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# Hadron Structure and Modern Spectroscopy

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**Abstract**

The colour, flavour, spin and $J^{PC}$ of glueballs and hybrid mesons and baryons are constructed in an intuitive manner in both the gluon counting and adiabatic definitions. Glueball decay, production and mixing and hybrid meson decay selection rules and production are clarified.

In the arena of strong nuclear interactions, there are three distinct levels of understanding. First there is *Quantum Chromodynamics* (QCD), the Lagrangian of which is relativistic and non-linear (containing three- and four-particle interactions). Because the lowest energy state (vacuum) is non-empty, it can be thought of as a many-body system of particles. In addition, QCD is a quantum field theory. All these features conspire to make the physical predictions in the regime of strong interactions largely intractable. There hence exists a second level of understanding, called *phenomenology*, which attempts to capture strong interaction phenomena by use of simplified pictures. Phenomenology receives "data" from the first level by virtue of QCD and its computational expression, called lattice QCD. Phenomenology also receives data from the third level of understanding: *experiment* on strongly

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interacting particles (hadrons), sometimes called "empirical hadron spectroscopy". These lectures concern the data stream between levels two and three. Phenomenology serves to guide and interpret calculations on level one, and observations on level three. It does not make a claim to precision, and that proviso should be kept in mind throughout our discussion.

However, these noble features are not enough to protect level two from extinction, and replacement by a sole data stream between levels one and three. So why do we study phenomenology? Ultimately it is because it provides a language in which to express strong interactions, so that the phenomenon can be comprehended by the human mind.

These lectures will present highlights on explicit excitations of the force carriers, i.e., the gluons. A bibliography of recent books and reviews is provided for further reference.

Towards this purpose, we briefly review the non-relativistic quark model of quark-antiquark pairs (mesons) and three-quark composites (baryons).

# 1 Quark Model

A meson at rest can be represented by

$$\psi(\mathbf{r})\delta_{c\bar{c}}\mathcal{F}_{f\bar{f}}\mathcal{S}_{s\bar{s}}q^{+}_{cfs}(\frac{\bar{m}\mathbf{r}}{m+\bar{m}})\bar{q}^{+}_{\bar{c}\bar{f}\bar{s}}(\frac{-m\bar{\mathbf{r}}}{m+\bar{m}})|0\rangle \tag{1}$$

Here implicit summation or integration over respectively discrete (subscripted) and continuous variables is implied.

The labels $\mathbf{r}$, $c$, $f$, $s$, and $m$ denote the position, colour, flavour, (non-relativistic) spin and mass of the quark, which is created by the operator $q^+$ from the vacuum $|0\rangle$. Accordingly for the antiquark. The spacial wave function is $\psi$, and the flavour and spin structures $\mathcal{F}$ and $\mathcal{S}$ respectively. The orbital angular momentum $\mathbf{L}$ is conserved with $\psi$ carrying the quantum numbers $L$, $L_z$. The spin $\mathbf{S}$ is conserved with $\mathcal{S}$ carrying the quantum numbers $S$, $S_z$. The total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is also conserved, with the meson carrying quantum numbers $J$, $J_z$. The coefficient that expresses this is suppressed in Eq. 1.

Under reflection through the origin (parity), $\mathbf{r} \to -\mathbf{r}$ in $q^+\bar{q}^+$, and an additional sign appears because the intrinsic parity of an antiquark is opposite to that of a quark. The latter property holds for fermions in field theory, *i.e.*, comes from the first level of understanding. Noting that $\psi(-\mathbf{r}) = (-1)^L\psi(\mathbf{r})$, the parity $P = (-1)^{L+1}$.

Particle-antiparticle exchange (charge conjugation) interchanges $q^+$ and $\bar{q}^+$. Assume for the purpose of this paragraph that the quark and the antiquark have the same flavour. Noting that fermionic creation operators anticommute; that if the quark and antiquark have the same flavour, $F_{\bar{f}f} = F_{f\bar{f}}$; and that $S_{\bar{s}s} = (-1)^{S+1} S_{s\bar{s}}$, one can conclude that the charge conjugation $C = (-1)^{L+1+S+1} = (-1)^{L+S}$. It follows that $CP = (-1)^{S+1}$.

Given the equations $\mathbf{J} = \mathbf{L} + \mathbf{S}$, $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ one can construct the $J^{PC}$ of all mesons. It can then be checked that the combinations $J^{PC} = 0^{--}$, $0^{+-}$, $1^{-+}$, $2^{+-}$, $3^{-+} \ldots$ are not allowed. These will be referred to as "$J^{PC}$ exotic".

When the up $u$ and down $d$ quarks can be treated the same, *i.e.*, when their different electric charges and masses are neglected, the isospin $\mathbf{I}$ is conserved with $\mathcal{F}$ carrying the quantum numbers $I, I_z$. $G$-parity is defined as two operations conjoined: charge conjugation and a $180^o$ degree rotation is isospin space. The latter is equivalent to the transformation $u \to d$, $d \to -u$. Consider an isospin multiplet built only from $u$ and $d$ quarks. It is possible to show that all states in the multiplet carry the same quantum number $G$ [1]. Consider the $I_z = 0$ member of the multiplet, which also carries the quantum number $C$. By the definition of $G$-parity, $G = (-1)^I C$, where under $u \to d$, $d \to -u$, $\mathcal{F}_{f\bar{f}} \to (-1)^I \mathcal{F}_{f\bar{f}}$.

Baryons can be constructed via three quark creation operators, as a straight-forward variation of the meson case, except that the colour changes from the Kronecker delta $\delta_{c\bar{c}}$ to the (totally antisymmetric) $\epsilon$-tensor $\epsilon_{c_1 c_2 c_3}$. All the quantum numbers remain conserved, except for charge conjugation, which changes a baryon into an antibaryon and hence does not correspond to a quantum number. Because charge conjugation does not correspond to a quantum number, the same is true for the derivative operation $G$-parity. One can show by enumerating all possibilities that all $J^P$ are possible for baryons, so that there are no exotic $J^P$. The Kronecker delta and $\epsilon$-tensor are the only tensors available in the fundamental representation of colour $SU(3)$ in which quarks live in QCD [1]. They are both employed in such a way as to force the meson or baryon to carry no colour labels, *i.e.*, to be white. This requirement arises from the third level of understanding, called *confinement* [2]: Since no free (colour carrying) quarks or gluons have ever been observed, all free particles are taken to be white.

## 2 New white particles

In 1972 Murray Gell-Mann and Harald Fritzsch realized that there is a zoo of new white particles, among them:

**Glueballs**: The colour structure $\delta_{\gamma_1\gamma_2}$, in the adjoint representation of colour $SU(3)$ in which gluons live in QCD, is overall white for *two gluons.* The colour structure given by the invariant $SU(3)$ tensors $f_{\gamma_1\gamma_2\gamma_3}$ and $d_{\gamma_1\gamma_2\gamma_3}$ in the adjoint representation is overall white for *three gluons* [1].

**Hybrid mesons**: The colour structure $\lambda^{\gamma}_{c\bar{c}}$ is overall white for a *quark, antiquark and a gluon*, where $\lambda$ is a Gell-Mann matrix [1].

**Hybrid baryons**: The colour structure $\lambda^{\gamma}_{c_1c'}\ \epsilon_{c'c_2c_3}$ is overall white for *three quarks and a gluon.*

**Four-quark states or "Meson molecules"**: The colour structures $\lambda^{\gamma}_{c_1\bar{c}_1}\lambda^{\gamma}_{c_2\bar{c}_2}$ and $\delta_{c_1\bar{c}_1}\delta_{c_2\bar{c}_2}$ are overall white for *two quarks and two antiquarks.*

These definitions of a glueball, hybrid meson and baryon where we have a specific number of gluons, will be referred to as *gluon counting.*

Glueballs, being composed only out of gluons, cannot carry any flavour, as this is a property of quarks. Particularly, this implies that they have $I = I_z = 0$. Hybrid mesons and baryons are respectively mesons and baryons with an additional gluon, so they have the same flavour structure. Four-quark states have a more complicated flavour structure.

Now we summarize some properties that follow from the first level of understanding. A gluon field has $J = 1$, which means that it is a four-vector with both a time-like and three space-like components. The time-like component has $P = 1$ and the space-like components $P = -1$. However, not all these components are dynamical. This is because QCD is invariant under local $SU(3)$ colour transformations, which transform the gluon field. Because all these transforms of the gluon field are equivalent, one uses only one version, called *gauge fixing.* This restricts the gluon field to have only three dynamical components. These can be thought of as the space-like components with $P = -1$. The photon field, which mediates the electromagnetic interaction, has identical properties. In addition it also has $C = -1$. Accordingly, it has been verified experimentally in electron-positron ($e^-e^+$) annihilation into a photon that the photon field has $J^{PC} = 1^{--}$. The charge conjugation for the gluon field is not so simple. This is because a blue-antired gluon would for example transform to a red-antiblue gluon. We shall loosely say that the gluon has $C = -1$, although there will be exceptions.

In free space a gluon can have a continuous range of momenta. When one puts the gluon inside an enclosure its momenta become discrete. The lowest two momenta are called "magnetic" (also called "transverse electric", TE), and "electric" (also called "transverse magnetic", TM). TE gluons have $J^{PC} = 1^{+-}$ and TM gluons $1^{--}$.

Let's build the $J^{PC}$ of our new white particles.

**Glueballs**: Two gluons together will hence have $J^{PC} = (0,1,2)^{++}$ when they have no orbital angular momentum relative to each other, called *S-wave*. With one unit of angular momentum relative to each other, called *P-wave*, corresponding to higher mass particles, the glueballs will have $J^{PC} = (0,1,2)^{++} \otimes 1^{-} = (0,1,2,3)^{-+}$.

Since the first level of understanding states that gluons are massless before any interactions, and using the *Yang-Landau theorem* that massless $J = 1$ particles do not couple to two identical massless $J = 1$ particles [1], we deduce that $J = 1$ glueballs are not allowed. Because the gluons are not massless after interactions one expects that the $J = 1$ glueballs would have a substantial mass. This is confirmed by lattice QCD [3, 4]. Hence the lightest glueballs are expected to be $0^{++}$ and $2^{++}$, with the next lightest $0^{-+}$, $2^{-+}$ and $3^{-+}$. This mass ordering is confirmed by lattice QCD [3, 4]. Some three-gluon composites have $C = -$ since there are an odd number of gluons. Because gluons do have some mass due to self-energy, these are expected to be heavier than the lowest two-gluon glueballs. This is indeed found in lattice QCD [3, 4].

**Hybrid Mesons**: The $J^{PC}$ can be obtained by adding the $J^{PC}$ of the lowest lying quark-antiquark composites in the quark model, $0^{-+}$ and $1^{--}$, corresponding to $S = 0$ and 1 respectively, to the $J^{PC}$ of the gluon. For TE gluons, this gives $(0^{-+}, 1^{--}) \otimes 1^{+-} = 1^{--}, (0,1,2)^{-+}$. One immediately notes that $1^{--}$, $0^{-+}$ and $2^{-+}$ have the opposite spin assignment $S$ to what they would have if they were mesons. The remaining $S = 1$ state $1^{-+}$ is $J^{PC}$ exotic.

For TM gluons, which are heavier than TE gluons in bag models [1], the low-lying hybrids have $J^{PC} = (0^{-+}, 1^{--}) \otimes 1^{--} = 1^{+-}, (0,1,2)^{++}$. These are identical to the $L = 1$ mesons, with the same spin assignments.

We hence expect the lightest $J^{PC}$ exotic hybrid to be $1^{-+}$, which is confirmed by lattice QCD [5].

**Hybrid Baryons**: One may think that the $J^P$ is found by adding the $J^P$ of the low-lying three-quark composites, $\frac{1}{2}^+$ and $\frac{3}{2}^+$, corresponding to $S = \frac{1}{2}$ and $\frac{3}{2}$ respectively, to the $J^P$ of the gluon. For TE gluons, this gives $\left(N\frac{1}{2}^+, \Delta\frac{3}{2}^+\right) \otimes 1^+ = N\left(\frac{1}{2}, \frac{3}{2}\right)^+$, $\Delta\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right)^+$.

More careful study, including constraints from the Pauli Principle that two fermions (quarks) cannot occupy the same state, implies that the $S = \frac{1}{2}$ hybrid baryons are $N\left(\frac{1}{2}, \frac{3}{2}\right)^+$ and $\Delta\left(\frac{1}{2}, \frac{3}{2}\right)^+$, and the $S = \frac{3}{2}$ hybrid baryons are $N\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right)^+$, so that there are seven low-lying TE hybrid baryons [6].

A TM gluon has the same quantum numbers as a TE one, except for parity. The quantum numbers of the TM hybrid baryons are accordingly identical to the previous paragraph, except that all states have $P = -$.

**Four-quark states or "Meson Molecules"**: By looking at composites of two mesons, with some orbital angular momentum between them, it is easily shown that all $J^{PC}$ are principle allowed.

The main feature of four-quark states is that they can fall apart into two mesons without inhibition by simply arranging their colour structure to that of two mesons. One should hence regard them as being too unstable to be observed in experiment unless specific dynamics dictate otherwise.

The above definitions of glueballs, hybrid mesons and baryons relied on the notion that the gluons can be enumerated. However, this is by no means clear, as non-interacting gluons are massless, which would make stochastic multigluonic configurations just as massive as the cases listed so far. An alternative approach is suggested by fixing the positions of all the quarks and antiquarks and calculating the energy of the system, called *adiabatic potential*, as a function of quark/antiquark positions. Because QCD is a quantum theory, there will not only be a ground state adiabatic potential but also excited adiabatic potentials. Allowing the quarks and antiquarks to be heavy but not fixed may conceivably allow the following *adiabatic approximation*.

First calculate the adiabatic potentials by fixing the quark and antiquark positions. Then allow the heavy quarks and antiquarks to move in the adiabatic potentials just calculated. If the masses thus obtained are identical to masses from first principles, we say that the adiabatic approximation is valid. This is dependent on whether the quarks and antiquarks can be regarded as moving slowly with respect to the gluons.

If the adiabatic approximation is valid, as can be shown for a quark-antiquark [7] or three quarks moving on the ground state adiabatic potential, one can *define* these systems as mesons or baryons respectively. The $J^{PC}$ of the potential is $0^{++}$, as verified by lattice calculations [3]. Such a potential will not change the quantum numbers previously calculated for mesons and baryons. If the adiabatic approximation is valid for the low-lying excited adiabatic potential one can *define* the low-lying hybrid mesons or baryons as a quark/antiquark or respectively, three-quarks, moving in this potential. This is referred to as the *adiabatic* definition.

**Hybrid Mesons:** When one fixes the quark and the antiquark it is clear that the system is invariant under rotations around the line between the quark and the antiquark (see problem 9). If the orbital angular momentum around this line is $\Lambda$, one can form degenerate states $|\Lambda\rangle$ and $|-\Lambda\rangle$. These states are degenerate since the energy cannot depend on whether the system rotates clockwise or anticlockwise. Any linear combination of $|\Lambda\rangle$ and $|-\Lambda\rangle$ has the same energy. The action of parity is to interchange $|\Lambda\rangle$ and $|-\Lambda\rangle$, since it interchanges clockwise and anticlockwise rotations. The same is true for charge conjugation, which interchanges the quark and antiquark, i.e. changes the direction of the rotation axis, and hence makes clockwise rotations anticlockwise, and vice versa. One can now construct the eigenstates of parity and change conjugation $\frac{1}{2}(|\Lambda\rangle \pm |-\Lambda\rangle)$.

Taking from lattice QCD that the potential has $|\Lambda| = 1$ and $C = -P$ [3], and using the eigenstates above, it follows that the $J^{PC}$ of the adiabatic potential is $1^{+-}$ or $1^{-+}$. Technically $J$ is not a quantum number of the adiabatic potential, but only $|\Lambda|$ (see problem 9). We loosely equate $J$ and $|\Lambda|$. The low-lying hybrid mesons are $(0^{-+}, 1^{--}) \otimes (1^{+-}, 1^{-+})$ $= 1^{--}, (0,1,2)^{-+}, 1^{++}, (0,1,2)^{+-}$. There is the same number of states as in the previous definition of a hybrid meson, with six having the same $J^{PC}$. Note that all non-exotic $J^{PC}$ adiabatic hybrids have the opposite spin $S$ than what they would have if they were conventional mesons. The states $1^{-+}, 0^{+-}$ and $2^{+-}$ are $J^{PC}$ exotic. Lattice QCD confirms that these are the three lightest $J^{PC}$ exotic hybrids [5].

Within the adiabatic definition of a hybrid, it is possible to specialize to the case of gluon counting, so that the two definitions does not have to be disjoint. An example is the adiabatic bag model where the hybrid is still defined as a quark-antiquark-gluon composite but studied using the adiabatic approximation. One finds that the TE hybrids have the same quantum numbers as outlined for adiabatic hybrids in the previous paragraph. There are hence eight of them, in contrast to the four TE hybrids originally discussed in the gluon counting definition!

**Hybrid Baryons:** The Isgur-Paton flux-tube model [8, 9] indications are that the low-lying excited adiabatic potential has $J^{PC} = 1^{++}$. This yields five hybrid baryons with $J^P =$ $(N\frac{1}{2}^+, \Delta\frac{3}{2}^+) \otimes 1^+ = N(\frac{1}{2}, \frac{3}{2})^+, \Delta(\frac{1}{2}, \frac{3}{2}, \frac{5}{2})^+$, with the former two states having spin $\frac{1}{2}$, just like the conventional $N$, and the latter three states having spin $\frac{3}{2}$, just like the conventional $\Delta$ [10]. The reason why the Pauli Principle does not change this simple argument is that the quark label exchange properties of the colour structure remain totally antisymmetric for hybrid baryons in the flux-tube model, as it is for the $\epsilon$-tensor of conventional baryons.

Note that four of the five low-lying hybrid baryons agree, as far was their flavour and $J^P$ are concerned, with the seven low-lying TE hybrid baryons according to the former definition. However, when spin $S$ is considered in addition, this is only true for two of the five hybrid baryons.

What about an adiabatic definition of glueballs? Conceptually, this is difficult because there are no heavy quarks that can be treated as moving adiabatically. Hence only hybrid mesons and baryons and four-quark states can possibly be described by the adiabatic definition.

The way glueballs, conventional and hybrid meson and baryons, and four-quark states were described sofar did not allow for the possibility of mixing between different types of states with the same quantum numbers $J^{PC}$ or $J^P$. The unmixed states are referred to as *primitive* (bare), and the mixed states as *physical* (dressed).

# 3 Decays

There is always the possibility that gluons will allow a quark-antiquark pair to be created, called *decay*, coming from the first level of understanding.

If initial state $A$ decays to final states $B$ and $C$, several quantum numbers are conserved. A straightforward example is the electric charge. For total angular momenta, $\mathbf{J}_A = \mathbf{J}_B + \mathbf{J}_C + \tilde{\mathbf{L}}$, where $\tilde{\mathbf{L}}$ is the relative orbital angular momentum between $B$ and $C$. Also, for parity, $P_A = (-1)^{\tilde{L}} P_B P_C$. When all the states have well-defined $C$, charge conjugation conservation gives $C_A = C_B C_C$. For isospin symmetry $\mathbf{I}_A = \mathbf{I}_B + \mathbf{I}_C$. For all states having well-defined G-parity $G$, $G_A = G_B G_C$.

I shall now discuss what is qualitatively known about decays of glueballs and hybrid mesons. Little is known about the decays of hybrid baryons and four-quark states.

**Glueballs:** Glueballs, in the limit where the $u, d$ and strange $s$ flavour quark behave the same, called $SU(3)$ ***flavour*** symmetry, are expected to decay to the $\pi$, $\eta$ and $\eta'$ as follows. We respectively use the $SU(3)$ flavour structures $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$. Then

$$\begin{aligned} \left\langle G|\pi^+\pi^-\right\rangle &= \left\langle 0|u\bar{d}d\bar{u}\right\rangle = 1 = \left\langle G|\pi^0\pi^0\right\rangle \\ \left\langle G|K^+K^-\right\rangle &= \left\langle 0|u\bar{s}s\bar{u}\right\rangle = 1 = \left\langle G|K^0\bar{K}^0\right\rangle \end{aligned}$$

| | Amplitude | Width | Final states |
|---|---|---|---|
| $G \to \pi\pi$ | 1 | 3 | $\pi^+\pi^-, \pi^-\pi^+, \pi^0\pi^0$ |
| $G \to K\bar{K}$ | 1 | 4 | $K^+K^-, K^-K^+, K^0\bar{K}^0, \bar{K}^0K^0$ |
| $G \to \eta\eta$ | 1 | 1 | $\eta\eta$ |
| $G \to \eta'\eta$ | 0 | 0 | $\eta'\eta, \eta\eta'$ |

Table 1: Ratios of intrinsic amplitudes to one final state, and widths to all final states.

$$\begin{aligned} \langle G|\eta\eta\rangle &= \left\langle 0|\frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)\frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)\right\rangle = \frac{1}{6}(1+1+4) = 1 \\ \langle G|\eta'\eta\rangle &= 0 \end{aligned} \quad (2)$$

This decay pattern is indicated in Table 1 and is called *flavour democratic* decay. The decay topology assumed for glueball decay is that of topology 4a in Fig. 1. This is called an *Okubo-Zweig-Iizuka (OZI) forbidden* decay, because the "half-doughnut" final state can be "pulled away" from the initial glueball, i.e. it is possible to cut through the topology without intersecting a quark line. Topology 4b is *double OZI forbidden*, because both of the final "raindrops" can be pulled away separately from the glueball. The (phenomenological) OZI rule states that the size of decay decreases as the number of components in a topology that can be pulled away from each other increases [11].

Flavour democratic decay was not confirmed in lattice QCD in the $SU(3)$ limit [4]. This invalidates the intuitive argument presented above. From a heuristic point of view, glueball decay includes two possibilities: Firstly, the glueball can decay directly to two mesons, in the sense that the two quark-antiquark pairs are created at a similar time. Secondly, the glueball can mix with a meson, and the meson then decays at a later time to two mesons. Here the idea is that the first quark-antiquark pair is created long before the second. The first possibility is called *primitive glueball decay*, while the second is due to *glueball-meson mixing*. Although it is not possible to rigorously separate these two notions, current modelling suggests that glueball-meson mixing can explain the lattice results without the need to invoke primitive glueball decay.

**Hybrids:** Consider topology 1 in Fig. 1. Each of the three participating quark-antiquark pairs are connected to each other, called *connected* decay. None can be pulled away from the other, i.e. the decay is *OZI allowed* and hence expected to be dominant. Note that the quark in the initial state ends up in the one final meson, and the antiquark in the other meson. Topologies 2 and 3b are single OZI forbidden, and topology 3a double

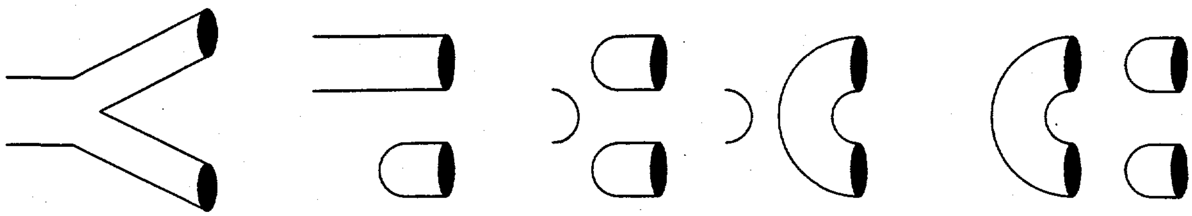



Figure 1: Decay topologies.

OZI forbidden.

Let us explore connected decay within the adiabatic definition of conventional and hybrid mesons. Under the adiabatic approximation one can fix the positions of all quarks and antiquarks participating in the decay. This means that there must be an amplitude for the gluons of the initial state to fold into the gluons of the two final states. This *flux-tube overlap* depends on the variables that specify the configuration: the distance between the initial quark and antiquark, and the vector from the midpoint of the initial quark-antiquark line to the pair creation position (see problem 6). The spacial orientation of the initial quark-antiquark line is irrelevant by rotational invariance. A flux-tube overlap will also exist for the decay of conventional or hybrid baryons.

Consider connected decay. Assume that the quark-antiquark pair creation is with spin $\tilde{S} = 1$. Then we deduce the following *spin selection rule*: Spin $S_A = 0$ mesons do not decay into two spin $S_B = S_C = 0$ mesons. This follows simply because the total spin in the initial state is 0, while the total spin in the final state is 1, because $\tilde{S} = 1$, so that spin is not conserved in the decay. This selection rule holds spectacularly better for conventional meson decay than one may expect. As recently measured by VES, the decay $\pi_2(1670) \rightarrow b_1\pi$, where each participating meson is spin 0, has a minute branching ratio of less than 0.2% at the $2\sigma$ confidence level.

Assuming the spin selection rule to also be valid for hybrid meson decay, one obtains important experimental implications. It has already been pointed out before that the low-lying non-exotic TE hybrid in the gluon counting definition, and all the low-lying non-exotic hybrids in the adiabatic definition, have the opposite spin assignment than their conventional meson partners with the same $J^{PC}$. Restrict the discussion of hybrid mesons in this paragraph to these hybrids. Consider a decay of an initial state to two final states

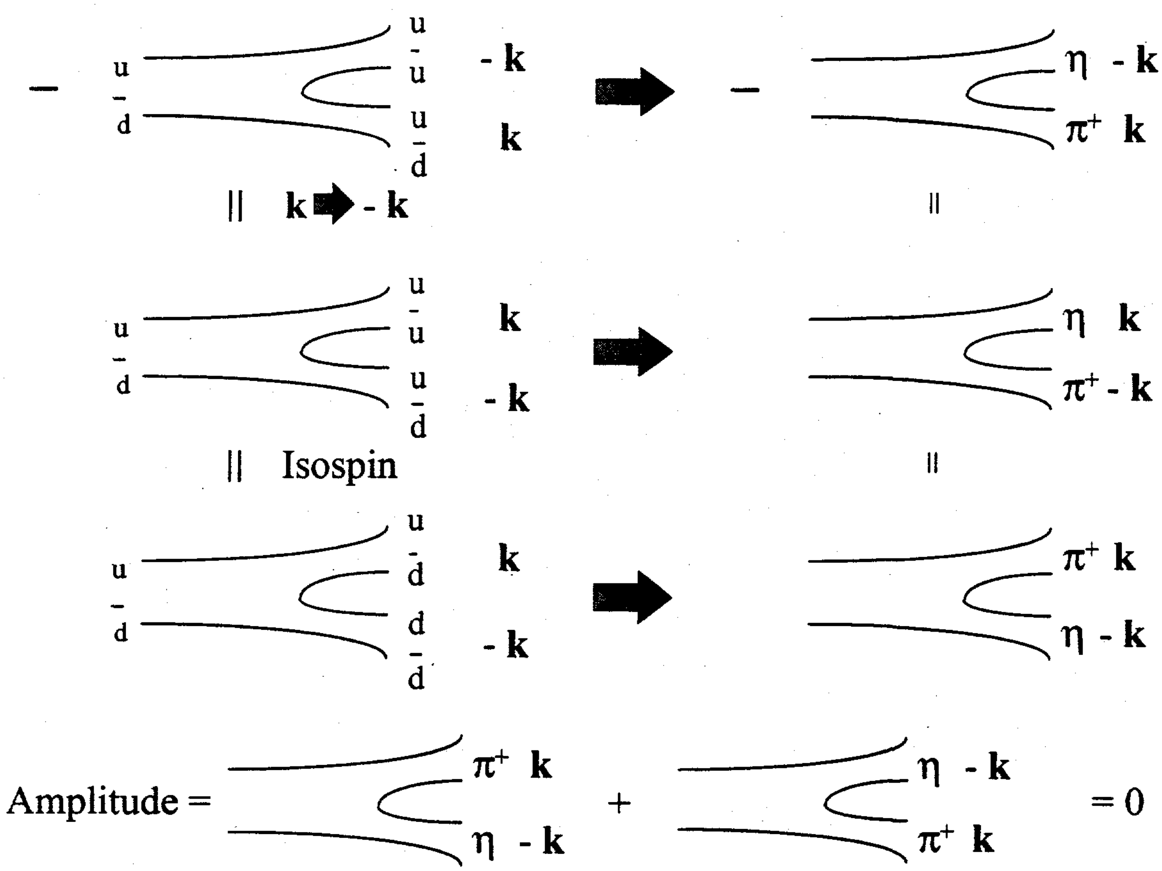



Figure 2: Selection Rule I.

where the spin selection rule is operative. Then it follows that if the nature of the initial state is interchanged between conventional and hybrid mesons, the spin selection rule will no longer be valid. For example, if $\pi_2(1670)$ was a hybrid meson, its decay to $b_1\pi$ would be uninhibited. This means that the conventional or hybrid meson nature of the initial state can be distinguished based on whether the width is suppressed or not.

There are two further selection rules which are more general than specific models:

I. $J^{PC} = 1^{-+},\ 3^{-+},\ \ldots$ *flavour structure $q\bar{q}$ hybrid mesons does not proceed via connected decay to $\eta\pi$.* Here $q\bar{q}$ refers to the initial state having the same flavour quarks and antiquarks. If isospin symmetry is assumed for a decay involving only $u, d$ quarks, the result can be extended to all members of the isomultiplet. This rule, originally noticed by Lipkin, LeYaouanc, Oliver, Pène and Raynal in 1988-89, does not follow from any standard conservation principle, and is specific to the connected topology, in the sense that it is known not to be valid for topology 2. The derivation does not depend on assuming non-relativistic behaviour, and can in fact be derived from the first level of understanding.

We outline an intuitive derivation for the decay of a positively charged $J^P = 1^-,\ 3^-,\ \ldots$ meson to $\eta\pi^+$ when isospin symmetry is assumed. This decay is allowed by all the conservation principles listed in the beginning of this section. Because G-parity conservation implies that the neutral isospin partner of the initial state is $J^{PC}$ exotic, the initial state must be a hybrid meson. The gluons in the connected decay (topology 1 in Fig. 1) are not indicated. The argument is depicted in Fig. 2. Taking the initial hybrid at rest, the $\eta$ and $\pi^+$ emerge with momenta $-\mathbf{k}$ and $\mathbf{k}$ respectively. First consider the three top left diagrams. The top diagram has a negative sign in front by convention. When the transformation $\mathbf{k} \leftrightarrow -\mathbf{k}$ is applied, the middle diagram is obtained, noting that an odd $\tilde{L}$ decay acquires an extra minus sign. This is a general property of odd $\tilde{L}$ decays. The bottom diagram is obtained by noting that the amplitude to create a $u\bar{u}$ pair is the same as for a $d\bar{d}$ pair within isospin symmetry. The three top right diagrams are now obtained from the three top left (quark) diagrams by attaching the initial hybrid to the initial $u\bar{d}$ quarks, and the final $\pi^+$ to the final $u\bar{d}$ quarks. Since the flavour wave function of the $\eta$ is proportional to $u\bar{u} + d\bar{d}$, it is attached to either $u\bar{u}$ or $d\bar{d}$, with a positive relative sign. Because each of the three top left diagrams are equal, it follows that each of the three top right diagrams are equal. The bottom diagrams depict the decay amplitude, taking into account that there are two possible ways for the final $\eta$ and $\pi^+$ to couple. Looking back at the top right diagrams one immediately notices that the decay amplitude vanishes. This is the selection rule.

**II.** *Flavour structure $q\bar{q}$ hybrid mesons does not proceed via connected decay to two $L_B = L_C = 0$ conventional mesons which are identical, except possibly for their flavour and spin, under $\tilde{S} = 1$ quark-antiquark pair creation* [5, 12, 13]. Here restrict the hybrid mesons to the four low-lying TE hybrids in the gluon counting definition, and all the eight low-lying hybrids in the adiabatic definition. Evidently, non-relativistic behaviour is assumed. The same comments about isospin symmetry made for the first rule applies here.

The general derivation of this rule is somewhat complicated, but a simple derivation obtains for hybrids in the adiabatic definition *if* the following ansatz is made: the $CP$ of the participating adiabatic potentials and the $CP$ of the created pair are conserved[1]. The ansatz means that $-1 = 1 \times 1 \times (-1)^{\tilde{S}+1}$. We used that the hybrid and conventional

[1]The general derivation is in P.R. Page, *Phys. Lett.* **B402** (1997) 183, and the ansatz in C. Michael, *8th Int. Symp. on Heavy Flavor Physics (Heavy Flavors 8)*, Southampton, UK, 25-29 July 1999; hep-ph/9911219.

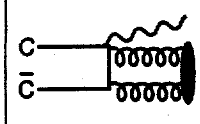
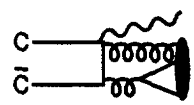
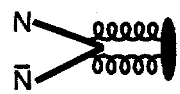
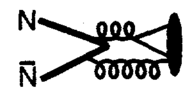
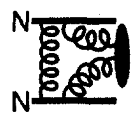
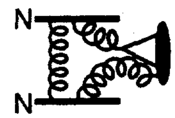
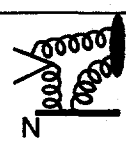
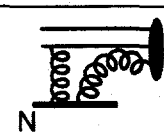
meson potentials have respectively negative and positive $CP$, and the created pair has $CP = (-1)^{\tilde{S}+1}$. When $\tilde{S} = 1$, the ansatz is not satisfied and the decay vanishes: thus the selection rule.

# 4 Production

In Fig. 3 we indicate the main production mechanisms relevant to spectroscopy. These are $\psi$ (charm-anticharm, $c\bar{c}$) radiative decay, proton-antiproton ($p\bar{p}$) annihilation, central and diffractive production, pion ($\pi$) and photon ($\gamma$) beams, two-photon production and $e^-e^+$ annihilation. Processes not listed that have yielded spectroscopical information include Primakoff production (an incoming particle in an electromagnetic field), jets, $\tau$, $D$, $D_s$, $B$ and $\psi$ hadronic decay, and $K$ beams. Current experiments are also listed: BES at the Beijing Electron-Positron Collider, CBAR (Crystal Barrel) and Obelix at the Low Energy Antiproton Ring at CERN, WA102 and LEP2 at CERN, VES at Serpukhov, E852 at the Alternating Gradient Synchrotron at Brookhaven, Hall B at Jefferson Lab, CLEO at the Cornell Electron Storage Ring and ARGUS at DORIS II at the Deutsche Electronen Synchrotron. Only the production of glueballs and hybrids are indicated in Fig. 3. The $f_0(1500)$, $f_J(1710)$ and $f_J(2220)$ are glueball candidates and the remainder of the states listed are hybrid meson candidates.

For the cross-sections of various production mechanisms, we perform a naïve counting in relative powers of the strong coupling constant $\alpha_S$ for light quarks. The first three processes in Fig. 3 are glue-rich: they prefer to produce glueballs, with hybrids suppressed at order $\alpha_S$. For these processes conventional mesons and four-quark states are only produced at order $\alpha_S^2$. Diffractive production prefers hybrid mesons, with glueballs, conventional mesons and four-quark states suppressed at order $\alpha_S$. The last four processes are glue-averse (glueballs at order $\alpha_S^2$, and hybrids at order $\alpha_S$), and prefer conventional meson production at order 1. Four-quark state production is order $\alpha_S^2$, except for two-photon production at order 1.

The naïve power counting in Fig. 3 corresponds narrowly to whether glueballs and hybrids are actually observed experimentally.

| Production | Experiment | Glueball Process | Glueball Order | Glueball Observed X | Hybrid Process | Hybrid Order | Hybrid Observed X |
|---|---|---|---|---|---|---|---|
| $\psi \to \gamma X$ | BES |  | 1 | $f_0(1500)$ $f_J(1710)$ $f_J(2220)$ |  | $\alpha_s$ | |
| $p\bar{p}$ bomb $\to X$ | CBAR OBELIX |  | 1 | $f_0(1500)$ $f_J(2220)$ |  | $\alpha_s$ | $\hat{\rho}(1405)$ $\rho(1450)$ |
| Central $pp \to ppX$ | WA102 |  | 1 | $f_0(1500)$ $f_J(1710)$ $f_J(2220)$ |  | $\alpha_s$ | |
| Diffractive (glueball exchange) | VES E852 Hall B |  | $\alpha_s$ | |  | 1 | $\pi(1800)$ |

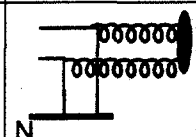
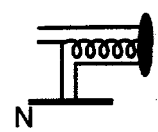
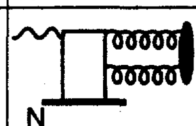
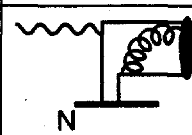
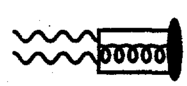
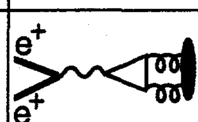
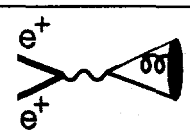
| Production | Experiment | Glueball Process | Glueball Order | Glueball Observed X | Hybrid Process | Hybrid Order | Hybrid Observed X |
|---|---|---|---|---|---|---|---|
| $\pi N \to XN$ (meson exchange) | VES E852 |  | $\alpha_s^2$ | $f_0(1500)$ $f_J(1710)$ $f_J(2220)$ |  | $\alpha_s$ | $\hat{\rho}(1405)$ $\hat{\rho}(1600)$ $\rho(1450)$ $\omega(1600)$ |
| $\gamma N \to XN$ (meson exchange) | Hall B |  | $\alpha_s^2$ | |  | $\alpha_s$ | $\rho(1450)$ |
| $\gamma\gamma \to X$ | CLEO LEP2 ARGUS |  | $\alpha_s^2$ | Not $f_0(1500)$ Not $f_J(2220)$ |  | $\alpha_s$ | |
| $e^+e^- \to X$ | BES |  | $\alpha_s^2$ | |  | $\alpha_s$ | $\rho(1450)$ $\omega(1420)$ $\omega(1600)$ $\psi(4040)$ $\psi(4160)$ |

Figure 3: Production processes. The nucleon $N$ is a proton or neutron.

# 5 Pivotal Experimental Results

The last decade marked the discovery of gluonic excitations, overturning the traditional taxonomy of all known hadrons as being either conventional mesons or baryons. The $J^{PC} = 0^{++}$ (scalar) glueball has been discovered, although its exact location in the spectrum has not yet been pinned down. This can be regarded as the only robust experimental result on gluonic excitations. Closely following is strong evidence for the existence of two $1^{-+}$ exotic $I = 1$ (isovector) states, something that could not be said a decade earlier. Three issues of significant current interest will not be covered: the ephemeral $2^{++}$ (tensor) cousin of the scalar glueball, the possible four-quark nature of the $f_0(980)$ and $a_0(980)$ and the existence or non-existence of a broad $\sigma$ resonance. Further information on these subjects, and other outstanding puzzles of hadron spectroscopy, can be found in detailed reviews [12, 14] . The search for hybrid baryons and four-quark states is still a nascent field, reviewed in refs. [6, 10, 12].

## 5.1 Scalar glueball

Significant advances have been made in clarifying the spectrum of $J^{PC} = 0^{++}$ $I = 0$ (isoscalar) states. The $f_0(980)$ and $f_0(1500)$ are today the best established scalar isoscalar states. The subscript is the total angular momentum $J$, and the argument the mass in MeV. Recently clear evidence for $f_0(1370)$ has emerged and a number of analyses are converging on the $J = 0$ assignment for $f_J(1710)$. There is a possible higher mass resonance, or resonances, $f_0(2000-2100)$. Debate is still raging about whether the low mass $\sigma$ phenomenon is resonant or not. Details can be found in refs. [14, 15, 16].

That the scalar resonances are too fecund is illustrated by the fact that the Isgur-Godfrey relativized quark model expects only two scalar resonances below 1.7 GeV, while probably more than two states are below this mass (Figs. 16 and 22 of ref. [12]). This suggests the possibility of additional four-quark or glueball states. A small subset of models does allow hybrid mesons in the correct mass range, but we shall exclude this possibility.

The argument for the presence of a glueball amongst the scalar states is firstly based on the convergence of lattice calculations on a primitive glueball mass of around $1.6 \pm 0.1$ GeV [12]. Note the proximity to $f_0(1500)$ and $f_J(1710)$. Secondly, glueball character is indicated by production in glue-rich processes and non-production in glue-averse processes, as well as the so-called Close-Kirk filter, as we shall now elaborate.

The $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are strongly produced in central production, where there are techniques to accertain that they are produced mostly via the glue-rich collision indicated in Fig. 3. The two gluons connecting to the proton are called the "pomeron", so that the process can be thought of as a double pomeron collision. Also, $f_0(980), f_0(1370)$ and $f_0(1500)$ are strongly produced in glue-rich $p\bar{p}$ annihilation. The $f_J(1710)$ is often at the edge of phase space in $p\bar{p}$ annihilation, so that its non-observation need not be significant. Glue-rich $\psi$ radiative decay also significantly produces $f_0(1500)$, $f_J(1710)$ and $f_0(2000-2100)$. Close, Farrar and Li have developed quantitive techniques to extract the gluon affinity for a state from $\psi$ radiative decay data. These techniques indicate that $f_0(1500)$ and $f_0(1710)$ have substantial glueball components.

Detailed analyses of the (mostly) double pomeron exchange process have been performed. Consider the two-dimensional momentum vectors $\mathbf{p}_T^1$ and $\mathbf{p}_T^2$ for the two pomerons, where "T" indicates that the vectors are the transverse components to beam pipe. Define the magnitude $dp_T \equiv |\mathbf{p}_T^1 - \mathbf{p}_T^2|$. Grouping together resonances according to their $dp_T$ behaviour yields that the $f_0(980)$, $f_0(1500)$ and $f_0(1710)$ behaves in the opposite way to all well-established conventional mesons. The observation that all conventional mesons behave in the same way is called the *Close-Kirk filter.* Also, $f_0(1370)$ has a behaviour somewhere between conventional mesons and $f_0(980)$, $f_0(1500)$ and $f_0(1710)$. The errant behaviour of the isoscalar scalar states is taken to mean that they contain something beyond conventional mesons. The higher mass $f_0(2000-2100)$ behaves like a conventional meson.

It is also instructive to look at the non-appearance of states in glue-averse two photon production. The ALEPH collaboration at LEP2 provided a restrictive bound on the two-photon width of the $f_0(1500)$. The $f_0(980)$ also has a small two-photon width [17]. On the other hand, $f_0(1370)$ has a two-photon width $5.4 \pm 2.3$ keV [17] which is perfectly consistent with expectations for a conventional $n\bar{n} \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ meson. There is currently no definitive measurement for $f_J(1710)$ where $J$ has been determined. As we shall see below, it is possible to explain the small two-photon width of $f_0(1500)$ without invoking a glueball.

It is clear that production processes indicate that the primitive glueball might be distributed over more than one physical state: notibly $f_0(1500)$ and $f_J(1710)$. This implies that there is significant mixing between primitive glueballs and mesons.

We now analyse the mixing mathematically. Assume that a glueball couples to a pair of primitive mesons, one with flavour $n\bar{n}$ and the other with flavour $s\bar{s}$. The coupling to intermediate decay channels is neglected for the purposes of this introductory orientation.

Then, we have the following $3 \times 3$ hermitian mass matrix, where, in addition to the meson mixing amplitude $A$, we have the amplitude for glueball-meson mixing which we denote by $g$:

$$\mathcal{M} = \begin{pmatrix} G & g & gr \\ g^* & S+A & Ar \\ g^* r^* & Ar^* & N+A|r|^2 \end{pmatrix}, \tag{3}$$

where $G, S, N$ indicates the (real) primitive masses. Here $\langle G|n\bar{n}\rangle = r\ \langle G|s\bar{s}\rangle$ and $\langle n\bar{n}|n\bar{n}\rangle = r\ \langle n\bar{n}|s\bar{s}\rangle = |r|^2\ \langle s\bar{s}|s\bar{s}\rangle$. In the $SU(3)$ limit one can use the methods of Eq. 2 to show that $r = \sqrt{2}$.

Note that $A$ must be real for the matrix to be hermitean. With $g$ and $r$ both real the matrix is the most general parametrization of $3 \times 3$ (real) symmetric matrix, since it contains six independent parameters.

The matrix is diagonalized $\mathcal{M} \Longrightarrow \text{diag}\left(\tilde{G}, \tilde{S}, \tilde{N}\right)$ by the masses of the three physical states, which are determined from the three eigenvalue ($\lambda$) equations (which follow from $\text{Det}\,(\mathcal{M} - \lambda I) = 0$). Eliminating $A$ and $g$ from the eigenvalue equations leads, upon some algebra, to the formula

$$\begin{aligned} &\left[(1+|r|^2)\tilde{G} - |r|^2 S - N\right]\left[(1+|r|^2)\tilde{S} - |r|^2 S - N\right]\left[(1+|r|^2)\tilde{N} - |r|^2 S - N\right] \\ &\qquad + \left[(1+|r|^2)G - |r|^2 S - N\right]\ |r|^2\left[S-N\right]^2 = 0 \end{aligned} \tag{4}$$

which is called the *generalized Schwinger mass formula.* In 1964 Julian Schwinger derived a simpler, phenomenologically successful, formula for the case where there is no glueball. His matrix is just the right-bottom $2 \times 2$ sub-matrix of the $3 \times 3$ matrix (Eq. 3), restricted to be real with $r = \sqrt{2}$. We note that the generalized Schwinger mass formula does not depend on the couplings $A$ or $g$. This is very useful, as they are difficult to extract from experiment.

Assume that there is no direct coupling between mesons, i.e. that $A = 0$. The coupling between mesons $A$ can be shown to be suppressed as $\frac{1}{\sqrt{N_c}}$ relative to the glueball-meson coupling $g$, where $N_c$ is the number of colours in QCD [18]. This result comes from the first level of understanding. When $A = 0$ one can combine Eq. 4 with the trace condition for the matrix (Eq. 3),

$$\tilde{G} + \tilde{S} + \tilde{N} = G + S + N \tag{5}$$

in order to determine *two* unknown masses. The strategy is to assume a value for $r$, and use four input masses to predict the remaining two masses.

Now specialize to real and positive $g$. Once all primitive and physical masses are known there are formulae that enable calculation of the coupling constant $g$, as well as the matrix that diagonalizes $\mathcal{M}$, called the *valence content* matrix. This matrix contains the eigenvectors of $\mathcal{M}$ in its columns. The formulae are now exhibited without proof.

The coupling can be calculated from the masses according to

$$\sqrt{-\frac{(S-\tilde{G})(S-\tilde{S})(S-\tilde{N})}{S-N}} = g = \sqrt{-\frac{(N-\tilde{G})(N-\tilde{S})(N-\tilde{N})}{r^2(N-S)}} \tag{6}$$

If we write the valence content of the physical state $X$, either the physical glueball, $s\bar{s}$ or $n\bar{n}$, as $|X\rangle = X_G|G\rangle + X_S|S\rangle + X_N|N\rangle$, one requires that $|X\rangle$ be normalized, i.e. that $X_G^2 + X_S^2 + X_N^2 = 1$. It is possible to show that the valence content can be explicitly calculated as:

$$X_G = \mathcal{N}_X \qquad X_S = \mathcal{N}_X \frac{g}{\tilde{X}-S} \qquad X_N = \mathcal{N}_X \frac{rg}{\tilde{X}-N} \tag{7}$$

where $\tilde{X}$ is the physical mass of state $X$ and

$$\frac{1}{\mathcal{N}_X} = \sqrt{1 + (\frac{g}{\tilde{X}-S})^2 + (\frac{rg}{\tilde{X}-N})^2} \tag{8}$$

Note that the valence contents are only specified up to an overall sign, i.e. one cannot distinguish between $X_G, X_S, X_N$ and $-X_G, -X_S, -X_N$. Eqs. 6 - 8 have been checked numerically.

We shall now consider four limiting scenarios, and study the valence content of the physical glueball in each case, taking $r = \sqrt{2}$ for simplicity.

$SU(3)$ **symmetry**: This arises in two cases.

First take the $SU(3)$ limit $S = N$ and $r = \sqrt{2}$. From Eq. 7 this implies that $X_S : X_N = 1 : \sqrt{2}$, i.e. that the physical glueball has flavour content proportional to $u\bar{u} + d\bar{d} + s\bar{s}$. This is an $SU(3)$ *singlet*: Since the primitive glueball carries no flavour, i.e. is an $SU(3)$ singlet, we expect that it should only mix with the $SU(3)$ singlet quark flavour combination.

Secondly consider a physical glueball much higher in mass than the primitive $s\bar{s}$ and $n\bar{n}$. Again $X_S : X_N = 1 : \sqrt{2}$, i.e. the physical glueball has the same flavour content as before.

**Midway**: Consider a physical glueball halfway between the primitive $s\bar{s}$ and $u\bar{u}$ states. Then $X_S : X_N = 1 : -\sqrt{2}$, i.e. the physical glueball has flavour content proportional to $u\bar{u} + d\bar{d} - s\bar{s}$. This is somewhere between the *ideal* mixing assignment $u\bar{u} + d\bar{d}$ and the

| | $SU(3)$ singlet | | Midway | | $SU(3)$ Octet | | $n\bar{n}$ | | $s\bar{s}$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathcal{A}$ | $\Gamma$ | $\mathcal{A}$ | $\Gamma$ | $\mathcal{A}$ | $\Gamma$ | $\mathcal{A}$ | $\Gamma$ | $\mathcal{A}$ | $\Gamma$ |
| $\pi\pi$ | 1 | 3 | 3 | 27 | 2 | 3 | 6 | 27 | 0 | 0 |
| $K\bar{K}$ | 1 | 4 | 0 | 0 | $-1$ | 1 | 3 | 9 | 3 | 9 |
| $\eta\eta$ | 1 | 1 | $-1$ | 1 | $-2$ | 1 | 2 | 1 | 4 | 4 |
| $\eta'\eta$ | 0 | 0 | $2\sqrt{2}$ | 16 | $2\sqrt{2}$ | 4 | $2\sqrt{2}$ | 4 | $-2\sqrt{2}$ | 4 |

Table 2: Amplitude $\mathcal{A}$ and width $\Gamma$ ratios of a physical glueball decaying to pseudoscalar final states.

$SU(3)$ *octet* $u\bar{u} + d\bar{d} - 2s\bar{s}$, and is, perfunctorily, a popular choice for the flavour content of the $\eta$.

$SU(3)$ **Octet**: With the physical glueball between the primitive $s\bar{s}$ and $n\bar{n}$ states, but two times further from the $n\bar{n}$ than from the $s\bar{s}$, one obtains flavour structure of the physical glueball proportional to $u\bar{u} + d\bar{d} - 2s\bar{s}$. This is an $SU(3)$ octet, indicating that $SU(3)$ symmetry is maximally violated.

$n\bar{n}$: When the physical glueball mass is near the primitive $n\bar{n}$ mass, $X_G : X_S : X_N = 0 : 0 : 1$. This means that the physical glueball undergoes very strong mixing and becomes the primitive $n\bar{n}$!

$s\bar{s}$: Similarly, for a physical glueball near the primitive $s\bar{s}$ mass, $X_G : X_S : X_N = 0 : 1 : 0$, so that the physical glueball is just the primitive $s\bar{s}$.

It is clear that one can consider the physical glueball at various places between the primitive $n\bar{n}$ and $s\bar{s}$, and obtain any desired ratio $X_S : X_N$ with the restriction that the sign of $X_S$ and $X_N$ is different.

On the other hand, if the physical glueball is either above or below *both* the $n\bar{n}$ and $s\bar{s}$ states, the sign of $X_S$ and $X_N$ will be the same.

Now consider decays. Assume that the primitive $n\bar{n}$ and $s\bar{s}$ decay to $\pi\pi$, $K\bar{K}$, $\eta\eta$ and $\eta'\eta$ via connected decay. Also assume that the primitive glueball does not decay to these final states, i.e. its decays are subdominant as expected from the OZI rule. If the total decay width is below expectations for conventional mesons, as is the case for the $f_0(1500)$ and $f_J(1710)$, that may indicate a substantial glueball valence content. However, the total decay widths are not small, as expected for an unmixed glueball.

The decay of the physical glueball can be calculated by considering the decay of its primitive $n\bar{n}$ and $s\bar{s}$ valence content (see problem 10). The amplitudes are obtained in the same manner as Eq. 2, yielding

| | $\sigma\sigma$ | $\rho\rho$ | $\pi(1300)\pi$ | $a_1(1260)\pi$ | $\text{Sum}_1$ |
|---|---|---|---|---|---|
| $f_0(1370)$ | $105.2 \pm 32.0$ | $76.8 \pm 37.0$ | $6.6 \pm 4.2$ | $37.2.0 \pm 16.3$ | 226 |
| $f_0(1500)$ | $15.6 \pm 9.2$ | $6.5 \pm 5.9$ | $9.8 \pm 7.7$ | $7.9 \pm 5.5$ | 40 |
| | $\pi\pi$ | $\eta\eta$ | $\eta\eta'$ | $K\bar{K}$ | $\text{Sum}_2$ |
| $f_0(1370)$ | $19.2 \pm 7.2$ | $0.4 \pm 0.2$ | | $7.0 \pm 1.6$<br>$18.8 \pm 4.0$ | 32.5 |
| $f_0(1500)$ | $24.6 \pm 2.7$ | $1.91 \pm 0.24$ | $1.61 \pm 0.06$ | $4.52 \pm 0.36$ | 32.6 |

Table 3: Crystal Barrel widths in MeV, ca. 2000.

$$
\begin{aligned}
\left\langle n\bar{n}|\pi^+\pi^-\right\rangle = \left\langle n\bar{n}|\pi^0\pi^0\right\rangle &= \sqrt{2} & \left\langle s\bar{s}|\pi^+\pi^-\right\rangle = \left\langle s\bar{s}|\pi^0\pi^0\right\rangle &= 0 \\
\left\langle n\bar{n}|K^+K^-\right\rangle = \left\langle n\bar{n}|K^0\bar{K}^0\right\rangle &= \frac{1}{\sqrt{2}} & \left\langle s\bar{s}|K^+K^-\right\rangle = \left\langle s\bar{s}|K^0\bar{K}^0\right\rangle &= 1 \\
\langle n\bar{n}|\eta\eta\rangle &= \frac{\sqrt{2}}{3} & \langle s\bar{s}|\eta\eta\rangle &= \frac{4}{3} \\
\langle n\bar{n}|\eta'\eta\rangle &= \frac{2}{3} & \langle s\bar{s}|\eta'\eta\rangle &= -\frac{2\sqrt{2}}{3}
\end{aligned} \tag{9}
$$

The amplitudes and widths are displayed in Table 2, up to an arbitrary normalization. It is evident that predictions for the widths vary widely, indicating the sensitivity of experimental widths to the valence content of the state. The reader is invited to determine which pattern of widths best correspond to the most recent experimental data from Crystal Barrel in Table 3. It is clear that the data for $f_0(1500)$ are not consistent with the $SU(3)$ singlet / unmixed glueball (flavour democratic) or the $s\bar{s}$ interpretation. The small two-photon width of $f_0(1500)$ also excludes the $n\bar{n}$ interpretation. However, there exists a valence content for the physical glueball that gives zero two-photon width (see problem 4). The above argues that mixing is needed to explain the decay pattern of $f_0(1500)$.

An ingredient which is currently missing in the description of the scalar isoscalar states is the consideration of, amongst others, radially excited quark model states. This would lead to at least $5 \times 5$ matrices. These, and higher dimensional analogues are known to obey generalized Schwinger formulae and a trace condition similar to the ones derived in this section.

Does the substantial scalar glueball-meson mixing imply that the same is true for glueballs with other $J^{PC}$? I.e. are other glueballs also not narrow and hence difficult to detect experimentally? It is clear that the higher the primitive glueball mass, the more conventional and hybrid mesons will have similar masses, since there is both a tower of radially and orbitally excited states, and a tower of different types of hybrid mesons. Can glueball-meson mixing be suppressed? There is currently no theoretical consensus on this issue.

## 5.2 Isovector $J^{PC} = 1^{-+}$ exotics

Evidence for an embarrassment of riches of isovector $J^{PC} = 1^{-+}$ exotic enhancements $\hat{\rho}(1405)$ at mass $1392^{+25}_{-22}$ MeV, width $333 \pm 50$ MeV [17] and $\hat{\rho}(1600)$ at mass $1593 \pm 8$ MeV, width $168 \pm 20$ MeV has recently emerged [12]. The former enhancement is observed by both E852 and Crystal Barrel in very different production processes decaying the $\eta\pi$.. The enhancement $\hat{\rho}(1600)$ was observed by E852 decaying to $\rho\pi$. There is also some weaker evidence from E852 and VES that it decays to $\eta'\pi$ and $b_1\pi$, but *not* to $\eta\pi$ and $f_2\pi$. Evidence for higher mass states is more tentative.

In experimental analyses the observed enhancements are described by complex amplitudes, with both a magnitude and a phase. The change of the phase as one moves from low to high four-momentum squared $(p_B + p_C)^2$ of the final decay channel, e.g. $\eta\pi$, is called *phase motion*. Here $p_B$ and $p_C$ denote the four-momenta of the final states. The phase motion is expected to go through $180^o$ for a simple resonance. This enables one to determine whether the observed enhancements are resonant or not. Let's take the $\hat{\rho}(1405)$ as an example. Crystal Barrel recently claimed that the phase motion in the $\eta\pi$ P-wave goes through $213^o \pm 5^o$, consistent with expectations for a resonance. At E852 there is a well-known resonance $a_2$ decaying to $\eta\pi$ which dominates the $\hat{\rho}(1405)$. This raises the prospect that experimental misidentification might lead to the $a_2$ appearing in the $J^{PC} = 1^{-+}$ amplitude. It is frequently argued that this circumstance would lead to a fake $J^{PC} = 1^{-+}$ amplitude having the same phase motion as the $a_2$ amplitude. This is based on the idea that experimental misidentification *cannot* by itself lead to phase motion. If one studies the relative phase motion between the $J^{PC} = 1^{-+}$ and $a_2$ amplitudes, and finds this to be constant, one should therefore conclude that the $J^{PC} = 1^{-+}$ amplitude is due to experimental misidentification. E852 did not observe this constancy, and hence concluded that the $\hat{\rho}(1405)$ was resonant. This interpretation might be overly simplistic in view of the fact that there is still the possibility of non-resonant $\eta\pi$ production, which can interfere with a resonant $\hat{\rho}(1600)$, and appear as an apparent resonance at the mass of the $\hat{\rho}(1405)$. This mechanism can in fact account for the E852 data [12]. However, from Occam's razor and the independent Crystal Barrel observation, I shall be predisposed towards the simpler

E852 interpretation for the remainder of this lecture.

Phase motion of the $\hat{\rho}(1600)$ against $\pi(1300)/\pi(1800)$, $a_1$, $a_2$ and $\pi_2(1670)$ has also been observed by E852, and was interpreted as evidence for the resonant nature of the enhancement.

Manifestly exotic $J^{PC}$ isovector quantum numbers immediately translate into either a hybrid meson or four-quark interpretation for the resonances.

Since $\hat{\rho}(1405)$ has only been observed in $\eta\pi$, it natural to assume that the decay has a substantial branching ratio. If this is the case, the observed decay is in contravention with selection rule I of section 3. This means that either the OZI rule is violated or that $\hat{\rho}(1405)$ is not dominantly a hybrid meson. This in itself would be an important result, signalling the observation of a four-quark state. The only other decay channels with substantial phase space are $\rho\pi$ and $\eta'\pi$.

The $\hat{\rho}(1600)$ has enough phase space to decay to $K^*K$, $b_1\pi$, $f_1\pi$, $f_2\pi$ and $\eta(1295)\pi$ in addition. Selection rule II of section 3 appears to say that decay should only be to non - $L_B = L_C = 0$ mesons, i.e. to $b_1\pi$, $f_1\pi$, $f_2\pi$ and $\eta(1295)\pi$. This observation has important experimental consequences, as $b_1$, $f_1$, $f_2$ and $\eta(1295)$ decay on a strong interaction time scale to other particles, so that the final state is complicated. This stands in marked contrast the final states $\eta\pi$, $\eta'\pi$ and $K^*K$, where $\eta$, $\eta'$ and $K^*$ are almost stable on the strong interaction time scale.

Selection rule II only holds if the final states can be regarded as the same, except for their flavour and spin. For example, in the decay to $\rho\pi$, the $\rho$ and $\pi$ clearly have different flavours and spins. This does not break the selection rule. However, $\rho$ and $\pi$ have different sizes, which does break the selection rule. Hence the selection rule is not exact. VES quotes the width ratios $\rho\pi : \eta'\pi : b_1\pi = 1.6 \pm 0.4 : 1.0 \pm 0.3 : 1$ for $\hat{\rho}(1600)$. E852 sees $\hat{\rho}(1600)$ in $\rho\pi$ and $\eta'\pi$, but not in $f_2\pi$. This appears to challenge the validity of the selection rule and hence current models which imply it [19].

# 6 No Conclusions

The ideas presented here constitute some of the phenomenologist's language to describe experiment, incorporating ideas about gluon excitations from QCD. It is possible that this whole beautiful structure will be swept away by a thunderbolt from lattice QCD or experiment.

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# 7 Problems

**Epicurean:**

**1** The hybrid meson candidate $\pi(1800)$ is strongly produced in diffractive $\pi N$ collisions. Based on this, which production process discussed is expected to copiously produce the $N\frac{1}{2}^+$ hybrid baryon?

**2** Assume that the $e^-e^+$ widths of $\psi(4040)$ and $\psi(4160)$ are approximately the same, and that hybrid mesons have negligible $e^+e^-$ widths. Explain how the two physical states can be constituted from a primitive conventional and hybrid meson. Why should the decay pattern to other final states of $\psi(4040)$ and $\psi(4160)$ be closely related?

**3** Can only glueballs decay flavour democratically? (Refer to Tables 1 and 2).

**4** Take into account that the $u, d, s$ quarks respectively have electric charges $\frac{2}{3}, -\frac{1}{3}$ and $-\frac{1}{3}$, and assume lowest order electromagnetic coupling of quarks and vanishing electromagnetic coupling of gluons. Show that for $X_S : X_N = 5 : -\sqrt{2}$ the two-photon decay of the physical glueball vanishes in the $SU(3)$ limit.

**Stoic:**

**5** By considering that a conventional baryon has two independent quark positions in its centre of mass frame, argue that the combination $L^P = 0^-$ is the only $L^P$ combination that cannot be constructed for baryons in the non-relativistic quark model, i.e. that it is "quark model exotic". Here $L$ is the total orbital angular momemtum of the baryon.

**6** Consider the connected decay of an adiabatic hybrid meson $A$ with $\Lambda = \Lambda_A$ to two adiabatic hybrid mesons $B$ and $C$ with $\Lambda = \Lambda_B$ and $\Lambda_C$ respectively. Denote the quark-antiquark line of $A$ by $\hat{\mathbf{r}}$. Decompose the pair creation position $\mathbf{y}$ from the midpoint of the quark-antiquark line of $A$ in polar coordinates. Defining $\phi$ to be the angle of $\mathbf{y}$ around the $\hat{\mathbf{r}}$-axis derive the following result related to the conservation of angular momentum around the $\hat{\mathbf{r}}$-axis: The most general form of the flux-tube overlap in the limit where pair creation is near to the initial quark-antiquark line is proportional to $e^{i(\Lambda_A-\Lambda_B-\Lambda_C)\phi}$.

**7** Why are $s\bar{s}$ excited conventional mesons rarely seen in production processes studied experimentally? Specifically, why are they suppressed in central production?

**Herculean**:

**8** List the decays of a $J^{PC} = 0^{+-}$ $c\bar{c}$ exotic below the $D^{**}D$ threshold, where $D^{**}$ denotes the $L = 1$ conventional charm-light mesons. Argue that decays to $D\bar{D}$, $D^*\bar{D}$ and $D^*D^*$ are forbidden. Which decay mode should $0^{+-}$ be searched in?

**9** Show that the gluons in adiabatic hybrid mesons with a fixed quark and antiquark are characterized by three conserved quantum numbers: (1) The magnitude $|\Lambda|$ of the angular momentum of the gluons projected onto the quark-antiquark line, (2) $CP$ around the midpoint between the quark and the antiquark, and (3) if $|\Lambda| = 0$, reflection in the plane containing the quark-antiquark line.

**10** Rewrite Eq. 3 as a hamiltonian quadratic in the fields corresponding to the primitive states. Introduce an additional term which describes the coupling of each primitive state to a specific decay channel. Now make a transformation from primitive fields to physical fields, equivalent to diagonalizing Eq. 3. Note that the unitary matrix that attains this is the valence content matrix. Show that in order to calculate the decay amplitude of a physical state to the decay channel, it is necessary to add the decay amplitudes of all its primitive states, weighted by their valence content.

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