INTRODUCTION
Raleigh-Taylor (RT) instabilities occur when a low-density fluid attempts to accelerate a high-density fluid. The instability manifests itself by forming bubbles of the low-density that percolate through the high-density fluid. Concurrently, spikes of the high-density fluid "jet" into the low-density fluid. This instability is seen in familiar occurrences such as the growth of icicles and the draining of sinks. If the interface between the two fluids remains perfectly smooth the instability will not occur; that is, it must be "seeded" by a wave-like perturbation. For the two cases mentioned above, a vibration can be sufficient to perform the "seeding".

The implosion of an Inertial Confinement Fusion (ICF) target often requires that a low-density fluid accelerate a high-density fluid. This leads to the possibility of developing RT instabilities that can produce a non-uniform implosion and drastically reduce the energy coming from the target. The "seed" for these instabilities comes from manufacturing defects such as surface finish or form errors. An illustration of manufacturing-induced RT instabilities is seen in figure 1[1] where the magnetic field acts as a low-density fluid that was imploding an aluminum cylinder. The high-density fluid in this case was the aluminum cylinder wall. The interface between the fluids was "seeded" by two different sine waves that were machined into the aluminum surface. The wavelength of the sine wave seen in the upper portion of the cylinder shown in figure 1 was 2 mm. The wavelength of the lower sine wave was 0.75 mm. Both sine waves had an amplitude of 25 μm. An examination of the data presented in figure 1 shows that as time increases the instabilities grow; however, the longer wavelength sine wave grows at a faster rate than the one with the shorter wavelength. These data, along with other data, have shown that there is both a wavelength and amplitude dependence on RT instability growth [2]. Since there will always be manufacturing defects in ICF targets and since RT instability growth is a rather complicated physical phenomena, a significant theoretical and experimental effort has been dedicated to understanding this phenomena.

TARGET FOR OBSERVING RT INSTABILITY GROWTH LATE IN TIME
A series of experiments have been performed at the Omega and Nova lasers to understand RT instability growth during the initial growth stages [2]. Since the laser used to illuminate the x-ray backlighters is only on for a time-duration of one nanosecond, observation of a fully developed instability can not be accomplished. Therefore, a target needed to be produced that would adequately simulate the geometry of a RT instability midway through its development. Drawings of the experimental package are shown in figure 2. It was anticipated that the 30 μm tall by 10 μm thick spikes and the 5 μm thick base would be difficult to fabricate. It was decided to begin two methods of production: machining and photolithography.

MACHINED TARGET PACKAGE
The machined targets were made according to the drawing in figure 2b. Approximately 10 targets of this style were needed for these experiments. It was decided to approximate the straight spikes by machining them on the rim a disc that was 70 mm in diameter. An aluminum disc was machined with the appropriate reference surfaces being diamond turned so that the tool location
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SNS SUPERCONDUCTING CAVITY MODELING
-ITERATIVE LEARNING CONTROL (ILC)

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Abstract

The SNS SRF system is operated with a pulsed beam. For the SRF system to track the repetitive reference trajectory, a feedback and a feedforward controllers has been proposed. The feedback controller is to guarantee the closed loop system stability and the feedforward controller is to improve the tracking performance for the repetitive reference trajectory and to suppress the repetitive disturbance. As the iteration number increases, the error decreases.

1 INTRODUCTION

The Spallation Neutron Source (SNS) Linac to be built at Oak Ridge National Laboratory (ORNL) consists of a combination of low energy normal conducting (NC) accelerating structures as well as higher energy superconducting RF (SRF) structures. In order to efficiently provide a working control system, a lot of modeling has performed. The modeling is used as a way to specify RF components; verify system design and performance objectives; optimize control parameters; and to provide further insight into the RF control system operation.

The modeling addressed in this note deals with the PI feedback controller and the plug-in feedforward controller (the iterative learning controller). The purpose of the PI feedback controller is to guarantee the robustness and the zero steady state error. However, the PI feedback controller does not yield the satisfactory transient performances for the RF filling and the beam loading. The feedforward controller proposed in this note takes a simple form and is effective. In order to generate the one step ahead feedforward control, the feedforward controller makes use of current error, the derivative of the current error and the integration of the current error. This PID-type feedforward controller is the natural consequence of the PI feedback control system where the inverse of the closed loop system transfer matrix has the same form as the transfer matrix of the PID system. The proposed feedforward controller achieves the better performance for the repetitive reference trajectory to be tracked by the system output and achieves the suppression of the repetitive disturbance such as the Lorentz Force Detuning.

2 SUPERCONDUCTING CAVITY MODEL

The modeling of a superconducting cavity is based on the assumption that the RF generator and the cavity are connected with a transformer. The equivalent circuit of the cavity is transformed to the equivalent circuit of RF generator with transmission line (wave guide) and the model is obtained[3]. A superconducting cavity is represented by the state space equation.

\[ \dot{x} = A(\Delta \omega_L)x + B(\Delta \omega_L)u + B_I(\Delta \omega_L)I \]  
\[ y = C(\Delta \omega_L)x \]

and the Lorentz force detuning is

\[ \dot{\Delta \omega_L} = -\frac{1}{\tau_m} \Delta \omega_L - \frac{2\pi}{\tau m} K_x \frac{2\pi}{\tau m} K_x \]  

where

\[ A(\Delta \omega_L) = \begin{bmatrix} \frac{-1}{\tau_L} & -2(\Delta \omega_m + \Delta \omega_L) \frac{1}{\tau_L} \\ (\Delta \omega_m + \Delta \omega_L) & \frac{1}{\tau_L} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ B(\Delta \omega_L) = \begin{bmatrix} 2c_1 & 2c_3 & 2c_0 \end{bmatrix}, \quad B_I(\Delta \omega_L) = \begin{bmatrix} -2x_1 & -2x_2 \end{bmatrix} \]

\[ c_1 = \frac{R_{cu}}{\tau}, \quad c_3 = \frac{R_{cu}}{2Q_o \tau}, \quad \kappa = \kappa \left[ \frac{E_o [MV/m]}{V_{gap}[V]} \right]^2 \]

\[ \zeta : \text{Transformation ratio}, \quad Q_o : \text{Unloaded } Q \]

\[ R_{cu} : \text{Resistance of the cavity equivalent circuit} \]

\[ \Delta \omega_m : \text{Detuning frequency}[\text{rad/s}] \]

\[ Z_o : \text{Transmission line impedance} \]
\( \tau_L \): Loaded cavity damping constant
\( \tau \): Unloaded cavity damping constant
\( \tau_m \): Mechanical time constant
\( K \): Lorentz Force Detuning Constant
\( u = [v_I, v_Q]^T \): forward Voltage in I/Q
\( I = [I_I, I_Q]^T \): Beam current in I/Q
\( x = [v_I, v_Q]^T \): Cavity Field in I/Q

The modeling of the cavity is based on the assumption that the exact characteristics, parameters of a cavity are known. When there are parameter perturbations, unknown deterministic disturbances and random noises in the input channels or measurement channels, those uncertainties are added to the state equation or the output equation. For the control of this uncertain system, modern robust controllers such as \( H_\infty \) controller, loop-shaping controller are applied. On the other hand, PI (PID) controllers are designed by using \( H_\infty \) controller, loop-shaping controller design techniques.

3 ITERATIVE LEARNING CONTROL

The SNS SRF system is operated with a pulsed beam. The period of the beam pulse is 16.67 msec (1/60 Hz). The objective of the SRF controller is to generate a periodic reference trajectory whose period is 16.67 msec (1/60 Hz) and to achieve a stable cavity field periodically so that the RF power is delivered to the periodic beam pulse safely. A control system that is suited for this type of applications is Iterative Learning Control (ILC) [1],[2].

Consider a controller at the \( k \)th iteration,
\[
\begin{align*}
    u^k &= u^k_C + u^k_F \\
    u^k_C &= A_c x^k + B_c r^k \\
    u^k_F &= S_e(s)B^{-1}(sl - A(\Delta \omega_L))R(s)
\end{align*}
\]

where \( A_c = A(\Delta \omega_L) - BK_P \). Since \( c_1 \gg c_3 \), with the proper diagonal terms and zero off-diagonal terms of the gain matrices \( K_P \) and \( K_I \) of the PI controller, the diagonal terms of the matrix \( A(\Delta \omega_L) - BK_P \) and the matrix \( BK_I \) are sufficiently large and so the I channel error and the Q channel error (4) are almost decoupled.

The Laplace transform of the error equation (4) yields
\[
E^k(s) = -S_e(s)U^k_F(s) - S_e(s)B^{-1}B_I B(s)
\]
\[
+ S_e(s)B^{-1}(sl - A(\Delta \omega_L))R(s)
\]
where
\[
S_e(s) = \left( sl - A_c + \frac{1}{s} BK_I \right)^{-1} B
\]

Define the learning control rule as follows.
\[
U^k_F(k+1) = Q(f \cdot U^k_F + \alpha \cdot LE^k)
\]

where \( f \), \( 0 < f < 1 \), is called the forgetting factor and \( \alpha \), \( 0 < \alpha < 1 \), is a design constant. The forgetting factor \( f \) and the constant \( \alpha \) are to guarantee the robust stability against uncertainties in the plant model and the nonlinearity of the klystron. They also allow for elimination of the influence of random noise, spikes and glitches. \( U^k_F \) is the Laplace transform of the feedforward signal in iteration \( k \) and \( E^k \) is the Laplace transform of the corresponding tracking error. Learning converges if the feedback loop is stable and the following condition holds. For \( \forall \omega \in \mathbb{R} \),
\[
\lim_{k \to \infty} \left\| U^k_F(j\omega) - U^k_F(j\omega) \right\|_\infty < \lim_{k \to \infty} \left\| U^k_F(j\omega) - U^k_F(j\omega) \right\|_\infty < 1
\]
which results in learning convergence condition
\[
\lim_{k \to \infty} \left\| Q(f \cdot I - \alpha \cdot LS_e) \right\|_\infty < 1
\]

The \( Q \)-filter is designed such that it suppresses the high frequency components at which the plant model is inaccurate and pass low frequency, at which the model is accurate. The \( Q \)-filter is either placed before the memory, or in the memory feedback loop. Thus, the bandwidth of the \( Q \)-filter should be chosen greater than or equal to the desired closed loop bandwidth. From the \( H_\infty \) controller design point of view, (8) interprets the \( Q \)-filter as a weighting function for learning performance, i.e.,
It seems natural that the Q-filter is viewed as a measure of learning performance and the cut-off frequency $\omega_c$ of the Q-filter is chosen as large as possible in order to guarantee zero tracking error up to frequency $\omega_c$.

To design a L-filter, detailed knowledge of the plant is required. For low frequency dynamics, a competent model of the plant often exists. However, identification and modeling of high frequency dynamics is difficult and may lead to an inadequate model. This could result in a learning filter $L$ that compensates well for low frequencies but does not compensate appropriately for all high frequencies and therefore causes unstable behavior. This unstable behavior is prevented by the Q-filter and to determine $\omega_c$, a trade-off between the performance and the robust stability is necessary. An intuitive synthesis of the learning L-filter for given Q-filter is as follow.

$$L(s) = S_e^{-1}(s) = \left( sI - A_c + \frac{1}{s}BK_I \right)B^{-1}$$  \(\text{(10)}\)

When the feedback PI controller gain matrix $K_I$ is defined as a diagonal matrix, then (10) is reduced to

$$L(s) = sB^{-1} - (A(\Delta \omega_L)B^{-1} - K_F) + \frac{1}{s}K_I$$  \(\text{(11)}\)

Equation (11) shows that the learning L-filter has the characteristics of PID.

3 SIMULATION

The closed loop system with PI feedback controller and iterative learning controller was simulated. Figure 1 and figure 2 show the field amplitude and the field phase, where the great improvement of the transient behaviors both in RF filling and in beam loading is observed as iteration number increases. Also, two figures show that the periodic Lorentz Force Detuning effect on the field amplitude and the field phase is suppressed gradually as the iteration number increases. Figure 3 shows the Lorentz Force Detuning. Note that the static value of the Lorentz Force Detuning calculated with the cavity data ($K = -2.0$ Hz/(MV/m)$^2$, $E_{acc} = 11.9$ MV/m) is -283 Hz. With the RF On period 1.3 msec (300 $\mu$sec field settling period + 1000 $\mu$sec beam period), the Lorentz Force Detuning is developed up to -200 Hz.

REFERENCES

