Modified Fragmentation Function and Jet Quenching at RHIC

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Medium modification of jet fragmentation functions and parton energy loss in cold and hot matter are reviewed. The predicted nuclear modification of the jet fragmentation function agrees well with the recent HERMES data with a resultant energy loss $dE/dx \approx 0.5$ GeV/fm. From the recent PHENIX data of high $p_T \pi_0$ spectra in central Au + Au collisions at $\sqrt{s} = 130$ GeV, one extracts an energy loss for a 10 GeV parton that is equivalent to $dE/dx = 7.3$ GeV/fm in a static medium with the same gluon density as in the initial stage of the collision at $\tau_0 = 0.2$ fm/c. Constraints on jet quenching by the central rapidity density of charged hadrons is also discussed.

1. INTRODUCTION

It is with a great respect that I present this talk on this special occasion of celebrating Helmut Satz's scientific career and his unwavering role in high-energy heavy-ion physics. I first met Helmut when I was still a graduate student at my first Quark Matter meeting in 1988. His name was already synonymous to quark-gluon plasma and $J/\Psi$ suppression for me at that time. Because of our common interests in hard processes in heavy-ion collisions, we started working together in 1994 to coordinate a working group called Hard Probes Collaboration. Since then this working group has become a forum where we meet regularly together with other colleagues to discuss problems related to hard processes in high-energy heavy-ion collisions. Helmut has been an inspiration for all of us in the working group and in the community of high-energy nuclear physics.

The subject of jet quenching and parton energy loss is one aspect of hard processes in heavy-ion collisions. However, with the advent of the RHIC facility and experiments, jets of high-energy partons will become an important and useful tool to the study of the properties of dense matter formed in high-energy heavy-ion collisions. Because large $p_T$ partons are produced very early in heavy-ion collisions and their production rates can be calibrated in $pp$ and $pA$ collisions at the same energy, they are ideal probes of the dense matter that is formed in the same reaction. What probes the dense medium is the scattering induced energy loss suffered by an energetic parton as it propagates through the matter. The parton energy loss is directly related to the parton density of the medium.

Theoretical studies of the parton energy loss in hot medium date back to the first attempt by Bjorken [1] to calculate elastic energy loss of a parton via elastic scattering in the hot medium. A simple estimate can be given by the thermal averaged energy transfer $\nu_\parallel \approx q_\perp^2/2\omega$ of the jet parton to a thermal parton with energy $\omega$, $q_\perp$ being the transverse
momentum transfer of the elastic scattering. The resultant elastic energy loss [2]

\[ \frac{dE_{\text{el}}}{dx} = C_2 \frac{3\pi \alpha_0^2}{2} T^2 \ln \left( \frac{3ET}{2\mu^2} \right) \]

is sensitive to the temperature of the thermal medium but is in general small compared to radiative energy loss. Here, \( \mu \) is the Debye screening mass and \( C_2 \) is the Casimir of the propagating parton in its fundamental presentation. The elastic energy loss can also be calculated within finite temperature QCD [3] with a similar result, but with a more careful and consistent treatment of screening effect.

Though there had been estimates of the radiative parton energy loss using the uncertainty principle [4], a first theoretical study of QCD radiative parton energy loss incorporating Landau-Pomeranchuk-Migdal interference effect [5] is by Gyulassy and myself [6] where multiple parton scattering is modeled by a screened Coulomb potential model. Bäier et al. (BDMPS) [7] later considered the effect of gluon rescattering which turned out to be very important for gluon radiation induced by multiple scattering in a dense medium. These two studies have ushered in many recent works on the subject, including a path integral approach to the problem [8] and opacity expansion framework [9,10] which is more suitable for multiple parton scattering in a thin plasma. The radiative parton energy loss to the leading order of the opacity \( \tilde{\alpha} = L/\lambda \) in the thin plasma of size \( L \) is estimated as [9,11]

\[ \frac{dE_{\text{rad}}}{dx} \approx C_2 \frac{\alpha_0 \mu^2 L}{4\lambda} \ln \left( \frac{2E}{\mu^2 L} \right), \]

where \( \lambda \) is the gluon’s mean-free-path in the medium. The unique \( L \) dependence of the parton energy loss is a consequence of the non-Abelian LMP interference effect in a QCD medium. It is also shown in a recent study [11] that thermal absorption and stimulated emission in a thermal environment can be neglected for high energy partons \( (E \gg \mu) \) while they are important for intermediate energy partons.

Using this latest result one can estimate the total energy loss for a parton with initial energy \( E = 40 \text{ GeV} \) to be about \( \Delta E \approx 10 \text{ GeV} \) after it propagates a distance of \( L = 6 \text{ fm} \) in a medium with \( \mu = 0.5 \text{ GeV} \) and \( \lambda = 1 \text{ fm} \). For an expanding system, the total energy loss is reduced by a factor of \( 2\tau_0/L \) from the static value [12,27]. Assuming that most of this energy loss is carried by gluons outside the jet cone [14], measuring the energy loss would require the experimental resolution \( \delta E \) to be much smaller than the total energy loss \( \Delta E \). With the measured total multiplicity density \( dN/d\eta \approx 900 \) [15] and energy density \( dE_T/d\eta \approx 500 \text{ GeV} \) [16] in central \( Au + Au \) collisions at \( \sqrt{s} = 130 \text{ GeV} \), one can estimate that the average total background energy within the jet cone (\( \delta\eta = 1 \) and \( \delta\phi = 1 \)) is about \( \Sigma E_T \approx 80 \text{ GeV} \) with a fluctuation of \( \delta E_T \approx 10 \text{ GeV} \). It is therefore very difficult, if not impossible, to determine the energy of a jet on a event-by-event base [17]. Since high \( p_T \) hadrons in hadron and nuclear collisions come from fragmentation of high \( E_T \) jets, energy loss naturally leads to suppression of high \( p_T \) hadron spectra. Miklos Gyulassy and I then proposed [18] that one has to reply on measuring the suppression of high \( p_T \) hadrons to study parton energy loss in heavy-ion collisions. Since inclusive hadron spectra is a convolution of jet production cross section and the jet fragmentation function in pQCD, the suppression of inclusive high \( p_T \) hadron spectra is a direct consequence of
the medium modification of the jet fragmentation function induced by parton energy loss. Assuming that jet fragmentation function is the same for the final leading parton with a reduced energy, the modified fragmentation function can be assumed as [19]

\[ \bar{D}(z) \approx \frac{1}{1 - \Delta E/E} \bar{D}\left(\frac{z}{1 - \Delta E/E}\right). \]  

(3)

Therefore, in this effective model the measured modification of fragmentation function can be directly related to the parton energy loss.

2. MODIFIED FRAGMENTATION FUNCTIONS

Since a jet parton is always produced via a hard process involving a large momentum scale, it should also have final state radiation with and without rescattering leading to the DGLAP evolution equation of fragmentation functions. Such final state radiation effectively acts as a self-quenching mechanism softening the leading parton momentum distribution. This process is quite similar to the induced gluon radiation and the two should have strong interference effect [9,10]. It is therefore natural to study jet quenching and modified fragmentation function in the framework of modified DGLAP evolution equations in a medium [20].

The simplest case for jet quenching is deeply inelastic scattering of an electron off a nucleon target where the virtual photon knocks one quark out of a nucleon inside a nucleus. The quark then will have to propagate through the rest of the nucleus and possibly scatter again with other nucleons with induced gluon radiation. The induced gluon radiation reduces the quark's energy before it fragments into hadrons with a modified fragmentation function. One can study the nuclear modification of the fragmentation function by comparing it with the same measurement in DIS with a nucleon target.

We work in an infinite momentum frame, where the photon carries momentum \( q = \{-x_Bp^+, q^-, 0, 0, \} \) and the momentum of the target per nucleon is \( p = [p^+, 0, 0, 0] \) with the Bjorken variable defined as \( x_B = Q^2/2q^-p^+ \). The differential semi-inclusive hadronic tensor in a collinear factorization approximation to the leading twist can be written as

\[ \frac{dW^S_{\mu\nu}}{dz_h} = \sum q \int dx f_q^A(x, \mu_F^2) H_{\mu\nu}(x, p, q) D_{q\to h}(z_h, \mu^2), \]  

(4)

where \( f_q^A(x, \mu_F^2) \) is the quark distribution of the nucleus, \( H_{\mu\nu}(x, p, q) \) is the hard part of \( \gamma^* + q \) scattering and \( D_{q\to h}(z_h, \mu^2) \) is the quark fragmentation function in vacuum as measured in the \( e^+e^- \) annihilation process. The scale \( \mu^2 \) dependence of the fragmentation function comes from the radiative correction as shown in Fig. 1. Taking into account of both the radiative (central cut diagram) and the virtual correction (virtual cut diagram), the renormalized fragmentation function is defined as

\[ D_{q\to h}(z_h, \mu^2) \equiv D_{q\to h}(z_h) + \int_0^{\mu^2} \frac{d\mu_F^2}{\mu_F^2} \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \times \left[ \gamma_{q\to qg}(z) D_{q\to h}(z_h/z) + \gamma_{q\to g}(z) D_{g\to h}(z_h/z) \right], \]  

(5)

which satisfies the DGLAP evolution equation,

\[ \frac{\partial D_{q\to h}(z_h, \mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[ \gamma_{q\to qg}(z) D_{q\to h}(z_h/z, \mu^2) + \gamma_{q\to g}(z) D_{g\to h}(z_h/z, \mu^2) \right]. \]  

(6)
Here $\gamma_{q\to qg}(z) = \gamma_{q\to qg}(1-z) = C_F[(1 + z^2)/(1 - z) + (3/2)\delta(1 - z)]$ is the splitting function.

In a nucleus target, the outgoing quark can scatter again with another parton from the nucleus. The additional scattering may induce additional gluon radiation and cause the leading quark to lose energy. Such induced gluon radiation will effectively give rise to additional terms in the evolution equation leading to modification of the fragmentation functions in a medium. Contributions from multiple parton scattering are always non-leading twist. However we will consider only those that are enhanced by the nuclear size $A^{1/3}$.

\begin{figure}[h]
\begin{center}
\includegraphics[width=0.4\textwidth]{fig1.pdf}
\end{center}
\caption{Diagram of final state radiation correction to the jet fragmentation function with central and virtual cuts.}
\end{figure}

\begin{figure}[h]
\begin{center}
\includegraphics[width=0.4\textwidth]{fig2.pdf}
\end{center}
\caption{One example diagram of gluon radiation associated with double scattering with different cuts (central, left, right and virtual).}
\end{figure}

We work in the LQS framework [21] in which the twist-four contributions can be expressed as the convolution of partonic hard parts and four-parton matrix elements. At the lowest order (processes without gluon radiation) in this framework, rescattering with collinear gluons gives the eikonal contribution to the gauge-invariant leading-twist and lowest-order result, assuming collinear factorization of the quark fragmentation function. The radiative correction to double scattering processes with another gluon from the nucleus, shown in Fig. 2 as an example, generally involves matrix elements of four parton fields and the cross section of four parton scattering. In the LQS framework, one makes collinear expansion of the four-parton scattering cross section in terms of the transverse momentum of the scattering gluons. The first term in the collinear expansion corresponds to rescattering with collinear gluons. Similar to the leading order case, it contributes only to the eikonal correction of the radiative processes at the leading twist as a consequence of the cancellation between different cut diagrams (central, right and left cut). The leading twist-four contribution then comes from the second derivative of the four-parton scattering cross section in the central-cut diagrams. With the quadratic term $k^\alpha_k k^\beta_{\perp}$, this contribution depends on the gluon field strength, because $k^\alpha_k A^\nu k^\beta_{\perp} A^+ \rightarrow F^{\alpha\nu} F^{\beta\perp}$ by partial integrations.

Considering all possible diagrams in addition to the example shown in Fig. 2, including
the virtual corrections (virtual cut), one get the leading twist-four contribution to the semi-inclusive cross section from double scattering,

\[
\frac{W^{D}_{\mu \nu}}{d \zeta k} = \sum_{q} \int dx \bar{H}^{(0)}_{\mu \nu}(x, p, q) \frac{2 \pi \alpha_s}{N_c} \int \frac{d \ell_T^2}{\ell_T^2} \int_{\zeta_k}^{1} \frac{dz \alpha_s}{z} C_A \left\{ D_{q \rightarrow h}(z_k/z) \left[ 1 + \frac{1}{(1 - z^2)} T_{qg}(x, x_L) + \delta(1 - z) \Delta T_{qg}^A(x, \ell_T^2) \right] \right. \\
+ \left. D_{g \rightarrow h}(z_k/z) \left[ \frac{1 + (1 - z)^2}{z} T_{qg}(x, x_L) + \delta(1 - z) \Delta T_{qg}^A(x, \ell_T^2) \right] \right\}, 
\]

where

\[
T_{qg}(x, x_L) = \int \frac{dy^-}{2\pi} dy^- dy^- e^{i(x + x_L) x} \psi_g(y^-) \psi_q(y^-) \frac{(1 - e^{-i x_L y^-})(1 - e^{-i x_L y^-}(y^- - y_-))}{2} \langle A|\gamma^+_{\nu q}(0) F_{\gamma^+}(y^-) F_{\gamma^+}(y_1^-) \psi_q(y^-)|A\rangle \theta(1 - y^-) \theta(y^- - y_-)
\]

is the quark-ghon correlation function which essentially contains four independent twist-four parton matrix elements in a nucleus. Here \(x_L = \ell_T^2 / 2p^+ q^- z(1 - z)\) and \(x_T = (k_T^2) / 2p^+ q^- z(1 - z)\). In the central-cut diagrams (Fig. 2 for example) where the leading higher-twist result comes from, there are typically four contributions from each cut diagram. The radiated gluon can either come as the final state radiation of the \(\gamma\)-quark scattering or as the initial state radiation of the secondary quark-ghon scattering. The amplitudes of these initial and final state radiation have opposite signs with also a phase difference \(x_L p^+ y^-\). The sum of these two radiation processes and their interferences gives rise to the dipole-like factor \((1 - e^{-i x_L y^-})(1 - e^{-i x_L y^-}(y^- - y_-))\) in the effective two-parton correlation function that enters into the double-scattering cross section with induced gluon radiation. This is exactly the LPM effect of bremsstrahlung in medium.

Summing up the single and double scattering contributions to the semi-inclusive process, one can define the effective modified fragmentation function as

\[
\frac{dW_{\mu \nu}}{d \zeta k} = \frac{dW^{S}_{\mu \nu}}{d \zeta h} + \frac{dW^{D}_{\mu \nu}}{d \zeta h} = \sum_{q} \int dx \bar{f}^A_q(x, \mu_F^2) H^{(0)}_{\mu \nu}(x, p, q) \bar{D}_{q \rightarrow h}(z_k, \mu^2),
\]

\[
\bar{D}_{q \rightarrow h}(z_h, \mu^2) \equiv D_{q \rightarrow h}(z_h, \mu^2) + \Delta D_{q \rightarrow h}(z_h, \mu^2)
\]

\[
\Delta D_{q \rightarrow h}(z_h, \mu^2) = \int_{0}^{\mu^2} \frac{d \ell_T^2}{\ell_T^2} \frac{\alpha_s}{2\pi} \int_{z_h}^{1} \frac{dz}{z} \left[ \Delta \gamma_{q \rightarrow g q}(z) D_{q \rightarrow h}(z_h) + \Delta \gamma_{q \rightarrow g q}(z) \bar{D}_{q \rightarrow h}(z_h) \right],
\]

where \(D_{a \rightarrow h}(z_h, \mu^2)\) is the leading-twist parton fragmentation function and

\[
\Delta \gamma_{q \rightarrow g q}(z) = \left[ \frac{1 + 2}{(1 - z)} T_{qg}(x, x_L) + \delta(1 - z) \Delta T_{qg}^A(x, \ell_T^2) \right] \frac{C_A 2\pi \alpha_s}{(\ell_T^2 + \langle k_T^2 \rangle) N_c f_q^A(x, \mu_F^2)}
\]

\[
\Delta \gamma_{q \rightarrow g q}(1 - z) = \Delta \gamma_{q \rightarrow g q}(1 - z)
\]

are the modified splitting functions for the induced gluon radiation. Similar as the vacuum case, the \(\delta\)-function part from the virtual correction contribution with \(\Delta T_{qg}^A(x, \ell_T^2)\) defined as

\[
\Delta T_{qg}^A(x, \ell_T^2) \equiv \int_{0}^{1} \frac{dz}{1 - z} \left[ 2T_{qg}^A(x, x_L)|_{z=1} - (1 + z^2)T_{qg}^A(x, x_L) \right].
\]
Such virtual corrections are important to ensure the infrared safety of the modified fragmentation function and the unitarity of the gluon radiation processes. This virtual correction is equivalent in nature to the absorption processes in some effective models [22].

To evaluate the modified fragmentation, one needs to know the two-parton correlation function $T_{qq}^A(x, x_L)$ which consists of both diagonal and off-diagonal twist-four parton matrices. We generalize the factorization assumption of LQS [21] to both types of matrices. Assuming a Gaussian nuclear distribution in the rest frame, $\rho(r) \sim \exp(-r^2/2R_A^2)$, $R_A = 1.12A^{1/3}$ fm, we express $T_{qq}^A$ in terms of single parton distributions,

$$T_{qq}^A(x, x_L) = \tilde{C}m_NR_A f^A_q(x)(1 - e^{-x_L^2/x_A^2}),$$

where $x_A = 1/m_NR_A$, and $m_N$ is the nucleon’s mass. The off-diagonal terms involve transferring momentum $x_L$ between different nucleons inside a nucleus and thus should be suppressed for large nuclear size or large momentum fraction $x_L$. Notice that $\tau_f = 1/x_LP^+$ is the gluon’s formation time. Thus, $x_L/x_A = L_A/\tau_f$ with $L_A = R_Am_N/p^+$ being the nuclear size in the infinite momentum frame.

Because of the LPM interference effect, the above effective parton correlation and the induced gluon emission vanishes when $x_L/x_A \ll 1$. Therefore, the formation time of the gluon radiation due to the LPM interference requires the radiated gluon to have a minimum transverse momentum $E_T \sim Q^2/MR_A \sim Q^2/A^{1/3}$. The nuclear corrections to the fragmentation function due to double parton scattering will then be in the order of $\alpha_sA^{1/3}/E_T^2 \sim \alpha_sA^{2/3}/Q^2$, which depends quadratically on the nuclear size. For large values of $A$ and $Q^2$, these corrections are leading and yet the requirement $E_T \ll Q^2$ for the logarithmic approximation in deriving the modified fragmentation function is still valid.

![Figure 3](image1.png)  
**Figure 3.** The predicted nuclear modification of jet fragmentation function is compared to the HERMES data [23].

![Figure 4](image2.png)  
**Figure 4.** Comparison of the calculated nuclear modification with the effective model in Eq. (3) with $\Delta z = 0.6\langle z_g \rangle$.

Shown in Fig. 3 are the calculated nuclear modification factor of the fragmentation function for $^{14}N$ and $^{84}Kr$ targets as compared to the recent HERMES data [23].
predicted shape of the $z$ dependence and the quadratic nuclear size dependence agrees well with the experimental data. The energy dependence of the suppression also has excellent agreement with our prediction [24]. What is amazing is the clear quadratic $A^{2/3}$ nuclear size dependence of the suppression which is a true QCD non-Abelian effect. In fitting the data of the overall suppression for $^{14}$N target we obtain the only parameter in our calculation, $\tilde{C} \alpha_s^2 = 0.00065$ GeV$^2$. This parameter is shown to be related to the transverse momentum nuclear broadening of Drell-Yan dilepton in $pA$ collisions [25], $\langle \Delta q^2 \rangle = \tilde{C} \pi \alpha_s / N_c x_A$. With an experimental value [25] of $\langle \Delta q^2 \rangle = 0.016 A^{1/3}$ GeV$^2$ and with $\alpha_s = 0.21$ (at $M^2 = 40$ GeV$^2$), one finds $\tilde{C} \alpha_s^2 = 0.00057$ GeV$^2$. This indicates that $\tilde{C}$ might have some $Q^2$ dependence. With this value of $\tilde{C}$ we can also predict the nuclear transverse momentum broadening of single jets in DIS [24].

With the modified fragmentation function in Eq. (10), one can calculate theoretically the average energy loss by the quark, which is the energy carried away by the radiated gluons,

$$\Delta E = \nu \langle \Delta z_g \rangle \approx \tilde{C} \alpha_s^2 m_N R_A^2 (C_A / N_c) 3 \ln(1/2 x_B).$$

With the value of $\alpha_s^2 \tilde{C}$, and $L_A = R_A \sqrt{2 \pi}$ one gets the quark energy loss $dE / dx \approx 0.5$ GeV/fm for a $Au$ nuclear target.

3. JET QUENCHING IN HEAVY-ION COLLISIONS

In high-energy heavy-ion collisions, the jet production rate is not affected by the formation of dense matter and the final state multiple scattering. One can assume that the high $p_T$ hadron spectra can then be given by the convolution of the jet production cross section and the medium modified jet fragmentation function $D_{h/\pi}(z_c, Q^2)$,

$$\frac{d\sigma_{AB}^h}{dy d^2 p_T} = K \int d^2 b d^2 t_A(r) t_B(|b - r|) \sum_{abcd} \int dx_a dx_b d^2 k_a d^2 k_b g_B(k_b, Q^2, |b - r|)$$

$$\times g_A(k_a, Q^2, r) f_{a/A}(x_a, Q^2, r) f_{b/B}(x_b, Q^2, |b - r|) \frac{D_{h/\pi}(z_c, Q^2)}{\pi z_c} \frac{d\sigma_{ab\rightarrow cd}}{dt},$$

where $t_A(r)$ is the thickness function of the nucleus $A$, $f_{a/A}(x_a, Q^2, r)$ is the parton distribution in a nucleus, $g_A(k_a, Q^2, r)$ is the distribution of parton intrinsic transverse momentum with nuclear broadening [26].

In principle, one should use the modified fragmentation function evaluated according to the pQCD calculation for a dense medium. However, before that can be done in a practical manner, we have used the effective approach in Eq. (3) by rescaling the fractional momentum by $1-\Delta z$ to take into account of the parton energy loss. To verify whether such an effective approach is adequate, we compare the two modified fragmentation functions in Fig. 4. We found that the effective model (dashed lines) can reproduce the pQCD result (solid lines) of Eq. (10) very well, but only when $\Delta z$ is set to be $\Delta z \approx 0.6(z_g)$. Therefore the actual averaged parton energy loss should be about 1.6 times of that used in the effective modified fragmentation function. This difference is caused by the absorptive processes or unitarity correction effect in the full pQCD calculation.

Unlike in DIS nuclear scattering, the dense medium in high-energy heavy-ion collisions is not static. It has to go through rapid expansion which should also affect the effective
total parton energy loss. The total energy loss extracted from experiments should be a quantity that is averaged over the whole evolution history of the expanding system. It is therefore useful to convert the averaged quantity to an energy loss in a static system that has the same parton density as the expanding system at its initial stage. If the averaged total parton energy loss in a longitudinally expanding system with a transverse size $R$ is $\Delta E_{1d}$, one finds [27] that the corresponding parton energy loss in a static system with the same initial parton density would be $\Delta E = \Delta E_{1d}(R/2\tau_0)$. Here $\tau_0$ is the initial formation time of the dense medium.

![Figure 5](image5.png)  
**Figure 5.** pQCD parton model calculation of the charged hadron and pion spectra in $p\bar{p}$ and central $Au+Au$ collisions compared with the experimental data [28,29]. The effective modified fragmentation function is used in the calculation.  

![Figure 6](image6.png)  
**Figure 6.** The energy dependence of the charged multiplicity density per participant nucleon pair in the two-component model with and without parton shadowing compared with the experimental data [15,32].

Comparing the recent PHENIX data [28] with Eq. (16) as shown in Fig. 5, one can extract a value of $dE/dx = 0.23 \ln E/\ln 5 \text{GeV}/\text{fm}$ with a logarithmic energy dependence that one needs to use in the effective modified fragmentation function in fitting the data. Taking into account the unitarity correction effect and the expansion, this corresponds to an effective energy loss $dE/dx = 1.6 \times 0.23(R/2\tau_0) \ln E/\ln 5$. In a static system with a density similar to the initial stage of the expanding system at $\tau_0$. With $R \sim 6 \text{ fm}$ and $\tau_0 \sim 1/\rho_0 = 0.2 \text{ fm}$, this would give $dE/dx \approx 7.3 \text{ GeV}/\text{fm}$ for a 10 GeV parton, which is about 15 times of that in a cold nuclear matter as extracted from the DIS processes. Since the parton energy loss is directly proportional to gluon density, this implies that the gluon density in the initial stage of $Au+Au$ collisions is about 15 times higher than
that inside a cold nucleus.

4. GLOBAL CONSTRAINTS OF PARTON ENERGY LOSS

When a fast parton experiences multiple scattering and induced gluon radiation, the radiated gluon would also contribute to the final parton production leading to the enhancement of modified fragmentation function at small $z$ as shown in Fig. 4. These partons might get thermalized in the medium, but will contribute to the final hadron production. In the HIJING model [30] with the default setting, a simple $dE/dx = 2 \text{ GeV/fm}$ is assumed for every jet with $p_T > 2 \text{ GeV/c}^2$. Such a default setting predicted an enhancement of the total hadron multiplicity in the central region [18] and a strong energy dependence [31] which is now excluded by the new PHOBOS data [15] of $Au + Au$ collisions at $\sqrt{s} = 200 \text{ GeV}$. This either implies that the $dE/dx = 2 \text{ GeV/fm}$ used in HIJING is too large or the onset of jet quenching is set at too low $p_T$ or both. According to the recent study of parton energy loss with detailed balance [11], gluon absorption by the fast parton in the thermal medium reduces the parton energy loss significantly for low energy partons. Taking into account this strong energy dependence of $dE/dx$, one might expect that the threshold for gluon radiation could be higher than what HIJING used. In fact, if one increases the threshold to 3 GeV in HIJING, one can still fit both $dN_{ch}/d\eta$ and the suppressed hadron spectra at high $p_T$. Since the production rate of mini-jets with $p_T > 3 \text{ GeV/c}^2$ is very small, their contribution due to induced bremsstrahlung to the total charged multiplicity is negligible at the RHIC energies. By similar argument, the effect of parton thermalization on the total hadron multiplicity might also be small.

Neglecting the effect of jet quenching on the total multiplicity, one can assume the final $dN_{ch}/d\eta$ to be proportional to the total number of mini-jets produced in addition to the soft particle production. Assuming that the mini-jet production is proportional to the number of binary collisions and the soft part is proportional to the number of participating nucleons, one get in this simple two-component model

$$\frac{dN_{ch}}{d\eta} = \frac{1}{2} \langle N_{part} \rangle_{s} \langle n \rangle_{s} + \langle n \rangle_{h} \langle N_{binary} \rangle \frac{\sigma_{jet}^{AA}(s)}{\sigma_{in}} ,$$

where $\sigma_{jet}^{AA}(s)$ is the averaged inclusive jet cross section per $NN$ in $AA$ collisions. The energy dependence of the above multiplicity is shown in Fig. 6. The parameters $\langle n \rangle_{s} = 1.6$ and $\langle n \rangle_{h} = 2.2$ is fixed by the $p + p(\bar{p})$ data. The average number of participant nucleons and number of binary collisions for given impact-parameters can be estimated using HIJING Monte Carlo simulation. If one assumes that the jet production cross section $\sigma_{jet}^{AA}(s)$ is the same as in $p + p$ collisions, the resultant energy dependence of the multiplicity density in central nuclear collisions is much stronger than the RHIC data. Therefore, one has to consider nuclear effects of jet production in heavy-ion collisions. Using a more recent parameterization [33], we found that the RHIC data requires a stronger nuclear shadowing of gluon distributions than quarks.

5. CONCLUSIONS

We have calculated the medium modification of the jet fragmentation functions due to gluon radiation induced by the multiple parton scattering. The predictions of the shape,
energy dependence, and the quadratic nuclear size $A^{2/3}$ dependence of the modification agree well with the recent HERMES data. The resultant parton energy loss in the cold nuclear medium is estimated to be about 0.5 GeV/fm inside $Kr$ nuclei. Comparing to the QCD result of the modification of fragmentation function, we found that the actual averaged energy loss is about 1.6 times that of the effective energy loss used in a earlier effective model for the same modification. Considering the effect of expansion, we found that the recent PHENIX data imply a medium induced energy loss in central $Au + Au$ collisions equivalent to 7.3 GeV/fm in a static medium with the same gluon density as in the initial stage of the collision. This is about 15 times larger than the energy loss in a cold nucleus. Due to detailed balance in induced radiation and absorption, we argue that parton energy loss for small and medium high energy jets is very small. Therefore, the contribution from their induced radiation to the hadron multiplicity is negligible.

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24. X.-N. Wang, to be published.
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