Title: TWO METHODS FOR A FIRST ORDER HARDWARE GRADIOMETER USING TWO HTS SQUIDS

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Two different systems for noise cancellation (first order gradiometers) have been developed using two similar high temperature superconducting (HTS) SQUIDs. "Analog" gradiometry is accomplished in hardware by either 1) subtracting the signals from the sensor and background SQUIDs at a summing amplifier (parallel technique) or 2) converting the inverted background SQUID signal to a magnetic field at the sensor SQUID (series technique). Balance levels achieved are 2000 and 1000 at 20 Hz for the parallel and series methods respectively. The balance level as a function of frequency is also presented. The effect which time delays in the two sets of SQUID electronics have on this balance level is presented and discussed.

I. INTRODUCTION

This paper presents two systems for first order gradiometry or noise cancellation in hardware using two similar High Temperature Superconducting (HTS) SQUID magnetometers. The first technique, parallel noise cancellation, involves subtracting the background from the sensor SQUID signal at a summing amplifier. In the second technique, referred to as series noise cancellation, the background sensor signal is converted to a magnetic field at the sensor SQUID, thus preserving the dynamic range of the sensor device. In the discussion of these two gradiometer techniques this paper explores a crucial difference between electronic gradiometers and wire-wound gradiometers; time delays in the SQUID electronics limit the balance levels achievable.

Balance levels of ~2000 and 1000 (at 20 Hz) were obtained for the parallel and series mode respectively. Balance level is defined as the ability of the gradiometer to reject a uniform background field, or as the ratio of the amplitudes of the uniform field signal in the sensor SQUID without and with the gradiometry. Also presented in this paper is the theoretical "ideal" balance level one may attain limited primarily by time delays in the electronics. It is shown that the behavior of both the parallel and series noise cancellation devices is that of a first order gradiometer with a balance level limited by these time delays.

A more complete discussion of the ideas put forth in this paper can be found in [1].

II. DESCRIPTION

Both gradiometers were constructed out of two HTS Conductus Mini-Mag SQUID magnetometers controlled via personal computer using Conductus PC SQUID electronics[2]. The two SQUIDs were mounted in an axial gradiometer configuration, with their central axes aligned along a common "z" axis. Both SQUIDs fit snugly inside a fiberglass tube for alignment. The tube was placed vertically inside a standard fiberglass liquid nitrogen dewar. The distance between the two SQUIDs was 1 cm, which is the effective baseline of the gradiometer.

For the parallel noise cancellation the output of both the background and sensor SQUIDs went to a summing amplifier where the gains were adjusted and the difference was taken.

For the series noise cancellation the output of the background SQUID was sent to the amplifier for gain adjustment and inversion and then summed with the modulation/feedback current of the sensor SQUID, effectively "nulling" the background field present there. The series technique realizes the goal of eliminating most of the background fields seen by the sensor SQUID and preserving the dynamic range.

III. TIME ($t_{\text{RELAy}}$) AND PHASE ($\phi$) DIFFERENCE: IN THEORY

Two main factors limit the absolute balance achievable in any two SQUID gradiometer, differing geometries and any time delays in each SQUID system. The geometrical considerations are largely the same as in a wire-wound gradiometer: the two SQUID pickup loops should be identical in area and alignment. The time delays in the SQUID systems are a problem unique to electronic gradiometers.

A finite amount of time is required for the magnetic field (source signal) detected by a SQUID to be converted to a voltage signal that can be read out at the SQUID electronics. The time delay results in a phase difference between the source signal and the SQUID response that is a function of the signal frequency (discussed below). In the case of the parallel gradiometer the presence of these time delays is not crucial, however they must be identical in order for the signals to be in phase (resulting in maximum cancellation) at the amplifier when subtracted. As the frequencies increase, a fixed difference in the time delay results in an increasing phase difference causing cancellation or balance level to deteriorate. For the series gradiometer any time delays in the electronics degrade gradiometer performance because the signal from the background SQUID has to propagate through the electronics to the sensor SQUID to cancel the real-time background field. This causes an inherently out-of-phase background cancellation. Thus the goals for the SQUID electronics time delay tuning are 1) that the time delays be identical for the parallel gradiometer and 2) minimized in the case of a series gradiometer.
For the theoretical estimation of the time delays and resulting phase differences consider the small signal closed-loop frequency response, \( A(f) \), for a flux-locked loop circuit with signal-lock feedback and a one-pole integrator \[3\]

\[
A(f) = \frac{G_f}{1 + G_f}, \tag{1}
\]

where \( G_f \) is the open loop gain defined as the complex number

\[
G_f = \frac{V_f G_f(f) M_{fb}}{R_{fb}} = \frac{f_1}{f}. \tag{2}
\]

\( V_f \) is the SQUID transfer function at the working point of operation, \( G_f(f) = 1/(2\pi R f C) \) is the gain of an ideal one pole integrator with resistance \( R \) and capacitance \( C \), \( i = \sqrt{-1} \), \( R_{fb} \) is the feedback resistance and \( M_{fb} \) is the modulation/feedback coil coupling. Using (2) \( f_1 \), the unity-gain frequency of the feedback loop, can be written

\[
f_1 = \frac{V_f M_{fb}}{(2\pi R C R_{fb})}. \tag{3}
\]

In this case, the closed loop frequency response \( A(f) \) with the one pole integrator is identical to that of a first-order low pass filter with a 3-dB cutoff frequency, \( f_c \), and \( f_1 = f_c \).

Using (1) and (2), the small signal phase shift, \( \theta \), is

\[
\theta = \arctan \left\{ \frac{\text{Im}[A(f)]}{\text{Re}[A(f)]} \right\} = \arctan \left[ \frac{f}{f_1} \right] = \frac{f}{f_1} \tag{4}
\]

at low frequencies.

The phase, \( \theta \), is related to time delays by

\[
\tau_{\text{delay}} = \frac{\theta}{(2\pi f)} = \frac{1}{(2\pi f_1)}. \tag{5}
\]

IV. MEASURED \( \theta \) AND \( \tau_{\text{DELAY}} \)

From (5) and (4) in the preceding section one can see that for any time delay in the system there is a corresponding phase difference that increases with increasing frequency. Thus, as noted above, the parallel and series two SQUID gradiometers are generally optimized by both matching and minimizing time delays for both SQUID electronics as much as possible. Matching the time delays is achieved by matching the small-signal cutoff frequency, \( f_1 \), for both background and sensor SQUID electronics. Minimizing the delays is achieved by making \( f_1 \) as large as possible.

For this experiment two HTS SQUIDs were used which were very much alike in their modulation/feedback coil coupling, and the peak-to-peak amplitudes of their V-\( \Phi \) curves. (3) illustrates that this similarity means the \( f_1 \) of the two SQUIDs will be as alike as possible. Both SQUIDs also had similar effective areas of \( \sim 0.08 \text{ mm}^2 \), and white noise levels \( <2 \times 10^{-13} \text{ T/Hz} \). The field sensitivity of either SQUID was \( \sim 25 \text{ nT}/\text{Hz} \).

The delay time was measured experimentally in the shielded can using a function generator to supply an excitation signal to the modulation/feedback coil. The phase difference between the excitation signal and the SQUID's response were measured as a function of frequency by a lock-in amplifier. The results are plotted in Fig. 1.

In the upper panel the phase data are shown along with the best least-square polynomial fits to the data (dotted lines). The low frequency approximation of (4) predicts the phase vs. frequency behavior should be a linear function of the frequency, and the slope should be equal to \( 1/f_1 \). The best fits to the data (dotted lines) give \( f_1 \), the unity-gain frequency of 25.8 kHz and 21.0 kHz for the background and sensor SQUIDs respectively. \( f_1 \) is clearly linear with frequency and the measured \( f_1 \) are consistent with the 20 kHz measured previously. The lower panel of Fig. 1 shows the time delays corresponding to the phase data and the \( f_1 \) obtained from best fitting (dotted lines). The data and best fit predictions are in good agreement. The measured time delays for the background and sensor SQUIDs are \( \sim 6.2 \mu \text{s} \) and \( \sim 7.6 \mu \text{s} \) respectively.

V. THEORETICAL LIMIT TO BALANCE DUE TO TIME DELAY

If we use the measured time delay for either SQUID, \( \tau_{\text{delay1}} = 6.2 \mu \text{s} \) and \( \tau_{\text{delay2}} = 7.6 \mu \text{s} \), then we can predict the balance level limit due to these time delays. Assuming a sine wave signal of amplitude, \( G \), one can predict the balance level, \( \gamma(\tau_{\text{delay}}) \) as
\[ \gamma(t_{\text{delay}}) = \frac{\Delta}{|\Delta|}, \quad (6) \]

where

\[ \Delta = G_1 \sin(\omega t - \omega t_{\text{delay}1}) - G_2 \sin(\omega t - \omega t_{\text{delay}2}) \quad (7) \]

and

\[ \Sigma = G_2 \sin(\omega t - \omega t_{\text{delay}2}), \quad (8) \]

In the above equations, \( \omega = 2\pi f \), where \( f \) is the frequency of interest. \( t_{\text{delay}} \) is the delay time.

If we assume that \( |G_1| = |G_2| \), then we can write

\[ \cos(\omega t_{\text{delay}}) = 1, \quad \sin(\omega t_{\text{delay}}) = \omega t_{\text{delay}}, \quad (9) \]

then we can write

\[ \gamma(t_{\text{delay}}) = \frac{1}{\omega |t_{\text{delay}1} - t_{\text{delay}2}|} = \frac{1}{\omega \delta t_{\text{delay}}}. \quad (10) \]

Another approach to compensate for our inability to precisely align and orient the SQUIDs identically in our apparatus is to consider small differences in the signal amplitudes, \( |G_1| = a |G_2| \), where \( a \) is a number close to 1.

In this case, again using the small angle approximation, we find the balance level to be

\[ \gamma(t_{\text{delay}}) = \frac{1}{\sqrt{(1-a)^2 + a^2 \delta t_{\text{delay}}^2}}. \quad (11) \]

VI. OPERATION IN A SHIELDED ENVIRONMENT

The balance levels for both techniques were measured with the SQUIDs inside the shielded can using an external excitation coil driven by a sine wave signal from a function generator. The magnetic signal was about \( 1/4 \Phi_0 \) at each SQUID. The measured balance level for both gradiometers are presented in Fig. 2 as data points. The balance levels predicted by a best fit to (11) are shown as dotted lines, with \( \delta t_{\text{delay}} \) allowed to vary. The balance levels predicted by a best fit to (12) are shown as dashed lines, where both \( a \) and \( \delta t_{\text{delay}} \) were allowed to vary. There is no quantitative way, a priori, to determine \( a \). It depends largely on how well the SQUIDs are aligned and oriented. However, because we can use the gains on the amplifier to account for most of the effects of geometrical mismatch (something one cannot do with a wire-wound gradiometer), one does qualitatively expect that a will not be more than \( \pm 10\% \) from unity.

For the parallel method (upper panel) fit to (11) (dotted line) we found \( \delta t_{\text{delay}} = 3.64 \pm 0.21 \mu s. \) For the fit to (12) (dashed line) the best fit to the data gave \( a = 1.0015 \pm 0.0002 \) and \( \delta t_{\text{delay}} = 1.55 \pm 0.11 \mu s. \) The latter is in good agreement with that expected from our phase measurements, \( \delta t_{\text{delay}} = (7.6 - 6.2) \mu s = 1.4 \mu s. \)

For the series method (lower panel) fit to (11) (dotted line) we found \( \delta t_{\text{delay}} = 5.95 \pm 0.32 \mu s. \) For the fit to (12) (dashed line) the best fit to the data gave \( a = 1.0011 \pm 0.0002 \) and \( \delta t_{\text{delay}} = 4.45 \pm 0.26 \mu s. \) The upper limit on \( \delta t_{\text{delay}} \) for the series method can be estimated by assuming \( \delta t_{\text{delay}} \) will be less than the sum of the delay for the background SQUID signal plus the delay from transit through the sensor SQUID test port. From this we expect \( \delta t_{\text{delay}} \leq 13.8 \mu s - 6.2 \mu s, \) or \( \delta t_{\text{delay}} \leq 7.6 \mu s. \) This is true for both fits. The fit to (12) is quite good.

Fig. 2 clearly shows that both the parallel and series gradiometers behave as devices with a balance level limited by the phase differences (time delays) in the electronics. Therefore reducing time delays in the electronics can attain even better balance levels than we report here. Another important aspect is that even small differences in the values of \( |G_1| \) and \( |G_2| \) (a not quite 1) can result in a large difference in the achievable balance level, especially at low frequencies. This restates the need for great care in the orientation and alignment of the SQUIDs.

VII. OPERATION IN AN UNSHIELDED ENVIRONMENT

The SQUID gradiometers were also characterized in the unshielded laboratory. The SQUIDs were dominated by 60 Hz and harmonics caused by the power lines in and around the room. The white noise floor for the background and sensor SQUIDs alone were \( 1.75 \times 10^{-13} \text{T/} \sqrt{\text{Hz}} \) and...
Fig. 3. Upper panel: Measured (data points) and predicted balance levels for parallel noise cancellation. Dotted lines are from a best fit to (11) and dashed lines are from a best fit to (12). Lower panel: Measured (data points) and predicted balance levels for series noise cancellation. Dotted and solid lines as in upper panel. Measurements were performed with the SQUIDs in the unshielded laboratory.

1.25x10^{-13} \text{T} \cdot \text{Hz} \text{ respectively, measured at 4.5 kHz. The white noise level using parallel and series noise cancellation were 1.9x10^{-13} \text{T} \cdot \text{Hz} and 2.3x10^{-13} \text{T} \cdot \text{Hz}, respectively. These both correspond to adding the white noise levels from both sensor and background SQUIDs in quadrature. As expected, the parallel method performed better, reducing the the 60 Hz peak by a factor of ~25 compared to the case with no gradiometry. The series method reduced the 60 Hz peak by ~9.[1]

Even with exceptionally high balance level, first order gradiometry is only effective for unshielded applications where the gradient of the ambient field noise is small.[4] Power line noise in many cases cannot be assumed to be uniform and therefore first order gradiometry may not be adequate. A second order gradiometer will likely be more suitable, however this is a more difficult device to realize with HTS SQUIDs and beyond the scope of this paper.[5]

The balance level for the unshielded environment was measured using a uniform field from a Helmholtz coil providing a similar amplitude signal as in the shielded case. In this case, a large ambient noise signal was superposed on the signal from the Helmholtz coil. We were interested in measuring the balance levels in the unshielded case because it is has been shown that the phase shift of the SQUIDs can change as a function of signal amplitude.[6] Such an additional phase shift would reduce the balance level observed in the shielded can. These levels are plotted (Fig. 3 solid lines) along with the predicted balance levels (dotted, dashed lines, same as Fig. 2). It can be seen that the balance levels are similar to those in the shielded case and very well fit by the theoretical predictions of (11) and (12). We conclude the additional phase shift is negligible.

For the parallel method (Fig. 3, upper panel) we find for the fit to (11) (dotted line) $\delta_{\text{delay}} = 3.29 \pm 0.22 \mu$s. For the fit to (12) (dashed line) the best fit to the data gave $a = 1.0008 \pm 0.0001$ and $\delta_{\text{delay}} = 0.95 \pm 0.07 \mu$s. We expect from our phase measurements that $\delta_{\text{delay}} \sim 1.4 \mu$s.

For the series method (lower panel) fit to (11) (dotted line) we found $\delta_{\text{delay}} = 9.24 \pm 0.39 \mu$s. For the fit to (12) (dashed line) the best fit to the data gave $a = 1.0011 \pm 0.0002$ and $\delta_{\text{delay}} = 4.85 \pm 0.28 \mu$s. We expect $\delta_{\text{delay}}$ to be $\leq 7.6 \mu$s. The balance levels and best fit parameters between the shielded and unshielded case are indistinguishable.

VIII. DISCUSSION

Our study investigated two different methods of creating a first order HTS SQUID gradiometer. Both the parallel and series techniques were implemented with commercial magnetometers and slightly modified feedback electronics.

Unlike conventional wire-wound gradiometers, time delays in the SQUID electronics affect the balance level by introducing different phases between the SQUID signals. Balance level can be optimized by maximizing the small-signal cut-off frequency, $f_1$, or minimizing the time delays (series method) and tuning $f_1$ to be as similar as possible (parallel and conventional electronic methods) for the two SQUIDs. These steps are crucial to obtaining a high balance level and maintaining it as frequencies increase. Our study found that the time delay was the limiting factor to the balance level of our two devices. A consequence of the fixed time delay was that the balance level deteriorated with increasing frequency.

The parallel gradiometer does not require a computer in the laboratory and one can balance the SQUIDs (adjust gains) in real time. The series gradiometer has the further advantage of the noise being "nulled" at the sensor, preserving much of the dynamic range of the sensor device. Excellent balance levels of $10^3$ can be achieved at 20Hz with off-the-shelf commercial devices, and better results could be expected with better matched electronics and reduced time delays in the electronics. Until the development of a readily available planar second order gradiometer (or HTS wire), one possible approach to a further reduction of the noise is to design a simple second order electronic HTS gradiometer. Critical to obtaining optimal performance of any order gradiometer utilizing more than one SQUID is 1) matching the electronics and 2) minimizing time delays in the electronics.

REFERENCES