Pointing Control Design for a High Precision Flight Telescope

Using Quantitative Feedback Theory

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Summary

A pointing control system is developed and tested for a flying gimballed telescope. The two-axis pointing system is capable of sub-microradian pointing stability and high accuracy in the presence of large host vehicle jitter. The telescope also has high agility—it is capable of a 50-degree retarget (in both axes simultaneously) in less than 2 seconds. To achieve the design specifications, high-accuracy, high-resolution, two-speed resolvers were used, resulting in gimbal-angle measurements stable to 1.5 microradians. In addition, on-axis inertial angle displacement sensors were mounted on the telescope to provide host-vehicle jitter cancellation. The inertial angle sensors are accurate to about 100 nanoradians, but do not measure low frequency displacements below 2 Hz. The gimbal command signal includes host-vehicle attitude information, which is band-limited. This provides jitter data below 20 Hz, but includes a variable latency between 15 and 25 milliseconds. One of the most challenging aspects of this design was to combine the inertial-angle-sensor data with the less perfect information in the command signal to achieve maximum jitter reduction. The optimum blending of these two signals, along with the feedback compensation were designed using Quantitative Feedback Theory.

Introduction

The two-axis gimballed telescope is shown below in Figure 1. This project was funded to retrofit the telescope (which was built over ten years ago) to incorporate the latest advances in servo technology and thereby achieve an "order-of-magnitude" improvement in pointing accuracy and line-of-sight stability. The telescope was retrofitted with precision resolvers and new (higher torque) motors. On-gimbal Inertial Angle Sensors were also added to enhance inertial pointing stability. A servo system was developed that blends feedback from the resolvers together with the inertial angle sensors to achieve less than one microradian line-of-sight pointing stability, ± 1.5 microradian pointing resolution and ± 30 -microradian accuracy.

This paper will describe in considerable detail, the new servo system including each sub-system, namely: the gimbal (including the motor, and bearing friction), Inertial Angle Sensors (IAS), resolvers (including the resolver-to-digital converters), Inertial Measurement Unit (IMU), feedback compensator, feedback sensor blending compensator, motor power amplifiers, and IAS notch filters. The pointing performance of the refurbished telescope is also presented.

Figure 1. Telescope gimbal

Two separate models were used depending on the operating mode of the telescope. For large re-targeting maneuvers, the telescope is a *Multiple-Input Multiple-Output* (MIMO) servo system with significant non-linear interactions between the two gimbal axes. However, for small maneuvers, gimbal interactions can be ignored—which reduces the model to two de-coupled *Multiple-Input Single-Output* (MISO) systems. The two inputs are: the reference input “R” and the disturbance “D,” while the output is the inertial pointing angle “Y.” (Although, the gimbal angle “ θ ,” is also an output, it will not be controlled independently of “Y.”) Axis de-coupling is possible because both axes of the telescope are balanced—the center of mass is very close to the center of rotation.

A simplified flow graph of the servo system is shown below in Figure 2. While only one axis is shown, with a few parameter substitutions the flow diagrams for both are identical. In Figure 2, and throughout this study, the telescope angle definitions are as follows. The gimbal angle of either axis with respect to its base is referred to as “ θ ,” the inertial gimbal angle is “Y,” and the angular position of the gimbal base is a disturbance and is therefore labeled “D.” The relationship between these three angles is: $\theta = Y - D$.

Figure 2. Single-axis (de-coupled) servo model

The “plant” in Figure 2, labeled $P(s)$, models the effect of gimbal inertia “J,” that is: $P(s) = 1/Js^2$. The input to $P(s)$ is torque “ τ ,” while the output is the inertial angle “Y.” The torque input to $P(s)$ comes from two sources namely: 1) $W(s)$ which collectively represents the motor windings, the power amplifiers (which deliver current to the motor), and a feedback compensator, and 2) $F(s)$ which represents gimbal friction. Of course, friction opposes the relative motion and thus carries a negative sign.

The gimbal angle θ is measured by two high-accuracy resolvers (a 1x and a 64x resolver). In Figure 2 above, the resolvers together with the *Resolver-to-Digital* (R/D) converters are lumped together and labeled $Q(s)$ —and for simplicity, will hereafter be referred to as the resolver. The inertial angle “Y” is measured with a Systron Donner *Inertial Angle Sensor*¹ (IAS) and is labeled $H(s)$ in Figure 2. The IAS is a *band-pass* device whose bandwidth is between 2 Hz and 10 kHz. The theory of operation, application and evaluation of the IAS is further explained in Reference 1.

Because $H(s)$ rolls-off at low frequencies, another sensor was needed to cancel the low-frequency components of the disturbance “D.” A connection to the Inertial Measurement Unit (IMU) on the host vehicle was added to the telescope pointing system to provide low-frequency *feed-forward* cancellation of the base disturbance. Of course, since the IMU is located off-gimbal, a transformation is necessary to convert host attitude data into gimbal coordinates. The frequency response of the IMU together with the coordinate transformation are lumped together in Figure 2, and labeled $B(s)$. The transformed host attitude data are subtracted from the desired pointing angle “R” to produce the commanded angle “C.” This calculation, together with the coordinate transformation are computed on a separate processor outside the servo controller system as described in this report, however, these components are shown in Figure 2 for completeness.

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Finally, there is the compensator $G(s)$. The function of $G(s)$ is to blend the low-frequency data coming from $B(s)$ and $Q(s)$ with the "high-frequency" feedback data coming from the inertial sensor $H(s)$ —producing a "smooth" transition between the two sensors.

System Identification of Fixed Components

In this section, we identify the *transfer functions* of those components in the servo that are *fixed* in the sense that they are not available for modification by the design process. We also quantify the uncertainties in their parameters. For example, elevation gimbal inertia is estimated at 86 oz-in-s². This estimate was made prior to the telescope being completed so for design purposes elevation inertia was allowed to vary between 75 and 100 oz-in-s². Azimuth gimbal inertia depends on elevation as per Equation 1 below. For an elevation angle of 90 degrees, azimuth inertia is approximated at 142 oz-in-s², for 180° it is estimated at 104 oz-in-s².

$$J_{az} \approx 50 + 54 \sin^2(\text{elevation}) + 92 \cos^2(\text{elevation}) \text{ oz-in-s}^2. \quad (1)$$

The nominal transfer function of the inertial angle sensor $H(s)$, given by Equation 2 below, has been found to vary slightly with temperature. The manufacturer provided four different transfer functions to represent $H(s)$ at various temperatures. This set of four was used to represent the variation in $H(s)$ as a function of temperature. A Bode plot of this set is shown in Figure 3.

$$H_n(s) \cong \frac{s^3 [s + 187][s + 7.6]}{[s + 201][s + 4.2][s + 1.1][s + 2(0.55)15 + 15^2]} \quad (2)$$

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Figure 3. Inertial Angle Sensor Frequency Response

A Bode plot of the IMU, $B(s)$ was provided from the host vehicle contractor (shown below in Figure 4.) Also shown in Figure 4 is the effect of a variable time delay in the host-vehicle attitude data (which alternates between 15 and 25 milliseconds). From this plot, a nominal transfer function $B_n(s)$ was empirically determined that matched the Bode plot. The transfer function, given by Equation 3 (excluding the time delay), is shown below. For design purposes, the dominant pole at 300 rad/sec was varied $\pm 20\%$, while its damping ratio of 0.6 was varied $\pm 10\%$. A sample space of sixteen variations on $B(s)$, plus the nominal, was used to represent the uncertainty in the IMU.

$$B_n(s) \cong \frac{300^2 (1000)[0.205s + 1]}{[0.201s + 1][s + 2(0.6)300 + 300^2][s + 1000]} \quad (3)$$

Figure 4. Inertial Measurement Unit (IMU) frequency response

The nonlinear behavior of bearing friction was simulated with the Dahl friction model²—whose differential equation is shown below in Equation 4. The telescope manufacturer estimated the Dahl friction model parameters as shown below in Table 1. A simulation of this equation is shown in Figure 5 as a plot of bearing friction versus angle.

$$\frac{dF}{dt} \cong \text{abs} \left\{ 1 - \text{sign} \left(\frac{d\theta}{dt} \right) \frac{F}{F_{\max}} \right\} (\text{slope}) \frac{d\theta}{dt} \quad (4)$$

Axis	Running Torque $F_{\max}$ (oz-in)			Slope (in-lb./rad.)		
	Min.	Nominal	Max.	Min.	Nominal	Max.
Elevation	0.75	1.50	3.0	30	60	120
Azimuth	3.00	6.00	12.0	230	460	920

Table 1. Dahl friction model parameters

In order to include the nonlinear effect of bearing friction into the design process it was convenient to convert the nonlinear Dahl effect into an equivalent frequency response. This was done using the *describing function technique*. That is, the response of the nonlinear model to various sinusoidal inputs was measured, and the input/output transfer function was derived from these data. Sinusoidal inputs to equation 4 produce outputs with the same fundamental frequency, plus higher harmonics due to the nonlinear nature of Equation 4. Describing function technique tests the nonlinear system at one frequency at a time. The transfer function is determined at each frequency by ignoring the harmonics and computing the input/output ratio for the fundamental.

Accordingly, the Dahl equation was simulated at various frequencies, under various conditions. (Four such simulations are shown in Figure 6.) When the data for these simulations were reduced, it was found that the Dahl frequency response varies as a function of both frequency, and input magnitude. The Dahl friction behaves much like a single-order low-pass filter—whose roll-off frequency is a function of not only the Dahl parameters of F_{max} and $Slope$, but also a function of input magnitude. That is, at low frequency and/or large input magnitudes, the bearing friction has a flat frequency

response, and the phase shift approaches zero. For small and/or high frequency inputs, the Dahl friction causes a 90° phase shift between input and output, and rolls off at a rate of 20 dB per decade. This is shown more clearly in Figure 7. From these data, it was empirically determined that the describing function equivalent of Equation 4 could be written as shown below in Equation 5. (Note that because the input to the Dahl equation is $d\theta/dt$ instead of θ , Equation 5 must be multiplied by “s” in order to yield the transfer function $F(s)$ with bearing friction as the output and θ as the input.)

Figure 5. Bearing friction versus gimbal angle

$$\left| \frac{F(s)}{s} \right| = \left| \frac{F}{s\theta} \right| = \left| \frac{\text{slope}}{s + a} \right| \quad \text{Where : } a = \frac{\pi \cdot \text{slope} \cdot \text{input magnitude}}{4F_{\max}} \quad (5)$$

Figure 6. Dahl bearing friction input/output simulations

Figure 7. Describing function analysis of bearing friction

The raw simulation data (shown in symbols) along with the interpolation of Equation 5 (shown in solid lines) are plotted in Figure 7. Note that the interpolation of Equation 5 perfectly matches the raw magnitude data, but does not match the raw phase data. In order to match Equation 5 to the phase data, four phase lag filters were added as shown below in Equation 6. The resulting phase plot is shown in Figure 8.

$$\text{phase} \left(\frac{F(s)}{s} \right) = \text{phase} \left(\frac{[s + 0.195f][s + 2.5f][s + 6.67f][s + 43f]}{[s + a][s + 0.18f][s + 2f][s + 6f][s + 22f]} \right) \quad (6)$$

$$\text{where } f = e^{[\ln(a) - 3.9045]}$$

Figure 8. Describing function analysis of bearing friction with extra phase shift

Design Procedure

Applying standard flow graph modeling techniques to Figure 2 yields the transfer function shown below in Equation 7 for the inertial angle "Y." The first term in Equation 7 is the response due to the control reference signal "R," while the second term represents the system response due to the disturbance "D."

$$Y(s) = \frac{RGWP}{1 + P[F + W(GQ + H)]} + \frac{DP[F + GW(Q - B)]}{1 + P[F + W(GQ + H)]} \quad (7)$$

The design goals for this project were: 1) 1.5 microradian pointing resolution, 2) 1 microradian line-of-site pointing stability in the presence of host vehicle attitude disturbance "D" and 3) high agility—that is, *fast* re-targeting capability. Note that the transfer function for the pointing accuracy is represented in the first term of Equation 7, while the stability performance is represented in the second term of Equation 7. The most challenging design goal was the stability criterion.

Resolver-to-Digital (R/D) Converter Design

The gimbal angle θ is measured by two high accuracy resolvers (a 1x and a 64x resolver). The resolver signals are converted into a 22-bit word by combining two 16 bit Resolver-to-Digital

converters (R/D)—see Table 2 below. In Figure 2, the resolvers together with the R/D converters are lumped together and labeled Q(s)—and for simplicity, will hereafter be referred to as the resolver.

Resolver	Binary	Degrees
1x (16-bit)	01 0101 1111 0010 10	123.453
64x (16-bit)	1111 0010 1010 0111	341.22986 / 64
Composite (22-bit)	01 0101 1111 0010 1010 0111	123.45672

Table 2. Bit relations of the resolver-to-digital converters

By observing the second term in Equation 7, it is clear that the system response to disturbance “d” can be significantly reduced by designing $Q(s) \approx B(s)$. That is, we want the frequency response of the resolvers to match as closely as possible that of the IMU shown in Figure 4, and Equation 3.

An Analog Devices (AD2S80A) chip was used for the R/D converter. It is a “type II feedback system” (using two integrators) that requires external compensation to achieve the desired closed-loop response. According to the design detail obtained from Analog Devices,³ the converter can be summarized as shown below in Figure 9. Using the design process detailed in the data book, one can select the parameters k, a, and b for a desired tracking rate and bandwidth f_{bw} . This will produce the closed-loop response shown in Equation 8. The parameters k, a, and b, affecting the closed-loop bandwidth f_{bw} of the R/D converter, are selected by the choice of external components. As stated

earlier, we want the resolver transfer function $Q(s)$ to closely match that of the IMU $B(s)$, in order to maximize disturbance cancellation. To accomplish this, another pole was added to the compensator, as shown in Figure 10, and the lead-lag filter was adjusted slightly. This produced the desired closed-loop response shown in Equation 3 and Figure 4 (without the variable time delay).

Figure 9. Block diagram of R/D converter

$$\frac{\theta_{out}}{\theta_{in}} = \frac{2.4^3 [s_n + 1]}{[s_n + 2.4][s_n^2 + 2(0.7)2.4s_n + 2.4^2]} \quad \text{where: } s_n = \frac{2s}{\pi f_{bw}} \quad (8)$$

Figure 10. Modified block diagram of resolver-to-digital converter

The compensation required to achieve this design is shown in Figure 11, with the extra pole drawn in gray. Assuming $R_3 \ll R_4$, the relationship between the component values and the parameters shown in Figure 10 are as follows: $a = 1/(R_2 C_2)$, $b = (C_1 + C_2)/(R_2 C_2 C_1)$, $c = 1/(R_3 C_3)$ and $k = 6.163 \times 10^6 / [R_1 C_1 (R_3 + R_4)]$. The component values are listed in Table 3 below. For servo design purposes, the gain parameter k, as well as all three capacitors, were varied independently $\pm 20\%$ of their nominal values. This produced a set of sixteen variations on $Q(s)$, plus the nominal, to quantify the uncertainty in the resolver $Q(s)$.

Figure 11. Schematic diagram of resolver-to-digital external compensation

R_1	R_2	R_3	R_4	C_1	C_2	C_3
8.25k Ω	205k Ω	23.7k Ω	681k Ω	2200pF	1 μ F	0.1 μ F

Table 3. Component values for Figure 11

One of the design goals of this project was to improve telescope agility. As mentioned earlier, the telescope was retrofit with larger motors with a maximum torque output of 190 ounce-inches. When making large maneuvers full torque is applied to the gimbal to maximize speed. For step commands greater than 15 degrees, the gimbal exceeds the tracking speed of the 64x Resolver-to-Digital converter, causing the measured gimbal angle to lag behind the true angle. When the error reaches 180° the converter makes a sudden correction to get the angle back on-track. These sudden corrections caused a serious problem for the large maneuver servo algorithm, so the 64x R/D converter had to be modified in order to work properly.

Of the two axes, elevation has the least inertia, as well as the largest range of travel, and thus will potentially see the highest angular velocities. Assuming a maximum step command of 100°, maximum torque of 190 in-oz and a gimbal inertia of 86 oz-in-s², the elevation gimbal would reach a maximum angular velocity of about 0.3 revolutions-per-second. Thus, the 64x R/D converter must have a maximum tracking speed of about 20 RPS—which corresponds to a 300 Hz bandwidth (see Equation 8). To implement the higher bandwidth, the standard design procedure was followed. Adding C4 and R6 and replacing R4 with R5 thus modified the circuit in Figure 11. An analog switch was also added. (In Figure 12, $C_4 = 0.039\mu$ F, $R_5 = 635$ k Ω , and $R_6 = 47.5$ k Ω .)

Figure 12. Schematic diagram of 64x resolver-to-digital external compensation

When making large maneuvers the normally-closed switch shown in Figure 12 is opened—effectively changing C_1 from 1 μ F to 0.038 μ F, while the normally-open switch is closed—bypassing the extra pole (shown in dashed lines in Figure 11). After the maneuver is completed, the switches return to their default positions which returns the resolver-to-digital converter to the frequency response required for maximum jitter cancellation. Note that the two switches shown in Figure 12 are from the same (Double-Pole-Single-Throw) DPST analog switch, so that only one control line is used to switch both. This circuit solved the above-mentioned problem for large maneuvers.

Design of Feedback Sensor Blending Compensator $G(s)$

The most challenging aspect of this design was that of the compensator $G(s)$. As previously mentioned, the function of $G(s)$ is to blend the low-frequency data coming from $B(s)$ and $Q(s)$ with the *high-frequency* feedback data coming from the inertial sensor $H(s)$. The effective feedback sensor is the composite $G(s)Q(s) + H(s)$. Since $H(s)$ rolls-off for frequencies below 2 Hz, we want to use the resolver feedback data from $Q(s)$ for all frequencies below 2 Hz, and use the inertial angle sensor $H(s)$ for all frequencies above 2 Hz. Therefore, ideally $G(s)$ should cut-off $Q(s)$ *sharply* at 1 Hz as shown in Figure 13, however, this presents a significant problem. Factoring the equation $G(s)Q(s) + H(s)$, reveals zeros in the right-half of the s-plane—resulting in unstable poles in the closed-loop system, (since according to root-locus analysis, closed-loop poles “seek” the open-loop zeros at high gain).

Figure 13. Bode plot of $H(s)$ and desired $G(s)Q(s)$

This can be more easily seen by studying the root-locus plot of $G(s)Q(s) + H(s)$. The root-locus for $G(s)$ as proposed in Figure 13 (a second-order low-pass at 1 Hz) is shown in Figure 14. Using the root-locus technique, it is possible to modify $G(s)$ to cause $G(s)Q(s) + H(s)$ to have all zeros (and

poles) in the left-half plane. However, this proved to be quite difficult when taking into account the divergence in $H(s)$ and $Q(s)$ due to temperature and component variations. That is, finding a compensator that meets this specification (no right-half-plane poles and zeros) is relatively simple for each combination of $H(s)$ and $Q(s)$ individually. But finding *one* compensator that satisfied this criterion for *all* combinations of H and Q in the set proved to be quite impractical using the root-locus technique. It was therefore necessary to find another method.

Figure 14. Root-locus of $G(s)Q(s) + H(s)$ as proposed in Figure 13

Dr. Isaac Horowitz, a consultant to Sandia on this project, suggested a simple technique that proved to be very effective. As it turned out, *Quantitative Feedback Theory* (QFT) is ideally suited for such a design challenge. Quantitative Feedback Theory greatly simplifies the task of designing control systems with nonlinear and/or time-varying behaviors to quantitative performance specifications. Typical QFT designs specify *upper* and *lower* performance bounds on the closed-loop frequency response. For example, in our case the design specifications on $G(s)Q(s) + H(s)$ were quite simple: 1) First we needed a *smooth transition* between the $Q(s)$ and $H(s)$ feedback sensors (that is, there should be no significant *peaking* over the desired operating range). 2) Secondly the composite sensor $GQ + H$ must be minimum phase (no right-half-plane zeros). To be more precise on the first specification, *smooth transition* was defined as $-2.3 \leq |GQ + H| \leq 2.3$ dB for frequencies less than 100 Hz. Here ± 2.3 dB was used for the upper and lower bounds which correspond to damping ratio of about 0.42—that is, the composite feedback sensor $GQ + H$ should have no poles or zeros with damping ratios less than 0.42. Designing to both of these specifications simultaneously, while taking into account plant uncertainty, turned out to be relatively simple using QFT.

With QFT, compensator design is typically performed using the Nichols chart. The Nichols chart is a convenient plotting technique that *quantitatively* relates the open-loop frequency response (L) to that of the closed-loop $L/(1+L)$. Similar “*Nichols charts*” can be created that simplify the design of other topologies such as $GQ + H$ as in our case. Designing to requirement 1) is straightforward using QFT. However, meeting criterion 2) required some problem manipulation. As pointed out by Dr. Horowitz, the sensor equation simply needed to be *rearranged* to fit the classical QFT approach as follows: $GQ + H = H(1 + GQ/H) = H(1 + L)$, where $L = GQ/H$.

If $H(s)$ and $Q(s)$ are both stable and minimum phase, (which is true in our case) and if $1 + L$ is also minimum phase and stable, then $GQ + H$ is also. To guarantee that $1 + L$ is minimum phase and stable, we need to satisfy the Nyquist criterion—that the number of critical-point encirclements of $L(s)$ is equal to the number of right-half-plane zeros minus the number of right-half-plane poles in $L(s)$. Since $L(s)$ has neither right-half plane poles nor zeros, we can simply design $L(s)$ to have no encirclements of the critical point. On the Nichols chart, the critical point is found at 0 dB and -180 degrees. Thus if $L(s)$ crosses 0 dB to the right of this critical point, then we will have no encirclements, and our system will be stable and minimum phase. Ideally, of course, we would want to have some phase margin when $L(s)$ crosses 0 dB, so we will stay a considerable distance away from the critical point—specifically, we will stay outside the 2.3 dB Nichols chart contour.

Figure 15 shows the Nichols plot of the compensated nominal plant $GL_n(s) = GQ_n/H_n$. While Figure 15 shows only the nominal, each point carries with it a *set of plants* (or template) that define an area of uncertainty. With QFT the closed-loop contours of the Nichols chart are replaced with *bounds* on $L_n(s)$. These bounds can be obtained for a set of frequencies by moving the plant templates around the Nichols chart to determine the regions on the chart where the open-loop variation can be accommodated into the closed-loop performance specification. These bounds are with respect to the nominal plant—that is, if the *nominal* plant satisfies the bounds at all frequencies, then *all* plants within the set will meet or exceed the design specifications for the system. In our case, the performance bounds were defined as $-2.3 \leq |GQ + H| \leq 2.3$ with respect to the nominal plant Q_n/H_n .

At each frequency where bounds were calculated, there are two bounds plotted in Figure 15. The dashed lines represent the upper bound of $|GQ + H| \leq 2.3$, while the solid lines represent the lower bound $-2.3 \leq |GQ + H|$. As long as the compensated nominal plant $GL_n(s)$ lies below the dashed lines, and above the solid lines at each frequency, then $-2.3 \leq |GQ + H| \leq 2.3$ for all combinations of Q and H in the set. A lead-lag compensator $G(s)$ was designed to meet the design specifications at all frequencies— $G(s)$ is given below in Equation 9. (The Quantitative Feedback Theory toolbox from *The Mathworks Inc.* was used in this design to calculate the bounds, and to shape the compensators—which greatly simplified the design process.)

Figure 16 shows a Bode plot of $GQ(s)$ along with $H(s)$, and a simulated disturbance $D(s)$ as measured by $B(s)$. This sample disturbance is taken as the absolute worst case that could be expected on the host vehicle. Note the large spike at 4.5 Hz. Figure 17 shows the Bode plot of the composite sensor $G(s)Q(s) + H(s)$.

$$G(s) = \frac{27[s+4]}{[s+9][s+12]} \quad (9)$$

Figure 15. Nichols plots of GQ_n/H_n with bounds on $GQ + H$.

Figure 16. Bode plot of $G(s)Q(s)$ and $H(s)$ along with $D(s)B(s)$.

Figure 17. Bode plot of $G(s)Q(s) + H(s)$

Figure 18. Simulated host vehicle attitude data $d(t)$

Figure 19. Simulated disturbance attenuation results (without attitude latency)

Figure 20. Disturbance attenuation with host vehicle attitude latency

Redesign of Feedback Sensor Blending Compensator $G(s)$

The simulated host vehicle pointing error data $d(t)$ is shown in Figure 18, which reveals a ± 6 microradian jitter at 4.5 Hz. The host vehicle attitude data from $B(s)$ is sampled at 40 Hz, and contains a variable latency. Every other sample is expected to have a maximum latency of 15 milliseconds, with the remaining samples having a latency of 25 milliseconds. Since this delay is not inside the feedback loop, it will not affect stability, but it severely restricts the disturbance rejection performance. Detailed simulations revealed that because of this variable latency, attenuation of the 4.5 Hz jitter was quite difficult. (See disturbance attenuation simulations in Figures 19 and 20.) The impasse is due to the fact that the 4.5 Hz disturbance is measured by both the inertial angle sensors $H(s)$ as well as IMU $B(s)$, but is corrupted in $B(s)$ by the variable latency. The delay becomes more significant at higher frequency, causing the two sensors $B(s)$ and $H(s)$ produce conflicting information on $D(s)$ at 4.5 Hz, which seriously limits disturbance attenuation.

Ideally, we would have rolled-off $B(s)$ sooner so it would not see the 4.5 Hz signal (as shown in Figure 13). However, as it turns out, a certain amount of over-lap between the two sensors is needed in order to satisfy the minimum phase requirement. Thus, another approach was needed to further attenuate $D(s)$. To accomplish this, $G(s)$ was modified (as shown in Equation 10) to purposely violate the upper bound $|GQ + H| \leq 2.3$ for frequencies local to 4.5 Hz. The idea was to allow the 4.5 Hz jitter to come through the feedback sensor with a higher gain so that it could be acted upon more aggressively than its neighboring frequencies. In an effort to emphasize the uncorrupted information coming from $H(s)$ with respect to $B(s)$, a gain of 3 was added to $H(s)$.

This significantly improved the disturbance attenuation at 4.5 Hz, however, when the servo was implemented on the actual telescope, large structural resonances in the inertial angle sensor mounts that attach the *Inertial Angle Sensors* to the telescope were discovered. The lowest of these resonances were too close to the system bandwidth to effectively notch-out, so it became necessary to sacrifice performance in order to avoid exciting the structural resonances. (These resonances will be discussed in detail, later in this report.) The gain on $H(s)$ was thus lowered from 3 to 1.5. The Bode plot of the composite sensor $GQ + H$ in Figure 21 includes the gain of 1.5 on the inertial angle sensor $H(s)$.

$$G(s) = \frac{42[s + 4]}{[s + 14][s + 12]} \quad (10)$$

Figure 21. Bode plot of composite sensor $G(s)Q(s) + 1.5 H(s)$

Design of Feedback Compensator $W(s)$

As mentioned in the system overview, $W(s)$ collectively represents the motor windings, the power amplifiers (which deliver current to the motor), and a feedback compensator. The power amplifiers (whose design will be detailed later in this report) were designed as current drivers with current feedback from the motor windings. This effectively removed the motor winding dynamics from the design process, and the bandwidth of these amplifiers was large enough to be ignored with respect to the design of $W(s)$. The driving factor in the design of $W(s)$ was that of disturbance attenuation. The desired pointing stability is ± 1 microradian. Simulated host vehicle pointing error data is shown in Figure 22.

Figure 22. Simulated host vehicle attitude data (worst-case)

Discounting the initial transient, the worst-case host-vehicle attitude data contains disturbances of about ± 250 microradians at low frequency and ± 6 microradians at 4.5 Hz. Of the two axes, the azimuth disturbance was the most challenging to attenuate, and since the host attitude is coupled to the telescope gimbals differently depending on gimbal angles, the azimuth disturbance was used for the design of both axes. For design purposes, it was assumed that the disturbance $D(s)$ would be no greater than the bound shown in Figure 23—derived by taking the Fourier transform of the azimuth data shown in Figure 22. To achieve a factor-of-two design margin, the system was designed to attenuate this disturbance down to ± 0.5 microradian. To accomplish this, an attenuation of 55 dB is needed at low frequencies and 22 dB is needed at the higher frequencies. These requirements are represented by the disturbance attenuation specification (W_d) shown in Figure 24. Also shown in Figure 24 is the effect of $W_d(s)$ on the disturbance $D(s)$.

The only other quantitative specification imposed on the design of $W(s)$ was the stability criterion—that the closed-loop system must have less than 2.3 dB of peaking. Converting these design specifications into bounds on the Nichols chart is a relatively straightforward task using the Quantitative Feedback Theory Toolbox.

Figure 23. Fourier transform of $D(s)$ with maximum disturbance limits

Figure 24. Disturbance attenuation specification with attenuated disturbance

The small-signal output equation (7) was made to match the *General Bound Manager* format of the QFT Toolbox (*QFT Toolbox manual* [1994], pp. 6-7) as follows:

$$Y(s) = \frac{R(s)C(s)W(s)}{K(s) + L(s)W(s)} + \frac{A(s) + E(s)W(s)}{K(s) + L(s)W(s)}, \quad (11)$$

where: $A(s) = D(s)P(s)F(s)$; $C(s) = G(s)P(s)$; $E(s) = D(s)C(s)[Q(s) - B(s)]$; $K(s) = 1 + P(s)F(s)$; $L(s) = P(s)[G(s)Q(s) + H(s)]$.

The stability criterion is given by Equation 12 and the disturbance attenuation specification is given by Equation 13. Nichols chart stability bounds were calculated from Equation 12 with respect to $L_n(s)$ using the QFT toolbox. Similarly, disturbance attenuation bounds were calculated from Equation 13 with respect to $L(s)$ also using the QFT toolbox. Figure 25 shows the intersection of the stability bounds with the disturbance attenuation bounds. Also shown in Figure 25 is the Nichols plot of $W(s)L_n(s) = W(s)P_n(s)[G(s)Q_n(s) + H_n(s)]$. (Only the azimuth axis is shown in Figure 25 because the elevation axis looks almost identical to azimuth.)

$$\left| \frac{C(s)W(s)}{K(s) + L(s)W(s)} \right| \leq 2.3 \text{ dB} \quad (12)$$

$$\left| \frac{A(s) + E(s)W(s)}{K(s) + L(s)W(s)} \right| \leq W_d(s) \quad (13)$$

To stabilize the servo loop, a lead-lag compensator $W(s)$ given by Equation 14 is included in Figure 25. Note that the servo system is stable, and meets the bounds at all frequencies. This is indicated by the fact that the plot of $L_n(s)W(s)$ —(Figure 25) falls above (or to the right) of all the solid bounds and below (or to the right) of all dashed bounds. Bode plots of the closed-loop system are shown in Figures 26 and 27. Figure 26 is the Bode plot of Equation 12 and shows the closed-loop response to the command signal $R(s)$, while Figure 27 shows the closed-loop disturbance attenuation (Equation 13). Also plotted in Figures 26 and 27 are the design criteria—which are met at all frequencies.

$$W(s) = \frac{k \times 10^6 b[s + a]}{a[s + b]} \quad \text{where,} \quad \begin{bmatrix} a \\ b \\ k \end{bmatrix} = \begin{bmatrix} 500 & 170 \\ 4400 & 2800 \\ 9 & 2 \end{bmatrix} \quad (14)$$

Figure 25. Nichols plot of $W(s)P(s) [G(s)Q(s) + H(s)]$ with bounds

Figure 26. Bode plot of closed-loop servo response (Equation 12)

Figure 27. Disturbance rejection Bode plot (Equation 13)

Design of the Power Amplifiers

The power amplifiers were designed to provide current to the motor windings and to have large enough bandwidth to be effectively ignored in the servo loop design of $W(s)$. Thus a bandwidth of no less than 200 Hz was needed. The schematic for the power driver is shown in Figure 28. The op-amp boosts the power to the motor windings from ± 15 volts up to a maximum of ± 25 volts at ± 1 amp. The motor current is measured across a 0.1Ω resistor in the return path. The measured current is subtracted from the input control signal and fed back into the power amplifier to close the loop. The motor has redundant windings that are normally used in parallel. Should one of the windings fail, it can be switched out with latching mechanical relays. When operating with only one winding, the feedback system automatically adjusts the current flowing in the remaining coil to make up the difference.*

* The maximum power is cut roughly in half when one winding is switched out. However, as long as the requested motor current is less than 0.4 amps, the amplifier adjusts the motor voltage so that the current (and thus the torque) remains the same regardless of the number of windings used. With only one winding energized, motor resistance is about 61Ω , which limits the maximum current to about 0.41 amp due to the 25 volt maximum output of the amplifiers.

Figure 28. Schematic diagram of power amplifier

Figure 29 shows the Bode plot of the power amp. The motor winding inductance was measured at about 39 mH, but due to mutual inductance, this varies between 29 and 49 mH as a function of gimbal angle. The motor resistance is constant with angle, but varies from one winding to the next between 58 and 65 ohms. Additional motor winding uncertainty was added to take into account the effect of operating on only one of the redundant windings. Figure 30 shows the Nichols plot of the open-loop current driver (which includes the power amp, the motor windings, and the current feedback measurement). The uncertainty of the motor windings is included in Figure 30. The closed-

loop amplifier was designed to have a maximum peaking of 3 dB (corresponding to a damping ratio of about 0.38), and to roll-off past 200 Hz. Each of the open-loop plots shown in Figure 30 is marked with a circle at $f = 200$ Hz. Note that at 200 Hz, all the plants in the set are above 0 dB—thus meeting the bandwidth requirement. Also, note that all plants lie outside the 3-dB contour—thus meeting the stability requirement.

Figure 29. Bode plot of power amplifier (w/o motor winding)

Figure 31 shows the Bode plot of the closed-loop power amplifier. Note that the current monitor circuit has an over-all gain of 10. Thus, the closed-loop gain of the power amplifier is 0.1 (or -20 dB)—that is, 10 volts input to the power amplifier results in 1 amp of current delivered to the motor. For this reason, the 3-dB design criterion is shown at -17 dB (-20 + 3 dB) in Figure 31.

Figure 30. Nichols plot of power amplifier loop (including motor winding)

Figure 31. Bode plot of closed-loop power amplifier

Design of the Coarse Mode Servo System

The feedback system described above was designed for the low-speed tracking required to observe a fixed point on the ground as the host vehicle travels along the flight path. This is referred to as the *fine* mode. While the telescope *appears* to be staring at a single point, it is actually tracking very slowly. In this mode, the telescope is expected to point to within 30 microradians of the target, while canceling out the host vehicle attitude jitter. Since the inertial angle sensors saturate at about 100 microradians, it is necessary for the pointing error to be less than 0.02 degrees (380 microradians) before the fine mode can be engaged. Thus, a *coarse* mode of operation was also needed to bring the telescope error into the capture range of the fine mode. The coarse mode could also be used as a back up in the event that one of the inertial angle sensors should fail. The design of the coarse mode will be described in the following section.

Figure 32. Coarse mode block diagram

Removing the inertial angle sensor from the block diagram of Figure 2 simplifies the feedback equation significantly. Figure 32 shows the simplified diagram while Equation 15 shows the feedback equation of Figure 32. In Figure 32 and Equation 15, $H(s)$ (inertial angle sensor) has been removed and the compensator $G(s)$ has been set to unity.

$$Y(s) = \frac{RWP}{1 + P[F + WQ]} + \frac{DP[F + W(Q - B)]}{1 + P[F + WQ]} \quad (15)$$

The design criteria were significantly relaxed for the coarse mode. There was no disturbance attenuation specification, and the stability criterion was the same as for the fine mode—less than 2.3 dB peaking in the closed-loop system. The other design requirement was that the steady-state error must be less than 0.02 degrees to allow the fine mode to switch on.

Because of the nonlinear behavior of the bearing friction, it is not possible to use the final value theorem to calculate steady-state error. Instead, we used a more simple approach to ensure that the servo gain was large enough to satisfy this requirement with a factor-of-4 error margin. With a steady-state gain of about 10^5 , an error as small as 100 microradians would result in about 10 oz-in of torque applied to the gimbal. Assuming that the maximum bearing friction is 9 oz-in, we should have enough torque to overcome the bearing torque and move the gimbal towards the control angle—allowing the fine mode to switch in. (An error of 90 microradians would produce just enough torque to counter-act the bearing friction, but not enough to move the gimbal. The fine mode minimum capture error of 380 microradians would produce 38 oz-inches of torque—four times more torque than what is needed to overcome 9 oz-in of bearing torque.) Thus, a coarse mode steady-state gain of 10^5 should be sufficient to ensure that the telescope pointing error can reach the fine mode capture angle of 0.02 degrees—with a gain margin of 12 dB.

The same design procedure that was used to design the fine mode was also used to design the coarse mode. Figure 33 shows the Bode plot of the closed-loop system. Note that due to the bearing friction, there is about 1 dB of low-frequency uncertainty in the closed-loop system. The lead-lag compensator $W(s)$ is shown below in Equation 16.

$$W(s) = \frac{10^5 a[s + 16]}{16[s + a]} \quad \text{where, } a = \begin{bmatrix} 2800 \\ 4400 \end{bmatrix} \begin{matrix} \text{(elevation)} \\ \text{(azimuth)} \end{matrix} \quad (16)$$

Figure 33. Closed-loop frequency response—coarse mode

Design of the Large-Step Algorithm

The capture angle for the coarse mode is about 1 degree, and thus a third servo mode was needed for large maneuvers with a steady-state error of less than 1 degree in order for the coarse mode to kick in. Because our microcontroller lacked the computing horsepower to use a time-optimal approach to large retarget maneuvers, a simple-minded approach was used. This was done with an algorithm that works as follows:

1. Whenever a pointing error of more than 0.3° is calculated, the servo is switched over to the *step* mode.
2. The initial pointing error is calculated and stored.
3. New pointing errors are constantly calculated and compared to the initial pointing error. If the new error is larger than the stored error (which would occur if the telescope were being pushed off target) the new error overwrites the stored value.
4. Full positive torque is applied to the gimbal to accelerate the gimbal towards the commanded angle, as long as the pointing error is more than half of the initial error.
5. As soon as the pointing error is less than half of the initial error, full negative torque is applied to start decelerating the gimbal.
6. Full deceleration torque is applied until gimbal velocity goes to zero, or changes sign.
7. The pointing error is then checked, if less than 0.3° , the servo system is switched back into the coarse mode. Otherwise, the algorithm loops back to step 1.
8. Once the coarse mode has stabilized the error to less than 0.02° , the servo is switched over to the fine mode.

This algorithm was tested extensively and found quite effective for rapid retarget maneuvers. In the absence of gimbal friction, this simple algorithm arrives at the commanded angle in just one pass through the above loop. However, when friction is included in the model, the gimbal takes less energy to decelerate than to accelerate—causing the gimbal to come to a stop slightly before arriving at the target angle. For smaller retarget maneuvers this effect is negligible and the gimbal arrives in just one pass. The largest retarget maneuver requires only two passes through the loop. Interaction between the two gimbals (which becomes significant only for larger retarget maneuvers) also adds to the large retarget error. However, the second pass through the above loop (being a much smaller maneuver) is sufficient to correct for these errors. See examples of small and large retarget maneuvers in Figures 34 through 36.

Figure 34. Maximum travel step response

Figure 35. Ten-degree step response

Figure 36. One-degree step response

Design of the Inertial Angle Sensor Notch Filters

The system was first tested on a single-axis gimbal with the same resolver, *Inertial Angle Sensor* (IAS) and roughly the same inertia, torque and bearing friction as the actual telescope. This provided an ideal test-bed for debugging servo hardware and software. After eliminating all software errors the system worked perfectly well. However, once implemented on the actual telescope structural resonances on the IAS mounts caused the telescope to oscillate in the (high-gain) fine mode. The single-axis test gimbal did not exhibit this phenomenon since the IAS was mounted rigidly at the center of the gimbal, while the makeshift telescope mounts, being retrofits, were significantly more compliant (see Figure 1). The open-loop frequency response of the fine mode servo system was measured using a network analyzer, and is shown below in Figure 37. This revealed several destabilizing resonances starting at about 200 Hz.

Figure 37. Open-loop frequency response showing IAS resonances

To eliminate this problem, a notch-filter board was added as a pre-filter for the IAS signals. The board provided a maximum of six, second-order active-notch filters, together with a third-order low-pass filter for each axis. The notch filter schematic is shown in Figure 38 along with the frequency response, while the low-pass filter is shown in Figure 39. The transfer function of the notch filter is shown in Equation 17, while that of the low-pass filters is shown in Equation 18. Note that for elevation, ω_3 is infinite. This is realized simply by removing R_1 and making R_2 zero. The specific notch frequency ω for each filter is given in table 5, while the specific values of R and C used to achieve each notch are not listed in this report.

$$N(s) = \frac{s^2 + \omega^2}{s^2 + 2\zeta\omega s + \omega^2} \quad \text{where, } \omega = \frac{1}{RC}, \quad \zeta = \frac{2R_1}{R_1 + R_2} = \frac{2k}{11k} \quad (17)$$

Figure 38. Notch filter schematic and frequency response

$$LP(s) = \frac{\omega_1^2 \omega_2 [s + \omega_3]}{\omega_3 [s^2 + \omega_1 s + \omega_1^2] [s + \omega_2]} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{matrix} \text{Az.} & \text{El.} \\ \begin{bmatrix} 5000 & 1500 \\ 10000 & 10000 \\ 1500 & \infty \end{bmatrix} \end{matrix} \quad (18)$$

$$\omega_1 = \frac{1}{RC}, \omega_2 = \frac{1}{100k \cdot 1000pF}, \omega_3 = \frac{R_1}{R_2} \omega_1$$

Figure 39. Low-pass schematic and frequency response

The frequency responses of Figure 37 provided a general idea of what notch filters would be needed. However, the process used to select the number of filters used as well as their frequencies (described below) was not based on this preliminary data—since it was gathered before the telescope was entirely assembled, and would most likely be different than the final configuration. Also, since each filter added unwanted phase lag, (see Figure 38) it was important to not use more notches than absolutely necessary to avoid eroding the phase margin. When the notch-filter board was designed, it was unclear how many filters would be needed, or what frequencies would be notched-out. Each filter block was thus designed to be "stuffed" as needed, and the board was designed with jumpers to bypass unused filter blocks.

The notch filters were selected as follows. The fine mode was engaged on the telescope while the dominant oscillation frequency for each axis was measured. A notch filter was then built to eliminate that frequency and inserted in front of the IAS signal. The fine mode was again enabled, while the next most dominant mode was identified. This continued until all oscillations were eliminated. The low-pass filter was then added to roll-off all high frequency resonances. The azimuth low-pass filter includes a zero at 1500 rad/sec needed to restore phase margin that was lost due to lower

frequency notches. See Table 4 below for the list of notch filters used for each axis. Also, note that the elevation axis needed one less notch filter than did azimuth.

Azimuth	264 Hz	335 Hz	406 Hz	504 Hz	2.5 kHz	5 kHz
Elevation		335 Hz	555 Hz	635 Hz	720 Hz	875 Hz

Table 4. Notch filter locations

Implementation Results

Figures 34 through 36 show the large step responses. Figures 40 through 42 show the performance of the coarse and fine modes as well as the transition between the three modes. Figure 40 shows a step response of 0.3 degrees in both axes simultaneously. The top graph in Figure 40 shows the gimbal angles in degrees. The second graph shows the output of the *Inertial Angle Sensor* (IAS) in microradians, while the bottom graph shows a close-up of the gimbal angles. The pointing system starts out in the big-step mode, and exits the big-step mode to begin coarse mode control at about $t = 0.3$ seconds. During this first segment, the IAS is "caged" to keep it out of saturation. While caged, the sensor behaves like an accelerometer. The sensor remains caged until about $t = 0.75$ seconds, to allow for the fluid inside the IAS to stabilize.* After about one second, the telescope is switched into the fine mode, which causes a small disturbance to the IAS signal. The higher gain of the fine mode allows the steady-state pointing error to be reduced down to a single bit (1.5 microradians).

Figure 41 shows a step response small enough to avoid the big-step algorithm. In this test the controller stayed in the coarse mode until about $t = 1$ second, then the fine mode was switched on. The sensors were uncaged at about $t = 0.7$ seconds. Figure 42 shows an even smaller step response that was executed entirely in the fine mode. Here the error remained just under 0.02 degrees, allowing the sensors to remain uncaged, and the controller to stay in the fine mode. Note that, the fine mode was optimized for maximum disturbance rejection—not minimum response time. Thus, the step response is significantly more "sluggish" than either the big-step mode or the coarse mode, however, because the sensors did not need to be caged, the pointing error stabilizes sooner than in the other modes (at about 0.6 seconds). Figure 43 shows actual in-flight pointing performance with typical host vehicle base motion disturbances. The azimuth pointing jitter as measured by the IAS's over this time interval is 295 nanoradians (rms) while the elevation pointing jitter is 385 nanoradians (rms). Note that both axes comfortably exceed the design goal of 1 microradian pointing stability.

* Since the sensors are caged prior to $t = 0.75$ seconds, the IAS data is not scaled to units of microradians until after $t = 0.75$ seconds. For this reason the data is shown in units of volts—which is equivalent to microradians once the sensors are uncaged.

Figure 40. Step response of 0.3 degrees in both axes simultaneously

Figure 41. Coarse mode step response (both axes)

Figure 42. Fine mode step response

Figure 43. In-flight tracking performance

Conclusions

Quantitative Feedback Theory (QFT) provided a simple, yet powerful design tool which resulted in a solution that exceeded the performance goal of 1 microradian pointing stability in spite of large host vehicle pointing errors. One of the most challenging aspects of this project was the blending of feedback and feed-forward sensors to achieve "optimum" performance. These feedback sensors included the gimbal resolvers and Inertial Angle Sensors (IAS), whose signals were blended with the variably-delayed host vehicle attitude data coming from the Inertial Measurement Unit (IMU). The blending of these sensors was greatly simplified using QFT. Using this technique, design trade-off options could be easily weighed against each other in terms of cost versus performance benefit.

Another significant challenge was overcoming the structural resonances in the telescope. The performance goal of 1 microradian pointing stability resulted in a relatively high gain/bandwidth controller. Because of the large bandwidth, several structural resonances were excited, causing pointing performance degradation. Several notch filters were added to the controller to eliminate the possibility of exciting these structural resonances. The extra phase lag introduced by these notch filters decreased the phase margin of the closed-loop system, which in turn further degraded pointing performance. Ultimately the "optimum" trade-off between gain and phase margin was obtained to produce a controller that exceeded the design goals without exciting structural resonances.

Because a suitable radiation-hardened floating-point CPU was not available to implement the control algorithms, all components of the feedback compensator and notch filters were implemented in analog hardware. Thus, the controller modifications, which were necessary to avoid exciting the structural resonances, were more difficult to create. Because of the short design cycle, it was not possible to fabricate new printed circuit boards in time to meet the delivery schedule. Therefore, all modifications had to be carefully reworked on the original flight hardware. This provided another significant challenge.

Yet despite these obstacles, the pointing control system was delivered on time, with measured in-flight performance that meets the design goal with significant margin.

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