy bag model (ICBM) [29,30], implemented by the QMC model [15–18]. The ICBM [30] includes a Peierls-Thouless projection to account for center of mass and recoil corrections, and a Lorentz contraction of the internal quark wave functions.

The electromagnetic current is given by the sum of the contributions from the quark core and the pion cloud,

$$j^\mu(x) = \sum_q Q_q \bar{\psi}_q(x) \gamma^\mu \psi_q(x) - i e [\pi^\dagger(x) \partial^\mu \pi(x) - \pi(x) \partial^\mu \pi^\dagger(x)].$$  

(19)
where $Q_q$ is the charge operator for a quark flavor $q$, and $\pi(x)$ destroys a negatively charged (or creates a positively charged) pion. Relevant diagrams included in the calculation of free space electromagnetic form factors are depicted in FIG. 5 [30].

![Diagrams](image)

**FIG. 5.** Diagrams included in the calculation of free space electromagnetic form factors in the ICBM [30]. The intermediate baryon states $B$ and $C$ are restricted to the $N$ and $\Delta$.

In the Breit frame the quark core contribution to the electromagnetic form factors of the bound nucleon is given by [15–18]:

\[
G_E^s(Q^2) = \eta^2 \ G_{E, s}^{\text{sph}}(\eta^2 Q^2), \tag{20a}
\]

\[
G_M^s(Q^2) = \eta^2 \ G_{M, s}^{\text{sph}}(\eta^2 Q^2), \tag{20b}
\]

where $Q^2 \equiv -q^2 = \vec{q}^2$, and the scaling factor $\eta = (M_N^*/E_N^*)$, with $E_N^* = \sqrt{M_N^{*2} + Q^2/4}$ the energy and $M_N^*$ the mass of the nucleon in medium, and $G_E^{s, s} (Q^2)$ are the form factors calculated with the static spherical bag wave function, and are subject to $Q \to \eta Q$:

\[
G_{E, s}^{\text{sph}}(Q^2) = \frac{1}{D} \int d^3 r \ j_0(Qr) \ f_q(r) \ K(r), \tag{21a}
\]

\[
G_{M, s}^{\text{sph}}(Q^2) = \frac{1}{D} \frac{2M_N^*}{Q} \ \int d^3 r \ j_1(Qr) \ \beta_q^* \ j_0(x_q R_N^*) \ j_1(x_q R_N^*) \ K(r). \tag{21b}
\]

Here $f_q(r) = j_0^2(x_q r/R_N^*) + \beta_q^2 \ j_1^2(x_q r/R_N^*)$, where $R_N^*$ is the nucleon bag radius in medium, $x_q$ the lowest eigenfrequency, and $\beta_q^2 = (\Omega_q^* - m_q^* R_N^*)/ (\Omega_q^* + m_q^* R_N^*)$, with $\Omega_q^* = \sqrt{x_q^2 + (m_q^* R_N^*)^2}$ and $m_q^* = m_q - g_\sigma^2 \sigma$. (See also Section II.) The recoil function $K(r) = \int d^3 \hat{x} \ f_q(\hat{x}) f_q(-\hat{x} - \hat{r})$ accounts for the correlation of the two spectator quarks, and $D = \int d^3 r \ f_q(r) \ K(r)$ is the normalization factor. The scaling factor $\eta$ in the argument of $G_{E, s}^{\text{sph}}$ arises from the coordinate transformation of the struck quark, and the prefactor in Eqs. (20) comes from the reduction of the integral measure of the two spectator quarks in the Breit frame.

The contribution from the pionic cloud is calculated along the lines of Ref. [15–18]. Although the pion mass would be slightly smaller in the medium than in free space, we use $m_\pi^* = m_\pi$, which is consistent with chiral expectations and phenomenological constraints. Furthermore, since the $\Delta$ isobar is treated on the same footing as the nucleon in the CBM, and because it contains three ground state light quarks, its mass should vary in a similar manner to that of the nucleon in the QMC model. As a first approximation we therefore take the in-medium and free space $N-\Delta$ mass splittings to be approximately equal, $M_\Delta^* - M_N^* \simeq M_\Delta - M_N$. 


The change in the ratio of the electric to magnetic form factors of the proton from free to bound,\[ R_{EM}^{p^*}(Q^2) / R_{EM}^{p}(Q^2) = \left( \frac{G_{E}^{p^*}(Q^2)}{G_{M}^{p^*}(Q^2)} \right) / \left( \frac{G_{E}^{p}(Q^2)}{G_{M}^{p}(Q^2)} \right), \] is illustrated in FIG. 6 for $^4$He, $^{16}$O (left panel) and for nuclear matter densities, $\rho = \rho_0$ and $\rho = \frac{1}{2} \rho_0$ (right panel) with $\rho_0 = 0.15 fm^{-3}$.

FIG. 6. The change in the ratio of the electric to magnetic form factors from free to bound protons, $R_{EM}^{p^*}/R_{EM}^{p} = (G_{E}^{p^*}/G_{M}^{p^*})/(G_{E}^{p}/G_{M}^{p})$, for $^4$He (1s$_{1/2}$ state) and $^{16}$O (1s$_{1/2}$ and 1s$_{3/2}$ states) (left panel) [18] and nuclear matter (right panel) [25]. $^{16}$O(B) stands for the results which allow changes of the bag constant according to the nuclear density [17]. ($\rho_0 = 0.15 fm^{-3}$.)

Because the average nuclear densities for all existing stable nuclei heavier than deuterium lie in the range $\frac{1}{2} \rho_0 \lesssim \rho \lesssim \rho_0$, we consider these two specific nuclear densities to give the upper and lower bounds for the change of the electromagnetic form factors (and structure functions at large $x$) of the bound nucleon. We emphasize that in the present analysis the absolute value of the proton magnetic form factor at $Q^2 = 0$ (the magnetic moment), which is enhanced in medium, plays an important role – as it did in the analysis of polarized $(\bar{e}, e' \bar{p})$ scattering experiments.

Because of charge conservation, the value of $G_{E}^{p}$ at $Q^2 = 0$ remains unity for any $\rho$. On the other hand, the proton magnetic moment is enhanced in the nuclear medium, increasing with $\rho$, so that $R_{EM}^{p^*} < R_{EM}^{p}$ at $Q^2 = 0$. In fact, the electric to magnetic ratio is $\sim 5\%$ smaller in medium than in free space for $\rho = \frac{1}{2} \rho_0$, and $\sim 10\%$ smaller for $\rho = \rho_0$. The effect increases with $Q^2$ out to $\sim 2$ GeV$^2$, where the (bound/free) ratio deviates by $\sim 20\%$ from unity.

The extension to the in-medium modification of the bound nucleon axial form factor $G_{A}^{*}(Q^2)$ has been made in a straightforward manner [27]. Since the induced pseudoscalar form factor $G_{P}(Q^2)$, is dominated by the pion pole, and can be derived using the PCAC relation [29], we do not discuss it here. The relevant axial current operator is then simply given by
\[ A_\mu^e(x) = \sum_q \bar{\psi}_q(x) \gamma^\mu \gamma_5 \frac{\tau_a}{2} \psi_q(x) \theta(R - r), \]  

where \( \psi_q(x) \) is the quark field operator for flavor \( q \).

Then, in the preferred Breit frame the resulting bound nucleon axial form factor is given [27] similarly to the electromagnetic form factors:

\[ G_A^*(Q^2) = \eta^2 G_A^{\text{sph}*}(\eta^2 Q^2), \]
\[ G_A^{\text{sph}*}(Q^2) = \frac{5}{3} \int d^3r \left\{ j_0(x_q r / R_N^*) - \beta^2 j_1(x_q r / R_N^*) \right\} j_0(Q r) \]
\[ + 2 \beta^2 j_1^2(x_q r / R_N^*) [j_1(Q r) / Q r] K(r) / D. \]

In FIG. 7 we show axial form factor \( G_A(Q^2) \) calculated in free space (normalized) by ICBM [27] together with the experimental data (left panel), and those calculated (space component) for nuclear densities, \( \rho = (0.5, 0.7, 1.0, 1.5) \rho_0 \) with \( \rho_0 = 0.15 \text{fm}^{-3} \).

**FIG. 7.** Axial form factor \( G_A(Q^2) \) in free space (normalized) calculated by ICBM (left panel) together with experimental data [31], and the ratio for in-medium to free [27] (right panel).

At \( Q^2 = 0 \), i.e., \( g_A^* \equiv G_A^*(Q^2 = 0) \) (space component), is quenched [4] about 10% at normal nuclear matter density. The modification calculated here may be corresponding to the "model independent part" in meson exchange language, where the axial current attaches to one of the two nucleon legs, but not the mesons exchanged [4]. This is because the axial current operator Eq. (23), is a one-body operator which operates on the quarks and pions belonging to a bound nucleon. The medium modification of bound nucleon axial form factor \( G_A^*(Q^2) \) may be observed e.g., in neutrino-nucleus scattering similar to that observed in EMC experiments, or in a similar experiment to the polarization transfer measurements performed on \(^4\text{He}\) [1,2,9] (if such kinds of experiment is possible). However, at present experimental uncertainties seem to be too large to detect such medium effect directly. Here, we should comment that we can extract the medium modification of structure functions e.g., \( F_3 \) of bound nucleon in deep inelastic neutrino induced reactions using the calculated in-medium axial form factor \( G_A^*(Q^2) \) and quark-hadron duality, as will be explicitly demonstrated in next section for the case of electromagnetic form factors and structure function \( F_2 \).
IV. QUARK-HADRON DUALITY AND NUCLEON STRUCTURE FUNCTIONS IN MEDIUM

The relationship between form factors and structure functions, or more generally between inclusive and exclusive processes, has been studied in a number of contexts over the years. Drell & Yan [32] and West [33] pointed out long time ago that, simply on the basis of scaling arguments, the asymptotic behavior of elastic electromagnetic form factors as $Q^2 \to \infty$ can be related to the $x \to 1$ behavior of deep-inelastic structure functions. In perturbative QCD language, this can be understood in terms of hard gluon exchange [34]: deep-inelastic scattering at $x \sim 1$ probes a highly asymmetric configuration in the nucleon in which one of the quarks goes far off-shell after the exchange of at least two hard gluons in the initial state; elastic scattering, on the other hand, requires at least two gluons in the final state to redistribute the large $Q^2$ absorbed by the recoiling quark [35].

More generally, the relationship between resonance (transition) form factors and the deep-inelastic continuum has been studied in the framework of quark-hadron, or Bloom-Gilman, duality: the equivalence of the averaged structure function in the resonance region and the scaling function which describes high $W$ data. The recent high precision Jefferson Lab data [23] on the $F_2$ structure function suggests that the resonance-scaling duality also exists locally, for each of the low-lying resonances, including surprisingly the elastic [24], to rather low values of $Q^2$.

A number of recent studies have attempted to identify the dynamical origin of Bloom-Gilman duality using simple models of QCD [37-39]. It was shown, for instance, that in a harmonic oscillator basis one can explicitly construct a smooth, scaling structure function from a set of infinitely narrow resonances [37,38]. Whatever the ultimate microscopic origin of Bloom-Gilman duality, for our purposes it is sufficient to note the empirical fact that local duality is realized in lepton-proton scattering down to $Q^2 \sim 0.5$ GeV$^2$ at the 10-20% level for the lowest moments of the structure function. In other words, here we are not concerned about why duality works, but rather that it works.

Motivated by the experimental verification of local duality, one can use measured structure functions in the resonance region to directly extract elastic form factors [36]. Conversely, empirical electromagnetic form factors at large $Q^2$ can be used to predict the $x \to 1$ behavior of deep-inelastic structure functions [22,34,40,41]. The assumption of local duality for the elastic case implies that the area under the elastic peak at a given $Q^2$ is equivalent to the area under the scaling function, at much larger $Q^2$, when integrated from the pion threshold to the elastic point [22]. Using the local duality hypothesis, de Rújula et al. [36], and more recently Ent et al. [24], extracted the proton's magnetic form factor from resonance data on the $F_2$ structure function at large $x$, finding agreement to better than 30% over a large range of $Q^2 (0.5 \lesssim Q^2 \lesssim 5$ GeV$^2$). In the region $Q^2 \sim 1-2$ GeV$^2$ the agreement was at the $\sim 10\%$ level. In FIG. 8 we show the extracted proton magnetic form factor $G_M^p$ using the quark-hadron local duality relation [42] and the $F_2^p(\xi)$ parametrization of Ref. [43], in order to give a feeling how better/bad quark-hadron duality works.
Applying the argument in reverse, one can formally differentiate the local elastic duality relation [22] with respect to $Q^2$ to express the scaling functions, evaluated at threshold, $x = x_{th} = Q^2/(W_{th}^2 - M_N^2 + Q^2)$, with $W_{th} = M_N + m_\pi$, in terms of $Q^2$ derivatives of elastic form factors. In Refs. [22,40] the $x \to 1$ behavior of the neutron to proton structure function ratio was extracted from data on the elastic electromagnetic form factors. Extending this to the case of bound nucleons, one finds that as $Q^2 \to \infty$ the ratio of bound to free proton structure functions is:

\[
\frac{F_2^p}{F_2^n} \to \frac{dG_M^{p*}/dQ^2}{dG_M^{n}/dQ^2}.
\]  

(26)

At finite $Q^2$ there are corrections to Eq. (26) arising from $G_E^p$ and its derivatives, as discussed in Ref. [40]. (In this analysis we use the full, $Q^2$ dependent expressions [40,41].) Note that in the nuclear medium, the value of $x$ at which the pion threshold arises is shifted:

\[
x_{th} \to x_{th}^* = \left( \frac{m_\pi(2M_N + m_\pi) + Q^2}{m_\pi(2(M_N^* + V_N) + m_\pi) + Q^2} \right) x_{th},
\]

(27)

where $V_N = 3g_\omega^2 \omega$ is the vector potential felt by the nucleon and (consistent with chiral expectations and phenomenological constraints) we have set $m_\pi^* = m_\pi$. (See also Eq. (16).) However, the difference between $x_{th}$ and $x_{th}^*$ has a negligible effect on the results for most values of $x$ considered.

Using the duality relations between electromagnetic form factors and structure functions, in FIG. 9 we plot the ratio $F_2^p/F_2^n$ as a function of $x$, with $x$ evaluated at threshold, $x = x_{th}$ (solid lines).
FIG. 9. The ratios of the bound to free proton structure functions $F_2^{p^*}/F_2^p$, calculated in the QMC and PLC models.

We emphasize that since we are interested in ratios of form factors and structure functions only, what is more relevant for our analysis is not the degree to which local duality holds for the absolute structure functions, but rather the relative change in the duality approximation between free and bound protons. Note that at threshold the range of $Q^2$ spanned between $x = 0.5$ and $x = 0.8$ is $Q^2 \approx 0.3$–1.1 GeV$^2$. Over the range $0.5 \lesssim x \lesssim 0.75$ the effect is almost negligible, with the deviation of the ratio from unity being $\lesssim 1\%$ for $\rho = \frac{1}{2}\rho_0$ and $\lesssim 2\%$ for $\rho = \rho_0$. For $x \gtrsim 0.8$ the effect increases to $\sim 5\%$, although, since larger $x$ corresponds to larger $Q^2$, the analysis in terms of the QMC model is less reliable here. However, in the region where the analysis can be considered reliable, the results based on the bound nucleon form factors inferred from the polarization transfer data [1,2] and local duality imply that the nucleon structure function undergoes very little modification in medium.

It is instructive to contrast this result with models of the EMC effect in which there is a large medium modification of nucleon structure. For example, let us consider the model of Ref. [26], where it is assumed that for large $x$ the dominant contribution to the structure function is given by the point-like configurations (PLC) of partons which interact weakly with the other nucleons. The suppression of this component in a bound nucleon is assumed to be the main source of the EMC effect. This model represents one of the extreme possibilities that the EMC effect is solely the result of deformation of the wave function of bound nucleons, without attributing any contribution to nuclear pions or other effects associated with nuclear binding [45].

The deformation of the bound nucleon structure function in the PLC suppression model is governed by the function [26]:

$$\delta(k) = 1 - 2(k^2/2M + \epsilon_A)/\Delta E_A,$$

where $k$ is the bound nucleon momentum, $\epsilon_A$ is the nuclear binding energy, and $\Delta E_A \sim 0.3$–0.6 GeV is a nucleon excitation energy in the nucleus. For $x \gtrsim 0.6$ the ratio of bound to free nucleon structure functions is then given by [26]:
\[
\frac{F_2^{N^*}(k, x)}{F_2^N(x)} = \delta(k).
\] (29)

The \(x\) dependence of the suppression effect is based on the assumption that the point-like configuration contribution in the nucleon wave function is negligible at \(x \lesssim 0.3\) \((F_2^{N^*}/F_2^N = 1)\), and for \(0.3 \lesssim x \lesssim 0.6\) one linearly interpolates between these values \([26]\). The results for \(^4\)He and \(^{16}\)O are shown in FIG. 9 (dashed lines) for the average values of nucleon momentum, \(\langle k^2 \rangle\), in each nucleus. The effect is a suppression of order 20% in the ratio \(F_2^{N^*}/F_2^N\) for \(x \sim 0.6-0.7\). In contrast, the ratios extracted on the basis of duality, using the QMC model constrained by the \(^4\)He polarization transfer data \([1,2]\), show almost no suppression \((\lesssim 1-2\%)\) in this region. Thus, for \(^4\)He, the effect in the PLC suppression model is an order of magnitude too large at \(x \sim 0.6\), and has the opposite sign for \(x \gtrsim 0.65\).

Although the results extracted from the polarization transfer measurements \([1,2]\) rely on the assumption of local duality, we stress that the corrections to duality have been found to be typically less than 20% for \(0.5 \lesssim Q^2 \lesssim 2\ \text{GeV}^2\) \([23,44]\). The results therefore appear to rule out large bound structure function modifications, such as those assumed in the point-like configuration suppression model \([26]\), and instead point to a small medium modification of the intrinsic nucleon structure, which is complemented by standard many-body nuclear effects.

As a consistency check on the analysis, one can also examine the change in the form factor of a bound nucleon that would be implied by the corresponding change in the structure function in medium. Namely, from the local duality relation \([36,41]\):

\[
\left[ G_M^p(Q^2) \right]^2 \approx \frac{2 - \xi_0}{\xi_0^2} \frac{(1 + \tau)}{(1/\mu_p^2 + \tau)} \int_{\xi_{\text{th}}}^{1} d\xi \ F_p^\pi(\xi),
\]

one can extract the magnetic form factor by integrating the \(F_p^\pi(\xi)\) structure function over \(\xi\) between threshold, \(\xi = \xi_{\text{th}},\) and \(\xi = 1\). Here \(\xi_0 = \xi(x = 1)\), \(\mu_p\) is the proton magnetic moment, and \(\xi = 2x/\left(1 + \sqrt{1 + x^2/\tau}\right)\) with \(\tau = Q^2/4M_N^2\) and \(x\) the Bjorken variable. In FIG. 10 we show the PLC model predictions for the ratio of the magnetic form factor of a proton bound in \(^4\)He to that in vacuum, derived from Eqs. (29) and (30), using the parameterization for \(F_p^\pi(\xi)\) from Ref. \([24]\), and an estimate for the in-medium value of \(\mu_p^\pi\) from Ref. \([15-18]\). Taking the average nucleon momentum in the \(^4\)He nucleus, \(k = \langle k \rangle\), the result is a suppression of about 20% in the ratio \(G_M^{p^*}/G_M^p\) at \(Q^2 \sim 1-2\ \text{GeV}^2\) (solid curve). Since the structure function suppression in the PLC model depends on the nucleon momentum (Eq. (28)), we also show the resulting form factor ratio for a momentum typical in the \((e, e'p)\) experiment, \(k = 50\ \text{MeV}\) (long dashed). As expected, the effect is reduced, however, it is still of the order 15% since the suppression also depends on the binding energy, as well as on the nucleon mass, which changes with density rather than with momentum. In contrast, the QMC calculation, which is consistent with the MAMI \(^4\)He quasi-elastic data, and Jlab polarization transfer measurements on \(^4\)He \([1,2]\), produces a ratio which is typically 5-10\% larger than unity (dashed). Without a very large compensating change in the in-medium electric form factor of the proton (which seems to be excluded by \(y\)-scaling constraints), the behavior of the magnetic form factor implied by the “PLC model + duality” would produce a large enhancement of the polarization transfer ratio, rather than the observed small suppression \([1,2]\). (See Eq. (1) and FIG. 4.)
FIG. 10. The ratios of the bound proton to free magnetic form factors $G_{M}^{P}/G_{M}^{P}$, calculated in the QMC and PLC models.

V. SUMMARY

In this article we have discussed medium modification of internal structure of bound nucleon due to the change of quark response to the nuclear environment. Especially, recent experimental results [1,2] from polarized proton knockout reactions off $^{4}$He nuclei may imply a first direct evidence for a small but nonzero modification of the proton electromagnetic form factors in nuclear medium. The analyses in Refs. [1,2] found that, compared with conventional nuclear calculations, the medium modifications observed in the $^{4}$He data could only be described within calculations which are implemented by a small modification of the nucleon form factors in medium calculated by the QMC model [15–21] (see also Ref. [46]).

We have also studied on the medium modification of the bound nucleon axial form factor $G_{A}(Q^{2})$, and have discussed an application in extracting the medium modification of bound nucleon structure function $F_{3}$, using quark-hadron duality. The modification of axial form factor may be observed e.g., in high precision neutrino-nucleus scattering experiments in future.

Finally, we have examined the consequences of quark-hadron duality applied to nucleons in the nuclear medium. Utilizing the experimental results [1,2] we use local duality to relate model-independently the medium modified electromagnetic form factors to the change in the intrinsic structure function of a bound proton. While the results rely on the validity of quark-hadron duality, the empirical evidence suggests that for low moments of the proton's $F_{2}$ structure function the duality violations due to higher twist corrections are $\lesssim 20\%$ for $Q^{2} \gtrsim 0.5$ GeV$^{2}$ [23], and decrease with increasing $Q^{2}$. In the context of the QMC model, the change in nucleon form factors allowed by the data imply a modification of the in-medium structure function of $\lesssim 1-2\%$ at $0.5 \lesssim x \lesssim 0.75$ for all nuclear densities between nuclear matter densities, $\rho = \rho_{0}$, and $\rho = \frac{1}{2} \rho_{0}$.

The results place rather strong constraints on models of the nuclear EMC effect, especially on models which assume that the EMC effect arises from a large deformation of the
nucleon structure in medium. Our findings appear to disfavor models with large medium modifications of structure functions as viable explanations for the nuclear EMC effect, although it would be desirable to have more data on a variety of nuclei and in different kinematical regions. There is a such proposed experiment at Jefferson Lab on $^{16}\text{O}$ [48] at $Q^2 = 0.8 \text{ GeV}^2$, which would make use of other, high precision cross section data at this momentum transfer, would have about 15 times the statistics of the original commissioning experiment [9]. This would enable a more thorough comparison of the medium dependence of form factors and structure functions for different nuclei.

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