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On $|V_{ub}|$ from the $\bar{B} \to X_u \ell \bar{\nu}$ dilepton invariant mass spectrum*

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The invariant mass spectrum of the lepton pair in inclusive semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decay yields a model independent determination of $|V_{ub}|$.[1] Unlike the lepton energy and hadronic invariant mass spectra, nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the $\bar{B} \to X_c \ell \bar{\nu}$ background. We discuss these backgrounds for q^2 slightly below $(m_B - m_D)^2$, and point out that instead of $q^2 > (m_B - m_D)^2 = 11.6\,\mathrm{GeV}^2$, the cut can be lowered to $q^2 \gtrsim 10.5\,\mathrm{GeV}^2$. This is important experimentally, particularly when effects of a finite neutrino reconstruction resolution are included.

A precise and model independent determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is important for testing the Standard Model at B factories via the comparison of the angles and the sides of the unitarity triangle. At the present time the allowed range for $\sin 2\beta$ in the SM is largely controlled by the model dependent theory error in $|V_{ub}|$.

If it were not for the huge background from decays to charm, it would be straightforward to determine $|V_{ub}|$. Inclusive B decay rates can be computed model independently in a series in $\Lambda_{\mathrm{QCD}}/m_b$ and $\alpha_s(m_b)$ using an operator product expansion (OPE),[2–5] and the result may schematically be written as

$$\mathrm{d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \frac{\alpha_s}{\pi}\,(\ldots) + \frac{\alpha_s^2}{\pi^2}\,(\ldots) + \ldots \right\}. \tag{1}$$

At leading order, the B meson decay rate is equal to the b quark decay rate. The leading nonperturbative corrections of order $\Lambda_{\mathrm{QCD}}^2/m_b^2$ are characterized by two heavy quark effective theory (HQET) matrix elements, usually called λ_1 and λ_2. These matrix elements also occur in the expansion of the B and B^* masses in powers of $\Lambda_{\mathrm{QCD}}/m_b$,

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots, \qquad m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} + \ldots. \tag{2}$$

Similar formulae hold for the D and D^* masses. The parameters $\bar{\Lambda}$ and λ_1 are independent of the heavy b quark mass, while there is a weak logarithmic scale dependence in λ_2. The measured $B^* - B$ mass splitting fixes $\lambda_2(m_b) = 0.12\,\mathrm{GeV}^2$, while $\bar{\Lambda}$ and λ_1 (or, equivalently, a short distance b quark mass and λ_1) may be determined from other physical quantities.[6–8] Thus, a measurement of the total $B \to X_u \ell \bar{\nu}$ rate would provide a $\sim 5\%$ determination of $|V_{ub}|$.[9,10]

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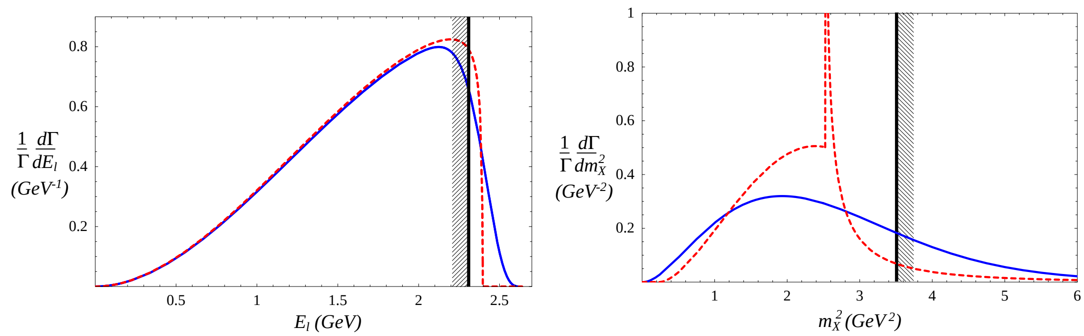

Figure 1: The shapes of the lepton energy and hadronic invariant mass spectra. The dashed curves are the b quark decay results to $\mathcal{O}(\alpha_s)$, while the solid curves are obtained by smearing with the model distribution function $f(k_+)$ in Eq. (5). The unshaded side of the vertical lines indicate the region free from charm background.

Unfortunately, the $\bar{B} \to X_u \ell \bar{\nu}$ rate can only be measured imposing cuts on the phase space to eliminate the ~ 100 times larger $\bar{B} \to X_c \ell \bar{\nu}$ background. The predictions of the OPE are only model independent for sufficiently inclusive observables, when hadronic final state with

$$m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2 \tag{3}$$

are allowed to contribute. Two kinematic regions for which the charm background is absent have received much attention: the large lepton energy region, $E_\ell > (m_B^2 - m_D^2)/2m_B$, and the small hadronic invariant mass region, $m_X < m_D$.[11,12] However, in both of these regions of phase space the $\bar{B} \to X_u \ell \bar{\nu}$ decay products are dominated by high energy, low invariant mass hadronic states, for which the inequality (3) is violated and the OPE breaks down. This occurs because the OPE includes the expansion parameter $E_X \Lambda_{\rm QCD}/m_X^2$ which becomes of order unity ($m_b \Lambda_{\rm QCD}/m_c^2 \sim 1$ numerically) for $E_X \sim m_b$ and $m_X \sim m_c$. To predict the rates in these regions, the complete series in $E_X \Lambda_{\rm QCD}/m_X^2$ must be resummed into a nonperturbative light-cone distribution function $f(k_+)$ for the b quark.[13] To leading order in $1/m_b$, the effects of the distribution function on various spectra[12,14] may be included by replacing m_b by $m_b^* \equiv m_b + k_+$ in the parton level spectrum, $\mathrm{d}\Gamma_{\rm p}$, and integrating over the light-cone momentum

$$\mathrm{d}\Gamma = \int \mathrm{d}k_+ \, f(k_+) \, \mathrm{d}\Gamma_{\rm p}\Big|_{m_b \to m_b^*} . \tag{4}$$

The situation is illustrated in Fig. 1, where we have plotted the lepton energy and hadronic invariant mass spectra in the parton model (dashed curves) and smeared with a simple one-parameter model for the distribution function (solid curves)[15]

$$f(k_+) = \frac{32}{\pi^2 \Lambda} \, (1-x)^2 \, \exp\left[-\frac{4}{\pi}(1-x)^2\right] \Theta(1-x) \,, \qquad x \equiv \frac{k_+}{\Lambda} \,, \qquad \Lambda = 0.48 \, \mathrm{GeV} \,. \tag{5}$$

While it may be possible to extract $f(k_+)$ from the $B \to X_s \gamma$ photon spectrum,[13,16] unknown order $\Lambda_{\rm QCD}/m_b$ corrections are left over, limiting the accuracy with which $|V_{ub}|$ may be obtained.

The situation is very different for the dilepton invariant mass spectrum. Decays with $q^2 \equiv (p_\ell + p_{\bar{\nu}})^2 > (m_B - m_D)^2$ must arise from $b \to u$ transition. Such a cut forbids the hadronic final state from moving fast in the B rest frame, and simultaneously imposes $m_X < m_D$ and $E_X < m_D$. Thus, the light-cone expansion which gives rise to the shape function is not relevant in this region of phase space.[12,17] This is also clear from Eq. (6): the contribution of the λ_1 term to the decay rate, which is the first term in the shape function, is suppressed compared to the lowest order term in the OPE for any value of q^2. The effect of smearing the q^2 spectrum with the model distribution function in Eq. (5) is illustrated in Fig. 2. It is clearly a subleading effect. The improved behavior of the q^2 spectrum over the E_ℓ and m_X^2 spectra is also reflected in

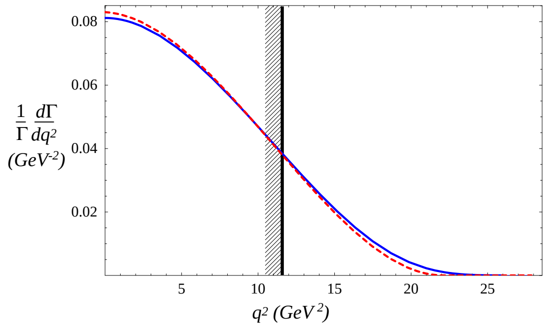

Figure 2: The dilepton invariant mass spectrum. The notation is the same as in Fig. 1.

the perturbation series. There are Sudakov double logarithms near the phase space boundaries in the E_ℓ and m_X^2 spectra, whereas there are only single logarithms in the q^2 spectrum.

The $\bar{B} \to X_u \ell \bar{\nu}$ decay rate with lepton invariant mass above a given cutoff can be reliably computed working to a fixed order in the OPE (i.e., ignoring the light-cone distribution function),

$$\frac{1}{\Gamma_0}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{q}^2} = \left(1 + \frac{\lambda_1}{2m_b^2}\right) 2\,(1-\hat{q}^2)^2\,(1+2\hat{q}^2) + \frac{\lambda_2}{m_b^2}\,(3 - 45\hat{q}^4 + 30\hat{q}^6) \\ + \frac{\alpha_s(m_b)}{\pi}\,X(\hat{q}^2) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \beta_0\,Y(\hat{q}^2) + \ldots\,, \tag{6}$$

where $\hat{q}^2 = q^2/m_b^2$, $\beta_0 = 11 - 2n_f/3$, and $\Gamma_0 = G_F^2\,|V_{ub}|^2\,m_b^5/(192\,\pi^3)$ is the tree level $b \to u$ decay rate. The ellipses in Eq. (6) denote terms of order $(\Lambda_{\rm QCD}/m_b)^3$ and order α_s^2 terms not enhanced by β_0. The function $X(\hat{q}^2)$ is known analytically,[18] whereas $Y(\hat{q}^2)$ was computed numerically.[19] The order $1/m_b^3$ nonperturbative corrections are also known,[20] as are the leading logarithmic perturbative corrections proportional to $\alpha_s^n \log^n(m_c/m_b)$.[21] The matrix element of the kinetic energy operator, λ_1, only enters the $\hat{q}^2$ spectrum in a very simple form, because the unit operator and the kinetic energy operator are related by reparameterization invariance.[22]

The relation between the total $\bar{B} \to X_u \ell \bar{\nu}$ decay rate and $|V_{ub}|$ is known at the $\sim 5\%$ level,[9,10]

$$|V_{ub}| = (3.04 \pm 0.06 \pm 0.08) \times 10^{-3} \left(\frac{\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})|_{q^2 > q_0^2}}{0.001 \times F(q_0^2)}\,\frac{1.6~\mathrm{ps}}{\tau_B}\right)^{1/2}\,, \tag{7}$$

where $F(q_0^2)$ is the fraction of $\bar{B} \to X_u \ell \bar{\nu}$ events with $q^2 > q_0^2$, satisfying $F(0) = 1$. The errors explicitly shown in Eq. (7) are the estimates of the perturbative and nonperturbative uncertainties in the upsilon expansion[9] respectively. At the present time the biggest uncertainty is due to the error of a short distance b quark mass, whichever way it is defined.[21] (This can be cast into an uncertainty in an appropriately defined $\bar{\Lambda}$, or the nonperturbative contribution to the $\Upsilon(1S)$ mass, etc.) By the time the q^2 spectrum in $\bar{B} \to X_u \ell \bar{\nu}$ is measured, this uncertainty should be reduced from extracting m_b from the hadron mass[6] or lepton energy[7] spectra in $\bar{B} \to X_c \ell \bar{\nu}$, or from the photon energy spectrum[8] in $B \to X_s \gamma$. The uncertainty in the perturbation theory calculation will be largely reduced by computing the full order α_s^2 correction in Eq. (7). The largest "irreducible" uncertainty is from order $\Lambda_{\rm QCD}^3/m_b^3$ terms in the OPE, the estimated size of which is shown in Fig. 3, together with our central value for $F(q_0^2)$, as functions of q_0^2.

There is another advantage of the q^2 spectrum over the m_X spectrum to measure $|V_{ub}|$. In the variable m_X, about 20% of the charm background is located right next to the $b \to u$ "signal region", $m_X < m_D$, namely $\bar{B} \to D \ell \bar{\nu}$ at $m_X = m_D$. In the variable q^2, the charm background just below $q^2 = (m_B - m_D)^2$ comes from the lowest mass X_c states. Their q^2 distributions are well understood based on heavy quark symmetry,[23] since this region corresponds to near

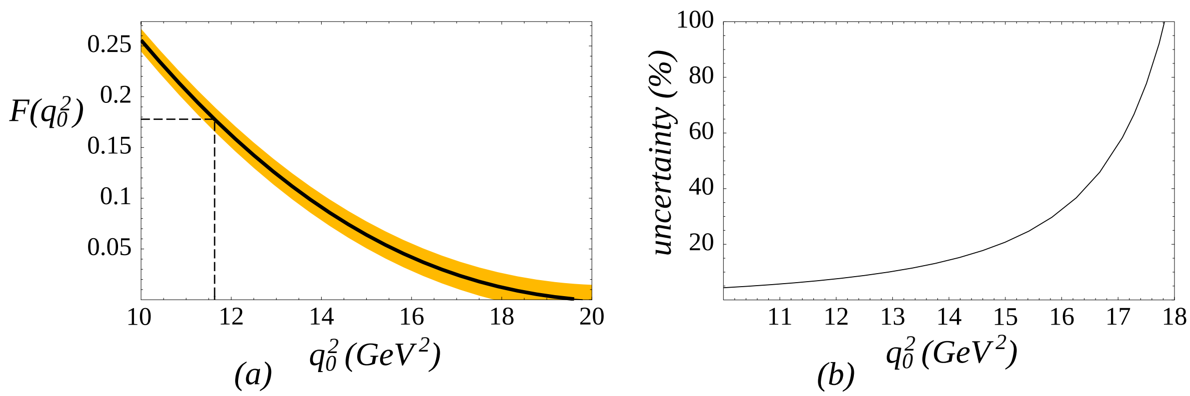

Figure 3: (a) The fraction of $\bar{B} \to X_u \ell \bar{\nu}$ events with $q^2 > q_0^2$, $F(q_0^2)$, in the upsilon expansion. The dashed line indicates the lower cut $q_0^2 = (m_B - m_D)^2 \simeq 11.6\,\mathrm{GeV}^2$, which corresponds to $F = 0.178 \pm 0.012$. The shaded region is the estimated uncertainty due to $\Lambda_{\mathrm{QCD}}^3/m_b^3$ terms; which is shown in (b) as a percentage of $F(q_0^2)$.

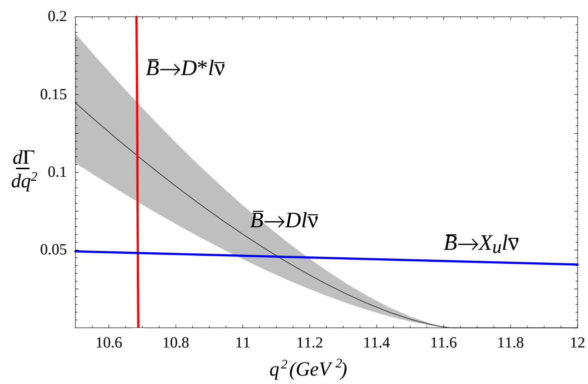

Figure 4: Charm backgrounds near $q^2 = (m_B - m_D)^2$. (Arbitrary units.)

zero recoil. Fig. 4 shows the $\bar{B} \to D\ell\bar{\nu}$ and $\bar{B} \to D^*\ell\bar{\nu}$ decay rates using the measured form factors[24] (and $|V_{ub}| = 0.0035$). The $\bar{B} \to X_u\ell\bar{\nu}$ rate is the flat curve. Integrated over the region $q^2 > (m_B - m_{D^*})^2 \simeq 10.7\,\mathrm{GeV}^2$, the uncertainty of the $B \to D$ background is small due to its $(w^2-1)^{3/2}$ suppression compared to the $\bar{B} \to X_u\ell\bar{\nu}$ signal. This uncertainty will be further reduced in the near future. This increases the $b \to u$ region relevant for measuring $|V_{ub}|$ by $\sim 1\,\mathrm{GeV}^2$. The $B \to D^*$ rate is only suppressed by $(w^2-1)^{1/2}$ near zero recoil, and therefore it is more difficult to subtract it reliably from the $b \to u$ signal. The nonresonant $D\pi$ final state contributes in the same region as $\bar{B} \to D^*$, and it is reliably predicted to be small near maximal q^2 (zero recoil) based on chiral perturbation theory.[25] The D^{**} states only contribute for $q^2 < 9\,\mathrm{GeV}^2$, and some aspects of their q^2 spectra are also known model independently.[26]

Concerning experimental considerations, measuring the q^2 spectrum requires reconstruction of the neutrino four-momentum, just like measuring the hadronic invariant mass spectrum. A lepton energy cut may be required for this technique, however, the constraint $q^2 > (m_B - m_D)^2$ automatically implies $E_\ell > (m_B - m_D)^2/2m_B \simeq 1.1\,\mathrm{GeV}$ in the B rest frame. Even if the E_ℓ cut has to be slightly larger than this, the utility of our method will not be affected, but a calculation including the effects of arbitrary E_ℓ and q^2 cuts would be required. If experimental resolution on the reconstruction of the neutrino momentum necessitates a significantly larger cut than $q_0^2 = (m_B - m_D)^2$, then the uncertainties in the OPE calculation of $F(q_0^2)$ increase. In this case, it may be possible to obtain useful model independent information on the q^2 spectrum in the region $q^2 > m_{\psi(2S)}^2 \simeq 13.6\,\mathrm{GeV}^2$ from the q^2 spectrum in the rare decay $\bar{B} \to X_s\ell^+\ell^-$, which may be measured in the upcoming Tevatron Run-II.

In conclusion, we have shown that the q^2 spectrum in inclusive semileptonic $\bar{B} \to X_u\ell\bar{\nu}$ decay gives a model independent determination of $|V_{ub}|$ with small theoretical uncertainty. Nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the charm background. This is a qualitatively better situation than other extractions of $|V_{ub}|$ from inclusive charmless semileptonic B decay.

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