Title: **Designing a Micro-Mechanical Transistor**

Author(s): **Ronnie Mainieri, T-13**

Submitted to: **DOE Office of Scientific and Technical Information (OSTI)**

RECEIVED

SFP 07 1999

STI

Los Alamos

NATIONAL LABORATORY

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Designing a Micro-Mechanical Transistor

Ronnie Mainieri*

Abstract

This is the final report of a three-year, Laboratory-Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). Micro-mechanical electronic systems are chips with moving parts. They are fabricated with the same techniques that are used to manufacture electronic chips, sharing their low cost. Micro-mechanical chips can also contain electronic components. By combining mechanical parts with electronic parts it becomes possible to process signal mechanically. To achieve designs comparable to those obtained with electronic components it is necessary to have a mechanical device that can change its behavior in response to a small input - a mechanical transistor. The work proposed will develop the design tools for these complex-shaped resonant structures using the geometrical ray technique. To overcome the limitations of geometrical ray chaos, the dynamics of the rays will be studied using the methods developed for the study of nonlinear dynamical systems. This leads to numerical methods that execute well in parallel computer architectures, using a limited amount of memory and no inter-process communication.

Background and Research Objectives

The proposed work was motivated by difficulties encountered in designing a mechanical filter for a micro-mechanical device. Micro-mechanical devices [1] are chips that have moving parts, such as motors, and electronic parts, such as amplifiers. Like purely electronic chips, they can be produced at very low cost. The most common use of a micro-mechanical chip is as an accelerometer in automobile air-bag systems [2]. But they have many other uses. At very high frequencies, such as those used in microwave communications and in the modulation of fiber optics, electronic processing of signals is less efficient than mechanical processing. Electronic circuits use more power and do not respond as fast. Today, most high frequency devices use one form or another of mechanical processing. Most televisions have comb filters [3], where two tuning forks with many prongs are intertwined, acting as a band-pass filter and removing ghost images. The same filters are used in cellular phones. Comb filters are modeled on filters using capacitors, inductors and resistors, and are not close in sophistication to filters built with transistors, operational amplifiers, or digital signal processing (DSP) computers.

To design mechanical devices that go beyond the simple filter design, one has to be able to design complicated resonant structures. If we want to achieve the goal of developing a mechanical transistor—a device that changes its response along one channel as a function of input in another channel—one has to be able to design and build complicated mechanical

structures. Designing with a computer reduces the development time and allows for a greater freedom in experimentation of alternative designs. The difficulty with this approach is that the reliable computation of resonances requires very large memories and fast computers to manipulate the arrays used. In favorable cases these calculations must be done on super computers, making them inaccessible to an engineer using a workstation. In typical cases, not even a super computer can carry out these calculations.

The objective of this work was to develop the mathematical techniques needed to handle the design of complicated resonant structures, such as the mechanical transistor. The main technique used to approach the problem was cycle expansions. Cycle expansions are a method for computing the properties of chaotic systems [5]; they provide the fastest known method to obtain the long-term averages of chaotic system and the resonances of systems that are related to chaotic systems. The basic idea used in cycle expansions is to consider the periodic solutions of the system, even if they are unstable, and use these orbits to construct other quantities of interest.

Importance to LANL's Science and Technology Base and National R&D Needs

The work proposed could be seen as part of a long-term effort at the Laboratory to develop techniques that are applicable to problems that are non-linear. These problems require a careful consideration of their topological properties and in many cases numerical simulations can be deceiving, as it is difficult to determine the quality of the solutions obtained.

The method proposed has the great advantage that it can use short-time behavior to estimate long-time behavior. This is a form of sub-grid modeling. These estimates respect all symmetries of the problem, are determined from the dynamics, and converge to the solution as the cut-off between short and long term behavior is moved towards longer periods. The estimates are possible because one computes averages and not individual solutions. The lack of ability to estimate individual solutions is not a disadvantage, as individual solutions are not very meaningful.

Scientific Approach and Accomplishments

There is a duality between the study of particle motion and that of wave propagation. This duality has been useful, as it has allowed me to transfer concepts that were developed for the study of chaotic dynamical systems to the study of partial differential equations. The duality was developed during the 1950s when there was a growing need to develop an understanding on how radar and sonar reflected off targets. For most targets, electromagnetic or acoustic fields are impossible to determine analytically, and numerical

methods on grids must be used. No computer then could handle these grids. As an alternative, Joseph Keller and collaborators [4] developed an asymptotic technique—the geometrical ray method—that allowed for the approximate solution of those problems. The ray method consists of assuming that the fields are a superposition of plane waves with position dependent velocity and amplitude. The partial differential equation then reduces to an ordinary differential equation for the velocity of the wave (the ray), with great savings in computation time. With the technique, approximate solutions were obtained quickly. If the geometrical ray method could be used for all resonant structures, then it would be possible to design resonant structures on a computer. The ray method was widely used until the late 1960's. While it worked very well with some shapes, it was a puzzle why it failed to converge for others. Problems that failed with the ray method were also hard to converge by other numerical methods. By the late 1960's computers became fast enough to be able to solve these problems by using fine enough grids and the ray method was abandoned as a numerical method. The reason why it failed remained unclear. In the simple problems that can be solved analytically one sees no reason for the failure. In more complicated problems, a series of explanations are put forth. None of these explanations can be applied to all cases, and they give little guidance to the numerical dilettante.

Why the ray method fails was discovered in the late 1960s by Gutzwiller [6]. He developed a method to extend the methods of old quantum mechanics to problems that were not separable. As Einstein had pointed out [7], the methods of Born and Sommerfeld for the quantization of Hamiltonian systems could not be applied in general and in particular it did not work for systems that are classically chaotic. Gutzwiller's discovery was that when periodic orbits are unstable and bunch up next to each other, one has to take into account their unstable behavior. Gutzwiller's method does not converge well, and this difficulty was corrected with the introduction of cycle expansions.

The circle of ideas that Keller, Gutzwiller, and Cvitanovic introduced are not limited to quantum mechanics and can be extended to the classical resonances of extended systems, as done in this project.

The results of the project were:

- a determination of how the error of numerically computed frequencies decreases as a function of the number of basis functions;
- a formalism for incorporating the effects of noise in the ray formalism; and
- a new theory for the synchronization of chaotic systems.

Error in resonances

As an example of computing resonances, let us suppose we were trying to solve the Schrödinger equation. To determine the chemical and physical properties of a molecule, one has to be able to determine the first few eigenvalues of the equation, at least as many levels as there are electrons in the system. Typical estimates of the number of basis functions required for the calculation say that the number of basis functions is proportional to the number of electrons N . The reason is that the N th eigenfunction will have N nodes, and within each node the solution looks like a sine wave; to approximate the sine wave one only needs a few points between nodes. The number of basis must then be proportional to the highest eigenvalue being computed. I discovered otherwise from the research of this project.

Suppose that we have computed k eigenvalues of the Schrödinger equation using n basis functions. We can then compute the same k eigenvalues with $n/2$ of the basis functions and compare the number of digits that remain the same. This gives an estimate of the quality of the eigenvalues. By carrying this procedure out in several systems, I was able to establish that the more chaotic the corresponding classical system is, the less digits one obtains. I was also able to verify that a universal curve characterizes the error of the eigenvalues.

Noise

I developed formalism for incorporating the effects of noise in calculations that involve chaotic dynamics. Because chaos is combined with noise, the calculation is very different from the case when there is no chaos. The main difficulty comes from having to incorporate in one term the contribution of noise to a periodic orbit and all its repeats. The main idea of the method is to use a smooth change of coordinates so that the action of the dynamics in the neighborhood of a periodic orbit is exactly the action of a linear map. These are the transformations that Sternberg studied in the late 1950s and that form the basis of the work of Moser and Arnold in their proofs of the KAM theorem. The developed formalism works best when the noise is weak. This is precisely the regime where numerical simulations are the hardest.

The work on noise also resulted in a new way to evaluate Feynman diagrams. The terms of the expansion are computed with the aid of Fourier transform and the number of terms computed is significantly less than required with other expansions.

Synchronization

An essential point in the design of a micro-mechanical transistor is the ability to use a chaotic resonator. The basic feature of the electronic transistor is that small changes in one of its channels generate large changes in another port. To produce large changes in a signal in the micro-mechanical transistor one uses the sensitive dependence of chaotic systems. Small changes at one point lead to large changes at another. To take advantage of this chaotic behavior I studied the notion of chaotic synchronization.

I was able to develop a general theory for the synchronization of chaotic systems. It is based on the topology of the dynamical system, that is, on periodic orbits and how their stable and unstable manifolds are arranged. Using this theory, I have been able to show that systems that were thought not to synchronize can be made to do so.

Publications

1. O. Bauer and R. Mainieri, The Convergence Of Chaotic Integrals, *Chaos* v. 7(#3) pp. 361-367 (1997).
2. P. Cvitanovic, C. P. Dettmann, R. Mainieri , G. Vattay. Trace formulas for stochastic evolution operators: Weak noise perturbation theory. *Journal of Statistical Physics*, v. 93, pp. 981 (1998).
3. P. Cvitanovic, C. P. Dettmann, R. Mainieri , G. Vattay. Trace formulas for stochastic evolution operators: smooth conjugations. *Nonlinearity*, accepted, to appear (1999).
4. Aronov, Beletskii, Berman, Campbell, Doolen, Dudiy, Mainieri. AC Transport and Collective Excitations in a Quantum Point-Contact. *Semiconductor Science and Technology* v. 13, pp. A104 (1998).
5. G. Berman, G. Doolen, R. Mainieri, et al. On Properties of Boundaries and Electron Conductivity in Mesoscopic Polycrystalline Silicon Films For Memory Devices. *Semiconductor Science and Technology* v. 13, pp. A33 (1998).
6. G. Berman, G. Doolen, R. Mainieri, V. Tsifrinovich. *Introduction to quantum computers*. Published by World Scientific, Singapore, 1998.
7. J. Rehacek and R. Mainieri. Projective synchronization in three-dimensional chaotic systems. *Physical Review Letters*, v. 82 pp. 3042 (1999).

References

- [1] R. Howe and K. Gabriel, "Silicon micromechanics: sensors and actuators on a chip," IEEE Spectrum, (1990). M. Mehregany, "Micromechanical systems," IEEE Spectrum, (1992).
- [2] R. E. Mihailovich, N. MacDonald and others, "Single crystal torsional resonators," page 184 of the Proc, of the IEEE MEMS workshop, (1993).
- [3] C. Lee and G. Dalman, "Microwave devices, circuits and their interaction," published by John Wiley in (1994).
- [4] J. Keller Journal of the Optical Society of America, v.**52**, p.116 (1962).
- [5] P. Cvitanovic et al. *Classical and Quantum Chaos*. Electronic book available at <http://www.nbi.dk/ChaosBook/>.
- [6] M. Gutzwiller. Journal of Mathematical Physics, v. **8**, p. 1979 (1967) and v.**10**, p. 1004, (1969) and also v. **11**, p. 1791 (1970).
- [7] A. Eienstein. Verh. Dtsch. Phys. Ges., v.**19**, p. 82 (1917).