Title:

Virtual Bandwidth via Stochastic Polyspectra

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Abstract

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This is the final report of a three-year, Laboratory-Directed Research and Development (LDRD) project at the Los Alamos National Laboratory (LANL). The work funded by this grant addresses theoretical issues pertaining to potential applications of higher-order spectra (polyspectra) for communications and measurement. Although the sensitivity to additive Gaussian noise provided a motive for our interest in the bispectrum, the research done here focused on the ability of the bispectrum to "see" outside the Nyquist interval. The main contribution of this work is a new understanding of the everyday phenomenon of *aliasing* that is described below. The second contribution this work makes is significant, though limited, progress in understanding the joint realizability conditions for spectra and bispectra. A third contribution, which arose in efforts to broaden our results on joint realizability, is the awareness that a very common inequality, used to normalize the bispectrum, is invalid. This result has considerable significance for the practical use of the bispectrum.

Background and Research Objectives

The ordinary power spectrum is the simplest of a series of so-called polyspectra. The next simplest polyspectrum is the bispectrum and is all that we need consider. The bispectrum is already a relatively unfamiliar tool for most people. A comparison of the power spectrum and the bispectrum may be helpful. (See also Figure 1.)

Power spectrum

- Function of a single frequency
- Measures the amount of power at that frequency
- Fourier transform of
$$c(\tau_1) = \langle x(t)x(t+\tau_1) \rangle$$
- Expected value of
$$\langle |\hat{x}(\omega_1)|^2 \rangle = \hat{x}(\omega_1)\hat{x}(\omega_2 = -\omega_1)$$

Bispectrum

- Function of two frequencies
- Measures the degree of interaction of frequencies
- Double Fourier transform of
$$c_3(\tau_1, \tau_2) = \langle x(t)x(t+\tau_1)x(t+\tau_2) \rangle$$
- Expected value of
$$\langle \hat{x}(\omega_1)\hat{x}(\omega_2)\hat{x}(\omega_3 = -\omega_1 - \omega_2) \rangle$$

The bispectrum is a welcome addition to the signal processor's toolkit. Again a comparison may be helpful.

Power spectrum

- Sensitive to additive Gaussian noise
- Information *at* frequency components
- Cannot see outside Nyquist band

Bispectrum

- Insensitive to additive Gaussian noise
- Information *between* frequency components
- Can see outside Nyquist band

Although the sensitivity to additive Gaussian noise provides a motive for our interest in the bispectrum (Figure 2), the research done here focuses rather on the ability of the bispectrum to see outside the Nyquist interval. The main contribution of this work is a new understanding of the everyday phenomenon of *aliasing* that is described below.

In order to use bispectra for signal conveyance, it is necessary to determine the amount of power required to transmit a given bispectrum. More generally, one would like to understand the conditions of *joint realizability* of power spectra and bispectra. The second contribution this work makes is significant, though limited, progress in this regard. This work is also described below.

A third contribution, which arose in efforts to broaden our results on joint realizability, is the awareness that a very common inequality, used to normalize the bispectrum, is invalid. This result has considerable significance for the practical use of the bispectrum.

The main results of this work, on aliasing, have been published and efforts to extend this work are underway. The remaining results are being currently being prepared for publication. We expect to present these results this year.

Importance to LANL's Science and Technology Base and National R&D Needs

Higher-order spectra (polyspectra) have unique applications to measurements of time series in the presence of noise and potentially for novel communications systems. The work done under this grant has clarified several theoretical issues of fundamental importance in the interpretation and use of polyspectra. The work on aliasing has led to new insights concerning a topic of fundamental importance in digital signal processing. Additionally, the usefulness of the bispectrum when applied to real-world measurements has been underestimated because of improper normalization: the work done here may lead to considerably greater employment of polyspectra.

Scientific Approach and Accomplishments

1. The Aliasing Results

It is a well-known, but none-the-less remarkable, fact that a continuous time signal can be perfectly reconstructed from a set of its (discretely) *sampled* values. This fact is the content of the Shannon reconstruction theorem. The condition that makes reconstruction possible is that the samples must be taken at a rate at least twice that of the highest frequency present in the signal.

If this condition is violated, i.e.; if the original signal is sampled at a rate less than the *Nyquist* frequency, then the samples do not contain all the information present in the original signal and the original signal can no longer be reconstructed. If one insists on reconstructing a continuous signal from these samples, one gets an *aliased* signal that differs from the original signal. However, this aliased signal is, in general, a perfectly reasonable signal that might have been the source of the original samples. For this reason (and others) aliasing is generally thought to be undetectable.

In 1999, we published a paper (and submitted another) that showed that, under conditions that are very common in practice, aliasing *can* be detected [1, 2]. That is, for the class of signals under consideration, aliased samples are measurably different than unaliased samples. Our first paper used the bispectrum and resolved a long-standing controversy regarding the use of the bispectrum for this purpose. The crucial insight is the role of stationarity as knowledge in addition to the sample values.

Under conditions that are carefully described in the references, the continuous-time signals reconstructed from aliased samples are non-stationary. Therefore detecting aliasing in (at least some) stationary continuous-time processes both makes sense and can be done. This work was extended in a second paper that confirmed, this time without using the bispectrum, that the detection of aliasing is possible. This second paper also introduced a more general notion, that of sample stationarity, which allows aliasing to be detected for signals that can not be said to be stationary in the usual sense. (Figure 3.)

2. The Joint Realizability Results

The detectability of aliasing is of considerable theoretical interest and suggests the possibility of novel communications schemes using the bispectrum. In order to understand what is possible here, it is necessary to determine the amount of power required to transmit a given bispectrum. The total power is the integral of the power spectrum over all frequencies. Thus one is led to consider the general question of the relationship between the power spectrum and the bispectrum of a signal. Given a power spectrum, there need be no bispectrum at all. However, what about the reverse? That is, given a bispectrum, what can one say about the power spectrum? This can be characterized as the issue of *joint realizability* of power spectra and bispectra. Limited, but significant, progress was made in this regard. We were able to obtain, for the first time, lower bounds on the power required to transmit a given bispectrum using a particular class of time series models. This work was done in collaboration with Mark Mineev-Weinstein at Los Alamos and has not yet been published. Efforts to generalize these results in order to obtain model-independent bounds have not yet been successful:

however, these investigations have borne other fruit. In particular, it was learned that a very common inequality, used to normalize the bispectrum, is invalid.

3. The Normalization Results

In practice, a normalization of the bispectrum is commonly used. To normalize the bispectrum, the a quantity called the bicoherence is calculated:

$$b(k,l) = \frac{|\hat{B}(k,l)|^2}{\hat{P}(k)\hat{P}(l)\hat{P}(k+l)}$$

This normalization, referred to as the bicoherence, is commonly believed to measure the percentage of power at frequency $f_m, m = k + l$ due to wave coupling. It is thought to be bounded between zero and one. This belief is unfounded: there are simple examples of signals for which the bound fails. This fact has significant implications for uses of the bispectrum in measurements. A paper describing this work is in progress and will be submitted for publication shortly. This work is being done in collaboration with Professor Melvin Hinich of the University of Texas at Austin.

Publications

1. Vixie, Kevin R., Murray Wolinsky and David E. Sigeti. The Bispectral Aliasing Test: A Clarification and Some Key Examples. *Proceedings of the Fifth Annual International Symposium on Signal Processing and Applications (ISSPA)*, Brisbane, Australia, July, 1999.
2. Vixie, Kevin R., David E. Sigeti, and Murray Wolinsky. Detection of Aliasing in Persistent Signals. Technical report, Los Alamos National Laboratory LA-UR-99-2126, May 1999. (submitted to IEEE Trans. Sig. Proc.)

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Figure 1. The fundamental domain for the power spectrum (left) and for the bispectrum (right).

Figure 2. Bispectrum of a synthetic signal in colored Gaussian noise, showing noise immunity of the bispectrum.

Figure 3. Detection of aliasing in a realistic signal. For an unaliased signal, the dotted curve would coincide with the solid. See reference [2] for further details.

Figures

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