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Author(s):

David G. Madland, James L. Friar

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Chiral Symmetry in Finite Nuclei

David G. Madland*, James L. Friar

Abstract

We have calculated the properties of finite nuclei over a wide mass range using chiral Lagrangians and dimensional-power-counting techniques to constrain the absolute and relative magnitudes of the various in-medium N-body forces that we have tested. Specifically, using power-counting techniques refined by Weinberg and later used by Lynn, we have investigated the physical content and convergence of the Weinberg-Lynn series for allowed terms for the first three orders in the large-mass QCD energy scale. These orders are Λ^0 , Λ^{-1} , and Λ^{-2} , where $\Lambda \sim 1$ GeV. This has been accomplished using a self-consistent procedure that solves the Dirac-Hartree equations for several nuclei simultaneously in a least-squares adjustment algorithm with respect to measured nuclear observables. The dimensionless coefficients of a physically complete and appropriately defined Lagrangian will all be of order unity if dimensional power counting holds in terms of the fundamental scales of QCD. A set of coefficients (coupling constants) of order unity from such a Lagrangian provides strong evidence that the chiral expansion works in nuclear physics. At this time we have been partially, but not completely, successful in the determination of this Lagrangian.

Background and Research Objectives:

A model-independent description of the properties of finite nuclei solely in terms of quantum chromodynamics (QCD), the underlying theory of the strong interactions, is one of the ultimate goals in nuclear physics. This is an enormous challenge, and is especially so because low-energy nuclear physics is that domain of QCD (small momenta or large distances) where perturbative calculations in terms of quarks and gluons are problematic. Accordingly, one turns to field-theoretic descriptions based on effective degrees of freedom - hadrons - that are constrained by the underlying symmetries of QCD. Chiral

symmetry is such an underlying symmetry of the QCD Lagrangian corresponding to massless u and d quarks, and even for small (finite) quark masses the (broken) symmetry produces strong constraints on the strong interaction dynamics. Effective chiral Lagrangians have proven to be a good approximation in hadronic physics (see, for example, the good agreement of measured and calculated pion-nucleon s-wave scattering lengths in the various isospin channels or nearly a dozen properties of pions and their decays).

In 1990, Weinberg introduced chiral perturbation theory into nuclear physics and showed that chiral Lagrangians predict the suppression of N-body forces through the use of the dimensional-power-counting scheme that he introduced and refined. He accomplished this by constructing the most general possible chiral Lagrangian involving pions and low-energy nucleons as an infinite series of allowed derivative and contact interaction terms and by using dimensional power counting to systematically categorize the terms of the series according to their characteristic (average) momentum or energy scales. Based on these

considerations alone, he concluded that N-body forces were a series in the ratio of a small momentum scale to a large one, leading to a systematic suppression of N-body forces.

The time was, and still is, therefore ripe to attempt a calculation of the properties of finite nuclei over a wide mass range using a chiral Lagrangian and dimensional-power-counting techniques to constrain the absolute and relative magnitudes of the various allowed in-medium N-body forces that are tested. A successful calculation performed in this way provides strong evidence that the chiral expansion works in nuclear physics and is a major step in achieving the goal of a model-independent description of the properties of finite nuclei in terms of QCD.

Importance to LANL's Science and Technology Base and National R&D Needs

Most of the advances made in nuclear physics over the past 50 years have been based on phenomenology at one level or another. No quantitatively successful solution to the nuclear many-body problem exists which has a direct link to the underlying symmetries of the fundamental dynamics. If we are successful in our attempt to reproduce measured nuclear observables using a chiral Lagrangian together with refined dimensional power counting, we will have established a linkage between the nuclear physics many-body

problem and QCD via chiral symmetry. This would place the nuclear physics discipline on more fundamental grounds and simultaneously would test QCD at its low-energy limits. Our results could lead to a new era in understanding and treating the nuclear many-body problem. Correspondingly, our ability to calculate nuclear observables pertinent to Laboratory as well as National needs and missions could be significantly enhanced, especially for those observables not accessible by experimental measurements. We mean here all nuclear observables that are calculated using ground-state Dirac single-particle wave functions determined for each nucleus of interest. At this time we have made considerable progress towards this goal, but more work needs to be done to achieve it.

Scientific Approach and Accomplishments

Our starting point was the relativistic Lagrangian used by Nikolaus, Hoch, and Madland[1] in a Dirac-Hartree calculation in mean-field approximation using a model consisting of four-, six-, and eight-fermion point couplings (contact interactions) which simulate the σ , ω , δ , and ρ meson interactions together with explicit derivative terms which simulate the finite ranges of the dominant σ and ω meson interactions. The nine coupling constants of the model were determined using a self-consistent procedure that solves the Dirac-Hartree equations for several nuclei simultaneously in a generalized nonlinear least-squares adjustment algorithm with respect to well-measured nuclear ground-state observables. With this procedure the nine coupling constants were determined so as to reproduce measured ground state masses (binding energies), mean-square charge radii, and spin-orbit splittings of closed major shell and closed subshell nuclei in nondeformed regions. The choice of the Skyrme-like (zero-range) interactions for this model was due to their many successes in previous phenomenological non-relativistic and relativistic Hartree calculations. The Lagrangian is given by

$$L = L_{free} + L_{4f} + L_{hot} + L_{der} + L_{em} , \quad (1)$$

where L_{free} and L_{em} are the kinetic and electromagnetic terms, respectively, and

$$\begin{aligned} L_{4f} = & -(1/2)\alpha_s(\bar{\varphi}\varphi)(\bar{\varphi}\varphi) - (1/2)\alpha_v(\bar{\varphi}\gamma_\mu\varphi)(\bar{\varphi}\gamma^\mu\varphi) \\ & -(1/2)\alpha_{ts}(\bar{\varphi}\tau\varphi) \cdot (\bar{\varphi}\tau\varphi) - (1/2)\alpha_{tv}(\bar{\varphi}\tau\gamma_\mu\varphi) \cdot (\bar{\varphi}\tau\gamma^\mu\varphi), \end{aligned} \quad (2)$$

$$L_{hot} = -(1/3)\beta_S(\bar{\varphi}\varphi)^3 - (1/4)\gamma_S(\bar{\varphi}\varphi)^4 \\ -(1/4)\gamma_V[(\bar{\varphi}\gamma_\mu\varphi)(\bar{\varphi}\gamma^\mu\varphi)]^2, \text{ and} \quad (3)$$

$$L_{der} = -(1/2)\delta_S(\partial_\nu\bar{\varphi}\varphi)(\partial^\nu\bar{\varphi}\varphi) \\ -(1/2)\delta_V(\partial_\nu\bar{\varphi}\gamma_\mu\varphi)(\partial^\nu\bar{\varphi}\gamma^\mu\varphi). \quad (4)$$

In these equations, φ is the nucleon field, the subscripts S and V refer to the isoscalar-scalar and isoscalar-vector densities, respectively, and the subscripts TS and TV refer to the isovector-scalar and isovector-vector densities, respectively, containing the nucleon isospin $\vec{\tau}$. Furthermore, L_{4f} contains the four-fermion point couplings with four coupling constants $\{\alpha_S, \alpha_V, \alpha_{TS}, \alpha_{TV}\}$, L_{hot} contains the higher order terms (hot) of six- and eight-fermion point couplings with three coupling constants $\{\beta_S, \gamma_S, \gamma_V\}$, and L_{der} contains derivatives of the isoscalar-scalar and isoscalar-vector densities with two coupling constants $\{\delta_S, \delta_V\}$. The final values of the nine coupling constants determined in Ref. [1] are given in Table 1. This set of coupling constants predicts accurately the properties of 54 other closed shell finite nuclei throughout the periodic table and also the “accepted” properties of saturated nuclear matter. These results are better than had been hoped for in a first attempt at a phenomenological relativistic mean-field model.

Table 1. Optimized Coupling Constants for the Relativistic Point Coupling Model and Corresponding Dimensional Power Counting Coefficients

Coupling Constant	Magnitude	Dimension	Weinberg-Lynn c_{lmn}
α_S	-4.508×10^{-4}	MeV^{-2}	-1.98
α_{TS}	7.403×10^{-7}	MeV^{-2}	0.0128
α_V	3.427×10^{-4}	MeV^{-2}	1.48
α_{TV}	3.257×10^{-5}	MeV^{-2}	0.56
β_S	1.110×10^{-11}	MeV^{-5}	0.28
γ_S	5.735×10^{-17}	MeV^{-8}	9.28
γ_V	-4.389×10^{-17}	MeV^{-8}	-7.10
δ_S	-4.239×10^{-10}	MeV^{-4}	-1.84
δ_V	-1.144×10^{-10}	MeV^{-4}	-0.49

The extracted coupling constants span 13 orders of magnitude in absolute units and we note that in the Dirac equation the terms in α are linear in the isoscalar and isovector densities, the term in β is quadratic in the isoscalar-scalar density, the terms in γ are cubic in the isoscalar densities, and the terms in δ contain derivatives of the isoscalar densities.

Now consider the generic Lagrangian for pions ($\vec{\pi}$) and nucleons (ϕ) and containing derivatives, (∂^μ), used indimensional power counting by Manohar and Georgi[2], and later refined by Weinberg[3] and Lynn[4]:

$$L \sim -c_{lmn} \left[\frac{\bar{\phi}\phi}{f_\pi^2 \Lambda} \right]^l \left[\frac{\vec{\pi}}{f_\pi} \right]^m \left[\frac{\partial^\mu, m_\pi}{\Lambda} \right]^n f_\pi^2 \Lambda^2 \quad (5)$$

where f_π and m_π are the pion decay constant, 92.4 MeV, and pion mass, 139.6 MeV, respectively. This Lagrangian should lead to dimensionless coefficients c_{lmn} of order unity[2] for each order in the QCD large-mass scale, Λ . The chiral constraint is given by[3]

$$\Delta = l + n - 2 \geq 0, \quad (6)$$

which guarantees that no Λ appears in the numerator of Eq.(5), and thus the Lagrangian is a series in Λ^{-1} . Inspection of Table 1 and Eqs.(1-5) demonstrates that the relativistic

mean-field phenomenology of Ref. [1] contains terms of order Λ^0 , Λ^{-1} , and Λ^{-2} , but the Weinberg-Lynn series is partly complete only for order Λ^0 and is mainly incomplete for the other two orders. Nevertheless, we have calculated[7] the corresponding dimensional power counting coefficients c_{lmn} from Eq. (5) and these are shown in the fourth column of Table 1. Five of the c_{lmn} are of order (1), one is of order (0.1), two are of order (10), and one is of order 10^{-2} .

Given these surprisingly good results obtained with an incomplete mix of terms from three different orders in Λ , it was concluded that the same process should be used to determine the values of the coupling constants, but to do so order by order in Λ for the Weinberg-Lynn series.

The simplest Lagrangian for order Λ^0 in the Weinberg-Lynn series is given by Eq. (2), that is,

$$L(\Lambda^0) = L_{4f}, \quad (7)$$

whereas that for order $L(\Lambda^{-1})$ in the Weinberg-Lynn series is given by that for order 0 plus the first term of Eq. (3), which is cubic in the isoscalar-scalar density, plus a term that is quadratic in the isovector-scalar density and linear in the isoscalar-scalar density. Thus, four coupling constants appear in the Lagrangian for order Λ^0 and six coupling constants appear in that for order Λ^{-1} . In both orders we were able to find converged sets of four and six coupling constants, respectively, when performing the nonlinear least squares adjustment (minimizing the total chi squared), but only for one or two of the nuclei out of the three to ten nuclei whose measured observables were used in constructing chi squared. Given that the sets of three to ten nuclei chosen for chi squared always covered a wide mass range we concluded that Lagrangians of order 0 and order -1 in the QCD large-mass scale Λ may not be applicable to a large range in nuclear ground-state mass (or binding energy). This speculation is supported by the fact that none of the sets of four or six coupling constants determined led to bound solutions for the complete test set of 54 closed shell nuclei used to study the mixed order Lagrangian of our original work [1]. Instead, bound solutions were obtained only for subsets of the 54 test nuclei and these subsets sometimes indicated a preferred mass region, but sometimes also did not. At this point we decided to return to the original approach [1] and consider all three orders in the QCD large-mass scale Λ simultaneously, namely, order 0, order -1, and order -2.

Using the original Nikolaus-Hoch-Madland [1] Lagrangian with nine coupling constants spanning three orders in Λ a total of three chi-squared minima were found: the original [1] and two others that differ from the original primarily in the isospin-dependent coupling constants α_{TS} and α_{TV} . The original set of coupling constants (a), Table 1, has six of the nine natural, the sum of two out of the remaining three natural ($c_{\gamma_S} + c_{\gamma_V}$), and has isospin-dependent coupling constants $\{0.0128, 0.56\}$. The second set of coupling constants (b), Table 2, has seven of the nine natural, the sum of the remaining two natural (the same two as in the original set), and has isospin-dependent coupling constants $\{-0.370, 0.916\}$. The predictive power of this set of coupling constants is only slightly less good than that of the original set, but its chi-squared is slightly better than that of the original set. The third set of coupling constants (c), not shown, also has seven of the nine natural, the sum of the same remaining two natural, and has isospin-dependent coupling constants $\{-1.504, 1.883\}$. The predictive power of this set of coupling constants is not as good as that of the original set (a) or the second set (b). In the latter two cases, (b) and (c), the individual isospin-dependent coupling constants are natural and their sums are natural whereas in the former case, (a), only one is natural, but the sum is natural. Note that the three sums are similar having the values, respectively, of 0.573, 0.546, and 0.379. Taking the average absolute deviations of calculated from measured binding energies and calculated from measured rms charge radii as a measure of predictive power, the sets (a), (b), and (c) give, respectively, the values $\{2.52 \text{ MeV}, 0.020 \text{ fm}\}$, $\{2.86 \text{ MeV}, 0.021 \text{ fm}\}$, and $\{4.33 \text{ MeV}, 0.031 \text{ fm}\}$. Thus, the set of coupling constants (b) from Table 2 yields the Lagrangian with the most naturalness and (nearly) the best predictive power.

Table 2. Optimized Coupling Constants for the Relativistic Point Coupling Model and Corresponding Dimensional Power Counting Coefficients for a Second Minimum in the Chi-Squared

Coupling Constant	Magnitude	Dimension	Weinberg-Lynn c_{lmm}
α_S	-4.517×10^{-4}	MeV^{-2}	-1.928
α_{TS}	2.168×10^{-5}	MeV^{-2}	-0.370
α_V	3.435×10^{-4}	MeV^{-2}	1.466
α_{TV}	5.365×10^{-5}	MeV^{-2}	0.916
β_S	1.137×10^{-11}	MeV^{-5}	0.276
γ_S	5.731×10^{-17}	MeV^{-8}	8.917
γ_V	-4.423×10^{-17}	MeV^{-8}	-6.881
δ_S	-4.282×10^{-10}	MeV^{-4}	-1.828
δ_V	-1.155×10^{-10}	MeV^{-4}	-0.493

We then reran this set of coupling constants in the least-squares adjustment algorithm with the constraint that the quartic terms in the scalar and vector densities (where the sum of the coupling constants is natural) have identical coupling constants. This resulted in a complete set of nine (really eight) natural coupling constants, but with very poor predictive power (a factor fourteen worse chi-squared). We then re-introduced additional isospin dependence via the term quadratic in the isovector-scalar density and linear in the isoscalar-scalar density leading to a Lagrangian with ten coupling constants. We found, similar

to the case of nine coupling constants, eight of ten coupling constants natural and the sum of the (same) remaining two natural, with predictive power almost identical {2.60 MeV, 0.022 fm} to that of the original Lagrangian (a). Again, if the coupling constants for the two quartic terms are constrained to be identical then all ten coupling constants are natural, but the predictive power is much worse (a factor fourteen worse chi-squared). An invited (plenary) talk and corresponding paper on these results was presented at an international conference [9].

At this point we concluded that we have not yet located the absolute minimum in total chi-squared for any of the sets of Lagrangian coupling constants that we have considered under the physical assumption that naturalness holds. A part of the problem here may be that whereas the measured observables used in the least-squares adjustment algorithm relate strongly to the vector (baryon) densities, they relate weakly to the scalar densities.

Saying this in a different way, the observables depend strongly upon the upper components of the Dirac single-particle wave functions and only weakly upon the corresponding lower components. Thus, an observable depending more strongly upon the lower components than the existing observables may lead to a more complete set of natural coupling constants. Such an observable exists: pseudospin doublets in nuclei where it has recently been demonstrated that the lower components of the corresponding Dirac single-particle

wave functions are nearly identical. This demonstration [11] utilized the coupling constants of Table 2. This past year we have used these Dirac single-nucleon wave functions to demonstrate the origin of (approximate) pseudospin symmetry in nuclei, namely, the conserved pseudospin of an observed pseudospin doublet is the orbital angular momentum of the lower components of the corresponding Dirac wave functions. We then introduced the least-bound proton and neutron experimental pseudospin doublets in 208-Pb as two new observables in our chi-square minimization program for testing various model Lagrangians and their corresponding coupling constants. The hope was that these new observables would be particularly sensitive to the lower components of the corresponding

wave functions in comparison to the other observables that we use. Unfortunately, this does not appear to be the case. While it is true that new minima were located none of them resulted in enhanced predictive power.

In summary, good evidence [Table 2] has been found that QCD and chiral symmetry apply to finite nuclei, but the evidence at this time remains only partly compelling. The goal is to construct a Lagrangian whose coupling constants are not only all natural, but whose predictive power is superior to the original Lagrangian of Nikolaus, Hoch, and Madland[1]. Currently, we are adding axial vector and tensor interactions, both isoscalar and isovector, and have excluded all pseudoscalar interactions on the basis of nucleon fields only and no explicit meson fields in our Lagrangians. We will also test derivative terms in the isovector-scalar and isovector-vector terms based upon our success with derivative terms of the corresponding isoscalar densities.

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