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X-ray Diffraction Optic**

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A Simple and Effective Modification of a Sagittally Focusing X-ray Diffraction Optic

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ABSTRACT

Some experiments performed at bending magnet beamlines at the Advanced Photon Source (APS) will benefit from a focused photon beam. A commercial sagittal focusing bender that uses a simple rectangular plate as the diffraction element could be utilized for this purpose. However, the thin rectangular diffracting plate, specified by the manufacturer, is susceptible to antilastic bending. A simple ribbed plate that utilizes this commercial bender that reduces antilastic bending is proposed as a solution. This design uses a pair of sufficiently stiff ribs on the diffracting surface, which substantially suppresses antilastic bending. The finite element analysis (FEA) method was used to predict the overall structural response of this sagittally bent plate and its antilastic distortion. Results are compared with unribbed plates showing an effective reduction in antilastic bending.

1. INTRODUCTION

Many experiments performed at a typical bending magnet beamline—in this case, at the Advanced Photon Source (APS) sector 2-BM—can benefit from a focused photon beam. Examples would include x-ray spectroscopy, x-ray scattering, x-ray diffraction, and x-ray microtomography. The normal incident dimensions of the bending magnet beam at the experiment station (2 mrad horizontal at 26 m from the source) are about 5 mm vertically by 50 mm horizontally. For microtomography, a 1-3 mm beam size is desired; for the others, focusing the beam to 10 to 200 microns in at least one dimension is desired. One of the approaches to focusing the beam is the use of a sagittally bent crystal. A commercially available four-point sagittal focusing bender that uses a simple rectangular plate as the diffracting element can be used. However, antilastic bending—the undesirable distortion in the tangential direction—degrades the focused beam. The overall dimensions of the focusing plate in this bender are 100 by 100 mm, with a thickness of about 2 mm.

A review of sagittal focusing designs shows a wide range of novel designs that have been proposed and tested by Sparks et al.¹⁻³ These designs rely on complex crystal geometries that require precision machining and etching. The designs range from a continuous array of thin stiffening ribs below the optical surface to discrete rectangular optical segments that are machined into the optical surface. The segmented-block designs included individual rectangular crystal blocks that are an integral part of a rectangular or triangular plate that is bent to the desired curvature.^{2,3} Each of these approaches reduces the antilastic bending. However, each was a piecewise, discontinuous representation of the desired cylindrical surface. The ideal solution is one that maintains the continuous cylindrical shape with minimal antilastic bending. It is known that reducing the width (sagittal dimension) to length (tangential dimension) of a sagittally bent rectangular plate reduces antilastic bending.⁴ We show that placing a pair of longitudinal ribs not only stiffens the structure but in a sense reduces the “effective” width (distance between the ribs, S) to length ratio, leading to reduced antilastic bending. The goal of the present work was to minimize the antilastic distortion while making use of an available four-point bender. This solution offers a simple and cost-effective solution.

2. SIMPLE BEAM THEORY

To better understand the overall structural response of the focusing plate to external loads and boundary conditions, e.g., the plate's radius of curvature, the plate was first analyzed as a beam using equations from beam theory.⁵ To simplify the analysis of the proposed four-point sagittal bender, the two external force couples, applied by the rollers, were replaced with an equivalent pair of external moments, M . For a beam bent by applying a moment at two ends, the governing second-order differential equation for the lateral deflection is given as

$$EI \frac{d^2 w}{dx^2} = -M, \quad (1)$$

where E is the modulus of elasticity, I is the constant area moment of inertia, x is the distance from one end of the beam, and $w(x)$ is the lateral displacement along the beam (Figure 1).

Figure 1. Simplified rectangular optical plate approximated as a simple beam.

To obtain $w(x)$, eq.1 is integrated twice. The two constants of integration are determined from the boundary conditions as zero lateral displacement at the ends of the beam. Thus,

$$w(x) = \frac{M}{2EI} (Lx - x^2), \quad (2)$$

where L is the length of the beam. The area moment of inertia for a plate with rectangular section, I , is given by

$$I = \frac{bt^3}{12}, \quad (3)$$

where b is the dimension into the paper and t is the thickness of the beam. The radius of curvature (sagittal radius), R_s , of the bent beam is given by

$$\frac{1}{R_s} = \frac{d^2 w}{dx^2} = \frac{M}{EI}. \quad (4)$$

For our bender, the moment needed to bend the silicon plate to a sagittal radius R_s of 26 m is determined to be about 0.5 N-m for a silicon plate ($E = 1.8E11$ N/m², $b = 100$ mm, $t = 2$ mm). For most engineering designs, the above relationships are adequate to describe the structural response of a simple beam or plate with a constant rectangular section and applied external load. However, this one-dimensional model cannot predict the anticlastic bending. A more general analytical treatment, one that includes the effects of Poisson's ratio, is required.

When a rectangular plate is bent into a cylindrical shape, e.g., by using two equal moments, a 'saddling' distortion occurs, due to the effects of Poisson's ratio. This saddling of the plate—known as anticlastic bending—causes a diffracted x-ray beam from this plate to diverge in the tangential dimension and reduce the intensity at the sample location. An exaggerated, pictorial illustration of the saddling is shown in Figure 2.

Figure 2. A pictorial representation of anticlastic distortion in a rectangular plate. The bending of the plate in the sagittal direction causes the “saddle-shaped” tangential distortion.

Our design approach has been to (i) utilize an available bender that accommodates the bending magnet (BM) beam dimensions, and (ii) use a simple design that minimizes anticlastic bending. The finite element analysis (FEA) method is used to analyze various designs. ANSYS,⁶ a general-purpose finite element computer code, is used to model and simulate the structural response of the plate. The distortion results for new designs are compared with unribbed plates.

3. PROCEDURE

3.1 Structural Analysis

A simple rectangular plate with two opposing bending moments was first analyzed. The size of this plate is determined by the available bender. The largest width and depth dimensions for this simple plate, as specified by the supplier of the mechanism, were 100 mm by 100 mm with a thickness of 2 mm. Due to the presence of the rollers, the actual dimensions of the usable area were about 80 mm for the width and 100 mm for the length (Figure 3). Since the normal incident dimensions of the BM beam are about 50 mm horizontal by 5 mm vertical. The bender will accommodate the full beam at typical incident angles greater than 10°. The coordinate system used in the structural analysis and the tangential and sagittal directions are shown in Figure 3.

Figure 3: Four-point sagittal-bending mechanism: (a) CAD model with beam, and (b) orientation.

A thin plate or beam with a pair of uniform moments, applied on two opposing simply supported sides will bend into a constant radius cylinder. Our bender creates these opposing moments by forming four uniformly distributed line-contact loads (forces) along the tangential direction of the plate (Figures 3b and 4). As shown in these diagrams, the center base has rotational joints for the mounting block for the rollers. This block is fixed, while the larger, lower plate is free to move vertically. This plate pushes on the mounting blocks for the rods, causing the blocks to rotate and the plate to bend.

Figure 4. Four-point line-contacts on the sagittal focusing plate.

Finite element model of a simple rectangular (100 mm x 100 mm x 2 mm) plate with four uniform line-contact loads was first created and solved. The results were then compared with the structural response of a similar plate with an equivalent applied moment (Figure 5). The sagittal and anticlastic results for both loading conditions differ by less than 2 percent. For simplicity, thus, the pair of opposing moments was used in all subsequent simulations.

Figure 5: Four-point bender with boundary conditions and loads : (a) with line-contact loads (F), (b) with equivalent moments (M).

The bending of a simple rectangular plate is shown in Figure 6. Anticlastic bending along the centerline near the upstream and downstream edges is clearly evident in this figure. The contours, representing out-of-plane deformations, are not parallel in the tangential direction. This is an indication of anticlastic distortion. Plots of the vertical displacement and slope along the centerline of the plate in the tangential direction are given in Figures 7a and 7b, respectively. There is considerable anticlastic bending along the tangential centerline. The addition of stiffening ribs to reduce the anticlastic bending was then investigated.

Figure 6. Simple rectangular sagittally bent plate. Contour lines for out-of-plane deformation are not straight, indicating anticlastic deformation.

Figure 7: Tangential centerline (a) displacement, and (b) slope for a simple plate, bent to a 26 m radius. The maximum slopes are $\pm 214 \mu\text{rad}$.

A plate with similar dimensions as before, but with a pair of ribs, is modeled and analyzed. A solid model of the four-point sagittal-bending mechanism with a ribbed plate is shown in Figure 8a. A FEA model for the ribbed plate was developed. The rib height is adjusted to minimize the anticlastic bending (Table 1). To accommodate the 50 mm wide BM beam, the minimum distance between ribs was 50 mm (Figure 8b). The width of the rib is fixed at 4 mm, the height is varied from 0 to 20 mm, and the distance between ribs is fixed at 56 mm. The vertical dimension of the BM beam is 5 mm and its vertical footprint on the crystal ($5/\sin\theta_B$, where θ_B is the incident Bragg angle) will extend to about 30 mm for $\theta_B > 10^\circ$.

Figure 8. Sagittal bender with stiffening ribs: (a) CAD model, and (b) FEA model with beam footprint.

Table 1. Ribbed plate maximum tangential slope along centerline at ± 25.0 mm from plate center.

Rib Height (mm)	Tangential Slope (μ rad)
0 - No Ribs	116.5
4	53.0
8	26.9
12	21.4
16	9.4
20	25.9

Figure 9. Sagittally bent ribbed plate. The contour lines are almost straight, indicative of reduced anticlastic bending (cf., Figure 6).

The results show that there may be an optimal rib height. A taller rib does not seem to result in reduced tangential distortion. The cause of this optimal height is not fully understood and could be due to a twisting of the ribs. A rib height of 16 mm was used for all subsequent simulations.

Graphical plots of the vertical displacements and slopes with a rib spacing of 56 mm and a pair of 4 mm x 16 mm ribs are shown in Figures 9 and 10. A comparison of these graphical plots with those for the simple plate show a significant reduction in the anticlastic bending. The almost straight contours in Figure 9 attest to this reduced anticlastic bending.

A study of the effect of varying the distance, S , between the ribs is made for a fixed 100 mm length. Anticlastic distortions along the tangential centerlines for all simulated cases are shown in Figure 11. From these results, it is clear that the addition of the stiffening ribs and a decrease in the rib spacing significantly reduces the anticlastic distortion.

Figures 10: Ribbed plate (a) displacements and (b) slopes

Figure 11. Effect of rib spacing on anticlastic slope.

The above results for ribbed plates are compared with similarly dimensioned unribbed plates of the same length (X) and width (Y) equal to the distance between ribs (S) of the ribbed plates. The efficacy of appropriately sized unribbed plates (having optimal length (X) to width (Y) aspect ratio) in reducing anticlastic bending is discussed by Kushnir et al.⁴ Figure 12 shows the effect of varying the aspect ratio for the unribbed plates and the additional reduction of anticlastic bending for the ribbed plates with equal effective areas.

Figure 12. Comparison of tangential (anticlastic) centerline slopes along ± 30 mm from center of the plate.

This comparison shows that, for a given (sagittal) width, the ribbed design performs better than a simple plate. In addition, the optimal aspect ratio for a simple plate often makes such a design suitable for undulator beams (due to the small horizontal dimension) rather than BM beams, which are much wider. A simple optimal plate design for a BM beam will be long, requiring a similarly long bender.

4. CONCLUSIONS

A simple sagittal focusing optic, utilizing an available four-point bender, was proposed and analyzed. Plates with and without stiffening ribs were analyzed and compared. Results indicate that a pair of stiffening ribs can substantially reduce anticlastic distortion in comparison with an unribbed plate of equivalent dimensions.

5. ACKNOWLEDGEMENTS

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