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IMPLICATIONS OF THE PSR INSTABILITY FOR THE SNS*

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Abstract

The Los Alamos Proton Storage Ring suffers from a violent, high frequency, transverse instability at high beam current. The Spallation Neutron Source will be similar to the PSR and one must insure that the PSR instability will not keep SNS from reaching its design goal. Efforts toward understanding the instability are described.

1 CHARACTERISTICS OF THE INSTABILITY

The coasting beam PSR instability has been observed in the PSR[1, 2, 3] and in the AGS Booster[4]. Using the Booster data and the cold, coasting beam approximation for the instabiltiy growth rate a transverse resistance of order 10 $\mathrm{M}\Omega/\mathrm{m}$ between 70 MHz and 120 MHz is expected. This is a large number for a ring with 200 m circumference and 6 cm pipe radius. In the PSR a broad band tranverse resistance of order 1 $\mathrm{M}\Omega/\mathrm{m}$ is needed to match the observed growth rates.

Transverse, bunched beam instability has been seen in the PSR. There are several curious features.

1. The central frequency of the instability f_ϵ increases with intensity.
2. For fixed bunch length τ_b the threshold intensity scales *linearly* with rf voltage, V_{rf}.
3. The threshold rf voltage for a given intensity is increased by injecting some unchopped beam.
4. A broad band tranverse resistance of order 1 $\mathrm{M}\Omega/\mathrm{m}$ is needed to match the observed growth rates.
5. For a fixed rf voltage the maximum number of stored protons can increase as the bunch length is reduced.
6. Near threshold, an intense electron flux at the wall is observed as the bunch passes.

The first two items are difficult to reconcile with an impedance driven instability since a fixed impedance should drive a given range of frequencies and given a fixed impedance the threshold intensity should scale linearly with momentum spread (or synchrotron frequency) and hence as $\sqrt{V_{rf}}$.

Item 3 would be relevant to an impedance driven instability if the beam in the gap was adequate to keep the offending resonator driven at a sufficient level. Data show that the threshold voltage doubles when $\sim 3\%$ of the injected turns are unchopped[3].

Item 4 is difficult to reconcile with a machine circumference of $C = 90$ m and an average pipe radius of 5 cm.

Item 5 is at odds with both narrow and broad band impedance driven stability models. As with item 3 it suggests that the gain of the unstable feedback depends on the characteristics of the gap.

Item 6 is a fairly recent observation[5, 6] and it is consistent with the model that has evolved to explain the previous 5, namely that the instability is driven by electrons[1, 2, 3].

2 THEORETICAL MODELS

Coasting beam models of the ep instability for transverse dipole oscillations with no electron secondary emission have been carefully explored [1], and higher order transverse modes are less important for present parameter ranges[7]. The model involves a coasting proton beam which traps electrons in its electrostatic potential well. For a uniform beam of radius a and current I the electron bounce frequency is

$$\omega_\epsilon = \sqrt{\frac{eZ_0 I}{2\pi\beta a^2 m_\epsilon}},$$

where $\beta = v/c$. Frequency spread in the electrons ($\delta\omega_e$) is due to variation in the proton beam size, via the lattice functions, as well as amplitude dependence. To model the electron centroid take

$$\frac{d^2\bar{y}_\epsilon}{dt^2} + 2\delta\omega_\epsilon\frac{d\bar{y}_\epsilon}{dt} = \omega_\epsilon^2(\bar{y}_p - \bar{y}_\epsilon), \qquad (1)$$

where $\bar{y}_\epsilon$ and $\bar{y}_p$ denote the electron and proton centroids, respectively. With no betatron frequency spread and neglecting wall effects, the proton centroid obeys

$$\frac{d^2\bar{y}_p}{dt^2} + \omega_\beta^2\bar{y}_p = \omega_p^2(\bar{y}_\epsilon - \bar{y}_p),$$

where $\omega_p^2 = f\omega_\epsilon^2 m_\epsilon/\gamma m_p$ and f is the fractional neutralization which is defined as the ratio of the total number of electrons to the total number of protons within the ring. Assuming $\delta\omega_\epsilon \gtrsim \omega_{rev}$ and $\omega_p << \omega_\beta$, the fastest growing mode of the proton beam has a coherent frequency shift of

$$\Delta\omega_\beta = i\frac{\omega_p^2\omega_\epsilon}{4\omega_\beta\delta\omega_\epsilon},$$

with carrier frequency ω_ϵ. The electron bounce frequencies tend to be $\gtrsim$100 MHz so the other main contribution to

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the coherent frequency shift is the space charge frequency shift, $(\Delta\omega_{sc})$. The total coherent frequency shift is then given by

$$\Delta\omega_\beta = \Delta\omega_{sc} + i\frac{\omega_p^2\omega_\epsilon}{4\omega_\beta\delta\omega_\epsilon}.$$

Betatron frequency spread in the proton beam is due to phase slip and chromaticity coupled with momentum spread,

$$\delta\omega_\beta = |\eta\omega_\epsilon + \xi\omega_0|\left(\frac{\delta p}{p}\right)_{hwhm},$$

where η is the frequency slip factor, ξ is the unnormalized chromaticity and the half width at half maximum momentum spread is used. With Chao's simplified criterion[8] the beam is stable if $|\Delta\omega_\beta| \leq \delta\omega_\beta/\sqrt{3}$. More refined estimates use the ratio $\Delta\omega_\beta/\delta\omega_\beta$ and standard dispersion diagrams[8].

Setting $\xi = 0$ and assuming $f\omega_\epsilon/\delta\omega_\epsilon$ is a constant one finds that the maximum number of stored protons scales as $N_p \propto (\delta p/p)^2 \propto V_{rf}$[6], which is the observed scaling. Uncertainties arise when one tries to match the formula to the data. In particular, the fractional neutralization is unconstrained. Assuming f and ξ are small, the threshold estimate is given by

$$\Delta\omega_{sc} \leq \omega_\epsilon\frac{|\eta|}{\sqrt{3}}\left(\frac{\delta p}{p}\right)_{hwhm}. \qquad (2)$$

Equation (2) is the threshold for coherent oscillations driven soley by space charge and electrons are required for growth. The implicit assumption is that some electrons will be present and the beam will have a, perhaps small, growth rate. Once the amplitude of the proton oscillations becomes large enough, electrons strike the walls and secondary emission can lead to a large increase in electrons[4]. The observed instability characteristics are during the second phase so that growth rates obtained from the linear theory are not applicable to the data.

For bunched beam threshold estimates it is assumed that a linear response model for the electrons and protons is adequate. This makes the problem similar to the usual transverse stability problem in bunched beams albeit with a non-standard wake field. If the number of electrons is fixed as the bunch passes the electron centroid is modelled using equation (1), though ω_p, ω_ϵ and $\delta\omega_\epsilon$ will depend on longitudinal position within the bunch through the instantaneous bunch current. Additionally, it is assumed that the electrons retain no memory across the gap so that the electrons may be taken as stationary with no net offset when the bunch arrives. More realistic models involving variable electron population are possible, but a theory is needed.

With electron frequencies $f_\epsilon = \omega_\epsilon/2\pi \gtrsim 100$ MHz and 200 ns bunch lengths the unstable modes have $\gtrsim$40 nodes within the bunch. The usual expansion technique[9] appears to require a very large number of modes. Therefore, we take an idealized view of the longitudinal dynamics and assume a square well longitudinal potential [10, 11, 12]. Experimentally, this is partially justified by the observation that both single and dual harmonic rf systems give comparable thresholds for similar τ_b and first harmonic voltage [6].

Use machine azimuth θ, as the time-like variable, and arrival phase with respect to the head of the bunch ϕ as the longitudinal spatial variable. One has $\theta = \omega_0 t - \phi$ where t is time measured on a clock in the lab frame. For zero chromaticity, the equation of motion for a single proton is

$$\frac{d^2y_p}{d\theta^2} = -Q_0^2y_p + 2Q_0\Delta Q_{sc}(y_p - \bar{y}_p) + Q_p^2\bar{y}_\epsilon \qquad (3)$$

where $\bar{y}_p$ and $\bar{y}_\epsilon$ are the instantaneous, transverse centroids of the protons and electrons, respectively. Also, frequencies have been replaced by their respective tunes. The electron centroid obeys

$$\frac{\partial^2\bar{y}_\epsilon(\phi,\theta)}{\partial\phi^2} + Q_\epsilon^2\bar{y}_\epsilon + 2\alpha\frac{\partial\bar{y}_\epsilon(\phi,\theta)}{\partial\phi} = Q_\epsilon^2\bar{y}_p(\phi,\theta), \quad (4)$$

where the boundary conditions are $\bar{y}_\epsilon(0,\theta) = 0$, and $\partial_\phi\bar{y}_\epsilon(0,\theta) = 0$. Integrating one obtains

$$\bar{y}_\epsilon(\phi,\theta) = \frac{Q_\epsilon^2}{\tilde{Q}}\int_0^\phi \bar{y}_p(\phi',\theta)\sin(\tilde{Q}[\phi-\phi'])e^{-\alpha[\phi-\phi']}d\phi' \qquad (5)$$

where $\alpha = \delta\omega_\epsilon/\omega_0$, and $\tilde{Q}^2 = Q_\epsilon^2 - \alpha^2$. To close the equations let $v = d\phi/d\theta$ and define the function $x(\phi,v,\theta)\exp(-iQ_0\theta)$ to be the transverse offset of the beam as a function of the phase space coordinates. In equation (3) one makes the substitution $d/d\theta \to \partial_\theta + v\partial_\phi$ and the coupling between the upper and lower betatron sidebands is neglected. This results in,

$$\begin{aligned}
&- 2iQ_0\left(\frac{\partial x(\phi,v,\theta)}{\partial\theta} + v\frac{\partial x}{\partial\phi} - \frac{dU(\phi)}{d\phi}\frac{\partial x}{\partial v}\right)\\
&= 2Q_0\Delta Q_{sc}[x(\phi,v,\theta) - \bar{x}(\phi,\theta)]\\
&+ Q_p^2\frac{Q_\epsilon^2}{\tilde{Q}}\int_0^\phi \bar{x}(\phi',\theta)\sin(\tilde{Q}[\phi-\phi'])e^{-\alpha[\phi-\phi']}d\phi',
\end{aligned}$$

where

$$\bar{x}(\phi,\theta) = \int_{-\infty}^{\infty} dv\rho(v)x(\phi,v,\theta),$$

with $\int dv\rho(v) = 1$, and $U(\phi)$ is the longitudinal potential associated with the square well.

To proceed consider the Hamiltonian for longitudinal motion, $H = v^2/2 + U(\phi)$[11]. This is put in action (I) angle (ψ) variable form using a canonical transformation $F_3(\psi,v) = -v\hat{\phi}s(\psi)/\pi$ where $\hat{\phi}$ is the full bunch length and the period 2π function is $s(\psi) = |\psi|$ for $|\psi| \leq \pi$. The old and new coordinates are related via $\phi = \hat{\phi}s(\psi)/\pi$ and $I = \hat{\phi}|v|/\pi$.

Assume a θ dependence $x = x(\psi, I)\exp(-i\Delta Q\theta)$ and substitute into the equation for $x(\phi, v, \theta)$. The resulting eigenvalue problem is given by

$$(\Delta Q + \Delta Q_{sc})x(\psi, I) + \frac{i\pi^2 I}{\phi^2}\frac{\partial x(\psi, I)}{\partial\psi}$$

$$= -\int \frac{dI'd\psi'}{2Q_0} W_\perp(\phi(\psi) - \phi(\psi'))x(\psi', I')\rho(v(I')), \quad (6)$$

where the total wake potential is given by

$$\begin{aligned} W_\perp(\phi) &= -2Q_0\Delta Q_{sc}\delta(\phi) \\ &+ Q_p^2\frac{Q_e^2}{\tilde{Q}}H(\phi)\sin(\tilde{Q}\phi)e^{-\alpha\phi}. \end{aligned} \quad (7)$$

In this expression $H(\phi)$ is one for positive arguments and zero for negative, and $\delta(\phi)$ is the delta function. Next expand $x(\psi, I)$ as

$$x(\psi, I) = \sum_{n=-\infty}^{\infty} x_n(I)e^{in\psi}$$

and substitute this into equation (6). Use Fourier orthogonality to isolate $x_n(I)$ and define

$$\dot{x}_n = \int_0^\infty dI' x_n(I')\rho(v(I')).$$

Since ϕ depends only on ψ and not I the second line of (6) depends only on the values of $\dot{x}_n$. Isolating the values of $\dot{x}_n$ on the first line of (6) and making the definition

$$A_k = \frac{\dot{x}_k + \dot{x}_{-k}}{1 + \delta_{k,0}}$$

where $\delta_{k,0}$ is the Kronecker delta, yields the final dispersion relation.

$$A_k(1 + \delta_{k,0}) = \frac{-\phi}{2\pi Q_0}D(\Delta Q + \Delta Q_{sc}, k)\sum_{m=0}^{\infty} R_{k,m}A_m \quad (8)$$

where the dispersion integral is given by

$$D(\Delta Q + \Delta Q_{sc}, k) = \int_{-\infty}^{\infty} \frac{dv\rho(v)}{\Delta Q + \Delta Q_{sc} - k\pi v/\phi} \quad (9)$$

and the impedance matrix is

$$R_{k,m} = \frac{2}{\pi}\int_0^\pi d\psi\cos(k\psi)\int_0^\pi d\psi'\cos(m\psi')W_\perp(\phi[\psi-\psi']/\pi). \quad (10)$$

Since $R_{k,m}$ is diagonal for space charge alone the analog of equation(2) may be obtained

$$1 = \Delta Q_{sc}\int \frac{dv\rho(v)}{\Delta Q + \Delta Q_{sc} - kv\pi/\phi}. \quad (11)$$

Setting $k = \omega_e\tau_b/\pi$ and using Chao's criteria in the dispersion integral, equation (2) is reproduced. If the factor of $\sqrt{3}$ in (2) is replaced with 0.6 one obtains the PSR threshold with $N_p = 4\times10^{13}$, $V_{rf} = 15$ kV, $\tau_b = 220$ ns, and $\epsilon_{rms} = 8.5\ \mu$m. Using the same constant, SNS would be stable for $N_p \lesssim 8\times10^{14}$; four times the design intensity.

As mentioned after equation (2), satisfying the dispersion relation (11) implies that coherent oscillations exist, but their growth rate is unspecified. For fast losses, the number of electrons must be sufficient to cause the proton beam to grow in amplitude, in the alotted time, to the point where a secondary emission cascade results. This appears to be a necessary consideration. Assume $(\delta p/p)/(\tau_b\sqrt{V_{rf}})$ and $\Delta Q_{sc}\tau_b a^2/N_p$ are machine dependent constants. The threshold is given by

$$N_p = KV_{rf}a^2\tau_b^3, \quad (12)$$

where K depends one beam energy, betatron tune and other machine constants. The observed threshold scales more like $\tau_b^{-0.5}$ than τ_b^3[1, 2], which shows that at least the gross characteristics of the electron population are needed to obtain thresholds. The threshold voltage doubles when $\sim 3\%$ of the injected turns are unchopped[3]. The few percent neutralization implied suggests strong secondary electron emission is needed to cause instability in the first place. Since SNS will have a TiN coating this too argues that SNS will be more stable than PSR.

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