Title: GLOBALLY OPTIMIZED FOURIER FINITE-DIFFERENCE MIGRATION IN THREE DIMENSIONS

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Globally optimized Fourier finite-difference migration in three dimensions
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ABSTRACT
A hybrid imaging approach — the globally optimized Fourier finite-difference method we developed recently — can accurately and efficiently image complex structures having not only large velocity variations but also steep interfaces. The method operates on data in the frequency domain and uses a combination of the split-step Fourier propagator implemented in the wavenumber and space domains and a second-order finite-difference propagator applied in the space domain. Within 1% wavefield phase error, the method can handle dip angles of up to nearly 90° for small lateral variations and approximately 65°–67° for strong lateral variations. In this paper, we extend the globally optimized Fourier finite-difference migration scheme to three dimensions. We make use of the four-way splitting approximation to the finite-difference operator used in the globally optimized method to alleviate the artificial anisotropic effects caused by the operator splitting. Migration of a common-shot dataset for the SEG/EAGE 3D salt model demonstrates the capability of the globally optimized method for accurately imaging complex structures. The method provides a promising tool for imaging 3D subsalt structures.

Key words: Finite-difference, Fourier transforms, hybrid, imaging, optimization, migration.

INTRODUCTION
Complex three-dimensional structures within the earth's subsurface often contain not only large velocity variations but also steep interfaces. Migration methods based on the one-way wave equation provide efficient imaging tool to obtain images of such structures. These methods recursively extrapolate the wavefield from one depth in the earth to another depth across an extrapolation interval followed by an imaging condition applied to the wavefield to obtain the migrated images. Migration methods that use finite-differences of the one-way wave equation (Claerbout, 1985) can in principle handle arbitrarily large velocity variations, but they are only accurate for dip angles — angles between the local propagation directions and the primary propagation direction (usually the z-axis) — of up to a fixed value even in a homogeneous region. The efficient phase-shift migration method (Gazdag, 1978) is accurate for dip angles of up to nearly 90°, but it cannot handle lateral (e.g. horizontal) velocity variations. The split-step Fourier (SSF) method (Stoffa et al., 1990) adds a phase-correction term in the frequency-space domain to the phase-shift method to handle lateral velocity variations. The accuracy of the SSF method rapidly decreases with increasing lateral velocity variation (Huang and Fehler, 1998; Huang and Fehler, 2000a). Within 1% wavefield phase error, the largest dip angle for the SSF method is small (less than 10°) for regions having large lateral velocity variations. Other Fourier transform-based methods, such as the extended local Born Fourier (Huang and Wu, 1996; Huang et al., 1999b), the extended local Rytov Fourier (Huang et al., 1999a), and the quasi-Born Fourier (Huang and Fehler, 2000b) methods, are more accurate than the SSF method and can accurately image complex structures with moderate lateral velocity variations. The phase-shift plus interpolation (PSPI) method (Gazdag and Sguazzero, 1984) uses several reference velocities for a given extrapolation interval to increase its accuracy for media with large lateral velocity variations. The multiple reference velocity logic of the PSPI can be applied to the SSF and the other Fourier transform-based methods to increase their ability to image structures with large velocity variations and steep interfaces (Kessinger, 1992; Huang et al., 1999b). The windowed Fourier transform-based method (Wu and Jin, 1997) is an alternative implementation of the multiple reference velocity logic of the PSPI. The computational cost of all multiple-reference-velocity-based methods increases proportionally to the number of reference velocities used, which depends on the complexity of a model. The high-order Fourier transform-based method (de Hoop et al., 1999; Le Rousseau and De Hoop, 1999) obtains increased accuracy at a cost of increasing the number of Fourier transforms used, which consequently increases its computational time. A hybrid approach, termed the Fourier finite-difference (FFD) method proposed by Ristow and Rühl (1994), has the advantage of both phase-shift and finite-difference methods, that is, it is exact in homogeneous regions and can handle strong lateral velocity variations. Within 1% wavefield phase error, the FFD method can handle dip angles of 45°–50° for large lateral velocity variations. Another hybrid method termed the wide-angle screen (WAS) method (Xie and Wu, 1998; Xie et al., 2000) is based on the approximation of the square-root operator using the first-order (one-term) Padé approximation, which is also referred to as Claerbout's 45° or Muir’s $R_2$ approximations (Claerbout, 1985). A first-order approximation is used when coming into one term rational approximation. This modified Fourier finite-difference scheme is accurate for dip angles of approximately 39° for regions with large lateral velocity variations.

To increase the accuracy of the FFD method while using only one term in the finite-difference operator, Ristow and Rühl (1994) proposed a locally optimized Fourier finite-difference (LOFFD) scheme in which they use one optimized coefficient that varies with velocity and hence varies throughout a heterogeneous model. The LOFFD scheme can handle approximately 16° larger dip angles than the FFD method. Because the coefficient is optimized for each individual velocity, the LOFFD method solves different equations for different velocities and hence, it introduces artificial discontinuities at positions of lateral velocity discontinuities. This artifact can be significant when the lateral velocity discontinuity is large. We numerically demonstrate that the LOFFD method can produce significant artifacts in images for structures having sharp lateral var-
ations in velocity. We have recently proposed a globally optimized Fourier finite-difference (GOFFD) method that uses two optimized coefficients fixed for the entire model (Huang and Fehler, 2000a). The method solves the same equation for the entire model and produces better images for models having sharp lateral variations in velocity than the LOFFD scheme. Within 1% wavefield phase error, the GOFFD method can handle dip angles of approximately 65°–67° for regions with large lateral velocity variations, which is approximately 28° larger than that for the wide-angle screen method and 16°–20° larger than that for the FFD method.

In this paper, we extend the globally optimized Fourier finite-difference method to three dimensions. We make use of the four-way splitting approximation (Ristow and Ruhf, 1997) to the finite-difference operator used in the globally optimized method to alleviate the artificial anisotropic effects caused by operator splitting. We present the impulse response of the 3D GOFFD method, and 3D GOFFD prestack migration image of a common-shot dataset for the SEG/EAGE 3D salt model. Numerical examples demonstrate that the GOFFD method is a promising imaging tool for efficiently and accurately imaging large three-dimensional complex structures.

METHOD

In three-dimensional space, the one-way wave equation in the frequency domain is given by

$$\frac{\partial P(x, y, z; \omega)}{\partial z} = iQ(x, y, z; \omega)P(x, y, z; \omega),$$

(1)

where \(P\) is pressure and the square-root operator \(Q\) is defined as

$$Q \equiv \sqrt{\frac{\omega^2}{v^2(x, y, z)} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}},$$

(2)

where \(\omega\) is the circular frequency and \(v\) is velocity. In the globally optimized Fourier finite-difference method (Huang and Fehler, 2000a), the operator \(Q\) is approximated by

$$Q \approx \sqrt{k_0^2 + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)} + k_0 \left(\frac{1}{m} - 1\right) + F,$$

(3)

with the finite-difference term \(F\) given by

$$F \equiv -k_0 \frac{a(m-1)X_0}{1 - b(1 + m^2)}X_0^2.$$  (4)

In equations (3) and (4), \(a\) and \(b\) are optimized coefficients (Huang and Fehler, 2000a), and velocity ratio \(m(x, y, z)\), wavenumber \(k_0\), and parameter \(X_0\) are, respectively, defined by

$$m(x, y, z) \equiv \frac{v(x, y, z)}{v_0(z)},$$

(5)

$$k_0(z) \equiv \frac{\omega}{v_0(z)},$$

(6)

$$X_0(z) \equiv -\frac{1}{k_0^2(z)}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right).$$

(7)

where \(v_0(z)\) denotes a reference velocity.

Extrapolation of pressure wavefields using eq. (1) and the combination of the first two terms on the right side of eq. (3) results in a 3D split-step Fourier propagator. The corresponding operator of the third term \(F\) in eq. (3) is a compensation operator for the split-step Fourier propagator to improve its reliability when lateral heterogeneity is strong. This operator can be implemented using an implicit finite-difference method (Claerbout, 1985). The direct implementation using the third term \(F\) in eq. (3) is too time-consuming (Claerbout, 1985). The term \(F\) (see equation (4)) must be approximated by splitting it into

$$F \approx -k_0 \frac{a(m-1)X_0^2}{1 - b(1 + m^2)}X_0^2 + \frac{a(m-1)}{1 - b(1 + m^2)}X_0^2,$$

(8)

where the terms \(X_0\) and \(X_0^2\) are given by

$$X_0^2(z) \equiv -\frac{1}{k_0^2(z)}\frac{\partial^2}{\partial x^2}, \quad X_0^2(z) \equiv -\frac{1}{k_0^2(z)}\frac{\partial^2}{\partial y^2}.$$  (9)

Equation (8) is the two-way splitting approximation of the finite-difference term \(F\).

Several different methods can be used to alleviate the artificial anisotropic effects caused by the operator splitting. For instance, one can use the Li’s correction (Li, 1991), or the multi-way splitting approximation (Ristow and Ruhl, 1997), or the helical transform (Claerbout, 1998; Rickett et al., 1998). The helical-transform-based method does not involve any artificial anisotropic effects but its implementation is more complicated than the other approximated methods. With the four-way splitting approximation (see Figure 1), the finite-difference term \(F\) is split into the combination of a two-way splitting approximation in the \(x-y\) coordinate and another two-way splitting approximation in the \(x'-y'\) coordinate that...
Fig. 2: Vertical slices of impulse responses for different migration operators in a homogeneous medium with a velocity of 4500 m/s. A reference velocity of 2250 m/s was used. The upper two panels (a1) and (b1) are for split-step Fourier method, the middle two panels (a2) and (b2) are for the globally optimized Fourier method with two-way splitting, and the lower two panels (a3) and (b3) are for the globally optimized Fourier method with four-way splitting. The panels in the left column are vertical slices along the y-direction and the source location at the center of the upper model boundary. Those in the right column are vertical slices along the diagonal y = x direction. The dashed line in each panel indicates the ideal location of the impulse response.

is a coordinate rotated 45° counterclockwise from the x-y coordinate (Figure 1). Therefore, the finite-difference term \( F \) given by equation (4) is approximated by

\[
P \approx -k_0 \left[ \frac{a (m - 1) X_{\delta z}}{1 - b (1 + m^2) X_{\delta y}} + \frac{a (m - 1) X_{\delta y}}{1 - b (1 + m^2) X_{\delta z}} \right]
\]

\[
+ \frac{a (m - 1) X_{\delta x}}{1 - b (1 + m^2) X_{\delta z'}} + \frac{a (m - 1) X_{\delta y'}}{1 - b (1 + m^2) X_{\delta x'}} \right].
\]

(10)

with

\[
X_{\delta x'}(z) \equiv -\frac{1}{k_0^2(z)} \frac{\partial^2}{\partial z'^2}, \quad X_{\delta y'}(z) \equiv -\frac{1}{k_0^2(z)} \frac{\partial^2}{\partial y'^2}.
\]

(11)

The four-way splitting approximation requires the grid spacings along the x- and y-directions be the same. Ristow and Rühl (1997) numerically demonstrated that the four-way splitting approximation (10) is almost identical to the combination of a two-way splitting approximation along the x-y coordinate when extrapolating wavefields from a given depth \( z_i \) to its next depth \( z_{i+1} \) and a two-way splitting approximation along the 45° counterclockwise rotated \( x'-y' \) coordinate when extrapolating wavefields from \( z_{i+1} \) to \( z_{i+2} \). The latter effective four-way splitting approximation can sig-
Fig. 3 Horizontal slices of impulse responses for different migration operators in a homogeneous medium with a velocity of 4500m/s. A reference velocity of 2250m/s was used. The slices shown are at the dip angles of 60°. The leftmost panel (a) is for split-step Fourier method, the middle panel (b) is for the globally optimized Fourier method with two-way splitting, and the rightmost panel (c) is for the globally optimized Fourier method with four-way splitting. The dashed circle in each panel indicates the ideal location of the impulse response.

If the velocity ratio $m$ is larger than 1.5, we investigate 3D impulse responses of different migration operators when the velocity ratio $m$ is 2. A homogeneous medium defined on a 3D grid $512 \times 512 \times 256$ was used. The source (or the input trace) was located at the center of the upper model boundary. Figure 2 shows vertical slices of the impulse responses of the split-step Fourier (SSF) method and globally optimized Fourier finite-difference (GOFFD) method with two-way splitting and four-way splitting. The impulse responses of the SSF method along different azimuthal directions (Figures 2 (a1) and (b1)) are very similar to each other but those for the two-way splitting GOFFD method (Figures 2 (a2) and (b2)) are different because of artificial anisotropic effects introduced by the two-way splitting approximation of the finite-difference term $F$ (see equation (8)). Making use of the four-way splitting approximation, the GOFFD method gives almost the same impulse responses along different azimuthal directions (Figures 2 (a3) and (b3)).

Figure 3 shows the horizontal slices of the impulse responses at dip angles of 60°. The SSF method does not have artificial anisotropic effects but the location of the impulse response differs significantly from the ideal location indicated by the solid circle (Figure 3a). The GOFFD method (Figures 3b and c) gives more accurate impulse responses than the SSF method (Figure 3a). However, the two-way splitting GOFFD method shows artificial anisotropic effects (Figure 3b) while the four-way splitting GOFFD method produces an impulse response with almost invisible artificial anisotropic effects (Figure 3c). From Figures 2a2 and a3 and Figures 3b and c, we notice that the largest dip angles for the four-way splitting GOFFD method along the $x$-direction and $y$-direction are slightly smaller than those for the two-way splitting GOFFD method.

IMPULSE RESPONSES

For the split-step Fourier method and other Fourier finite-difference methods, the formal error analysis performed by Huang and Fehler (2000a) demonstrates that their largest dip angles vary insignificantly when the velocity ratio $m$ is larger than 1.5. We investigate 3D impulse responses of different migration operators when the velocity ratio $m$ is 2. A homogeneous medium defined on a 3D grid $512 \times 512 \times 256$ was used. The source (or the input trace) was located at the center of the upper model boundary. Figure 2 shows vertical slices of the impulse responses of the split-step Fourier (SSF) method and globally optimized Fourier finite-difference (GOFFD) method with two-way splitting and four-way splitting. The impulse responses of the SSF method along different azimuthal directions (Figures 2 (a1) and (b1)) are very similar to each other but those for the two-way splitting GOFFD method (Figures 2 (a2) and (b2)) are different because of artificial anisotropic effects introduced by the two-way splitting approximation of the finite-difference term $F$ (see equation (8)). Making use of the four-way splitting approximation, the GOFFD method gives almost the same impulse responses along different azimuthal directions (Figures 2 (a3) and (b3)).

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MIGRATION EXAMPLES

To verify the capability of the globally optimized Fourier finite-difference method for imaging 3D complex structures, we performed a prestack depth migration of a dataset consisting of 45 common-shot gathers extracted from the synthetic dataset that was generated using a finite-difference scheme to solve the acoustic-wave equation for the SEG/EAGE 3D salt model (Ober et al., 1997; Xie et al., 2000). We compare the migration image with those obtained using the split-step Fourier method (Stoffa et al., 1990) and the extended local Rytov Fourier method (Huang et al., 1999a). The SEG/EAGE salt model was defined on a 3D grid $676 \times 676 \times 210$ with grid spacings of 20 m along the $x$-, $y$- and $z$-directions. Figure 4a illustrates the positions of sources located at the upper model boundary and Figure 4b shows the areas of the receivers, finite-difference modeling, and migration at the upper model boundary for a given shot. There are no any in-coming waves from the boundaries of a sub-volume used in the finite-difference modeling. For migration of each common-shot gather, we used a sub-volume that is slightly larger than that used in the finite-difference modeling so as to cover all illuminating regions within the 3D salt model for the common-shot gather. A Ricker's time history with a principal frequency of 15 Hz was used as the source function. Migration was performed for a frequency range of 2–30 Hz. For comparison, prestack depth migrations of the dataset were also carried out using the split-step Fourier method and the extended local Rytov Fourier method. We first compare images for a vertical slice along the $y$-direction at $x = 8180$ m. Figure 5a is the velocity model of the slice, where the upper boundary of the salt body is rough (see location A) and, therefore, this region cannot be imaged correctly using the SSF migration (see A in Figure 5b) and the image of the lower boundary of
Fig. 4: Illustration of the source locations (a) of the 45 common-shot gathers for the SEG/EAGE 3D salt model and the migration area (b) for a given shot. The "+" signs indicate the source locations at the upper boundary of the model defined on a 3D grid 676 × 676 × 210. Each common-shot gather involves 201 × 201 receivers centered around the shot location within the dark area in (b). Each common-shot gather was generated using a finite-difference scheme within a sub-volume of 328 × 328 × 210 with the shot located at the center of the upper boundary of the sub-volume (area within the dashed lines in (b)). Migration for each common-shot gather was performed within a sub-volume of 360 × 360 × 210 with the shot located at the center of the upper boundary of the sub-volume (see b).

Fig. 5: Velocity of a vertical slice of the SEG/EAGE 3D salt model (a) and images obtained using the split-step Fourier method (b), extended local Rytov Fourier method (c), and globally optimized Fourier finite-difference method with effective four-way splitting (d). The vertical slice is along the y-direction at x = 8180 m.
the salt body beneath this region is broken (see Figure 5b). The extended local Rytov Fourier method improves the image at location C but not those at locations A and B (Figure 5c). The regions around location A, B, and C are all well imaged by the globally optimized Fourier finite-difference method with effective four-way splitting (Figure 5d). For detailed comparison, the images within the rectangular areas in Figure 5 are shown in Figure 6 where Figure 6a is the velocity within that rectangular area.

Next, we compare images for a horizontal slice at depth \( z = 2100 \) m. Figure 7 shows the velocity of the slice (a) and its migration images (b–d). The extended local Rytov Fourier migration (Figure 7c) produces improved images of the boundaries of the salt bodies compared to those obtained using the split-step Fourier method (Figure 7b). Those boundaries are well imaged by the globally optimized Fourier finite-difference migration method (Figure 7c).

On a SGI’s Origin 2000 with a clock rate of 250 Mhz, the split-step Fourier migration took 310 CPU Hours, while the globally optimized Fourier finite-difference migration took 391 CPU Hours, which is approximately 26% more than that for the split-step Fourier migration. The extended local Rytov Fourier migration took 790 CPU Hours.
Fig. 7: Velocity of a horizontal slice of the SEG/EAGE 3D salt model (a) and images obtained using the split-step Fourier method (b), extended local Rytov Fourier method (c), and globally optimized Fourier finite-difference method with effective four-way splitting (d). The horizontal slice is at the depth of $z = 2100$ m.
CONCLUSIONS AND DISCUSSION

We have extended the globally optimized Fourier finite-difference method to three dimensions. To alleviate the artificial anisotropic effects caused by operator splitting, we have adopted the effective four-way splitting approximation to the finite-difference term used in the globally optimized Fourier finite-difference method. Prestack depth migration of a common-shot synthetic dataset for the SEG/EAGE 3D salt model demonstrate that the globally optimized Fourier finite-difference method can produce accurate image of complex structures at a computational cost of 26% more than that for the split-step Fourier method. The globally optimized method significantly improves the quality of images obtained using the split-step Fourier method.

The extended local Rytov Fourier migration improves the quality of images obtained using the split-step Fourier migration but its computational time significantly increases relative to the split-step Fourier migration because the Rytov-based method needs to use variable extrapolation intervals for such kind of salt model with strong lateral velocity variations. The extended local Rytov Fourier method is more suitable for models having very complex structures with moderate lateral velocity variations, such as the Marmousi model. For such models, the method minimizes the numerical dispersion because it is a purely Fourier transform-based method that numerically solves the lateral derivatives exactly. The method does not have any artificial anisotropic effects which occurs only to the finite-difference-based method.

The values of coefficients $a$ and $b$ we used in the globally optimized Fourier finite-difference method in 3D are those optimized for the 2D cases. The (effective) four-way splitting approximation introduces additional error to the globally optimized Fourier finite-difference method and consequently, it slightly reduces the accuracy of the method in 3D compared to its accuracy in 2D. We will investigate to optimize the method in 3D under the four-way splitting approximation so as to select the values of optimized coefficients $a$ and $b$ that may be different from those for 2D cases. On the other hand, we will apply two-way splitting approximation together with the Li's correction (1991) rather than the four-way splitting approximation to the globally optimized Fourier finite-difference method in three dimensions so that we can use the values of coefficients $a$ and $b$ optimized for 2D cases without decreasing the accuracy of the method. Finally, the helical transform is another promising tool worthwhile to be introduced into the globally optimized Fourier finite-difference method.

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