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SENSITIVITY OF PROBABILISTIC MCO WATER CONTENT ESTIMATES TO KEY ASSUMPTIONS

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Abstract: Sensitivity of probabilistic multi-canister overpack (MCO) water content estimates to key assumptions is evaluated with emphasis on the largest non-cladding film contributors, water borne by particulates adhering to damage sites, and water borne by canister particulate. Calculations considered different choices of damage state degree of independence, different choices of percentile for reference high inputs, three types of input probability density function (pdfs): triangular, log-normal, and Weibull, and the number of scrap baskets in an MCO.

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***SENSITIVITY OF PROBABILISTIC MCO
WATER CONTENT ESTIMATES TO
KEY ASSUMPTIONS***

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**Sensitivity of Probabilistic MCO Water
Content Estimates to Key Assumptions**

1.0 PURPOSE

The purpose of this document is to explore sensitivity of bounding multi-canister overpack (MCO) water inventory estimates to these three issues: (1) Degree of independence for choice of input parameters values, (2) Percentile values of reference high input parameter values, and (3) Selection of extreme values for input parameters.

2.0 SUMMARY

Sensitivity of probabilistic MCO water content estimates to key assumptions is evaluated with emphasis on the largest non-cladding film contributors, water borne by particulate adhering to damage sites and water borne by canister particulate. Previous documents present "high" values for MCO particulate and water content found by applying high estimates for damage state and water fraction to all fuel elements in an MCO, Sloughter, 1998a. That procedure is known to be conservative and this document presents calculations to quantify the degree of conservatism and sensitivity to underlying assumptions. The analyses concludes that the high, or bounding, values are conservative.

It should be noted that values used for canister particulate and bound water do not reflect the latest issued values, Sloughter, 1998b. The results from using the values given in Sloughter, 1998a for this analyses is the introduction of a small degree of conservatism since they are slighter lower than those in Sloughter, 1998b.

3.0 REFERENCES

Sloughter, J. P., 1998a, "Estimates of Particulate Mass in Multi-Canister Overpacks," HNF-1527 Rev 1, Duke Engineering & Services Hanford, Richland, Washington.

Sloughter, J. P., 1998b, "Estimates of Particulate Mass in Multi-Canister Overpacks," HNF-1527, Rev 2, Duke Engineering & Services Hanford, Richland, Washington.

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1.0 SUMMARY

Sensitivity of probabilistic multi-canister overpack (MCO) water content estimates to key assumptions is evaluated here with emphasis on the largest non-cladding film contributors, water borne by particulate adhering to damage sites, and water borne by canister particulate. Previous documents present "high" values for MCO particulate and water content found by applying high estimates for damage state and water fraction to all fuel elements in an MCO [Slougher, et al., 1998; and Duncan and Ball, 1998]. That procedure is known to be conservative, and this document presents calculations to quantify the degree of conservatism and sensitivity to underlying assumptions. Cladding surface films are discussed in [Slougher, et al., 1998].

Key technical issues addressed in this calculation note are:

1. Damage State Degree of Independence. Because an MCO is comprised of fuel from about 18 canister pairs, and the number of damaged fuel assemblies and the state of damage vary from canister to canister, the degree of independence, or number of independent choices of damage state in an MCO, must be considered.
2. Percentile of Reference High Inputs. The percentile assigned to reference high inputs [Slougher, et al., 1998; and Duncan and Ball, 1998] must be explored. The reference definition of a high input value is one that is exceeded occasionally, but is an unreasonable representation of an entire MCO.
3. Extreme Input Values. Reasonable extrema must be chosen for input parameters, if the previously used high values are to be exceeded.

Probabilistic calculations were performed to determine distributions for total water borne by particulate adhering to damage sites and canister particulate adhering to any part of a fuel assembly. In all, one hundred eighty (180) calculations were performed to explore the three issues described above. Calculations considered different choices of damage state degree of independence, different choices of percentile for reference high inputs, three types of input probability density functions (pdfs): triangular, log-normal, and Weibull, and the number of

scrap baskets in an MCO. The number of independent damage state choices was varied between 4 and 18, where 4 corresponds to each fuel basket plus associated scrap having independent damage states, and 18 corresponds to each canister pair (14 assemblies) having independent damage states. A degree of independence of at least 10 is judged to be realistic based on observations [Pitner, 1995]. For triangular distributions, percentile values for reference high input parameters were 100%, 97.5%, 95%, and 90%. For log-normal and Weibull distributions, these percentiles were 99.9%, 97.5%, 95%, and 90%. A percentile of at least 95% is judged to represent the reference choice of high values. Cases were replicated to consider MCO configurations with zero, one, and two scrap baskets.

Results of probabilistic calculations show the following trends:

- the calculated mean particulate mass and water content increase as the percentile of reference bounding input parameters decreases,
- the calculated particulate mass and water content standard deviation decrease as the number of independent damage states increases,
- output distributions are generally well-approximated as log-normal,
- the "incredible value" (the value with an exceedance probability of 1×10^{-6}) of particulate and water mass decreases as the number of independent damage state choices increases, and
- incredible values are comparable for cases with triangular, log-normal, and Weibull input pdfs.

Calculated mean values for water content depend on the number of scrap baskets, damage state independence, and percentile applied to reference high input parameters, but may be summarized as follows:

- For no scrap baskets, the mean value for water content was between 0.1 and 0.4 kg,
- For a single scrap basket, the mean value for water content was between 0.2 and 0.5 kg, and

- For two scrap baskets, the mean value for water content was between 0.2 and 0.6 kg.

Reference high water content values were only exceeded for extreme combinations of the 90th percentile assigned to reference input high quantities, and a low degree of damage state independence, less than 10 independent selections. For the case of reference high inputs representing the 95th percentile, and at least $n = 10$ for independent choices of damage level, incredible values are calculated to be roughly 2/3 the reference high water contents:

- For zero scrap baskets, about 0.8 kg are calculated to be incredible, versus the reference value of 1.2 kg,
- For one scrap basket, 0.75 to 0.83 kg are calculated to be incredible, versus the reference value of 1.37 kg, and
- For two scrap baskets, about 1.0 kg is calculated to be incredible, versus the reference value of 1.55 kg.

2.0 PURPOSE

The purpose of this work is to explore sensitivity of bounding multi-canister overpack (MCO) water inventory estimates to these three issues: (1) Degree of independence for choice of input parameter values, (2) Percentile values of reference high input parameter values, and (3) Selection of extreme values for input parameters. This calculation focuses on the most important two line item contributors to the MCO water inventory, excluding cladding surface films, water associated with particulate clinging to fuel assemblies at damage sites and water associated with canister particulate adhering to assembly surfaces after fuel cleaning. Cladding film inventory is discussed by [Sloughter, et al., 1998] and its water content is discussed by [Duncan and Ball, 1998].

3.0 SOURCES

High, nominal, and low values for the post-cold vacuum drying (CVD) MCO particulate inventory are provided by [Sloughter, et al., 1998], and similar values for the post-CVD MCO water inventory are provided by [Duncan and Ball, 1998]. Both documents used a framework with seven basic particulate/water source categories, as shown in Table 3-1. Cladding film, oxide film, and adhering particulate may be further broken down into scrap basket and fuel basket line item contributions. The largest contribution to water inventory is from a cladding film composed of aluminum oxide hydrates for those fuel assemblies stored in aluminum containers at K West; this contribution is considered by [Sloughter, et al., 1998].

The largest contributors remaining are water associated with particulate adhering to fuel assemblies at damage sites for an MCO with no scrap baskets, and water associated with canister particulate for an MCO with two scrap baskets. The particulate masses per assembly and water mass fractions were assigned values as shown in Table 3-2.

Canister particulate water was calculated by multiplying high, nominal, and low values for particulate per fuel element by the number of fuel elements in fuel baskets in an MCO, and multiplying the result by corresponding high, nominal, and low water fractions:

$$m_{wcp} = n_f n_{af} m_{cpf} f_{wcp} \quad (3-1)$$

where m_{wcp} = mass of canister particulate water in an MCO,

n_f = number of fuel baskets in an MCO,

n_{af} = number of fuel assemblies in a fuel basket, 54,

m_{cpf} = mass of canister particulate per fuel assembly, and

f_{wcp} = water fraction of canister particulate.

Note that canister particulate is only associated with assemblies in fuel baskets. Table 3-2 contains the high, nominal, and low values for the evaluation. For example, the nominal water

Table 3-1: Summary of Reference Water Mass Categories and Values Rounded From [Plys, et al., 1998].

Source	Zero Scrap, Five Fuel Baskets			One Scrap, Four Fuel Baskets			Two Scrap, Three Fuel Baskets		
	High	Nominal	Low	High	Nominal	Low	High	Nominal	Low
Al(OH) ₃ Cladding Film	3.72	0.931	0	3.52	0.88	0	3.31	0.83	0
Uranium Oxide Hydrate Cladding Film	0	0.04	0.06	0	0.04	0.05	0	0.03	0.05
Oxide Film	0.02	<0.001	<0.001	0.04	0.002	<0.001	0.06	0.004	0.001
Adhering Particulate	0.30	0.08	0.005	0.67	0.10	0.008	1.0	0.11	0.01
Canister Particulate	0.91	0.049	0	0.73	0.039	0	0.55	0.029	0
Generated Particulate	0.03	0	0	0.05	0	0	0.08	0	0
Free Water	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
TOTAL	5.2	1.30	0.26	5.2	1.25	0.26	5.2	1.20	0.26

Table 3-2: Calculation of Particulate Water Mass

Source	Low	Nominal	High
Adhering Particulate Mass per Assembly, Scrap, gram	6	20	200
Adhering Particle Mass per Assembly, Fuel, gram	1.3	7.6	20
Adhering Particulate Water Content	1.5%	4. %	5.5%
Canister Particulate Mass per Fuel Assembly, gram	0	2.22 (1)	13.0
Canister Particulate Water Content	2. %	7.5%	26. %
Example for MCO With One Scrap Basket:			
Canister Particulate Water Mass, kg	0	0.036	0.73
Adhering Particulate Water Mass, Scrap (2), kg	0.004	0.031	0.43
Adhering Particulate Water Mass, Fuel (2), kg	0.004	0.066	0.24
(1) This value is 2.41 grams in the final version of [Sloughter, et al., 1998]. For the methodology to follow, the difference between the draft and final values is negligible.			
(2) 39.1 assemblies for scrap, 216 assemblies for fuel.			

mass borne by canister particulate in an MCO with one scrap basket is $m_{wep} = 4 * 54 * 0.00241 * 0.075 = 0.039$ kg. Note that the nominal canister particulate mass changed slightly between draft and final versions of [Sloughter, et al., 1998]. The draft value in Table 3-2 was used for analyses in Section 5, and Appendix I contains calculations with the final value, and demonstrates that there is no impact on conclusions.

Adhering particulate water was calculated by summing contributions from fuel and scrap basket assemblies, where each contribution again is a product of mass and water content values assigned to all fuel assemblies of each basket type:

$$m_{wap} = n_f n_{af} m_{apf} f_{wap} + n_s n_{as} m_{aps} f_{wap} \quad (3-2)$$

where m_{wap} = mass of adhering particulate water in an MCO,

m_{apf} = mass of adhering particulate per fuel assembly,

m_{aps} = mass of adhering particulate per scrap assembly,

f_{wap} = water fraction of adhering particulate,

n_s = number of scrap baskets, and

n_{as} = number of equivalent fuel assemblies in a scrap basket, 39.1.

Table 3-2 also contains values needed to calculate the adhering particulate water mass. For example, the nominal water mass borne by adhering particulate in an MCO with one scrap basket is $m_{\text{wap}} = (4 * 54 * 0.0076 * 0.04) + (1 * 39.1 * 0.02 * 0.04) = 0.066 + 0.031 = 0.097 \text{ kg}$.

The technique described above is conservative because high values for each input parameter are combined and assumed to apply to all fuel elements. Clearly, it is improbable to load an MCO with fuel assemblies that are all at extreme damage levels and whose corrosion product contains the highest water content. Calculations in this report are motivated by these concerns.

4.0 TECHNICAL BASES

This assessment explores sensitivity to the degree of independence of key input parameter values and the shape and extrema of their probability density functions (pdfs).

A best judgement of the degree of independence of damage states is based on the K-East top view fuel survey which assigned damage states to every fuel assembly inner and outer element in every canister [Pitner, 1995]. As a technical basis for conclusions, the degree of independence of fuel damage levels is expected to be close to the canister pair level, i.e., the damage level will vary randomly from canister pair to canister pair, or for groups of a few canister pairs. In contrast, it is not expected that the degree of independence exists at the fuel basket level (a fuel basket holds 54 assemblies, or about 4 canister pairs), and it is certainly inappropriate to assume that all fuel assemblies in an MCO can have the same damage level. Note that this assumption applies to fuel damage levels and not to the cladding film; it is possible to construct an MCO comprising assemblies which all have or all lack an aluminum oxide hydrate cladding film.

Reference particulate [Slougher, et al., 1998] and water content [Duncan and Ball, 1998] documents provide technical bases for bounding input parameter values. High values are such that they may be occasionally exceeded, but are not a realistic value for an entire MCO. Here, percentiles are assigned to these values. For example, the 95-th percentile implies a 5% chance that the bound may be locally exceeded. The 95-th percentile is judged to represent the intent of the references.

The reference documents provide three points: low, nominal, and high values for input parameters. These values form the technical basis for selection of pdf's used here. The chosen pdf's, triangular, log-normal, and Weibull, capture the reference data and provide a detailed pdf shape not explicitly given by the references. Obviously, a triangular distribution can be created from the three reference points. A log-normal distribution has the advantage of allowing a longer tail for extreme values, which is useful for sensitivity analysis. The Weibull distribution

provides an alternative to the log-normal for additional sensitivity analysis. Other probability distributions can be used to describe the results noted in Tables 3-1 and 3-2, but with little realistic gain.

5.0 CALCULATION

5.1 Technical Issues

Probabilistic calculations were performed to calculate the particulate mass and water content associated with particulate clinging to fuel assemblies of a single MCO. These calculations use a range of input parameter values selected to address three key issues of a probabilistic assessment.

1. Damage State Degree of Independence. An MCO is loaded with fuel assemblies from about 18 canister pairs (assuming each pair contains 14 full assemblies). High estimates of particulate mass [Slougher, et al., 1998] were previously made by applying bounding values per assembly simultaneously to all assemblies loaded into an MCO. In practice, the damage state is variable from canister pair to canister pair, so as many as 18 independent choices of damage state are implicitly made as an MCO is loaded. This implies a very low probability that all loaded assemblies have damage states near the high value.

Thus the first key issue for a probabilistic assessment is the number of times a given damage state is independently chosen when loading an MCO. This issue is addressed by varying the number of independent damage state choices from 4 to 18 with an equal number of assemblies per choice.

2. Probability of Exceeding High Input Parameter Values. Input parameters required to estimate the particulate inventory and water content were assigned low, nominal, and high values in references. The stated interpretation of the high values is: The high value may occasionally be locally exceeded, but it would be inappropriate to apply this value for an entire MCO. From this statement, particulate and water content estimates were implicitly defined as incredible because high input parameter values were indeed applied to the entire contents of an MCO. Authors of the particulate and water content estimates did not have a good knowledge base to assign a probability that the selected high value might be locally exceeded.

Hence, the second key issue for a probabilistic assessment is to explore the impact of various probability levels that the high value for input parameters might be locally exceeded, in essence, to assign a percentile

value in a cumulative pdf to the previously used high parameter values. This issue is addressed by assigning percentile values of 100% (99.9% in the case of a log-normal or Weibull), 97.5%, 95%, and 90%.

3. Extreme Values of Input Parameters. When probability density functions (pdf's) are chosen for input parameter values, a key issue is the physical meaning of the highest values that are implied by the distribution. On one hand, it is physically necessary that a meaningful upper limit be used to define the distribution or be implied by the distribution; for example, the amount of particulate on a fuel assembly cannot physically exceed some fraction of the assembly mass. On the other hand, in the absence of a large number of measurements, a physical upper bound can be argued to be somewhat subjective. Expanding the previous example, the maximum fraction of an assembly that is particulate should not exceed a value at which the assembly would no longer have structural integrity -- but selection of the fraction is somewhat subjective.

Hence, the third key issue for a probabilistic assessment is the selection of physically possible extrema for input parameter pdf's. This issue is addressed by considering triangular, log-normal, and Weibull distributions. The log-normal and Weibull are useful for sensitivity because they exhibit long tails and allow large values.

Note that the issues are framed to resolve the question of high bounding results. If the concern were shifted to the expected range of particulate during normal operation, issues such as the choice of pdf for input parameters would also become important. However, the three issues described above suffice for exploration of bounding results.

5.2 Model Equations and Method

The mass of particulate adhering to damage sites in a single MCO can be found from:

$$m_{ap} = \sum_{i=1}^n \frac{n_s n_{as}}{n} m_{apsi} + \frac{n_f n_{af}}{n} m_{apfi} \quad (5-1)$$

where m_{ap} = mass of adhering particulate in an MCO,
 n = degree of independence,
 n_s = number of scrap baskets,

- n_{as} = number of assemblies per scrap basket,
 m_{apsi} = mass of adhering particulate per scrap assembly, random sample i,
 n_f = number of fuel baskets,
 n_{af} = number of assemblies per fuel basket, and
 m_{apfi} = mass of adhering particulate per fuel assembly, random sample i.

This equation is an extension of the reference calculations (see Section 3, Sources) to allow n independent damage level choices among 39.1 scrap and 216 fuel assemblies. If there is no independence between canister pairs, $n = 1$. If canister pairs are completely independent, $n = 18$. Calculations were performed here with $n = 4, 7, 10, 14$, and 18 to span the range of interest and demonstrate the impact of variation in this parameter. Each independent group of assemblies is of equal size because the point is to illustrate the impact of having $n > 1$.

Similarly, water content can be found from:

$$m_{wap} = \sum_{i=1}^n \left(\frac{n_s n_{as}}{n} m_{apsi} + \frac{n_f n_{af}}{n} m_{apfi} \right) x_{wapi} \quad (5-2)$$

where x_{wapi} = water fraction of adhering particulate on random sample i.

Similarly, the mass of canister particulate and the mass of water borne by canister particulate are given by

$$m_{cp} = \sum_{i=1}^n \frac{n_f n_{af}}{n} m_{cpfi} \quad (5-3)$$

$$m_{wcp} = \sum_{i=1}^n \frac{n_f n_{af}}{n} m_{cpfi} x_{wcpi} \quad (5-4)$$

- where m_{cp} = mass of canister particulate in an MCO,
 m_{cpfi} = mass of canister particulate per fuel assembly, random sample i, and
 x_{wcpi} = water fraction of canister particulate, random sample i.

Total water in an MCO for a particular trial is therefore

$$m_w = m_{wap} + m_{wcp} \quad (5-5)$$

A Monte Carlo method was employed using various probability density functions (pdf's) for inputs, described in Section 5.3, to generate pdf's for particle masses and water contents. Simulations using Crystal Ball™ [Decisioneering, 1996] generated thousands of random samples for the five random variables m_{apfi} , m_{apfi} , x_{wapi} , m_{cpfi} , and x_{wcpi} (equations (5-1) and (5-4) above), then evaluated m_{cp} , m_{wcp} , m_{ap} , m_{wap} , and m_w , and lastly, tallied the output.

Probability density functions used for the five random variables can be described as follows:

Triangular

Parameters: Minimum, Likeliest, and Maximum.

$$\text{Formula: } f(x) = \begin{cases} \frac{h(x - \text{Min})}{\text{Likeliest} - \text{Min}} & \text{if } \text{Min} < x < \text{Likeliest} \\ \frac{h(x - \text{Likeliest})}{\text{Likeliest} - \text{Max}} + h & \text{if } \text{Likeliest} < x < \text{Max} \\ 0 & \text{otherwise} \end{cases} \quad (5-6)$$

$$\text{where } h = \frac{2}{\text{Max} - \text{Min}}$$

Weibull

Parameters: Location (L), Scale (α), and Shape (β).

$$\text{Formula: } f(x) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{x - L}{\alpha} \right)^{\beta-1} e^{-\left(\frac{x-L}{\alpha} \right)^\beta} & \text{if } x > L \\ 0 & \text{if } x < L \end{cases} \quad (5-7)$$

Log-Normal

Parameters: Log mean (μ), and Log Standard Deviation (σ).

$$\text{Formula: } f(x) = \frac{1}{x \sqrt{2\pi} \sigma} e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}} \quad \text{for } 0 < x < \infty \quad (5-8)$$

As discussed in Section 5.4, generated output distributions require curve-fitting to evaluate the incredible (10^{-6} probability of exceeding) water content. For log-normally distributed results, the sample mean μ_s and standard deviation σ_s are related to the log-normal parameters for log mean μ and log deviation σ via [Miller and Freund, 1997]:

$$\sigma^2 = \ln \left[\left(\frac{\sigma_s}{\mu_s} \right)^2 + 1 \right] \quad (5-9a)$$

$$\mu = \ln \mu_s - \frac{1}{2} \sigma^2 \quad (5-9b)$$

5.3 Probability Density Function and Case Inputs

Probability distributions for the five random variables of equations (5-1) to (5-4) and discrete values for the number of independent choices n were selected as described here. Three sets of calculations were performed: (1) a set using triangular distributions for the particulate masses and water percentages, (2) a set using log-normal distributions instead, and (3) a set using Weibull distributions instead. For each type of distribution, four cases were defined by assuming a different percentile for the bounding values. In all, twelve cases were defined. For each case, five values of the degree of independence ($n = 4, 7, 10, 14$, and 18) were explored. Each case was repeated for configurations with zero, one, and two scrap baskets.

As shown in Table 5-1, the first four cases used triangular distributions for the particulate mass per assembly and water content. In Case 1, the high value from the references is the maximum value of each distribution. In Cases 2, 3, and 4, the reference high values are the

Table 5-1: Case Definitions

Case No.	Distribution Shape	Percentile of Reference High Value	Degree of Independence
1	Triangular	100%	4, 7, 10, 14, 18
2	Triangular	97.50%	4, 7, 10, 14, 18
3	Triangular	95.00%	4, 7, 10, 14, 18
4	Triangular	90.00%	4, 7, 10, 14, 18
5	Log-Normal	99.9%	4, 7, 10, 14, 18
6	Log-Normal	97.50%	4, 7, 10, 14, 18
7	Log-Normal	95.00%	4, 7, 10, 14, 18
8	Log-Normal	90.00%	4, 7, 10, 14, 18
9	Weibull	99.9%	4, 7, 10, 14, 18
10	Weibull	97.5%	4, 7, 10, 14, 18
11	Weibull	95.00%	4, 7, 10, 14, 18
12	Weibull	90.00%	4, 7, 10, 14, 18

97.5%, 95.0%, and 90.0% values of the input pdf's, respectively. Another way of looking at it is that the exceedance probabilities for the reference high values are 0.0%, 2.5%, 5.0%, and 10.0%, respectively, in Cases 1 through 4.

Table 5-2 lists triangular distribution parameters in Cases 1 through 4. Appendix A illustrates the triangular distributions. The triangular distribution for Case 1 has the following interpretation: (1) the minimum is the "low" value from the references, (2) the likeliest value is the "nominal" value, and (3) the reference high value is the maximum. In all four cases, the minima and likeliest values were preserved. For cases 2, 3, and 4, the reference high values have the listed percentiles, and the pdf maxima are shifted higher as a consequence. Derivation of maxima is explained in Appendix A.

Table 5-2: Parameters For Triangular Distributions

Case No.	Low (1)	Most Likely (1)	High (1)	Percentile of Reference Bound (2)	Maximum (3)
Adhering Particulate, Scrap: Mass/Assy., kg					
1	0.006	0.02	0.2	100%	0.200
2	0.006	0.02	0.2	97.50%	0.235
3	0.006	0.02	0.2	95%	0.254
4	0.006	0.02	0.2	90%	0.286
Adhering Particulate, Fuel: Mass/Assy., kg					
1	1.30E-03	7.60E-03	2.00E-02	100%	0.020
2	1.30E-03	7.60E-03	2.00E-02	97.50%	0.023
3	1.30E-03	7.60E-03	2.00E-02	95%	0.024
4	1.30E-03	7.60E-03	2.00E-02	90%	0.027
Adhering Particulate Water Content, %					
1	1.5	4	5.5	100%	5.500
2	1.5	4	5.5	97.50%	5.969
3	1.5	4	5.5	95%	6.225
4	1.5	4	5.5	90%	6.677
Canister Particulate: Mass/Assy., kg					
1	0.0	2.20E-03 (4)	1.30E-02	100%	1.3E-2
2	0.0	2.20E-03 (4)	1.30E-02	97.50%	1.52E-2
3	0.0	2.20E-03 (4)	1.30E-02	95%	1.63E-2
4	0.0	2.20E-03 (4)	1.30E-02	90%	1.84E-2
Canister Particulate Water Content, %					
1	2.0	7.5	26.0	100%	26.00
2	2.0	7.5	26.0	97.50%	29.97
3	2.0	7.5	26.0	95%	32.08
4	2.0	7.5	26.0	90%	35.0
(1) Values from [Sloughter, et al., 1998] for mass/assy. and from [Duncan and Ball, 1998] for water content. (2) Probability of a value less than the listed bound. (3) Value used for the triangular distribution. (4) 2.41E-3 in final version of [Sloughter, et al., 1998].					

Cases 5 through 8 used the log-normal distributions shown in Table 5-3 and Appendix B. Log-normal distributions do not have a practical maximum, and a 99.9 percentile value is used for Case 5 as the reference high value. Similarly, reference low values were assigned a probability of 0.1%. In Cases 6, 7, and 8, the high values are the 97.5%, 95.0%, and 90.0% values, respectively; i.e., the exceedance probabilities for the high values are 2.5%, 5.0%, and 10.0%, respectively, in Cases 6 through 8.

For adhering particulate mass, log-normal distribution parameters were selected to preserve the minimum value (at the 0.1% percentile) and to ensure the desired percentile for the reference high value. The mean of the resulting distributions (see Table 5-3) shifted to successively higher values compared to the reference nominal value. This approach allows the goal - a test of sensitivity to the assumed distribution - to be attained in a conservative manner.

For adhering particulate water content, a different approach was required to avoid physically unrealistic values above 11% water. Here, the reference nominal was preserved as the median of the input pdf, and values for the standard deviation were chosen to recover the desired percentile for the reference high value. As seen in Table 5-3, input pdf's resulting from this method had only trivial probability of unphysically large values.

For canister particulate mass, log-normal parameters were selected to preserve the reference nominal values as the median, and to match the desired percentile for the reference high value. This is because following the method used for adhering particulate resulted in mean and median values below the reference nominal. With these assumptions, final results are relatively insensitive to minor variations in the nominal values. If the nominal value (the median) changes from the draft value of 2.22 grams/assembly to the final value of 2.41 grams/assembly, the mean of the log-normal distribution changes from 3.93 grams to 4.05 grams, for a 3% increase. Of course, the canister particulate mass is but one part of the calculation for total water. For cases with one or two scrap baskets, the adhering particulate contributes the preponderance of water mass. Considering the random variations associated with a limited number of Monte Carlo trails, such variations in input are not significant.

Table 5-3: Parameters For Log-Normal Distributions						
Case No.	Low (1)	Mean	Std. Dev.	High (2)	Percentile (3)	
Adhering Particulate, Scrap: Mass/Assy., kg						
Ref. (4)	0.006	0.020	-	0.2	100%	
5	0.006	0.041	0.025	0.2	99.90%	
6	0.006	0.065	0.051	0.2	97.50%	
7	0.006	0.078	0.067	0.2	95.00%	
8	0.006	0.099	0.094	0.2	90.00%	
Adhering Particulate, Fuel: Mass/Assy., kg						
Ref. (4)	0.0013	0.0076	-	0.02	100%	
5	0.0013	0.0056	0.0026	0.02	99.90%	
6	0.0013	0.0080	0.0047	0.02	97.50%	
7	0.0013	0.0091	0.0058	0.02	95.00%	
8	0.0013	0.0111	0.0076	0.02	90.00%	
Canister Particulate: Mass/Assy., kg						
Case No.	Median		High		Percentile	
5	2.22E-3 (7)		1.29E-2		99.9%	
6	2.22E-3 (7)		1.29E-2		97.5%	
7	2.22E-3 (7)		1.29E-2		95.0%	
8	2.22E-3 (7)		1.29E-2		90.0%	
Canister Particulate Water Content, %						
5	7.5		26.0		99.9%	
6	7.5		26.0		97.5% (6)	
7	7.5		26.0		95.0% (6)	
8	7.5		26.0		90.0% (6)	
Adhering Particulate Water Content, %						
Case No.	Low (1)	Mean	Std. Dev.	High (2)	Percentile (3)	P(H ₂ O) > 11%
Ref. (5)	1.56	4	-	5.5	100%	0%
5	1.56	4	0.43	5.5	99.90%	< 1E-7%
6	1.56	4	0.68	5.5	97.50%	1E-7%
7	1.56	4	0.82	5.5	95.00%	2E-5%
8	1.56	4	1.07	5.5	90.00%	5E-3%
(1) Low = 0.1 percentile of Log-Normal. (2) High of [Slougher, et al., 1998].						
(3) Probability of a value less than listed high. (4) [Slougher, et al., 1998] values.						
(5) [Duncan and Ball, 1998] values. (6) Clipped at 35.0%.						
(7) 2.41E-3 in final version of [Slougher, et al., 1998].						

For canister particulate water, the reference nominal value was preserved as the median of the input pdf and values for the standard deviation were selected to recover the desired percentile for the reference high value, as was done for adhering particulate. In addition, the output value was truncated at 35% corresponding to a physical bound for pure $Al(OH)_3$.

Log-normal pdf's are plotted in Appendix B. As discussed, particulate distributions are a conservative interpretation of reference inputs, and water content distributions are both consistent and physically reasonable.

Weibull distributions were created by manipulating the location, scale, and shape parameters to reflect reference estimates. The distributions used are summarized in Table 5-4 and Appendix C. For both particulate and water, the location parameter was set equal to the low value of the reference estimate. Weibull distributions have the property that no values below the location can be generated. The most likely value, as defined by reference estimate, was set equal to the mode (most probable value of the Weibull distribution), and the bound value was set equal to the particular percentile point used in that case. Mathematical details are in Appendix C.2.

5.4 Results

Each simulation produced a mean, standard deviation and incredible value (1×10^{-6} exceedance probability) for the output particulate and water mass distributions. The simulations used 1,000 Monte Carlo trials. For each case, the simulation produced 1,000 random deviates for total water contact.

Output distributions were identified by curve fitting of the output random deviates. Random deviates for particulate mass and water content were tested against a variety of continuous distributions (normal, log-normal, Weibull, extreme value, etc.) using the Anderson-Darling goodness-of-fit test [Decisioneering, 1996]. The Anderson-Darling test was preferred in this instance because it is most useful when the extreme tails of a distribution are of concern.

Table 5-4: Parameters For Weibull Distributions					
Case No.	Low (Location) (1)	Scale	Shape	High (2)	Percentile (3)
Adhering Particulate, Scrap: Mass/Assy., kg					
Ref. (4)	0.006	-	-	0.2	100%
9	0.006	0.0439	1.30	0.2	99.90%
10	0.006	0.0648	1.19	0.2	97.50%
11	0.006	0.0760	1.16	0.2	95.00%
12	0.006	0.0929	1.13	0.2	90.00%
Adhering Particulate, Fuel: Mass/Assy., kg					
Ref. (4)	0.0013	-	-	0.02	100%
9	0.0013	0.0081	2.30	0.02	99.90%
10	0.0013	0.0094	1.89	0.02	97.50%
11	0.0013	0.0101	1.77	0.02	95.00%
12	0.0013	0.0113	1.64	0.02	90.00%
Adhering Particulate Water Content, %					
Ref. (5)	1.50	-	-	5.5	100%
9	1.50	2.63	4.60	5.5	99.90%
10	1.50	2.73	3.43	5.5	97.50%
11	1.50	2.83	3.16	5.5	95.00%
12	1.50	2.94	2.71	5.5	90.00%
Canister Particulate: Mass/Assy., kg					
9	0.00	3.96E-3	1.63	0.013	99.9%
10	0.00	5.19E-3	1.42	0.013	97.5%
11	0.00	5.82E-3	1.36	0.013	95.0%
12	0.00	6.84E-3	1.30	0.013	90.0%
Canister Particulate Water Content, %					
9	2.0	8.40	1.843	26.0	99.9%
10	2.0	10.46	1.571	26.0 (6)	97.5%
11	2.0	11.52	1.495	26.0 (6)	95.0%
12	2.0	13.26	1.407	26.0 (6)	90.0%
(1) Location parameter equal to Low. (2) High of [Sloughter, et al., 1998]. (3) Probability of a value less than the listed high. (4) [Sloughter, et al., 1998] values. (5) [Duncan and Ball, 1998] values. (6) Clipped at 35%.					

The Anderson-Darling test is similar to the Kolmogorov-Smirnov test in that both consider the differences between the two cumulative distributions (i.e., the data and the test distribution). The Anderson-Darling test places greater weight on the differences between the data and test distribution at the tails than the differences at the mid-range, making it a good choice for computing an incredible water content value [Kotz, et al., 1982].

Resulting water content output deviates were tested against one of the following distributions: normal, log-normal, Weibull, gamma, and beta. Inspection of the output histograms showed that other distributions would not provide good fits. Crystal Ball™ ranked the distributions according to the Anderson-Darling statistic, with the best distribution having the lowest Anderson-Darling statistic (A^2). Not every set of output deviates was tested because there are 180 of them all told. For each case, several sets of output deviates were tested until it became clear that the outputs for that case could or could not be described by an assumed distribution.

The Anderson-Darling statistic is part of the Cramer-von Mises family given by [Kotz, et al., 1982],

$$Q = n \int_{-\infty}^{\infty} (F_n(x) - F(x))^2 \psi(x) dF(x) \quad (5-10)$$

where Q is a statistic of the Cramer-von Mises family, n is number of samples, $F_n(x)$ is the empirical distribution function (EDF), $F(x)$ is the specified (cumulative) distribution, and ψ is a weighing function. An empirical distribution function is defined by,

$$F_n(x) = \frac{\text{number of observations} \leq x}{n} \quad \text{for } -\infty < x < \infty \quad (5-11)$$

In other words,

$$F_n(x) = 0 \quad \text{if } x < X_1 \quad (5-12a)$$

$$F_n(x) = \frac{i}{n} \quad X_i < x < X_{i+1} \quad \text{if } i = 1 \dots n-1 \quad (5-12b)$$

$$F_n(x) = 1 \quad X_n \leq x \quad (5-12c)$$

for a random sample of size n and $X_1, \dots, X_n$. A plot of an EDF will resemble a number of step functions. When the weighing function is chosen as

$$\psi = \frac{1}{F(x) [1 - F(x)]} \quad (5-13)$$

it is clearly large at the tails when $F(x) \rightarrow 0$ or $F(x) \rightarrow 1$, and the resulting statistic Q is the Anderson-Darling statistic, A^2 .

The following procedure is used to test a sample population against a specified distribution with the Anderson-Darling statistic [D'Agostino and Stephens, 1986]:

- The null hypothesis is that the sample population $X_1, X_2, \dots, X_n$ comes from $F(X, \theta)$, which is a completely specified continuous distribution.
- Place the sample population in ascending order $X_1 < X_2 < \dots < X_n$.
- Calculate $Z_i = F(X_i, \theta)$, $i = 1, 2, \dots, n$.
- Calculate the Anderson-Darling statistic using,

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^n (2i-1) [\ell \log Z_i + \ell \log (1 - Z_{n+1-i})] \quad (5-14)$$

- If the statistic exceeds the value at level α , the null hypothesis is rejected at significant level α .

In some instances, more than one distribution was acceptable; i.e., the null hypothesis (the output deviates are from that particular distribution) could not be rejected. In these instances, a log-normal distribution was preferred because it is simple, has a longer tail, and results in a more conservative incredible value.

For triangular inputs, water mass output was adequately described by a log-normal distribution. Depending on the case, Anderson-Darling statistics were between 0.2 and 0.6,

denoting levels of significance (α) (Appendix H) between 94% and 65%, respectively, for not rejecting the null hypothesis [D'Agostino and Stephens, 1986].

For Weibull distribution inputs, water mass output is well described by a log-normal distribution. Anderson-Darling statistics were generally less than 0.5. In the Weibull input cases, the incredible content was calculated by taking the inverse of the log-normal distribution with an exceedance probability of 1×10^{-6} .

For log-normal input, output deviates were adequately described by a log-normal distribution, with the exception of a few cases where none of the continuous distributions available to Crystal Ball™ are a good fit to the data. These exceptions are the case with one fuel basket and high values at 95.0% or 90.0%. In these instances, the Anderson-Darling statistic was much greater than one and the log-normal distribution was a poor fit. This is probably due to the fact that these distributions are highly skewed, with very long tails. This is evidenced by the incredible value calculated using a log-normal distribution with an exceedance of 1×10^{-6} . For the exceptional cases, the maximum output deviate, based on 1,000 trials, exceeded the incredible value assuming a log-normal distribution and 1×10^{-6} exceedance probability. Clearly, a log-normal distribution, or any other standard distribution, cannot be used to calculate an incredible value. Two of these cases are shown in Figures 5-1 and 5-2.

For log-normal input and one scrap basket, the simulation was re-run using the Latin Hypercube Sampling (LHS) technique available with Crystal Ball™. This was an attempt to obtain output distributions that could be described by any of the standard, continuous distributions, and better estimates for the incredible value. With an LHS scheme, an input distribution is divided into intervals of equal probability. Crystal Ball™ then generates one random value for each interval. This is more precise than a direct Monte Carlo technique because the entire distribution is sampled more evenly. The LHS scheme does not produce significantly better results. For log-normal input with high value equal to 90th or 95th percentile, output distributions do not fit any of the standard, continuous distributions, regardless of the sampling scheme. Means, standard deviations, incredible values, etc., did not change significantly by using a LHS technique.

Figure 5-1:

**Output Water Content Curve Fit for Case 8 (log-normal, 95 percentile) and 4 Groups,
Log-Normal Inputs, One Scrap Basket.**

For a log-normal distribution, Figure 5-1 shows the fit to the generated output. The fit is poor, based on an Anderson-Darling statistic of 1.5792 (refer to Appendix H). Also, the maximum value of the 1,000 output deviates (4.562) exceeds the incredible value (4.369) based on a log-normal distribution with 10^{-6} exceedance probability (see Appendix F).

Figure 5-2:

**Output Water Content Curve Fit for Case 8 (log-normal, 95 percentile) and 18 Groups,
Log-Normal Input, One Scrap Basket.**

For a log-normal distribution, Figure 5-2 shows the fit to the generated output. The fit is poor, based on an Anderson-Darling statistic of 5.6398 (refer to Appendix H). Also, the maximum value of the 1,000 output deviates (1.338) exceeds the incredible value (1.117) based on a log-normal distribution with 10^{-6} exceedance probability (see Appendix F).

A simulation using 1,000 trials was performed for each case identified in Table 5-1, with $n = 4, 7, 10, 14$, and 18 . Results are listed in Tables 5-5 through 5-13, and complete summary statistics are shown in Appendix F. Cumulative distributions are shown in Appendix G.

Results may be summarized as follows:

- For a given case, the calculated mean remains the same as the degree of damage state independence increases, but the standard deviation and incredible value decrease. These results are expected by theory (see Appendix D).
- The calculated mean increases as the percentile assigned to the reference high value increases; i.e., the mean is lowest for Case 1 (Case 5 or Case 9) and highest for Case 4 (Case 8 or Case 12). This is expected because the input distribution means increase.
- Log-normal distributions generally result in lower means but higher standard deviations than triangular cases, as expected based on inputs. As a result, the incredible values for the two sets of calculations are comparable.
- Results are relatively insensitive to input values for canister particulate mass/assembly (see Appendix I).
- Weibull distributions generally result in mean and incredible values that fall somewhere between mean and incredible values produced by triangular and log-normal distributions for the corresponding case.
- Reference high values for water content (1.21, 1.37, and 1.55 kg for zero, one, and two scrap baskets, respectively) are only exceeded for extreme combinations of the sensitivity parameters: 90th percentiles assigned to reference high values and a low degree of damage state independence. When log-normal input distributions are used, most of the 90th percentile case results exceeded the reference high water content, and in two cases at the 95th percentile and $n = 4$ independent choices, the result exceeded the reference bound.

For the case of reference high inputs representing the 95th percentile, and at least $n = 10$ for independent choices of damage level, incredible values are calculated to be roughly 2/3 the reference high water contents:

Table 5-5: Triangular Results Summary: No Scrap Basket (Particulate Water Content, kg)

Groups	Case 1: Bounds @ 100%			Case 2: Bounds @ 97.5%			Case 3: Bounds @ 95%			Case 4: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.260	0.065	0.816	0.317	0.082	1.031	0.343	0.093	1.176	0.412	0.121	1.566
7	0.259	0.049	0.618	0.318	0.063	0.798	0.348	0.073	0.921	0.408	0.092	1.149
10	0.259	0.041	0.537	0.317	0.053	0.691	0.347	0.063	0.804	0.406	0.076	0.965
14	0.258	0.034	0.481	0.317	0.045	0.613	0.347	0.053	0.707	0.406	0.064	0.843
18	0.258	0.030	0.443	0.316	0.040	0.570	0.347	0.047	0.654	0.405	0.057	0.780

Table 5-6: Triangular Results Summary: One Scrap Basket (Particulate Water Content, kg)

Groups	Case 1: Bounds @ 100%			Case 2: Bounds @ 97.5%			Case 3: Bounds @ 95%			Case 4: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.312	0.062	0.787	0.387	0.085	1.059	0.420	0.090	1.131	0.492	0.116	1.450
7	0.313	0.049	0.648	0.384	0.064	0.835	0.420	0.069	0.905	0.491	0.086	1.109
10	0.314	0.041	0.578	0.384	0.053	0.729	0.420	0.061	0.826	0.491	0.072	0.974
14	0.313	0.035	0.527	0.384	0.044	0.662	0.420	0.052	0.748	0.492	0.061	0.880
18	0.314	0.031	0.497	0.383	0.040	0.623	0.419	0.045	0.691	0.491	0.053	0.817

Table 5-7: Triangular Results Summary: Two Scrap Baskets (Particulate Water Content, kg)

Groups	Case 1: Bounds @ 100%			Case 2: Bounds @ 97.5%			Case 3: Bounds @ 95%			Case 4: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.369	0.081	1.010	0.445	0.102	1.281	0.492	0.113	1.412	0.575	0.140	1.756
7	0.369	0.061	0.795	0.446	0.077	0.995	0.495	0.084	1.092	0.575	0.102	1.314
10	0.371	0.050	0.698	0.448	0.065	0.886	0.495	0.070	0.958	0.576	0.085	1.151
14	0.371	0.042	0.629	0.449	0.055	0.796	0.496	0.057	0.851	0.575	0.074	1.049
18	0.371	0.038	0.598	0.450	0.047	0.738	0.495	0.052	0.808	0.576	0.067	0.993

Table 5-8: Log-Normal Results Summary: No Scrap Basket (Particulate Water Content, kg)

Groups	Case 5: Bounds @ 100%			Case 6: Bounds @ 97.5%			Case 7: Bounds @ 95%			Case 8: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.117	0.026	0.329	0.167	0.059	0.816	0.193	0.091	1.479	0.270	0.230	6.980 (1)
7	0.118	0.020	0.262	0.168	0.048	0.616	0.159	0.073	1.023	0.264	0.153	2.970 (1)
10	0.117	0.017	0.232	0.168	0.041	0.515	0.195	0.059	0.764	0.263	0.145	2.678 (1,2)
14	0.117	0.014	0.208	0.167	0.034	0.427	0.196	0.049	0.616	0.261	0.116	1.798 (1,2)
18	0.118	0.013	0.197	0.167	0.030	0.382	0.196	0.044	0.546	0.260	0.098	1.381 (1,2)
(1) No acceptable fit to any standard continuous distribution.												
(2) Maximum value of 1,000 random deviates exceeds incredible value based on a log-normal distribution (Appendix F)												

Table 5-9: Log-Normal Results Summary: One Scrap Basket (Particulate Water Content, kg)

Groups	Case 5: Bounds @ 100%			Case 6: Bounds @ 97.5%			Case 7: Bounds @ 95%			Case 8: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.158	0.031	0.387	0.237	0.071	0.915	0.281	0.095	1.274	0.370	0.220	4.369 (1,2)
7	0.158	0.023	0.310	0.235	0.051	0.634	0.281	0.072	0.899	0.370	0.153	2.281 (1,2)
10	0.157	0.019	0.279	0.235	0.043	0.546	0.280	0.060	0.748	0.370	0.130	1.776 (1,2)
14	0.158	0.017	0.259	0.236	0.035	0.473	0.278	0.051	0.649	0.370	0.107	1.368 (1,2)
18	0.158	0.015	0.243	0.236	0.032	0.441	0.279	0.045	0.590	0.368	0.089	1.117 (1,2)
(1) No acceptable fit to any standard continuous distribution.												
(2) Maximum value of 1,000 random deviates exceeds incredible value based on a log-normal distribution (Appendix F).												

Table 5-10: Log-Normal Results Summary: Two Scrap Baskets (Particulate Water Content, kg)

Groups	Case 5: Bounds @ 100%			Case 6: Bounds @ 97.5%			Case 7: Bounds @ 95%			Case 8: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.194	0.042	0.522	0.307	0.095	1.242	0.362	0.123	1.664	0.479	0.224	3.612
7	0.195	0.030	0.404	0.307	0.073	0.915	0.365	0.098	1.245	0.474	0.164	2.231
10	0.195	0.025	0.360	0.306	0.062	0.775	0.364	0.082	1.024	0.476	0.135	1.725
14	0.196	0.022	0.334	0.308	0.053	0.683	0.365	0.068	0.865	0.472	0.111	1.389
18	0.196	0.019	0.311	0.307	0.046	0.619	0.364	0.062	0.805	0.475	0.098	1.235

Table 5-11: Weibull Results Summary: No Scrap Basket (Particulate Water Content, kg)

Groups	Case 9: Bounds @ 100%			Case 10: Bounds @ 97.5%			Case 11: Bounds @ 95%			Case 12: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.179	0.040	0.504	0.251	0.074	0.962	0.287	0.095	1.275	0.358	0.127	1.742
7	0.178	0.032	0.413	0.248	0.054	0.675	0.290	0.075	0.951	0.359	0.097	1.233
10	0.178	0.026	0.356	0.247	0.046	0.586	0.288	0.062	0.780	0.358	0.082	1.030
14	0.178	0.022	0.321	0.247	0.040	0.521	0.289	0.052	0.671	0.355	0.068	0.862
18	0.178	0.019	0.297	0.248	0.035	0.479	0.289	0.046	0.606	0.355	0.061	0.784

Table 5-12: Weibull Results Summary: One Scrap Basket (Particulate Water Content, kg)

Groups	Case 9: Bounds @ 100%			Case 10: Bounds @ 97.5%			Case 11: Bounds @ 95%			Case 12: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.215	0.045	0.565	0.307	0.076	0.952	0.351	0.089	1.118	0.443	0.128	1.642
7	0.216	0.034	0.446	0.303	0.056	0.718	0.353	0.070	0.884	0.441	0.097	1.216
10	0.216	0.027	0.387	0.302	0.047	0.624	0.354	0.060	0.774	0.440	0.079	1.010
14	0.215	0.022	0.347	0.302	0.039	0.555	0.353	0.049	0.678	0.439	0.067	0.896
18	0.215	0.020	0.333	0.301	0.035	0.518	0.352	0.044	0.633	0.437	0.060	0.832

Table 5-13: Weibull Results Summary: Two Scrap Baskets (Particulate Water Content, kg)

Groups	Case 9: Bounds @ 100%			Case 10: Bounds @ 97.5%			Case 11: Bounds @ 95%			Case 12: Bounds @ 90%		
	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible	Mean	Std. Dev.	Incredible
4	0.251	0.059	0.735	0.357	0.096	1.213	0.414	0.123	1.587	0.515	0.148	1.901
7	0.252	0.044	0.564	0.360	0.074	0.925	0.414	0.093	1.160	0.516	0.111	1.391
10	0.251	0.036	0.490	0.357	0.062	0.797	0.415	0.078	0.992	0.514	0.094	1.200
14	0.250	0.030	0.437	0.357	0.054	0.719	0.415	0.064	0.853	0.516	0.082	1.086
18	0.251	0.027	0.413	0.356	0.047	0.656	0.415	0.057	0.791	0.516	0.074	1.010

- For zero scrap baskets, about 0.8 kg are calculated to be incredible, versus the reference value of 1.2 kg,
- For one scrap basket, 0.75 to 0.83 kg are calculated to be incredible, versus the reference value of 1.37 kg, and
- For two scrap baskets, about 1.0 kg is calculated to be incredible, versus the reference value of 1.55 kg.

6.0 REFERENCES

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APPENDIX A
TRIANGULAR INPUT DISTRIBUTIONS

A.1 Crystal Ball™ Input

Scrap Mass/Assy, kg: Bound = 100%ile

Triangular distribution with parameters:

Minimum	6.00E-03
Likeliest	2.00E-02
Maximum	2.00E-01

Scrap Mass/Assy, kg: Bound = 97.5%ile

Triangular distribution with parameters:

Minimum	6.00E-03
Likeliest	2.00E-02
Maximum	2.35E-01

Scrap Mass/Assy, kg: Bound = 95.0%ile

Triangular distribution with parameters:

Minimum	6.00E-03
Likeliest	2.00E-02
Maximum	2.54E-01

Scrap Mass/Assy, kg: Bound = 90.0%ile

Triangular distribution with parameters:

Minimum	6.00E-03
Likeliest	2.00E-02
Maximum	2.86E-01

Fuel Mass/Assy, kg: Bound = 100%ile

Triangular distribution with parameters:

Minimum	1.30E-03
Likeliest	7.60E-03
Maximum	2.00E-02

Fuel Mass/Assy, kg: Bound = 97.5%ile

Triangular distribution with parameters:

Minimum	1.30E-03
Likeliest	7.60E-03
Maximum	2.29E-02

Fuel Mass/Assy, kg: Bound = 95.0%ile

Triangular distribution with parameters:

Minimum	1.30E-03
Likeliest	7.60E-03
Maximum	2.44E-02

Fuel Mass/Assy, kg: Bound = 90.0%ile

Triangular distribution with parameters:

Minimum	1.30E-03
Likeliest	7.60E-03
Maximum	2.71E-02

Water Content, %: Bound=100%ile

Triangular distribution with parameters:

Minimum	1.50
Likeliest	4.00
Maximum	5.50

Water Content, %: Bound=97.5%ile

Triangular distribution with parameters:

Minimum	1.50
Likeliest	4.00
Maximum	5.97

Water Content, %: Bound=95%ile

Triangular distribution with parameters:

Minimum	1.50
Likeliest	4.00
Maximum	6.23

Water Content, %: Bound=90%ile

Triangular distribution with parameters:

Minimum	1.50
Likeliest	4.00
Maximum	6.68

A.2 Derivation of Triangular Distribution Parameters

Consider the two triangular pdfs shown in Figure A-1. The first pdf has minimum "a", most likely value "b", and bounding value "c". The second pdf has the same minimum and most likely values as the first, but the maximum is c', which is greater than c. The second pdf illustrates how the minimum and most likely values have been preserved, but the maximum has been shifted so that the bounding value now has some exceedance probability, P. Because the area of both triangles must be one, the value of p is shown by the shaded area.

If p is known or specified, the value of c' can be found using the relations for similar triangles. That is,

$$\frac{h}{p_{12}} = \frac{c' - c}{c' - b} \quad (\text{A-1})$$

where p_{12} is the probability at the most likely value of the second pdf. Recall that the probability of the most likely value can be expressed as

$$p_{12} = \frac{2}{c' - a} \quad (\text{A-2})$$

for the second triangle. Because the shaded area is equal to P,

$$\frac{(c' - c) h}{2} = P \quad (\text{A-3a})$$

or

$$h = \frac{2P}{c' - c} \quad (\text{A-3b})$$

Then,

$$\frac{c' - c}{c' - b} = \frac{P(c' - a)}{c' - c} \quad (\text{A-4})$$

Figure A-1: Derivation of New Triangular Distribution Maximum.

which can be rearranged for c' to yield:

$$c' = \frac{-(aP + bP - 2c) + \sqrt{(aP + bP - 2c)^2 - 4(1 - P)(c^2 - Pab)}}{2(1 - P)} \quad (A-5)$$

This solution was checked using Crystal Ball™ to verify that indeed, the calculated c' produced an exceedance probability P , with triangular distribution parameters a , b , and c' .

Sludge Mass per Fuel Assembly, kg: High = 100%

Triangular distribution with parameters:

Minimum	0.00E+00
Likeliest	2.20E-03
Maximum	1.30E-02

Selected range is from 0.00E+0 to 2.00E-2

Sludge Mass per Fuel Assembly, kg: High = 97.5%

Triangular distribution with parameters:

Minimum	0.00E+00
Likeliest	2.20E-03
Maximum	1.52E-02

Selected range is from 0.00E+0 to 2.33E-2

Sludge Mass per Fuel Assembly, kg: High = 95%

Triangular distribution with parameters:

Minimum	0.00E+00
Likeliest	2.20E-03
Maximum	1.63E-02

Selected range is from 0.00E+0 to 2.50E-2

Sludge Mass per Fuel Assembly, kg: High = 90%

Triangular distribution with parameters:

Minimum	0.00E+00
Likeliest	2.20E-03
Maximum	1.84E-02

Selected range is from 0.00E+0 to 2.82E-2

Sludge Water Content: High = 100%

Triangular distribution with parameters:

Minimum	2.00
Likeliest	7.50
Maximum	26.00

Selected range is from 2.00 to 35.00

Sludge Water Content: High = 97.5%

Triangular distribution with parameters:

Minimum	2.00
Likeliest	7.50
Maximum	29.97

Selected range is from 2.00 to 40.46

Sludge Water Content: High = 95%

Triangular distribution with parameters:

Minimum	2.00
Likeliest	7.50
Maximum	32.08

Selected range is from 2.00 to 43.36

Sludge Water Content: High = 90%

Triangular distribution with parameters:

Minimum	2.00
Likeliest	7.50
Maximum	35.00

Selected range is from 2.00 to 47.37

End of Assumptions

APPENDIX B
LOG-NORMAL INPUT DISTRIBUTIONS

Scrap Mass/Assy, kg: Bound = 100%ile

Lognormal distribution with parameters:

Log Mean -3.36E+00

Log Std. Dev. 5.67E-01

Scrap Mass/Assy, kg: Bound = 97.5%ile

Lognormal distribution with parameters:

Log Mean -2.97E+00

Log Std. Dev. 6.94E-01

Scrap Mass/Assy, kg: Bound = 95.0%ile

Lognormal distribution with parameters:

Log Mean -2.83E+00

Log Std. Dev. 7.41E-01

Scrap Mass/Assy, kg: Bound = 90.0%ile

Lognormal distribution with parameters:

Log Mean

-2.64E+00

Log Std. Dev.

8.02E-01

Fuel Mass/Assy, kg: Bound = 100%ile

Lognormal distribution with parameters:

Log Mean -5.28E+00

Log Std. Dev. 4.42E-01

Fuel Mass/Assy, kg: Bound = 97.5%ile

Lognormal distribution with parameters:

Log Mean -4.97E+00

Log Std. Dev. 5.41E-01

Fuel Mass/Assy, kg: Bound = 95.0%ile

Lognormal distribution with parameters:

Log Mean $-4.86\text{E}+00$
Log Std. Dev. $5.77\text{E}-01$

Fuel Mass/Assy: Bound = 90.0%ile

Lognormal distribution with parameters:

Log Mean $-4.71\text{E}+00$
Log Std. Dev. $6.25\text{E}-01$

Water Content %, Bound = 100%

Lognormal distribution with parameters:

Log Mean 1.38

Log Std. Dev. 0.11

Water Content %: Bound = 97.5%

Lognormal distribution with parameters:

Log Mean 1.38

Log Std. Dev. 0.17

Water Content %: Bound = 95.0%

Lognormal distribution with parameters:

Log Mean	1.38
Log Std. Dev.	0.20

Water Content, %: Bound = 90.0%

Lognormal distribution with parameters:

Log Mean	1.38
Log Std. Dev.	0.26

Sludge Mass/ Assy., kg,: High = 99.9%ile

Lognormal distribution with parameters:

Geometric Mean	2.22E-03
99.9% - tile	1.29E-02

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/ Assy., kg,: High = 97.5%ile

Lognormal distribution with parameters:

Geometric Mean	2.22E-03
97.5% - tile	1.29E-02

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/ Assy., kg.: High = 95.0%ile

Lognormal distribution with parameters:

Mean	3.94E-03
Standard Dev.	5.75E-03

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/ Assy., kg.: High = 90.0%ile

Lognormal distribution with parameters:

Geometric Mean	2.22E-03
90% - tile	1.29E-02

Selected range is from 0.00E+0 to +Infinity

Sludge Water Content, %: High = 99.9%

Lognormal distribution with parameters:

Geometric Mean	7.50
99.9% - tile	26.00

Selected range is from 0.00 to +Infinity

Sludge Water Content, %: High = 97.5%

Lognormal distribution with parameters:

Geometric Mean	7.50
97.5% - tile	26.00

Selected range is from 0.00 to 35.00

APPENDIX C
WEIBULL INPUT DISTRIBUTIONS

11/3/98

11/3/98

C.1 Weibull Input

Scrap Mass/Assy, kg: Bound = 99.9%ile

Weibull distribution with parameters:

Location	6.00E-03
Scale	4.39E-02
Shape	1.30E+00

Mean value in simulation was 4.65E-2

Scrap Mass/Assy, kg: Bound = 97.5%ile

Weibull distribution with parameters:

Location	6.00E-03
Scale	6.48E-02
Shape	1.19E+00

Mean value in simulation was 6.70E-2

Scrap Mass/Assy, kg: Bound = 95.0%ile

Weibull distribution with parameters:

Location	6.00E-03
Scale	7.56E-02
Shape	1.16E+00

Mean value in simulation was 7.77E-2

Scrap Mass/Assy, kg: Bound = 90.0%ile

Weibull distribution with parameters:

Location	6.00E-03
Scale	9.29E-02
Shape	1.13E+00

Mean value in simulation was 9.48E-2

Fuel Mass/Assy: Bound = 99.9%ile

Weibull distribution with parameters:

Location	1.30E-03
Scale	8.07E-03
Shape	2.30E+00

Mean value in simulation was 8.45E-3

Fuel Mass/Assy: Bound = 97.5%ile

Weibull distribution with parameters:

Location	1.30E-03
Scale	9.36E-03
Shape	1.89E+00

Mean value in simulation was 9.61E-3

Fuel Mass/Assy: Bound = 95%ile

Weibull distribution with parameters:

Location	1.30E-03
Scale	1.01E-02
Shape	1.77E+00

Mean value in simulation was 1.02E-2

Fuel Mass/Assy: Bound = 90%ile

Weibull distribution with parameters:

Location	1.30E-03
Scale	1.13E-02
Shape	1.64E+00

Mean value in simulation was 1.13E-2

Water Content, %: Bound = 99.9%ile

Weibull distribution with parameters:

Location	1.50
Scale	2.63
Shape	4.6

Mean value in simulation was 3.90

Water Content, %: Bound = 97.5%ile

Weibull distribution with parameters:

Location	1.50
Scale	2.73
Shape	3.43

Mean value in simulation was 3.96

Water Content, %: Bound = 95.0%ile

Weibull distribution with parameters:

Location	1.50
Scale	2.83
Shape	3.16

Mean value in simulation was 4.03

Water Content, %: Bound = 90.0%ile

Weibull distribution with parameters:

Location	1.50
Scale	2.94
Shape	2.71

Mean value in simulation was 4.11

C.2 Derivation of Weibull Distribution Parameters

Recall that the Weibull distribution is given by:

$$f(x) = \frac{\beta}{\alpha} \left(\frac{x - L}{\alpha} \right)^{\beta-1} \exp \left(- \left(\frac{x - L}{\alpha} \right)^{\beta} \right) \quad (C-1)$$

where L is the location, α is the scale, and β is the shape. The cumulative distribution is

$$F(x) = 1 - \exp \left(- \left(\frac{x - L}{\alpha} \right)^{\beta} \right) \quad (C-2)$$

Weibull distribution parameters were specified in the following manner:

1. The output deviates were greater than the low value from the reference estimate,
2. The mode (most probable value), x_m , was set equal to the most likely value from the reference estimate, and
3. The bound value, x_b , corresponded to the particular percentile point defined for the case (i.e., 99.9%, 97.5%, 95%, or 90%).

The properties of the three-parameter Weibull distribution are such that the output deviates are always greater than the location parameter, L , which allows us to simply set L equal to the low value from the reference estimate.

To find the mode, x_m , of the Weibull distribution, differentiate equation (C-1) and set it equal to zero to obtain:

$$\frac{x_m - L}{\alpha} = \left(\frac{\beta - 1}{\beta} \right)^{\frac{1}{\beta}} \quad (C-3)$$

The bound value, x_b , is easily determined using the cumulative distribution (equation (C-2)),

$$\frac{x_b - L}{\alpha} = [-\ln(1 - F(x_b))]^{\frac{1}{\beta}} \quad (C-4)$$

where $F(x_b)$ is the desired percentile point.

The scale parameter (α) can be eliminated to obtain an equation for the shape parameter (β);

$$\frac{x_b - L}{x_m - L} = \left[\frac{-\ln(1 - F(x_b))}{\frac{\beta - 1}{\beta}} \right]^{\frac{1}{\beta}} \quad (C-5)$$

because x_b , x_m , L , and $F(x_b)$ are specified. Once β is known, α can be found from equation (C-3) or (C-4).

This solution was checked using Crystal Ball™. A Weibull distribution with specified location, scale, and shape was used to generate output deviates. Output histograms verified that the x_m corresponded to the mode and x_b corresponded to the selected percentile point.

C3. Consister Particulate Input for Crystal Ball**Sludge Mass/Assy., kg:: High = 99.9%ile**

Weibull distribution with parameters:

Location	0.00E+00
Scale	3.96E-03
Shape	1.63E+00

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/Assy., kg:: High = 97.5%ile

Weibull distribution with parameters:

Location	0.00E+00
Scale	5.19E-03
Shape	1.42E+00

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/Assy., kg:: High = 95.0%ile

Weibull distribution with parameters:

Location	0.00E+00
Scale	5.82E-03
Shape	1.36E+00

Selected range is from 0.00E+0 to +Infinity

Sludge Mass/Assy., kg:: High = 90.0%ile

Weibull distribution with parameters:

Location	0.00E+00
Scale	6.84E-03
Shape	1.30E+00

Selected range is from 0.00E+0 to +Infinity

Sludge Water Content, %:: High = 99.9%

Weibull distribution with parameters:

Location	2.00
Scale	8.40
Shape	1.843

Selected range is from 2.00 to +Infinity

Sludge Water Content, %:: High = 97.5%

Weibull distribution with parameters:

Location	2.00
Scale	10.46
Shape	1.571

Selected range is from 2.00 to 35.00

Sludge Water Content, %:: High = 95.0%

Weibull distribution with parameters:

Location	2.00
Scale	11.52
Shape	1.495

Selected range is from 2.00 to 35.00

Sludge Water Content, %:: High = 90.0%

Weibull distribution with parameters:

Location	2.00
Scale	13.26
Shape	1.407

Selected range is from 2.00 to 35.00

APPENDIX D

VALIDATION OF MONTE CARLO CALCULATIONS

D.1 Particulate Mass Hand Calculation

Any sum S_n of random variables x_i , $i = 1$ to n ,

$$S_n = x_1 + x_2 + x_3 \dots x_n \quad (D-1)$$

with the same underlying distribution of mean μ and variance σ^2 has a mean of $n\mu$ and variance $n\sigma^2$. Linear combinations, such as the MCO particulate mass, have similarly weighted parameter values:

$$\mu_{MCO} = 39.1 \mu_{scrap} + 216 \mu_{fuel} \quad (D-2)$$

$$\sigma_{MCO}^2 = n \left[\left(\frac{39.1}{n} \right)^2 \sigma_{scrap}^2 + \left(\frac{216}{n} \right)^2 \sigma_{fuel}^2 \right] \quad (D-3)$$

These formulas are used to check the particulate mass for Case 7 (log-normal inputs), 95-th percentile for the reference bound, and $n = 10$ independent choices:

μ_{scrap}	0.0778
μ_{fuel}	0.00914
μ_{MCO} (equation (D-2))	5.016
μ_{MCO} (Appendix E)	5.006
σ_{scrap}^2	4.42e-3
σ_{fuel}^2	3.31e-5
σ_{MCO}^2 (equation (D-3))	0.830
σ_{MCO}^2 (Appendix E)	0.839

D.2 Independent Calculation

Attached is an independent evaluation for water content results for Case 7 and $n = 10$ independent choices using MATHCAD. In Section 1 of the calculation, values from Appendix B for the mean and standard deviations of particulate and water pdf's are assigned, and in Section 2 these are used to derive the log-normal distribution parameters. Input pdf's are then plotted to demonstrate that the cpdf's agree with the 95-th percentile for the reference bounding values. In Section 3, a Monte Carlo method is used with 10,000 trials to generate histograms for MCO particulate and water. The plots in these sections compare the local Monte Carlo results with those using the derived log-normal parameters, and agreement is excellent.

A technical concern noted in this review is that the output distribution for water mass may not be well-characterized by a log-normal. It is possible for the log-normal parameters derived from output distribution sample statistics to imply a water content at the 10^{-6} probability level which is exceeded once or a few times during 10,000 trials. Thus, the procedure to fit log-normal distributions and extrapolate the 10^{-6} or "incredible" water content may not in all cases be conservative.

INDEPENDENT CHECK OF TENACIOUS PARTICULATE & CANISTER PARTICULATE WATER

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Purpose: For Quality Assurance, Independent check of 'incredible' water mass loaded into an MCO associated with water of hydration on tenaciously clinging particulate and sludge.

Concern: Must address the following 3 key points:

- 1) Level of independence. Fuel assemblies are selected independently at no finer grain than the canister level.
- 2) Percentile represented by "high" value. The "high" value represents some percentile of possible values, not literally the 100% upper bound.
- 3) Value of Extreme Parameter. Extent of the 'tail' on the pdf for a parameter.

CASE OBJECTIVE : Reproduce particulate and water distributions for 95th percentile, 10 choices

Input and derived values for particulate (kg):

	Low @ 0.1% (in)	Mean & Sigma (out)	Bound @ 95% (in)
Scrap	0.0014	0.0778, 0.0665	0.2
Fuel	0.0064	0.0914, 0.00575	0.02

Input and derived values for water: Mean 4.04%, Sigma=0.82, yields 5.5% @ 95%.

Note: here mean and sigma are sample values; log-normal parameters must be derived.

Expected output (Sample statistics) to agree with Crystal Ball:

	Mean, kg	Sigma
Particulate	5.0	0.916
Water	0.20	0.040

Input and derived values for sludge - which only exists on fuel assemblies

	Low @ 0.1% (in)	Mean & Sigma (out)	Bound @ 95% (in)
Fuel	2.e-5	0.00394, 0.00575	1.29e-2 (0.0022 kg per element)
Water	n/a	12.35%, 7.21%	26.0%

Expected output (Sample statistics) to agree with Crystal Ball:

	Mean, kg	Sigma
Sludge	0.848	0.387
Water	0.078	0.048

Total expected water output:

Mean	0.280
Std Deviation	0.062

Incredible (10^{-6}) value using lognormal: 0.747

Note: Crystal ball model truncates the sludge water percentage, resulting in lower output standard deviation and lower "incredible" value.

1.0 Assign inputs

$m_{bs} := 0.2$	Scrap particulate bound	$P_b := 0.95$	Percentile of reference bound
$m_{bf} := 0.02$	Fuel particulate bound	$N := 10$	Independent choices
$\mu_{ss} := 0.0778$	Scrap particulate mean	$A_f := 216$	Fuel assemblies
$\mu_{fs} := 0.00914$	Fuel particulate mean	$A_s := 39.1$	Scrap assemblies
$\mu_{ws} := 4.04$	Particulate water fraction mean	$M := 10 \cdot 10^3$	Number of Monte Carlo trials, i.e. # of MCOs filled
$\mu_{gs} := 0.00394$	Sludge particulate mean		
$\mu_{gws} := 9.98$	Sludge water mean		
$\sigma_{ss} := 0.0665$	Scrap particulate sigma	$\sigma_{gs} := 0.00575$	Sludge particulate sigma
$\sigma_{fs} := 0.00575$	Fuel particulate sigma	$\sigma_{gws} := 8.76$	Sludge water sigma
$\sigma_{ws} := 0.82$	Water fraction sigma		

Subscript 's' is used for mu and sigma above to designate sample (not log-normal) values

2.0 Derive input distributions and plot

$f\sigma(\mu_s, \sigma_s) := \left[\ln \left(\left(\frac{\sigma_s}{\mu_s} \right)^2 + 1 \right) \right]^{-\frac{1}{2}}$	Function for log-normal parameter sigma given sample mean and standard deviation
$f\mu(\mu_s, \sigma) := \ln(\mu_s) - \frac{1}{2} \cdot \sigma^2$	Function for log-normal parameter mu given sample mean and log-normal parameter sigma
$g\sigma(x_b, \mu, \sigma, P_b) := \text{root}(\text{plnorm}(x_b, \mu, \sigma) - P_b, \sigma)$	Function to get log-normal parameter sigma given log-normal parameter mu and value x with given probability P

Log-normal parameters, subscript 'l' for log normal input distributions:

$\sigma_{sl} := f\sigma(\mu_{ss}, \sigma_{ss})$	$\sigma_{sl} = 0.741$	Scrap particulate sigma
$\sigma_{fl} := f\sigma(\mu_{fs}, \sigma_{fs})$	$\sigma_{fl} = 0.577$	Fuel particulate sigma
$\sigma_{wl} := f\sigma(\mu_{ws}, \sigma_{ws})$	$\sigma_{wl} = 0.201$	Particulate Water sigma
$\mu_{sl} := f\mu(\mu_{ss}, \sigma_{sl})$	$\mu_{sl} = -2.828$	Scrap particulate mu
$\mu_{fl} := f\mu(\mu_{fs}, \sigma_{fl})$	$\mu_{fl} = -4.862$	Scrap particulate mu
$\mu_{wl} := f\mu(\mu_{ws}, \sigma_{wl})$	$\mu_{wl} = 1.376$	Particulate water mu

$\sigma_{gi} := \sqrt{\sigma_{gs}^2 + \sigma_{gws}^2}$	$\sigma_{gi} = 1.068$	Sludge particulate sigma
$\sigma_{gwi} := \sqrt{\sigma_{gws}^2 + \sigma_{gwi}^2}$	$\sigma_{gwi} = 0.756$	Sludge Water sigma
$\mu_{gi} := \sqrt{\mu_{gs}^2 + \mu_{gws}^2}$	$\mu_{gi} = -6.107$	Sludge particulate mu
$\mu_{gwi} := \sqrt{\mu_{gws}^2 + \mu_{gwi}^2}$	$\mu_{gwi} = 2.015$	Sludge water mu

Check of desired percentile values:

$qnorm(0.001, \mu_{si}, \sigma_{si}) = 5.997 \cdot 10^{-3}$	scrap low bound check at 0.1% level
$qnorm(0.001, \mu_{fi}, \sigma_{fi}) = 1.299 \cdot 10^{-3}$	fuel low bound check at 0.1% level
$qnorm(0.95, \mu_{si}, \sigma_{si}) = 0.2$	scrap upper bound check at 95% level
$qnorm(0.95, \mu_{fi}, \sigma_{fi}) = 0.02$	fuel upper bound check at 95% level
$qnorm(0.95, \mu_{wi}, \sigma_{wi}) = 5.51$	water upper bound check at 95% level
$qnorm(0.95, \mu_{gi}, \sigma_{gi}) = 0.013$	sludge upper bound check at 95% level
$qnorm(0.95, \mu_{gwi}, \sigma_{gwi}) = 26.001$	sludge water upper bound check at 95% level

Loop over set of particulate and water masses to plot pdf's and cpdf's:

$x_{pmax} := 0.4$	$x_{wmax} := 12$	$i := 0..1000$	$x_{pi} := \frac{i \cdot x_{pmax}}{1000}$	$x_{wi} := \frac{i \cdot x_{wmax}}{1000}$
$pps_i := dlnorm(x_{pi}, \mu_{si}, \sigma_{si})$	$PPS_i := plnorm(x_{pi}, \mu_{si}, \sigma_{si})$			Scrap particulate p and cp
$ppf_i := dlnorm(x_{pi}, \mu_{fi}, \sigma_{fi})$	$PPF_i := plnorm(x_{pi}, \mu_{fi}, \sigma_{fi})$			Fuel particulate p and cp
$pw_i := dlnorm(x_{wi}, \mu_{wi}, \sigma_{wi})$	$PW_i := plnorm(x_{wi}, \mu_{wi}, \sigma_{wi})$			Water p and cp
$ppg_i := dlnorm(x_{pi}, \mu_{gi}, \sigma_{gi})$	$PPG_i := plnorm(x_{pi}, \mu_{gi}, \sigma_{gi})$			Sludge particulate p and cp
$x_{gwi} := i \cdot 40 \cdot 0.001$				Different range for sludge h2o
$pgw_i := dlnorm(x_{gwi}, \mu_{gwi}, \sigma_{gwi})$	$PGW_i := plnorm(x_{gwi}, \mu_{gwi}, \sigma_{gwi})$			Sludge Water p and cp

Scrap Particulate:

CPDF= dash
pdf= solid
95% value OK
at 0.2 kg
Median conservative
about 0.06 kg
Mode about 0.033
vs 0.02 kg reference
best-estimate

Fuel Particulate:

CPDF= dash
pdf= solid
95% value OK
at 0.02 kg
Median OK
about 0.008 kg
Mode about 0.005 kg
vs 0.01 kg reference
best-estimate

Water %:

CPDF= dash
pdf= solid
95% value OK
at 5.5%
50% value OK
about 4%
Mode about 3.75%
vs 4% reference

Sludge Particulate:

CPDF= dash
 pdf= solid
 95% value OK
 at 0.013 kg
 Median OK
 0.0022 kg

Sludge Water %:

CPDF= dash
 pdf= solid
 95% value OK
 at 26%
 50% value perfect
 at 7.5%

** Conclusion: Input distributions are reasonable.

3.0 Get MCO loading for N=10 choices using input parameters / assembly defined above

Function to fill M MCO's. Output matrix has 1 column/MCO, and rows for scrap particulate, fuel particulate, scrap water, and fuel water. Uses function rlnorm which yields a vector of N values distributed log-normally with the input mu's and sigma's.

```

fMCO( $\mu_s, \mu_f, \mu_w, \mu_g, \mu_{gw}, \sigma_s, \sigma_f, \sigma_w, \sigma_g, \sigma_{gw}, N, M$ ) :=
  a ←  $A_s N^{-1}$ 
  b ←  $A_f N^{-1}$ 
  for k ∈ 0..M-1
    S ← rlnorm( $N, \mu_s, \sigma_s$ ) · a
    F ← rlnorm( $N, \mu_f, \sigma_f$ ) · b
    W ← rlnorm( $N, \mu_w, \sigma_w$ ) · 0.01
    G ← rlnorm( $N, \mu_g, \sigma_g$ ) · b
    GW ← rlnorm( $N, \mu_{gw}, \sigma_{gw}$ ) · 0.01
     $V_{0,k} \leftarrow S$ 
     $V_{1,k} \leftarrow S \cdot W$ 
     $V_{2,k} \leftarrow S \cdot F$ 
     $V_{3,k} \leftarrow F \cdot W$ 
     $V_{4,k} \leftarrow S \cdot G$ 
     $V_{5,k} \leftarrow G \cdot GW$ 
  V

```

Test for 10 MCO's: $V := \text{fMCO}(\mu_s, \mu_f, \mu_w, \mu_g, \mu_{gw}, \sigma_s, \sigma_f, \sigma_w, \sigma_g, \sigma_{gw}, N, 10)$

												Rows contain:
V =	2.576	3.219	2.32	3.317	2.728	5.468	2.728	2.461	3.173	3.888		0 Scrap particulate
	0.123	0.129	0.1	0.127	0.121	0.229	0.099	0.108	0.112	0.158		1 Scrap water
	1.878	1.554	2.617	1.657	2.478	1.943	2.29	1.97	2.392	1.798		2 Fuel particulate
	0.09	0.061	0.116	0.066	0.103	0.08	0.084	0.089	0.091	0.072		3 Fuel water
	0.71	0.672	0.458	1.059	2.288	1.008	0.795	0.601	1.913	0.435		4 Sludge mass
	0.063	0.056	0.059	0.092	0.219	0.078	0.074	0.1	0.213	0.032		5 Sludge water

Expected results:

scrap particulate $39 \cdot 0.0778 = 3$, scrap water $3 \cdot 0.04 = 0.12$

fuel particulate $216 \cdot 0.00914 = 2$, fuel water $0.04 \cdot 2 = 0.08$

sludge 0.48, sludge water 0.036. So sludge water and mass mean values exceed the median.

GENERATE MCO's: $V := \text{fMCO}(\mu_{si}, \mu_{fi}, \mu_{wi}, \mu_{gi}, \mu_{gwi}, \sigma_{si}, \sigma_{fi}, \sigma_{wi}, \sigma_{gi}, \sigma_{gwi}, N, M)$

Sum particulate and its water per MCO: $j := 0..M-1$ $DP_{j,j} := V_{0,j} + V_{2,j}$ $DW_{j,j} := V_{1,j} + V_{3,j}$

$DWT_{j,j} := V_{1,j} + V_{3,j} + V_{5,j}$ $DG_{j,j} := V_{4,j}$ $DGW_{j,j} := V_{5,j}$

Sample statistics:

$\mu_{sp} := \text{mean}(DP)$	$\mu_{sp} = 5.019$	$\mu_{sw} := \text{mean}(DW)$	$\mu_{sw} = 0.203$	Particulate and its water
$\sigma_{sp} := \text{stdev}(DP)$	$\sigma_{sp} = 0.903$	$\sigma_{sw} := \text{stdev}(DW)$	$\sigma_{sw} = 0.039$	
$\mu_{sg} := \text{mean}(DG)$	$\mu_{sg} = 0.845$	$\mu_{sgw} := \text{mean}(DGW)$	$\mu_{sgw} = 0.083$	Sludge and its water
$\sigma_{sg} := \text{stdev}(DG)$	$\sigma_{sg} = 0.392$	$\sigma_{sgw} := \text{stdev}(DGW)$	$\sigma_{sgw} = 0.053$	
		$\mu_{swt} := \text{mean}(DWT)$	$\mu_{swt} = 0.286$	Total water
		$\sigma_{swt} := \text{stdev}(DWT)$	$\sigma_{swt} = 0.065$	

Lognormal parameters:

$\sigma_{lp} := \text{f}\sigma(\mu_{sp}, \sigma_{sp})$	$\mu_{lp} := \text{f}\mu(\mu_{sp}, \sigma_{sp})$	$\sigma_{lpw} := \text{f}\sigma(\mu_{sw}, \sigma_{sw})$	$\mu_{lpw} := \text{f}\mu(\mu_{sw}, \sigma_{sw})$
$\sigma_{lg} := \text{f}\sigma(\mu_{sg}, \sigma_{sg})$	$\mu_{lg} := \text{f}\mu(\mu_{sg}, \sigma_{sg})$	$\sigma_{lgw} := \text{f}\sigma(\mu_{sgw}, \sigma_{sgw})$	$\mu_{lgw} := \text{f}\mu(\mu_{sgw}, \sigma_{sgw})$
		$\sigma_{ltw} := \text{f}\sigma(\mu_{swt}, \sigma_{swt})$	$\mu_{ltw} := \text{f}\mu(\mu_{swt}, \sigma_{swt})$

Gamma parameters for total water:

$\beta_{tw} := \sigma_{swt}^2 \cdot \mu_{swt}^{-1}$ $\alpha_{tw} := \mu_{swt} \cdot \beta_{tw}^{-1}$ $\beta_{tw} = 0.015$ $\alpha_{tw} = 19.347$

Incredible values
using lognormal
fit parameters:

$\text{qlnorm}(0.999999, \mu_{lp}, \sigma_{lp}) = 11.54$	Incredible particulate and its water
$\text{qlnorm}(0.999999, \mu_{lpw}, \sigma_{lpw}) = 0.498$	
$\text{qlnorm}(0.999999, \mu_{lg}, \sigma_{lg}) = 6.249$	Incredible sludge and its water
$\text{qlnorm}(0.999999, \mu_{lgw}, \sigma_{lgw}) = 1.104$	
$\text{qlnorm}(0.999999, \mu_{ltw}, \sigma_{ltw}) = 0.811$	Incredible total water
$1 - \text{plnorm}(0.825, \mu_{ltw}, \sigma_{ltw}) = 6.883 \cdot 10^{-7}$	Value checks for total water

Gamma for total water: $\text{qgamma}(0.999999, \alpha_{tw}) \cdot \beta_{tw} = 0.708$ Gamma model yields lower incredible water content

Range of sample values: $\min(DGW) = 0.012$ $\max(DGW) = 0.844$

*** Very important: the maximum of from trials may exceed the 'incredible' value from the curve fit. This does not occur every time this worksheet is invoked because of the random selection of values.

*** CONCLUSION: CRYSTAL BALL RESULTS ARE INDEPENDENTLY CHECKED AND ARE CORRECTLY CALCULATED, but BEWARE OF CURVE FITTING THE OUTPUT DISTRIBUTIONS TO DERIVE INCREDIBLE VALUES. ***

For plots, generate histograms of total particulate and water:

$$n := 200 \quad P_{\max} := 20 \quad W_{\max} := 1.0 \quad \delta p := \frac{P_{\max}}{n} \quad \delta w := \frac{W_{\max}}{n} \quad \text{Bounds and intervals of histograms}$$

$$k := 0..n \quad Pb_k := k \cdot \delta p \quad Wb_k := k \cdot \delta w \quad \text{Histogram boundary assignment}$$

$$MP := \text{hist}(Pb, DP) \quad MW := \text{hist}(Wb, DW) \quad \text{Frequency assignment}$$

$$MG := \text{hist}(Pb, DG) \quad MGW := \text{hist}(Wb, DGW)$$

$$MTW := \text{hist}(Wb, DWT)$$

$$k := 0..n-1 \quad Pc_k := 0.5 \cdot (Pb_k + Pb_{k+1}) \quad Wc_k := 0.5 \cdot (Wb_k + Wb_{k+1}) \quad \text{Histogram centers}$$

For particulate and its water, get normalized histograms and lognormal distributions:

$$\text{norp} := (M \cdot \delta p)^{-1} \quad NP := MP \cdot \text{norp} \quad \text{norw} := (M \cdot \delta w)^{-1} \quad NW := MW \cdot \text{norw} \quad \text{Normalized histograms}$$

$$\text{np}_k := \text{dlnorm}(Pc_k, \mu_{lp}, \sigma_{lp}) \quad \text{nw}_k := \text{dlnorm}(Wc_k, \mu_{lpw}, \sigma_{lpw}) \quad \text{Derived distributions}$$

For sludge and its water, get normalized histograms and gamma distributions:

$$\text{norg} := (M \cdot \delta p)^{-1} \quad NG := MG \cdot \text{norg} \quad \text{norgw} := (M \cdot \delta w)^{-1} \quad NGW := MGW \cdot \text{norgw} \quad \text{Normalized histograms}$$

$$\text{ng}_k := \text{dlnorm}(Pc_k, \mu_{lg}, \sigma_{lg}) \quad \text{ngw}_k := \text{dlnorm}(Wc_k, \mu_{lgw}, \sigma_{lgw}) \quad \text{Derived distributions}$$

For total water get its normalized histograms and lognormal and gamma distributions:

$$\text{nortwg} := \left(\frac{M \cdot \delta w}{\beta_{tw}} \right)^{-1} \quad \text{NTWG} := MTW \cdot \text{nortwg} \quad \text{ntwg}_k := \text{dgamma} \left(\frac{Wc_k}{\beta_{tw}}, \alpha_{tw} \right)$$

$$\text{nortwl} := (M \cdot \delta w)^{-1} \quad \text{NTWL} := MTW \cdot \text{nortwl} \quad \text{ntwl}_k := \text{dlnorm}(Wc_k, \mu_{ltw}, \sigma_{ltw})$$

Adhering particulate (left) and its water (right): Normalized histograms (solid) compared to derived distributions (dashed):

Canister particulate (left) and its water (right): Normalized histograms (solid) compared to derived distributions (dashed):

Gamma distribution for total water, histogram vs fit. The gamma distribution is not good because its tail does not extend as far as the histogram.

Lognormal distribution for total water, histogram vs fit. When this worksheet is invoked, sometimes the extreme value of the samples exceeds the tail of the lognormal; sometimes not.

Conclusion: The large skew in the output distribution makes it hard to fit with a lognormal, so that predictions for extreme values are difficult.

APPENDIX E
DETAILED RESULTS FOR MONTE CARLO VALIDATION

lognormal_rev3_one_base_stats.xls

Statistics	Particulate Mass, kg 10, 95%	Particulate Water Mass,kg 10, 95%	Sludge Mass, kg 10, 95%	Sludge Water Mass,kg 10 95%	Total Water Mass, 10, 95%
Trials	10000	10000	10000	10000	10000
Mean	5.006	0.202	0.848	0.078	0.280
Median	4.914	0.198	0.769	0.066	0.271
Standard Deviation	0.916	0.040	0.387	0.048	0.062
Variance	0.839	0.002	0.150	0.002	0.004
Skewness	0.858	0.827	1.827	3.063	1.534
Kurtosis	4.773	4.457	9.166	22.488	9.351
Coeff. of Variability	0.183	0.196	0.456	0.613	0.221
Range Minimum	2.717	0.101	0.191	0.012	0.139
Range Maximum	11.359	0.435	4.537	0.673	0.886
Range Width	8.642	0.334	4.346	0.661	0.747

APPENDIX F
COMPLETE RESULTS FOR TOTAL WATER CONTENT

Statistics	Triangular, Zero, 100%, 4	Triangular, Zero, 100%, 7	Triangular, Zero, 100%, 10	Triangular, Zero, 100%, 14	Triangular, Zero, 100%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.260	0.259	0.259	0.258	0.258
Median	0.253	0.255	0.257	0.257	0.257
Standard Deviation	0.065	0.049	0.041	0.034	0.030
Variance	0.004	0.002	0.002	0.001	0.001
Skewness	0.505	0.385	0.289	0.251	0.233
Kurtosis	3.244	2.837	2.874	3.114	3.265
Coeff. of Variability	0.250	0.188	0.157	0.133	0.115
Range Minimum	0.110	0.147	0.163	0.168	0.158
Range Maximum	0.518	0.426	0.404	0.383	0.366
Range Width	0.408	0.279	0.241	0.215	0.208
Incredible	0.816	0.618	0.537	0.481	0.443
Statistics	Triangular, Zero, 97.5%, 4	Triangular, Zero, 97.5%, 7	Triangular, Zero, 97.5%, 10	Triangular, Zero, 97.5%, 14	Triangular, Zero, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.317	0.318	0.317	0.317	0.316
Median	0.309	0.312	0.316	0.315	0.316
Standard Deviation	0.082	0.063	0.053	0.045	0.040
Variance	0.007	0.004	0.003	0.002	0.002
Skewness	0.531	0.318	0.280	0.305	0.261
Kurtosis	3.349	2.809	3.344	3.223	3.175
Coeff. of Variability	0.258	0.199	0.167	0.141	0.126
Range Minimum	0.119	0.154	0.169	0.180	0.203
Range Maximum	0.662	0.554	0.593	0.500	0.494
Range Width	0.543	0.400	0.424	0.320	0.291
Incredible	1.031	0.798	0.691	0.613	0.570
Statistics	Triangular, Zero, 95%, 4	Triangular, Zero, 95%, 7	Triangular, Zero, 95%, 10	Triangular, Zero, 95%, 14	Triangular, Zero, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.343	0.348	0.347	0.347	0.347
Median	0.339	0.342	0.341	0.343	0.345
Standard Deviation	0.093	0.073	0.063	0.053	0.047
Variance	0.009	0.005	0.004	0.003	0.002
Skewness	0.483	0.531	0.440	0.306	0.248
Kurtosis	3.167	3.650	3.514	3.151	3.056
Coeff. of Variability	0.271	0.211	0.181	0.153	0.135
Range Minimum	0.129	0.195	0.201	0.214	0.224
Range Maximum	0.725	0.738	0.662	0.565	0.518
Range Width	0.596	0.543	0.461	0.350	0.295
Incredible	1.176	0.921	0.804	0.707	0.654
Statistics	Triangular, Zero, 90%, 4	Triangular, Zero, 90%, 7	Triangular, Zero, 90%, 10	Triangular, Zero, 90%, 14	Triangular, Zero, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.412	0.408	0.406	0.406	0.405
Median	0.398	0.399	0.401	0.402	0.403
Standard Deviation	0.121	0.092	0.076	0.064	0.057
Variance	0.015	0.008	0.006	0.004	0.003
Skewness	0.617	0.591	0.425	0.398	0.345
Kurtosis	3.326	3.577	3.286	3.220	3.350
Coeff. of Variability	0.295	0.225	0.186	0.157	0.140
Range Minimum	0.153	0.137	0.225	0.246	0.226
Range Maximum	0.949	0.812	0.731	0.639	0.617
Range Width	0.796	0.675	0.506	0.393	0.392
Incredible	1.566	1.149	0.965	0.843	0.780

Statistics	Triangular, One, 100%, 4	Triangular, One, 100%, 7	Triangular, One, 100%, 10	Triangular, One, 100%, 14	Triangular, One, 100%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.312	0.313	0.314	0.313	0.314
Median	0.308	0.311	0.313	0.311	0.312
Standard Deviation	0.062	0.049	0.041	0.035	0.031
Variance	0.004	0.002	0.002	0.001	0.001
Skewness	0.325	0.313	0.154	0.196	0.206
Kurtosis	3.002	2.935	2.816	2.731	2.817
Coeff. of Variability	0.200	0.156	0.130	0.111	0.098
Range Minimum	0.133	0.181	0.198	0.225	0.231
Range Maximum	0.550	0.492	0.450	0.418	0.408
Range Width	0.418	0.311	0.251	0.192	0.177
Incredible Value	0.787	0.648	0.578	0.527	0.497

Statistics	Triangular, One, 97.5%, 4	Triangular, One, 97.5%, 7	Triangular, One, 97.5%, 10	Triangular, One, 97.5%, 14	Triangular, One, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.387	0.384	0.384	0.384	0.383
Median	0.383	0.380	0.380	0.382	0.382
Standard Deviation	0.085	0.064	0.053	0.044	0.040
Variance	0.007	0.004	0.003	0.002	0.002
Skewness	0.290	0.245	0.301	0.156	0.140
Kurtosis	2.723	2.770	3.079	3.203	3.079
Coeff. of Variability	0.218	0.167	0.137	0.116	0.104
Range Minimum	0.141	0.216	0.257	0.255	0.265
Range Maximum	0.638	0.618	0.617	0.550	0.526
Range Width	0.497	0.402	0.360	0.295	0.261
Incredible Value	1.059	0.835	0.729	0.662	0.623

Statistics	Triangular, One, 95%, 4	Triangular, One, 95%, 7	Triangular, One, 95%, 10	Triangular, One, 95%, 14	Triangular, One, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.420	0.420	0.420	0.420	0.419
Median	0.413	0.416	0.416	0.418	0.416
Standard Deviation	0.090	0.069	0.061	0.052	0.045
Variance	0.008	0.005	0.004	0.003	0.002
Skewness	0.400	0.415	0.384	0.329	0.226
Kurtosis	3.319	3.453	3.222	3.010	2.913
Coeff. of Variability	0.215	0.165	0.144	0.123	0.106
Range Minimum	0.203	0.225	0.270	0.287	0.286
Range Maximum	0.797	0.728	0.664	0.615	0.557
Range Width	0.595	0.503	0.394	0.328	0.271
Incredible Value	1.131	0.905	0.826	0.748	0.691

Statistics	Triangular, One, 90%, 4	Triangular, One, 90%, 7	Triangular, One, 90%, 10	Triangular, One, 90%, 14	Triangular, One, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.492	0.491	0.491	0.492	0.491
Median	0.486	0.487	0.485	0.489	0.488
Standard Deviation	0.116	0.086	0.072	0.061	0.053
Variance	0.013	0.007	0.005	0.004	0.003
Skewness	0.427	0.356	0.306	0.220	0.223
Kurtosis	3.267	3.197	3.010	2.840	2.947
Coeff. of Variability	0.236	0.175	0.147	0.124	0.108
Range Minimum	0.179	0.253	0.294	0.321	0.351
Range Maximum	0.989	0.855	0.737	0.700	0.682
Range Width	0.810	0.602	0.443	0.379	0.330
Incredible Value	1.450	1.109	0.974	0.880	0.817

Statistics	Triangular, Two, 100%, 4	Triangular, Two, 100%, 7	Triangular, Two, 100%, 10	Triangular, Two, 100%, 14	Triangular, Two, 100%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.369	0.369	0.371	0.371	0.371
Median	0.365	0.365	0.369	0.372	0.370
Standard Deviation	0.081	0.061	0.050	0.042	0.038
Variance	0.007	0.004	0.003	0.002	0.001
Skewness	0.342	0.343	0.214	0.143	0.128
Kurtosis	3.109	3.166	2.770	2.922	2.959
Coeff. of Variability	0.219	0.165	0.135	0.112	0.101
Range Minimum	0.153	0.196	0.217	0.265	0.275
Range Maximum	0.658	0.592	0.522	0.508	0.502
Range Width	0.505	0.396	0.304	0.243	0.227
Incredible	1.010	0.795	0.698	0.629	0.598

Statistics	Triangular, Two, 97.5%, 4	Triangular, Two, 97.5%, 7	Triangular, Two, 97.5%, 10	Triangular, Two, 97.5%, 14	Triangular, Two, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.445	0.446	0.448	0.449	0.450
Median	0.432	0.440	0.445	0.448	0.448
Standard Deviation	0.102	0.077	0.065	0.055	0.047
Variance	0.010	0.006	0.004	0.003	0.002
Skewness	0.356	0.250	0.267	0.234	0.166
Kurtosis	2.811	2.861	3.042	2.925	2.901
Coeff. of Variability	0.230	0.173	0.146	0.122	0.105
Range Minimum	0.181	0.238	0.237	0.286	0.318
Range Maximum	0.774	0.705	0.667	0.653	0.605
Range Width	0.593	0.467	0.429	0.367	0.287
Incredible	1.281	0.995	0.886	0.796	0.738

Statistics	Triangular, Two, 95%, 4	Triangular, Two, 95%, 7	Triangular, Two, 95%, 10	Triangular, Two, 95%, 14	Triangular, Two, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.492	0.495	0.495	0.496	0.495
Median	0.485	0.490	0.493	0.492	0.493
Standard Deviation	0.113	0.084	0.070	0.057	0.052
Variance	0.013	0.007	0.005	0.003	0.003
Skewness	0.452	0.280	0.243	0.187	0.201
Kurtosis	3.418	2.849	2.800	2.908	2.990
Coeff. of Variability	0.229	0.170	0.141	0.115	0.104
Range Minimum	0.166	0.262	0.319	0.347	0.356
Range Maximum	1.008	0.761	0.721	0.693	0.697
Range Width	0.842	0.499	0.402	0.346	0.341
Incredible	1.412	1.092	0.958	0.851	0.808

Statistics	Triangular, Two, 90%, 4	Triangular, Two, 90%, 7	Triangular, Two, 90%, 10	Triangular, Two, 90%, 14	Triangular, Two, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.575	0.575	0.576	0.575	0.576
Median	0.564	0.572	0.574	0.573	0.577
Standard Deviation	0.140	0.102	0.085	0.074	0.067
Variance	0.020	0.010	0.007	0.005	0.004
Skewness	0.616	0.257	0.200	0.051	0.058
Kurtosis	4.052	3.294	2.881	2.830	2.942
Coeff. of Variability	0.244	0.178	0.148	0.128	0.116
Range Minimum	0.237	0.286	0.338	0.373	0.383
Range Maximum	1.305	1.014	0.871	0.803	0.804
Range Width	1.067	0.728	0.533	0.430	0.422
Incredible	1.756	1.314	1.151	1.049	0.993

Statistics	Log, Zero, 99.9, 4	Log, Zero, 99.9, 7	Log, Zero, 99.9, 10	Log, Zero, 99.9, 14	Log, Zero, 99.9, 18
Trials	1000	1000	1000	1000	1000
Mean	0.117	0.118	0.117	0.117	0.118
Median	0.113	0.115	0.116	0.116	0.117
Standard Deviation	0.026	0.020	0.017	0.014	0.013
Variance	0.001	0.000	0.000	0.000	0.000
Skewness	0.773	0.723	0.685	0.473	0.469
Kurtosis	4.036	3.960	3.869	3.374	3.308
Coeff. of Variability	0.224	0.172	0.145	0.122	0.110
Range Minimum	0.058	0.074	0.078	0.080	0.082
Range Maximum	0.243	0.212	0.200	0.182	0.171
Range Width	0.185	0.138	0.122	0.101	0.089
Incredible	0.329	0.262	0.232	0.208	0.197

Statistics	Log, Zero, 97.5, 4	Log, Zero, 97.5, 7	Log, Zero, 97.5, 10	Log, Zero, 97.5, 14	Log, Zero, 97.5, 18
Trials	1000	1000	1000	1000	1000
Mean	0.167	0.168	0.168	0.167	0.167
Median	0.158	0.158	0.162	0.162	0.162
Standard Deviation	0.059	0.048	0.041	0.034	0.030
Variance	0.004	0.002	0.002	0.001	0.001
Skewness	1.743	1.627	1.788	1.307	1.022
Kurtosis	8.266	8.245	10.698	7.015	5.183
Coeff. of Variability	0.355	0.287	0.244	0.204	0.179
Range Minimum	0.048	0.081	0.090	0.098	0.098
Range Maximum	0.555	0.539	0.519	0.407	0.348
Range Width	0.507	0.458	0.429	0.309	0.249
Incredible	0.816	0.616	0.515	0.427	0.382

Statistics	Log, Zero, 95.0, 4	Log, Zero, 95.0, 7	Log, zero, 95%, 10	Log, Zero, 95, 14	Log, Zero, 95.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.193	0.195	0.195	0.196	0.196
Median	0.173	0.179	0.183	0.187	0.188
Standard Deviation	0.091	0.073	0.059	0.049	0.044
Variance	0.008	0.005	0.003	0.002	0.002
Skewness	2.728	2.933	2.299	1.706	1.428
Kurtosis	17.791	23.175	15.096	9.258	6.777
Coeff. of Variability	0.471	0.373	0.303	0.251	0.223
Range Minimum	0.054	0.089	0.094	0.107	0.105
Range Maximum	1.114	1.024	0.763	0.583	0.478
Range Width	1.060	0.934	0.669	0.476	0.373
Incredible	1.479	1.023	0.764	0.616	0.546

Statistics	Log, Zero, 90.0, 4	Log, Zero, 90.0, 7	Log, Zero, 90, 10	Log, Zero, 90, 14	Log, Zero, 90.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.270	0.264	0.263	0.261	0.260
Median	0.214	0.225	0.234	0.235	0.238
Standard Deviation	0.230	0.153	0.145	0.116	0.098
Variance	0.053	0.023	0.021	0.013	0.010
Skewness	6.482	4.408	6.970	5.542	4.305
Kurtosis	72.704	39.946	94.689	67.843	45.789
Coeff. of Variability	0.853	0.579	0.551	0.443	0.377
Range Minimum	0.054	0.083	0.094	0.116	0.117
Range Maximum	3.626	2.180	2.699	2.051	1.623
Range Width	3.572	2.097	2.604	1.935	1.506
Incredible	6.980	2.970	2.678	1.798	1.381

Statistics	Log, One, 99.9, 4	Log, One, 99.9, 7	Log, One, 99.9, 10	Log, One, 99.9, 14	Log, One, 99.9, 18
Trials	1000	1000	1000	1000	1000
Mean	0.158	0.158	0.157	0.158	0.158
Median	0.155	0.157	0.158	0.156	0.156
Standard Deviation	0.031	0.023	0.019	0.017	0.015
Variance	0.001	0.001	0.000	0.000	0.000
Skewness	0.580	0.533	0.573	0.548	0.463
Kurtosis	3.481	3.614	3.966	3.436	3.366
Coeff. of Variability	0.194	0.144	0.122	0.106	0.092
Range Minimum	0.085	0.106	0.108	0.119	0.120
Range Maximum	0.274	0.254	0.253	0.234	0.218
Range Width	0.189	0.148	0.145	0.115	0.099
Incredible	0.387	0.310	0.279	0.259	0.243
Statistics	Log, One, 97.5, 4	Log, One, 97.5, 7	Log, One, 97.5, 10	Log, One, 97.5, 14	Log, One, 97.5, 18
Trials	1000	1000	1000	1000	1000
Mean	0.237	0.235	0.235	0.236	0.236
Median	0.224	0.227	0.231	0.233	0.235
Standard Deviation	0.071	0.051	0.043	0.035	0.032
Variance	0.005	0.003	0.002	0.001	0.001
Skewness	1.463	1.011	0.812	0.605	0.546
Kurtosis	7.332	4.776	4.319	3.664	3.598
Coeff. of Variability	0.299	0.215	0.182	0.149	0.134
Range Minimum	0.097	0.120	0.133	0.155	0.165
Range Maximum	0.701	0.498	0.442	0.391	0.360
Range Width	0.604	0.379	0.309	0.235	0.196
Incredible	0.915	0.634	0.546	0.473	0.441
Statistics	Log, One, 95.0, 4	Log, One, 95.0, 7	Log, one, 95%, 10	Log, One, 95, 14	Log, One, 95.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.281	0.281	0.280	0.278	0.279
Median	0.260	0.267	0.273	0.272	0.273
Standard Deviation	0.095	0.072	0.060	0.051	0.045
Variance	0.009	0.005	0.004	0.003	0.002
Skewness	1.749	1.465	1.284	1.266	1.003
Kurtosis	9.456	7.088	6.413	6.949	5.313
Coeff. of Variability	0.338	0.255	0.213	0.182	0.161
Range Minimum	0.109	0.135	0.150	0.166	0.179
Range Maximum	0.935	0.694	0.612	0.615	0.528
Range Width	0.825	0.559	0.462	0.449	0.349
Incredible	1.274	0.899	0.748	0.649	0.590
Statistics	Log, One, 90.0, 4	Log, One, 90.0, 7	Log, One, 90, 10	Log, One, 90, 14	Log, One, 90.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.353	0.352	0.354	0.352	0.353
Median	0.321	0.334	0.341	0.343	0.346
Standard Deviation	0.140	0.106	0.090	0.075	0.066
Variance	0.019	0.011	0.008	0.006	0.004
Skewness	1.752	1.396	1.345	1.035	0.812
Kurtosis	8.590	6.288	6.735	4.986	4.094
Coeff. of Variability	0.395	0.301	0.253	0.214	0.187
Range Minimum	0.125	0.162	0.194	0.196	0.208
Range Maximum	1.385	0.956	0.971	0.764	0.651
Range Width	1.260	0.794	0.777	0.568	0.443
Incredible	2.021	1.373	1.127	0.944	0.841

Statistics	Log, Two, 99.9, 4	Log, Two, 99.9, 7	Log, Two, 99.9, 10	Log, Two, 99.9, 14	Log, Two, 99.9, 18
Trials	1000	1000	1000	1000	1000
Mean	0.194	0.195	0.195	0.196	0.196
Median	0.189	0.192	0.193	0.195	0.194
Standard Deviation	0.042	0.030	0.025	0.022	0.019
Variance	0.002	0.001	0.001	0.000	0.000
Skewness	0.741	0.448	0.371	0.380	0.316
Kurtosis	3.860	3.180	3.408	3.301	3.255
Coeff. of Variability	0.214	0.156	0.131	0.114	0.098
Range Minimum	0.107	0.120	0.125	0.129	0.144
Range Maximum	0.376	0.324	0.306	0.300	0.279
Range Width	0.269	0.204	0.181	0.171	0.135
Incredible	0.522	0.404	0.360	0.334	0.311
Statistics	Log, Two, 97.5, 4	Log, Two, 97.5, 7	Log, Two, 97.5, 10	Log, Two, 97.5, 14	Log, Two, 97.5, 18
Trials	1000	1000	1000	1000	1000
Mean	0.307	0.307	0.306	0.308	0.307
Median	0.290	0.298	0.300	0.301	0.301
Standard Deviation	0.095	0.073	0.062	0.053	0.046
Variance	0.009	0.005	0.004	0.003	0.002
Skewness	1.304	0.882	0.699	0.525	0.584
Kurtosis	5.762	4.367	3.710	3.170	3.659
Coeff. of Variability	0.310	0.238	0.201	0.171	0.150
Range Minimum	0.122	0.142	0.169	0.173	0.177
Range Maximum	0.815	0.650	0.555	0.517	0.528
Range Width	0.693	0.508	0.386	0.345	0.350
Incredible	1.242	0.915	0.775	0.683	0.619
Statistics	Log, Two, 95.0, 4	Log, Two, 95.0, 7	Log, Two, 95.0, 10	Log, Two, 95.0, 14	Log, Two, 95.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.362	0.365	0.364	0.365	0.364
Median	0.341	0.354	0.358	0.357	0.356
Standard Deviation	0.123	0.098	0.082	0.068	0.062
Variance	0.015	0.010	0.007	0.005	0.004
Skewness	1.242	1.098	1.057	0.801	1.060
Kurtosis	5.586	5.199	5.194	4.007	6.165
Coeff. of Variability	0.341	0.269	0.225	0.186	0.171
Range Minimum	0.118	0.141	0.159	0.201	0.218
Range Maximum	1.037	0.834	0.790	0.643	0.803
Range Width	0.919	0.693	0.631	0.442	0.586
Incredible	1.664	1.245	1.024	0.865	0.805
Statistics	Log, Two, 90.0, 4	Log, Two, 90.0, 7	Log, Two, 90.0, 10	Log, Two, 90.0, 14	Log, Two, 90.0, 18
Trials	1000	1000	1000	1000	1000
Mean	0.479	0.474	0.476	0.472	0.475
Median	0.436	0.442	0.458	0.457	0.461
Standard Deviation	0.224	0.164	0.135	0.111	0.098
Variance	0.050	0.027	0.018	0.012	0.010
Skewness	2.881	2.161	1.503	1.160	0.942
Kurtosis	20.551	12.105	7.310	5.497	4.564
Coeff. of Variability	0.467	0.346	0.284	0.235	0.207
Range Minimum	0.143	0.210	0.210	0.227	0.266
Range Maximum	2.820	1.732	1.349	1.115	0.944
Range Width	2.677	1.522	1.139	0.889	0.678
Incredible	3.612	2.231	1.725	1.389	1.235

Statistics	Weibull, One, 99.9%, 4	Weibull, One, 99.9%, 7	Weibull, One, 99.9%, 10	Weibull, One, 99.9%, 14	Weibull, One, 99.9%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.215	0.216	0.216	0.215	0.215
Median	0.212	0.214	0.214	0.213	0.215
Standard Deviation	0.045	0.034	0.027	0.022	0.020
Variance	0.002	0.001	0.001	0.000	0.000
Skewness	0.507	0.463	0.330	0.265	0.148
Kurtosis	3.183	3.098	2.987	3.228	3.086
Coeff. of Variability	0.209	0.155	0.124	0.101	0.093
Range Minimum	0.106	0.138	0.147	0.145	0.151
Range Maximum	0.388	0.347	0.312	0.296	0.284
Range Width	0.282	0.209	0.164	0.151	0.134
Incredible	0.565	0.446	0.387	0.347	0.333
Statistics	Weibull, One, 97.5%, 4	Weibull, One, 97.5%, 7	Weibull, One, 97.5%, 10	Weibull, One, 97.5%, 14	Weibull, One, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.307	0.303	0.302	0.302	0.301
Median	0.299	0.298	0.298	0.300	0.298
Standard Deviation	0.076	0.056	0.047	0.039	0.035
Variance	0.006	0.003	0.002	0.002	0.001
Skewness	0.688	0.577	0.442	0.358	0.407
Kurtosis	3.872	3.498	3.401	3.404	3.300
Coeff. of Variability	0.247	0.186	0.156	0.130	0.116
Range Minimum	0.133	0.163	0.157	0.172	0.201
Range Maximum	0.655	0.531	0.479	0.452	0.417
Range Width	0.522	0.368	0.322	0.281	0.216
Incredible	0.952	0.718	0.624	0.555	0.518
Statistics	Weibull, One, 95%, 4	Weibull, One, 95%, 7	Weibull, One, 95%, 10	Weibull, One, 95%, 14	Weibull, One, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.351	0.353	0.354	0.353	0.352
Median	0.343	0.347	0.351	0.352	0.350
Standard Deviation	0.089	0.070	0.060	0.049	0.044
Variance	0.008	0.005	0.004	0.002	0.002
Skewness	0.575	0.443	0.362	0.264	0.329
Kurtosis	3.492	3.316	3.164	3.275	3.484
Coeff. of Variability	0.253	0.198	0.168	0.140	0.125
Range Minimum	0.140	0.168	0.205	0.215	0.213
Range Maximum	0.744	0.671	0.587	0.566	0.520
Range Width	0.604	0.502	0.382	0.351	0.307
Incredible	1.118	0.884	0.774	0.678	0.633
Statistics	Weibull, One, 90%, 4	Weibull, One, 90%, 7	Weibull, One, 90%, 10	Weibull, One, 90%, 14	Weibull, One, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.443	0.441	0.440	0.439	0.437
Median	0.428	0.432	0.433	0.433	0.433
Standard Deviation	0.128	0.097	0.079	0.067	0.060
Variance	0.016	0.009	0.006	0.005	0.004
Skewness	0.795	0.601	0.521	0.552	0.569
Kurtosis	4.499	3.623	3.543	3.912	3.791
Coeff. of Variability	0.289	0.220	0.179	0.153	0.137
Range Minimum	0.128	0.203	0.233	0.276	0.282
Range Maximum	1.158	0.870	0.783	0.744	0.690
Range Width	1.029	0.667	0.550	0.468	0.408
Incredible	1.642	1.216	1.010	0.896	0.832

Statistics	Weibull, Two, 99.9%, 4	Weibull, Two, 99.9%, 7	Weibull, Two, 99.9%, 10	Weibull, Two, 99.9%, 14	Weibull, Two, 99.9%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.251	0.252	0.251	0.250	0.251
Median	0.244	0.248	0.248	0.248	0.249
Standard Deviation	0.059	0.044	0.036	0.030	0.027
Variance	0.003	0.002	0.001	0.001	0.001
Skewness	0.809	0.491	0.316	0.209	0.184
Kurtosis	4.696	3.322	3.034	2.955	2.785
Coeff. of Variability	0.234	0.173	0.143	0.119	0.106
Range Minimum	0.105	0.143	0.143	0.153	0.182
Range Maximum	0.551	0.438	0.376	0.345	0.336
Range Width	0.445	0.295	0.233	0.192	0.154
Incredible	0.735	0.564	0.490	0.437	0.413

Statistics	Weibull, Two, 97.5%, 4	Weibull, Two, 97.5%, 7	Weibull, Two, 97.5%, 10	Weibull, Two, 97.5%, 14	Weibull, Two, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.357	0.360	0.357	0.357	0.356
Median	0.350	0.356	0.352	0.353	0.352
Standard Deviation	0.096	0.074	0.062	0.054	0.047
Variance	0.009	0.005	0.004	0.003	0.002
Skewness	0.507	0.483	0.447	0.456	0.324
Kurtosis	3.161	3.235	3.304	3.288	3.036
Coeff. of Variability	0.268	0.204	0.173	0.150	0.131
Range Minimum	0.123	0.149	0.166	0.221	0.238
Range Maximum	0.691	0.605	0.586	0.557	0.515
Range Width	0.568	0.456	0.420	0.336	0.277
Incredible	1.213	0.925	0.797	0.719	0.656

Statistics	Weibull, Two, 95%, 4	Weibull, Two, 95%, 7	Weibull, Two, 95%, 10	Weibull, Two, 95%, 14	Weibull, Two, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.414	0.414	0.415	0.415	0.415
Median	0.397	0.404	0.408	0.410	0.411
Standard Deviation	0.123	0.093	0.078	0.064	0.057
Variance	0.015	0.009	0.006	0.004	0.003
Skewness	0.608	0.460	0.503	0.417	0.398
Kurtosis	3.144	3.052	3.284	3.179	2.998
Coeff. of Variability	0.297	0.224	0.188	0.154	0.138
Range Minimum	0.148	0.188	0.199	0.250	0.259
Range Maximum	0.847	0.753	0.696	0.633	0.613
Range Width	0.700	0.565	0.497	0.384	0.353
Incredible	1.587	1.160	0.992	0.853	0.791

Statistics	Weibull, Two, 90%, 4	Weibull, Two, 90%, 7	Weibull, Two, 90%, 10	Weibull, Two, 90%, 14	Weibull, Two, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.515	0.516	0.514	0.516	0.516
Median	0.506	0.509	0.511	0.511	0.510
Standard Deviation	0.148	0.111	0.094	0.082	0.074
Variance	0.022	0.012	0.009	0.007	0.006
Skewness	0.623	0.544	0.535	0.489	0.346
Kurtosis	3.918	3.760	3.735	3.446	2.965
Coeff. of Variability	0.288	0.215	0.183	0.160	0.144
Range Minimum	0.163	0.243	0.284	0.329	0.332
Range Maximum	1.250	1.039	0.962	0.873	0.775
Range Width	1.087	0.796	0.678	0.544	0.444
Incredible	1.901	1.391	1.200	1.086	1.010

Statistics	Weibull, Zero, 99.9%, 4	Weibull, Zero, 99.9%, 7	Weibull, Zero, 99.9%, 10	Weibull, Zero, 99.9%, 14	Weibull, Zero, 99.9%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.179	0.178	0.178	0.178	0.178
Median	0.174	0.175	0.177	0.177	0.177
Standard Deviation	0.040	0.032	0.026	0.022	0.019
Variance	0.002	0.001	0.001	0.000	0.000
Skewness	0.694	0.554	0.340	0.413	0.374
Kurtosis	3.491	3.363	3.074	3.651	3.836
Coeff. of Variability	0.226	0.181	0.148	0.125	0.109
Range Minimum	0.081	0.107	0.111	0.117	0.116
Range Maximum	0.331	0.321	0.283	0.304	0.283
Range Width	0.251	0.214	0.171	0.187	0.167
Incredible	0.504	0.413	0.356	0.321	0.297
Statistics	Weibull, Zero, 97.5%, 4	Weibull, Zero, 97.5%, 7	Weibull, Zero, 97.5%, 10	Weibull, Zero, 97.5%, 14	Weibull, Zero, 97.5%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.251	0.248	0.247	0.247	0.248
Median	0.240	0.242	0.244	0.243	0.245
Standard Deviation	0.074	0.054	0.046	0.040	0.035
Variance	0.006	0.003	0.002	0.002	0.001
Skewness	0.790	0.600	0.649	0.544	0.415
Kurtosis	3.743	3.688	4.350	3.637	3.153
Coeff. of Variability	0.297	0.217	0.187	0.160	0.141
Range Minimum	0.089	0.115	0.111	0.140	0.162
Range Maximum	0.586	0.493	0.512	0.421	0.377
Range Width	0.498	0.377	0.401	0.280	0.215
Incredible	0.952	0.675	0.586	0.521	0.479
Statistics	Weibull, Zero, 95%, 4	Weibull, Zero, 95%, 7	Weibull, Zero, 95%, 10	Weibull, Zero, 95%,14	Weibull, Zero, 95%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.287	0.290	0.288	0.289	0.289
Median	0.273	0.278	0.283	0.285	0.284
Standard Deviation	0.095	0.075	0.062	0.052	0.046
Variance	0.009	0.006	0.004	0.003	0.002
Skewness	0.983	0.831	0.678	0.581	0.471
Kurtosis	4.520	4.062	3.554	3.694	3.234
Coeff. of Variability	0.333	0.261	0.216	0.181	0.159
Range Minimum	0.084	0.123	0.151	0.163	0.172
Range Maximum	0.753	0.624	0.556	0.514	0.457
Range Width	0.669	0.501	0.404	0.351	0.285
Incredible	1.275	0.951	0.780	0.671	0.606
Statistics	Weibull, Zero, 90%, 4	Weibull, Zero, 90%, 7	Weibull, Zero, 90%, 10	Weibull, Zero, 90%, 14	Weibull, Zero, 90%, 18
Trials	1000	1000	1000	1000	1000
Mean	0.358	0.359	0.358	0.355	0.355
Median	0.342	0.346	0.347	0.348	0.349
Standard Deviation	0.127	0.097	0.082	0.068	0.061
Variance	0.016	0.009	0.007	0.005	0.004
Skewness	0.862	0.550	0.641	0.533	0.477
Kurtosis	4.184	3.173	3.572	3.412	3.318
Coeff. of Variability	0.355	0.271	0.230	0.191	0.171
Range Minimum	0.107	0.149	0.181	0.201	0.211
Range Maximum	1.073	0.781	0.762	0.647	0.583
Range Width	0.966	0.632	0.582	0.446	0.372

APPENDIX G

TOTAL WATER CONTENT DISTRIBUTION FUNCTIONS

For a given distribution type and a given number of fuel baskets, there are four sets of curves on a plot. Each set corresponds to a percentile point for the high values. Starting from the left and proceeding to the right, the four sets of curves show the results with the high value set to the 100th percentile (99.9%, actually), 97.5 percentile, 95.0 percentile, and 90.0 percentile, respectively. Each set consists of five cumulative distribution functions representing the five levels of independence. The curves show that the mean value of the simulation increases and the high value percentile decreases. It also shows that as the level of independence decreases, the S-shaped curves become flatter; i.e., the standard deviation increases.

Crystal Ball Report

Simulation started on 9/22/98 at 15:06:50

Simulation stopped on 9/22/98 at 15:12:24

Crystal Ball Report
Simulation started on 9/22/98 at 14:35:45
Simulation stopped on 9/22/98 at 14:42:23

Crystal Ball Report
Simulation started on 9/24/98 at 9:54:08
Simulation stopped on 9/24/98 at 10:00:57

Crystal Ball Report

Simulation started on 9/25/98 at 14:24:17

Simulation stopped on 9/25/98 at 14:36:02

Crystal Ball Report

Simulation started on 9/25/98 at 14:24:17

Simulation stopped on 9/25/98 at 14:36:02

Crystal Ball Report

Simulation started on 9/25/98 at 15:13:18

Simulation stopped on 9/25/98 at 15:19:04

Crystal Ball Report

Simulation started on 9/25/98 at 15:36:48

Simulation stopped on 9/25/98 at 15:42:28

Crystal Ball Report

Simulation started on 9/24/98 at 11:32:22

Simulation stopped on 9/24/98 at 11:39:11

Crystal Ball Report
Simulation started on 9/24/98 at 11:46:43
Simulation stopped on 9/24/98 at 11:53:48

APPENDIX H
ANDERSON-DARLING STATISTIC SIGNIFICANCE LEVELS

Table H-1: Anderson-Darling Statistic Distribution $q = P(A^2 < z)$ [D'Agostino and Stephens, 1986]	
z	q ⁽¹⁾
0.100	0.0000
0.200	0.0096
0.300	0.0618
0.400	0.1513
0.500	0.2532
0.600	0.3520
0.700	0.4412
0.800	0.5190
0.900	0.5858
1.000	0.6427
1.100	0.6912
1.200	0.7324
1.300	0.7677
1.400	0.7973
1.500	0.8253
(1) α (or p-level) = 1 - q.	

APPENDIX I
WATER CONTENT USING FINAL VALUE
FOR CANISTER PARTICULATE MASS

A limited set of cases was re-run using the final nominal value for canister particulate mass per assembly (2.41 grams/assy). Recall that our method preserved the nominal value as the median of the log-normal distribution. For canister particulate mass/assembly, the *median* value was shifted from 2.22 grams to 2.41 grams, which shifted the *mean* value from 3.94 grams to 4.05 grams, which is a 2.8% increase in the mean. Of course, as the mean shifts upward slightly, the standard deviation must be reduced slightly to fit the percentile for the high value, be it 95% or 90%.

Table I-1 shows results and comparisons for six cases:

1. Log-Normal, Zero Scrap Baskets, 95%, 10 Groups
2. Log-Normal, Zero Scrap Baskets, 90%, 10 Groups
3. Log-Normal, One Scrap Basket, 95%, 10 Groups
4. Log-Normal, One Scrap Basket, 90%, 10 Groups
5. Log-Normal, Two Scrap Baskets, 95%, 10 Groups
6. Log-Normal, Two Scrap Baskets, 90%, 10 Groups

Differences are noticeable only for the case with no scrap baskets. In cases with one or two scrap baskets, the adhering particulate contributes the preponderance of the total water mass, and slight changes in the canister particulate are not important. Given the nature of our Monte Carlo calculations, where slight deviations occur from one run to the next regardless of the input, a minor (~3%) change in one of several input variables does not produce significant change.

Table I-1: Comparison of Results Using Draft and Final Inputs
for Canister Particulate Mass per Assembly.

Final Inputs for Canister Particulate						
Statistics	Lognormal, Zero, 95%, 10	Lognormal, Zero, 90%, 10	Lognormal, 95%, 10, one	Lognormal, 90%, 10, one	Lognormal, 95%, 10, two	Lognormal, 90%, 10, two
Trials	1000	1000	1000	1000	1000	1000
Mean	0.2001	0.2632	0.2847	0.3702	0.3650	0.4754
Median	0.1879	0.2347	0.2788	0.3494	0.3525	0.4481
Standard Deviation	0.0591	0.1153	0.0605	0.1190	0.0798	0.1365
Variance	0.0035	0.0133	0.0037	0.0142	0.0064	0.0186
Skewness	1.8443	3.0844	1.2993	3.6059	0.7822	1.4470
Kurtosis	9.8413	21.3441	8.3347	32.6794	3.7279	6.8920
Coeff. of Variability	0.2953	0.4382	0.2124	0.3215	0.2185	0.2872
Range Minimum	0.0962	0.1013	0.1576	0.1861	0.1821	0.2391
Range Maximum	0.6338	1.4934	0.7707	1.8264	0.6918	1.2888
Range Width	0.5376	1.3921	0.6132	1.6404	0.5096	1.0497
Incredible	0.762	1.778	0.758	1.572	0.999	1.749
Draft Slougher Inputs for Canister Particulate						
Statistics	Lognormal, Zero, 95%, 10	Lognormal, Zero, 90%, 10	Lognormal, 95%, 10, one	Lognormal, 90%, 10, one	Lognormal, 95%, 10, two	Lognormal, 90%, 10, two
Trials	1000	1000	1000	1000	1000	1000
Mean	0.195	0.263	0.280	0.370	0.364	0.476
Median	0.183	0.234	0.273	0.343	0.358	0.458
Standard Deviation	0.059	0.145	0.060	0.130	0.082	0.135
Variance	0.003	0.021	0.004	0.017	0.007	0.018
Skewness	2.299	6.970	1.284	4.653	1.057	1.503
Kurtosis	15.096	94.689	6.413	47.758	5.194	7.310
Coeff. of Variability	0.303	0.551	0.213	0.351	0.225	0.284
Range Minimum	0.094	0.094	0.150	0.194	0.159	0.210
Range Maximum	0.763	2.699	0.612	2.138	0.790	1.349
Range Width	0.669	2.604	0.462	1.944	0.631	1.139
Incredible	0.764	2.678	0.748	1.776	1.024	1.725

APPENDIX J
QUALITY ASSURANCE DOCUMENTS

Included here are:

1. Calculation Note Cover Sheet.
2. Calculation Note Methodology Checklist.

CALCULATION NOTE COVER SHEET

SECTION TO BE COMPLETED BY AUTHOR(S):

Page <u>i to J-4</u>	
Calc Note Number <u>FAI/98-1</u>	Revision Number <u>4</u>
Title <u>Sensitivity of Probabilistic MCO Water Content Estimates to Key Assumptions</u>	
Project <u>Hanford Spent Nuclear Fuel</u>	
Project Number or Shop Order <u>FDN001-14</u>	
Purpose: <u>Probabalistic assessment of bounding MCO water content, with sensitivity.</u>	
Results Summary: <u>At 95th percentile for high inputs and n = 10 independent damage level choices, 0.75 kg H₂O for one scrap basket, vs. 1.4 kg reference high bound.</u>	
References of Resulting Reports, Letters, or Memoranda (Optional)	
Author(s): (See Page I-3)	
Name (Print or Type)	Signature
<u>Martin G. Plys</u>	<u>Martin Plys</u>
<u>Boro Malinovic</u>	<u>Boro Malinovic</u>
	Completion Date
	<u>10/05/98</u>
	<u>10/05/98</u>

SECTION TO BE COMPLETED BY VERIFIER(S):

Verifier(s): (See Page I-3)	
Name (Print or Type)	Signature
<u>Martin G. Plys</u>	<u>Martin Plys</u>
<u>Boro Malinovic</u>	<u>Boro Malinovic</u>
	Completion Date
	<u>10/05/98</u>
	<u>10/05/98</u>
Independent Review or	
Method of Verification: Design Review <u> </u> , Alternate Calculations <u>X</u> , Testing <u> </u>	
Other (specify) <u> </u>	

SECTION TO BE COMPLETED BY MANAGER:

Responsible Manager:	
Name (Print or Type)	Signature
<u>Martin G. Plys</u>	<u>Martin Plys</u>
	Approval Date
	<u>10/05/98</u>

Authorship and Review of this Calculation Note is as Follows:

Section	Author	Reviewer
1	M.G. Plys	B. Malinovic
2	M.G. Plys	B. Malinovic
3	M.G. Plys	B. Malinovic
4	M.G. Plys	B. Malinovic
5.1	M.G. Plys	B. Malinovic
5.2	B. Malinovic	M.G. Plys
5.3	B. Malinovic	M.G. Plys
5.4	B. Malinovic	M.G. Plys
6	M.G. Plys	B. Malinovic
App. A	B. Malinovic	M.G. Plys
App. B	B. Malinovic	M.G. Plys
App. C	B. Malinovic	M.G. Plys
App. D	M.G. Plys	B. Malinovic
App. E	B. Malinovic	M.G. Plys
App. F	B. Malinovic	M.G. Plys
App. G	B. Malinovic	M.G. Plys
App. H	B. Malinovic	M.G. Plys
App. I	B. Malinovic	M.G. Plys
App. J	N/A	N/A

CALC NOTE NUMBER FAI/98-1 REV. 4 PAGE i to J-4

CALCULATION NOTE METHODOLOGY CHECKLIST

CHECKLIST TO BE COMPLETED BY AUTHOR(S) (CIRCLE APPROPRIATE RESPONSE)

1. Is the subject and/or the purpose of the design analysis clearly stated? YES . NO
2. Are the required inputs and their sources provided? YES . NO . N/A
3. Are the assumptions clearly identified and justified? YES . NO . N/A
4. Are the methods and units clearly identified? YES . NO . N/A
5. Have the limits of applicability been identified? YES . NO . N/A
(i.e., Is the analysis for a 3 or 4 loop plant or for a single application.)
6. Are the results of literature searches, if conducted, or other background data provided? YES . NO . N/A
7. Are all the pages sequentially numbered and identified by the calculation note number? YES . NO
8. Is the project or shop order clearly identified? YES . NO
9. Has the required computer calculation information been provided? YES . NO . N/A
10. Were the computer codes used under configuration control? YES . NO . N/A
11. Was the computer code(s) used applicable for modeling the physical and/or computational problems identified? YES . NO . N/A
(i.e., Is the correct computer code being used for the intended purpose.)
12. Are the results and conclusions clearly stated? YES . NO
13. Are Open Items properly identified? YES . NO . N/A
14. Were approved Design Control practices followed without exception? YES . NO . N/A
(Approved Design Control practices refers to guidance documents within NSD that state how the work is to be performed, such as how to perform a LOCA analysis.)
15. Have all related contract requirements been met? YES . NO . N/A

NOTE: If NO to any of the above, Page Number containing justification _____

DISTRIBUTION SHEET

To	From	Page 1 of 1
Distribution	Process Engineering	Date 10/21/98
Project Title/Work Order		EDT No. N/A
Sensitivity of Probabilistic MCO Water Content Estimates to Key Assumptions		ECN No. 648615

Name	MSIN	Text With All Attach.	Text Only	Attach./ Appendix Only	EDT/ECN Only
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