

RELATIVISTIC QUANTUM DYNAMICS OF MANY-BODY SYSTEMS

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Relativistic quantum dynamics requires a unitary representation of the Poincaré group on the Hilbert space of states. The Dynamics of many-body systems must satisfy cluster separability requirements. In this paper we formulate an abstract framework of four-dimensional Euclidean Green functions that can be used to construct relativistic quantum dynamics of N -particle systems consistent with these requirements. This approach should be useful in bridging the gap between few-body dynamics based on phenomenological mass operators and on quantum field theory.

1 Introduction

The superposition principle and the space-time symmetry are realized in relativistic quantum mechanics by a Hilbert space of states with a unitary representation of the inhomogeneous Lorentz group (Poincaré group).¹ Various representations of single particle states are well known.² Since the components of the four-momentum p are constrained by the mass, $p^2 = -m^2$, there is a choice of convenient independent momentum variables in the wave function: For instance the components \vec{p} orthogonal to some fixed time-like vector, or the components p^+ , p_{\perp} orthogonal to some null-vector. For particles with spin the functions representing state vectors are functions of spin variables undergoing Wigner rotations. All these representations are equivalent. States of noninteracting particles are represented by tensor products of single particle states.

In quantum mechanics the Hilbert space of state vectors is the same for free and interacting particles. The interactions are implemented by modifications of the Poincaré generators. Following Bakamjian and Thomas³ this has been done modifying the mass operator, leaving the spin operator independent of interactions. The Poincaré generators obtain as functions of kinematic components of the four momentum operator and canonically conjugate positions, the mass operator, the spin operator. The choice of these kinematic components determines the "form of dynamics".⁴ The principal difficulty in this approach is the realization of cluster separability.⁵ The properties of any isolated cluster of particles should not depend on the presence or absence of other clusters. The solution involves the recursive construction of appropriate many-body interactions in the many-body mass operators.⁶ There are no theorems defining minimal many-body interactions.

Alternatively, single-particle states can be represented by equivalence classes of covariant functions of the four-momentum with the positive semi-definite inner

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product measure $d\mu(p) := d^4p\delta(p^2 + m^2)\theta(p^0)$. For particles with spin the functions representing state vectors are Lorentz covariant functions of spinor indices with a semi-definite inner product measure. Starting from kinematically covariant functions representing multi-particle states, interactions can be introduced modifying the semi-definite inner product measure.⁷

For free fields (free particles) Poincaré generators obtain by integration of the energy-momentum tensor over a three-dimensional manifold in the Minkowski space.⁸ Interactions in the energy-momentum tensor require local commutation relations and infinitely many degrees of freedom. The action of these Poincaré generators on Fock-space vectors produces linear functionals over the Fock space, not vectors in Fock-space. The Hilbert spaces of free and interacting fields are necessarily inequivalent.

Minkowski-Green functions are defined by vacuum expectation values of time-ordered products of local renormalized Heisenberg fields. Using assumed spectral properties of the intermediate states and the asymptotic properties of the field operators there are simple relations to observable bound-state masses, scattering amplitudes and form factors. The principal problem is to establish a quantitative relation of "approximations" to a theory local operators.

A central feature in the formulation of relativistic quantum theory is the absence of finite-dimensional unitary representations of the Lorentz group, $O(1,3)$. This is the reason the representation of states by Lorentz covariant functions requires a semidefinite inner product measure. However, the Lorentz group is related, by complexification, to the orthogonal group in four dimensions, $O(4)$ which does have finite dimensional unitary representations. The unitary representations of the real Euclidean group $E(4)$ together with invariant Green operators are useful in the formulation of Poincaré invariant dynamics. This connection has been exploited extensively in quantum field theory. The equivalence of the Wightman axioms with the Osterwalder-Schrader axioms⁹ establishes that Euclidean Green functions (Schwinger functions) satisfying the Osterwalder-Schrader axioms imply the existence of field operators. In the context of Lagrangean field theory the Schwinger functions obtain as moments of the functional measure defined by the Lagrangean. In the Euclidean formulation the locality axiom is independent of the axioms which establish unitary Poincaré representations with the appropriate spectral properties. This feature is essential for the formulation of Poincaré covariant dynamics dynamics finite many-body systems.

The purpose of this paper is to explore the formulation of relativistic quantum dynamics based on Euclidean invariant Green functions. Ordinary quantum mechanics provides some heuristic indications. The Hilbert space, \mathcal{H} of states of N particles is independent of the dynamics. It is the same for free and interacting particles. The dynamics is specified by the invariant Casimir Hamiltonian, $h := H - \vec{P}^2/2M$ or the resolvent operator $G(E) := 1/(h - iE)$. Interactions are introduced by invariant additions to the Casimir Hamiltonian $h = h_0 + V$, or $G(E)^{-1} = G_0(E)^{-1} + V$. While the approach explored here should be applicable to many qualitatively different physical systems and illuminate relations to quantum field theory the focus of this exploratory study is limited systems with a fixed number of particles, for instance nucleons. The abstract framework of the dynamics

to be explored is formulated in Section 2. Section 3 provides a realization for single particles. Two-body interactions are formulated in Section 4. The many-body dynamics with the realization of cluster separability is discussed in Sec. 5.

2 The Abstract Framework

2.1 The Auxiliary Hilbert Space.

Since there are no finite dimensional unitary representations of the Lorentz group $SO(1,3)$ we assume the representatives of physical states to be a subset of a larger space, subject to the following assumptions.

- H1. Physical states are a linear subset of vectors Ψ in an auxiliary Hilbert space \mathcal{H}_a with the norm $\|\Psi\|_a^2 = \langle \Psi, \Psi \rangle$.
- H2. There is a unitary representation $U(\mathcal{R}, a)$ of the Euclidean group $E(4)$ with $\mathcal{R} \in O(4)$ on the Hilbert space \mathcal{H}_a . The self-adjoint generators of $E(4)$ are denoted by P^μ and $J^{\mu\nu} = -J^{\nu\mu}$.
- H3. There is a self-adjoint, unitary operator Θ , on \mathcal{H}_a , which is invariant under the 3-dimensional Euclidean subgroup

$$[\Theta, P^k] = 0, \quad [\Theta, J^{ik}] = 0 \quad (1)$$

and satisfies

$$\Theta P^0 = -P^0 \Theta, \quad \Theta J^{0k} = -J^{0k} \Theta \quad (2)$$

The two Casimir operators \tilde{j}_\pm^2 of $O(4)$ are functions of the generators $J^{\mu\nu}$,¹⁰

$$j_\pm^k := \frac{1}{\sqrt{2}} \left(\sum_{\mu < \nu} \epsilon^{0k\mu\nu} J^{\mu\nu} \pm J^{0k} \right) \quad (3)$$

The spectra of the Casimir operators are $\sigma(j_\pm^2) = s_\pm(s_\pm + 1)$ with non-negative integer or half-odd integer values of s_\pm .

The operators

$$P^0 := iP^0, \quad P^k := P^k, \quad J^{0k} := iJ^{0k}, \quad J^{ik} := J^{ik} \quad (4)$$

satisfy the Poincaré Lie algebra, and the spectrum of P^2 is the real line. $-\infty < \sigma(P^2) < \infty$. The operators

$$\exp(iP^0 t) \equiv \exp(-P^0 t) \quad \text{and} \quad \exp(iJ^{0k} \chi) \equiv \exp(-J^{0k} \chi) \quad (5)$$

are self-adjoint. Together with the unitary representations of the 3-dimensional Euclidean group they define a non-unitary representation of the Poincaré group. The inner product $(\Psi, \Psi) := \langle \Theta \Psi, \Psi \rangle \equiv \langle \Psi, \Theta \Psi \rangle$ defines a pseudo-Hilbert space¹¹ of the vectors in \mathcal{H}_a . The inner product (Ψ, Ψ) of the pseudo-Hilbert space is Poincaré invariant, $(U(\Lambda, a)\Psi, U(\Lambda, a)\Psi) = (\Psi, \Psi)$.

2.2 Green Operators and the Physical Hilbert Space.

The representation of physical states and the dynamical properties of the system are specified by a Green operator, G , with the following properties.

- G1. The Green operator G is a bounded normal operator on \mathcal{H}_a with an inverse defined on a dense set.
- G2. The Green operator G commutes with P^μ and $J^{\mu\nu}$ and hence with P^μ , $J^{\mu\nu}$.
- G3. The operator ΘG is Hermitean, $\Theta G = G^\dagger \Theta$.
- G4. There is a Poincaré invariant linear manifold \mathcal{S} of vectors $\Psi \in \mathcal{H}_a$ that all $\Psi \in \mathcal{S}$ satisfy the inequalities

$$0 \leq (e^{-iP^0\tau}\Psi, Ge^{-iP^0\tau}\Psi) \equiv (\Psi, Ge^{-2P^0\tau}\Psi) \leq (\Psi, G\Psi) \quad \forall \tau \geq 0 \quad (6)$$

By assumption G4 the inner product $(\Psi, G\Psi)$ of vectors in this manifold is semi-definite. Physical states are represented by equivalence classes of vectors. Two vectors Ψ_a and Ψ_b are equivalent, $\Psi_a \sim \Psi_b$ iff

$$\|\Psi_a - \Psi_b\|^2 := ((\Psi_a - \Psi_b), G(\Psi_a - \Psi_b)) = 0 \quad (7)$$

The physical Hilbert space \mathcal{H} is equipped with a unitary representation of the Poincaré group. Single-particle states Ψ_M of mass M , elementary or composite, satisfy $(P^2 + M^2)\Psi_M \sim 0$.

2.3 Perturbations of Green Operators

A perturbation $G_0 \rightarrow G$ may be defined by

$$G^{-1} := G_0^{-1} + \mathcal{U} \quad (8)$$

where \mathcal{U} is an $E(4)$ invariant, pseudo-Hermitean operator with domain $D(\mathcal{U}) \supset D(G_0^{-1})$. By assumption \mathcal{U} is bounded relative to G_0^{-1} ,

$$\|\mathcal{U}\Psi\|_a \leq a\|G_0^{-1}\Psi\|_a + b\|\Psi\|_a, \quad (9)$$

with $0 \leq a < 1$ and $0 \leq b$. The operators $\mathcal{U}G_0$, $G_0\mathcal{U}$ and are bounded with a bound less than 1. It follows that $G^{-1}G_0$, and G_0G^{-1} are bounded operators with bounded inverses.

3 Realization for Single Particles

3.1 Single Particles with Spin 0

The auxiliary Hilbert space \mathcal{H}_a of a spin-zero single particle is realized by square integrable functions $\Psi(x)$ with $x := \{x^0, x^1, x^2, x^3\}$ with the inner product

$$\langle \Psi, \Psi \rangle = \int d^4x |\Psi(x)|^2 \quad (10)$$

Schwartz functions $f(x)$ are dense in this Hilbert space. The involution operator Θ is defined by

$$\Theta\Psi(x) := \Psi(-x^0, \vec{x}) \quad (11)$$

The self-adjoint generators of the real Euclidean group $E(4)$ are

$$P^\mu := \frac{1}{i} \frac{\partial}{\partial x^\mu}, \quad J^{\mu\nu} := \frac{1}{i} \left(x^\mu \frac{\partial}{\partial x^\nu} - x^\nu \frac{\partial}{\partial x^\mu} \right) \quad (12)$$

The associated Poincaré generators are then defined by eq. (4). The Green operator is represented by the Green function $G(x - x')$

$$\begin{aligned} G(x - x') &:= \left(\frac{1}{2\pi} \right)^4 \int d^4 p \frac{\exp[ip(x - x')]}{p^2 + m^2} \\ &= \left(\frac{1}{2\pi} \right)^3 \int d^3 \vec{p} \frac{\exp\left(i\vec{p}(\vec{x} - \vec{x}') - \omega(p)|x^0 - x^0|\right)}{2\omega(p)}. \end{aligned} \quad (13)$$

where $\omega(p) := \sqrt{m^2 + \vec{p}^2}$. Schwartz functions $f(x)$ with support restricted to positive values of x^0 represent a linear manifold of vectors in \mathcal{H}_a which satisfies the the requirement G4. With the support restriction it follows that

$$(f, Gf) = \langle \tilde{f}, \tilde{f} \rangle := \int \frac{d^3 p}{2\omega(p)} |\tilde{f}(\vec{p})|^2 = \int d^4 x |\tilde{f}(x)|^2 \quad (14)$$

where

$$\begin{aligned} \tilde{f}(\vec{p}) &:= (2\pi)^{-\frac{3}{2}} \int d^4 x e^{-i\vec{p}\cdot\vec{x}} e^{-\omega(p)x^0} f(x), \\ \tilde{f}(x) &:= (2\pi)^{-\frac{3}{2}} \int d^3 p e^{i\vec{p}\cdot\vec{x} - \omega(p)x^0} \theta(x^0) \tilde{f}(\vec{p}) \end{aligned} \quad (15)$$

The Hilbert space \mathcal{H} is constructed by the usual procedure of moding out zero-norm vectors and adding Cauchy sequences. The equivalence classes of Schwartz functions are dense in \mathcal{H} . Two functions f_1 and f_2 are equivalent, $f_1 \sim f_2$ iff

$$\|f_1 - f_2\| = 0. \quad (16)$$

It follows that two functions f_1 and f_2 are in the same equivalence class, $f_1 \sim f_2$ iff $\tilde{f}_1 = \tilde{f}_2$.

Since

$$\theta(x^0)\Theta G(x' - x)\theta(x^0) = \theta(x^0)\theta(x^0) \left(\frac{1}{2\pi} \right)^3 \int d^3 p \frac{e^{-\omega(p)(x'^0 + x^0)} e^{i\vec{p}\cdot(\vec{x}' - \vec{x})}}{2\omega(p)}. \quad (17)$$

it follows that the inner product $(f_a, G f_b)$ of functions satisfying the support condition is manifestly Lorentz invariant,

$$(f_b, G f_a) = \int \frac{d^3 p}{\omega(p)} \tilde{f}_b(\vec{p})^* \tilde{f}_a(\vec{p}) = \int d^4 p \delta(p^2 + m^2) \theta(p^0) f_b(p)^* f_a(p) \quad (18)$$

with

$$f(p) := \int d^4 x \exp(-i\vec{p}\cdot\vec{x} - p^0 x^0) f(x). \quad (19)$$

The time evolution evolution $f_a(t) := e^{-iP^0 t} f_a$ is given explicitly by

$$(f_b, G e^{-iP^0 t} f_a) = \langle \tilde{f}_b, \tilde{f}_a(t) \rangle \quad (20)$$

with $\tilde{f}_a(t, p) := e^{-i\omega(p)t} \tilde{f}_a(\vec{p})$.

3.2 Single Particles with Spin $\frac{1}{2}$

For a single spin 1/2 particle the Green function is

$$G(x-y) := \frac{1}{(2\pi)^4} \int d^4 p e^{ip \cdot (x-y)} \left(\frac{p \cdot \gamma_e + m}{p^2 + m^2} \right). \quad (21)$$

where the spinor matrices $\gamma_e := i\beta, \beta\vec{\alpha}$ with $\vec{\alpha} := \gamma_5 \vec{\sigma}$ satisfy

$$\frac{1}{2} \{ \gamma_{e\mu}, \gamma_{e\nu} \} = -\delta_{\mu\nu}, \quad (22)$$

The involution operator Θ must also act on the spinor indices,

$$(\Theta f)(x) := \beta f(-x^0, \vec{x}). \quad (23)$$

As in the case of spin 0 it is easy to verify positivity of the inner product (f, Gf) for Schwartz functions with support restricted to positive values of x^0 .

$$\begin{aligned} (f, Gf) &:= \int d^4 x d^4 y f^\dagger(x) \Theta G(x-y) f(y) = \int d^4 x d^4 y f^\dagger(x) \beta G(-x^0 - y^0, \vec{x} - \vec{y}) f(y) \\ &= \int d^3 p \tilde{f}^\dagger(p) \frac{\omega(p) + \vec{\alpha} \cdot \vec{p} + \beta m}{2\omega(p)} \tilde{f}(p) = \int d^4 x \tilde{f}^\dagger(x) \tilde{f}(x) \geq 0, \end{aligned} \quad (24)$$

where

$$\tilde{f}(x) := (2\pi)^{-\frac{3}{2}} \int d^3 p e^{i\vec{p} \cdot \vec{x} - \omega(p)x^0} \theta(x^0) \frac{\omega(p) + \vec{\alpha} \cdot \vec{p} + \beta m}{2\omega(p)} \tilde{f}(\vec{p}). \quad (25)$$

4 Two-Body Dynamics

The auxiliary Hilbert space \mathcal{H}_a is the tensor product of the single-particle auxiliary Hilbert spaces. The involution operator Θ is the tensor product of single particle involution operators. Schwartz functions $f(x_1, x_2) \equiv f(X, x)$ with $X := \frac{1}{2}(x_1 + x_2)$ and $x := x_1 - x_2$ are dense in this Hilbert space. For spin 1/2 particles these functions depend on spinor variables as well. The $E(4)$ generators are additive.

$$P^\mu = \frac{1}{i} \sum_{n=1}^2 \frac{\partial}{\partial x_n^\mu}, \quad J^{\mu\nu} = \frac{1}{i} \sum_{n=1}^2 \left(x_n^\mu \frac{\partial}{\partial x_n^\nu} - x_n^\nu \frac{\partial}{\partial x_n^\mu} + \frac{1}{4} [\gamma_{ne}^\mu, \gamma_{ne}^\nu] \right). \quad (26)$$

In general Green operators are realized by $E(4)$ invariant tempered distributions $G(x_1, x_2; x'_2, x'_1)$. For free particles the Green function is the product of single particle Green functions, Interactions are added according to eq. (8). A simple example of a nucleon-nucleon interaction is of the form

$$U\Psi(X, x) = \left\{ V_S(x) + \gamma_5^{(1)} \gamma_5^{(2)} V_P(x) + \sum_{\mu, \nu} [\gamma_{e\mu}^{(1)}, \gamma_{e\nu}^{(1)}] [\gamma_{e\mu}^{(2)}, \gamma_{e\nu}^{(2)}] V_T(x) \right\}$$

$$+ (\gamma_e^{(1)} \cdot \gamma_e^{(2)}) V_V(x) + (\gamma_5^{(1)} \gamma_e^{(1)} \cdot \gamma_e^{(2)} \gamma_5^{(2)}) V_A(x) \} \Psi(x, x) \quad (27)$$

Scattering wave functions obtain by the weak time limits

$$(\Psi_b, G\Omega_{a\pm}) = \lim_{t=\pm\infty} (\Psi_b, Ge^{iP^0 t} \tilde{\Psi}_a(t)) \quad (28)$$

where $\tilde{\Psi}_a(t) := \tilde{\Psi}_a^{(1)}(t) \times \tilde{\Psi}_a^{(2)}(t)$ is the tensor product of single particle states.¹² The S -matrix obtains in the limit

$$S_{ba} = \lim_{t'=\pm\infty} \lim_{t=-\infty} (\tilde{\Psi}_b(t'), Ge^{iP^0(t-t')} \tilde{\Psi}_a(t)) \quad (29)$$

5 Many-Body Dynamics

The auxiliary Hilbert space \mathcal{H}_a is the N -fold tensor product of the single-particle auxiliary Hilbert spaces. The involution operator Θ is the outer product of single particle involution operators. Schwartz functions, $f(x_1, \dots, x_N)$, of N points and N spinor indices are dense in this Hilbert space. The $E(4)$ generators are additive.

$$P^\mu = \frac{1}{i} \sum_{n=1}^N \frac{\partial}{\partial x_n^\mu}, \quad J^{\mu\nu} = \frac{1}{i} \sum_{n=1}^N \left(x_n^\mu \frac{\partial}{\partial x_n^\nu} - x_n^\nu \frac{\partial}{\partial x_n^\mu} + \frac{1}{4} [\gamma_{ne}^\mu, \gamma_{ne}^\nu] \right). \quad (30)$$

In general Green operators are realized by $E(4)$ invariant tempered distributions $G_N(x_1, \dots, x_N; y_N, \dots, y_1)$. The cluster properties can be conveniently realized using formal annihilation operators $a(x)$, $b(x)$ and creation operators $a^\dagger(x)$, $b^\dagger(x)$ which satisfy the commutation relations¹³

$$\{a(x), a^\dagger(y)\} = \delta^{(4)}(x-y), \quad \{b(x), b^\dagger(y)\} = \delta^{(4)}(x-y) \\ \{a(x), b(y)\} = \{a(x), b^\dagger(y)\} = 0. \quad (31)$$

The cluster structure of the general N -particle Green functions is realized by the expression¹⁴

$$G_N(x_1, \dots, x_N; y_N, \dots, y_1) = \langle 0 | a(x_1) \cdots a(x_N) b(y_N) \cdots b(y_1) \exp \left(\sum_n S_n \right) | 0 \rangle. \quad (32)$$

where $|0\rangle$ is the cyclic vector that is annihilated by $a(x)$, $b(x)$, and

$$S_n := \frac{1}{n!^2} \int d^4 x_1 \cdots \int d^4 y_1 \cdots \\ \times a^\dagger(x_n) \cdots a^\dagger(x_1) b^\dagger(y_1) \cdots b^\dagger(y_n) S_n(x_1, \dots, x_n; y_n, \dots, y_1) \quad (33)$$

The functions S_n vanish for separation of the points into widely separated clusters. Representations of the inverse Green operators have the same cluster structure.

$$G^{-1} = \exp \left[\int d^4 x \int d^4 y a^\dagger(x) b^\dagger(y) S^{-1}(x-y) + \sum_{n \geq 2} U_n \right] \quad (34)$$

A many-body Green operator can be completely determined by two-body interactions, $U_n \equiv 0$ for $n > 2$,

$$U_2 = \frac{1}{4} \int d^4x_1 \int d^4x_2 \int d^4y_1 \int d^4y_2 a^\dagger(x_1) a^\dagger(x_2) b^\dagger(y_2) b^\dagger(y_1) U(x_1, x_2; y_2, y_1) \quad (35)$$

6 Conclusions

Kinematic Poincaré covariance of state vectors of many-body systems requires a dynamically determined semi-definite inner product measure. An effective realization is based on an auxiliary Hilbert space endowed with a unitary representation of the four-dimensional Euclidean group. A self-adjoint unitary involution operator provides a Poincaré invariant indefinite inner product. The Euclidean invariant Green operator specifies the Poincaré invariant semi-definite inner product of the subspace of physical states. In this framework two-body Green operators are sufficient to determine many-body Green functions satisfying cluster separability.

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