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**A MICROWAVE INVERSE CERENKOV ACCELERATOR
(MICA)**

FINAL REPORT

on

research carried out under Phase II SBIR grant
DE FG02-95-ER-82047
from US Department of Energy

Submitted by:

J. L. Hirshfield, Principal Investigator

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DOE SBIR PHASE II GRANT DE FG02-95-ER-82047**

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FINAL REPORT

INTRODUCTION

The objective of this Phase II SBIR research program was to complete the final design originated during Phase I for a prototype Microwave Inverse Cerenkov Accelerator (MICA), to fabricate the prototype MICA, and to test its performance as an electron accelerator. This report contains details of the design, predictions of accelerator performance, results of cold tests on the MICA structure, and details of the installation of MICA on the Yale Beam Physics Laboratory 6-MeV beamline. Discussion of future work is also included.

MICA is a slow-wave device, where a dielectric element is inserted into a waveguide to reduce the phase velocity of an electromagnetic wave to match the velocity of the electrons undergoing acceleration. Acceleration is provided by appropriate phasing of the RF wave with respect to the phase of a short injected electron bunch from an RF gun, so that a continuous accelerating force is experienced by all the particles. The acceleration mechanism is thus similar to that in a conventional RF linear accelerator (linac), but with a continuous rather than periodic loading structure in the waveguide. If necessary, a variation in phase velocity of the RF guided wave in MICA can be provided by a slow taper or stepped variation in the dimensions of the dielectric loading element, so as to preserve matching to the electron velocity. But unlike Microwave Inverse Free Electron Laser Accelerator (MIFELA), which uses a tapered wiggler and a guide magnetic field, MICA requires neither of these. Hence the overall structure is much simpler (and less expensive) than either MIFELA or a conventional RF linac; the orbital motion of the particles during acceleration is nearly one-dimensional; and there is reason to expect the normalized transverse emittance of a MICA beam to be lower than that of a MIFELA beam.

MICA is the inverse of stimulated Cerenkov radiation, a mechanism for generating coherent emission]. The name "Cerenkov" is appropriated to signify that radiating particles move at speeds greater than light speed in the surrounding medium. In the linearized analysis of this interaction, a self-consistent solution for particle motions and RF field amplitudes leads to a dispersion relation governing the spatial rate of growth or decay for allowed electromagnetic modes in the system. Typically, three roots of the dispersion relation exist for modes that co-propagate with the beam, one corresponding to a growing wave and one corresponding to a damped wave; the latter represents stimulated absorption and thus electron acceleration. In MICA, one exploits the latter mode, albeit into the fully nonlinear regime.

To our knowledge, there has only been one experimental approach taken so far to study the inverse Cerenkov acceleration mechanism, namely in a program at Brookhaven National Laboratory using a CO₂ laser and an axicon to interact with an electron beam of 40 MeV initial energy. In the Brookhaven work, slowing of the laser beam phase velocity to below c , the velocity of light in vacuum, is brought about by the introduction of hydrogen as a filling gas. In published results, inverse Cerenkov acceleration of up to 3.7 MeV was reported, corresponding to an acceleration gradient of 31 MeV/m. In the MICA experiment discussed here, no filling gas is required; thus beam scattering on the gas atoms that would increase beam emittance is absent. Furthermore, since the experiment uses an injected short bunch of ~6 MeV electrons from an RF gun driven from the 2.856 GHz XK-5 SLAC klystron that also drives the Cerenkov accelerator section, phase synchronism between the wave and all the particles can be arranged *a priori*. This allows nearly full trapping of the injected electrons in a narrow phase window and a nearly mono-energetic bunch at the MICA output. On the other hand, use of a dielectric lining in the MICA waveguide can introduce problems of breakdown and/or multipactoring that will limit the acceleration gradient one is able to achieve.

The following three sections comprise the bulk of this report. First is a collection of reprints of published papers that contain theory and computations that underlie the physics of MICA, and that lead to specific design criteria. Second are design details for the MICA structure that has been fabricated, including photographs of essential components of MICA and of its installation in the Yale Beam Physics Laboratory. Third are several reprints of work published on wake fields in dielectric-loaded waveguides and their possible use in particle acceleration; this work was initiated during the Phase II MICA project to address questions relating to beam stability in MICA, but has since been expanded during a Phase I SBIR project under DoE grant DE-FG02-98ER 82631.

THEORY AND COMPUTATIONS THAT UNDERLIE MICA DESIGN

Several publications contain details of the theory of MICA operation, including numerical computations that predict MICA performance. These details underlie the design that was perfected and carried forward into the fabrication of a prototype MICA structure that will be described in the next section of this report. The publications included here are the following:

"Microwave inverse Cerenkov accelerator," by T. B. Zhang, T. C. Marshall, M. A. LaPointe, J. L. Hirshfield, and Amiran Ron, *Phys. Rev. E* **54**, pp. 1918-1929 (1996).

"A microwave inverse Cerenkov accelerator," by T. B. Zhang and T. C. Marshall, *Nucl. Instr. and Meth. in Phys. Res. A* **375**, pp. 614-617 (1996).

"A survey of microwave inverse FEL and inverse Cerenkov accelerators," by T. C. Marshall and T. B. Zhang, CP396, *New Modes of Particle Acceleration: Techniques and Sources*, edited by Z. Parsa (AIP Press, New York, 1997). pp. 105-119.

"Microwave inverse Cerenkov accelerator," by T. B. Zhang, T. C. Marshall, M. A. LaPointe, and J. L. Hirshfield, CP398, *Advanced Accelerator Concepts*, edited by S. Chattopadhyay, J. McCullough and P. Dahl (AIP Press, New York, 1997). pp. 618-628.

Microwave inverse Čerenkov accelerator

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Analysis and experimental tests have been carried out on a dielectrically lined waveguide, which appears to be a suitable structure for accelerating electrons. From the dispersion relation for the TM_{01} mode, inner and outer radii of a copper-clad alumina pipe ($\epsilon=9.40$) have been determined such that the phase and group velocities are $0.9732c$ and $0.1096c$, respectively. Analysis and particle simulation studies for the injection of 6-MeV microbunches from a 2.856-GHz rf gun, and subsequent acceleration by the TM_{01} fields, predict that an acceleration gradient of 6.3 MV/m can be achieved with a traveling-wave power of 15 MW applied to the structure. Synchronous injection into a narrow phase window is shown to allow trapping of all injected particles. The rf fields of the accelerating structure are shown to provide radial focusing, so that longitudinal and transverse emittance growth during acceleration is small and that no external magnetic fields are required for focusing. The acceleration mechanism is the inverse of that in which electrons radiate as they traverse a waveguide at speeds exceeding the phase velocity of the microwaves (Čerenkov radiation) and is thus referred to as a microwave inverse Čerenkov accelerator. For 0.16-nC, 5-psec microbunches, the normalized emittance of the accelerated beam is predicted to be less than 5π mm mrad. Experiments on sample alumina tubes have been conducted that verify the theoretical dispersion relation for the TM_{01} mode over a two-to-one range in frequency. No excitation of axisymmetric or nonaxisymmetric competing waveguide modes was observed. High power tests showed that tangential electric fields at the inner surface of an uncoated sample of alumina pipe could be sustained up to 8.4 MV/m without breakdown. These considerations suggest that a microwave inverse Čerenkov test accelerator can be built to examine these predictions using an available rf power source, a 6-MeV rf gun, and an associated beam line. [S1063-651X(96)07408-9]

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I. INTRODUCTION

The stimulated Čerenkov effect is a well-understood mechanism for generating coherent radiation from an energetic electron beam [1–3]. The radiating electrons move at speed greater than that of the velocity of light in the structure (hence the name Čerenkov); although there are several ways to slow light waves, as a general rule the term is used when the slowing is caused by a dielectric element. When one does a linearized treatment of the fields and the self-consistent motion of the particles, a dispersion relation is obtained for growth or decay of radiation in the system. One of the three roots obtained corresponds to a damped wave; this we identify with the mechanism of stimulated absorption, whereby an electron will gain energy at the expense of the rf field. In the discussion that follows we consider the application of stimulated absorption in the nonlinear regime of particle trapping, which applies to an electron accelerator device. This we refer to as a microwave inverse Čerenkov accelerator (MICA) [4].

Acceleration of the electron is done by appropriate phasing of a 6-MeV electron bunch that is emitted from a thermionic cathode rf gun, so that a continuous accelerating force is applied to all the electrons, which move synchronously with the slow rf wave. Variation of the wave speed, if necessary, can be done by using a small taper in the filling factor of the dielectric element. Thus the device resembles a rf linear accelerator but without the periodic loading structures

in the waveguide. As the MICA is a smooth bore and the motion of the particles is rather one dimensional, we expect that the quality of the electron beam produced will be attractive. The MICA under consideration will use a SLAC (Stanford Linear Accelerator) source of microwave power at 2.856 GHz, and with a bunch length of only 5 psec compared to the rf period of 350 psec, we can expect excellent trapping and acceleration of a monoenergetic bunch of electrons. Another approach [5–8], the inverse Čerenkov accelerator experiment at Brookhaven National Laboratory, uses a CO_2 laser and an axicon to accelerate an electron beam at 40 MeV energy; the light wave is slowed by introducing hydrogen gas into the beam line. The gas contributes to some electron scattering, and the main disadvantage of the short laser wavelength is that electrons interact with the wave over the full range (2π) of phase; that is, the bunch length is long compared to the rf wavelength. In the MICA, the electrons move down a 1-cm-diam hole in an alumina dielectric liner as a filamentary beam of under 1 mm diameter. The main limitation here is that of the maximum axial field gradient (120–160 kV/cm [9]) along the dielectric surface. Shown in Fig. 1 is a schematic layout of the MICA.

In this paper we describe first the analysis of a wave inside a dielectric annular cylinder fitted into a cylindrical waveguide. We find dispersion relations [10] for the axisymmetric TM_{0n} -like mode of the system, that is, the mode that gives a maximum electric field along the axis. This calculation provides both the slowing factor and the distribution of

FIG. 1. (a) Schematic diagram of the MICA layout. A preaccelerated short bunch of ~ 6 -MeV electrons from a rf gun is injected in the appropriate accelerating phase into the MICA, so that trapping and further acceleration of the entire bunch can occur. (b) MICA accelerating section.

fields. In Sec. III we analyze the power flow and energy attenuation of the rf field in the loaded waveguide and find the group velocity of the field. Based on the single-particle dynamics, we then, in Sec. IV, model the acceleration and motion of electrons in the vacuum fields of this device. We will show that the beam off-axis effect is not important in the MICA. We anticipate that the electron beam self-field effect is negligible and the normalized emittance is constant throughout the acceleration of the bunch, and this is established by making use of the detailed accelerator code PARMELA in Sec. V. Knowledge of the rf breakdown limits of the dielectric relates to the maximum acceleration gradient of the beam, and therefore we design and construct a cavity resonator with an alumina liner for the breakdown limit test, which is described in Sec. VI. We conclude with a summary in Sec. VII. The objective of this effort is to determine whether a compact high-quality accelerator of this type is feasible, and we find that the answer is positive.

II. DISPERSION RELATION AND FIELD DISTRIBUTION OF THE DIELECTRIC-LOADED WAVEGUIDE

The MICA configuration is a circular waveguide loaded by high- ϵ dielectric material, with a small hole on axis for passage of the beam as shown in Fig. 2. Before selecting this configuration we explored another dielectric structure, a rectangular waveguide loaded by a dielectric slab or slabs. No conventional TM or TE modes exist in the latter, but rather longitudinal-section electric modes or longitudinal-section magnetic modes with either zero electric or zero magnetic field normal to the dielectric surface [11]. The maximum

FIG. 2. Longitudinal cross section of the loaded waveguide showing the axial electric field distribution as well as electron phase positions along the waveguide.

axial field strength occurs inside the vacuum-dielectric interface and so this configuration is not appropriate for electron acceleration. However, the use of a high- ϵ annulus inside a circular waveguide maintained a large uniform E_z field inside the hole, which is ideal for an accelerator.

A. Dispersion relation and eigenmodes

The axisymmetric modes of this cylindrical dielectric-loaded system are either TE or TM, and we consider the TM_{0n} -like mode of this system, i.e., the mode with finite axial electric field on axis, no azimuthal variations, and one radial maximum for the axial electric field. Using the appropriate boundary conditions at the interface of two differing media as well as at the outer metallic conducting wall, we solve the Maxwell equations using standard procedures [10] and arrive at a dispersion relation of the system for TM_{0n} eigenmodes, namely,

$$\frac{I_1(k_{11}a)}{I_0(k_{11}a)} = \frac{\epsilon_2 k_{11} J_1(k_{12}a) N_0(k_{12}R) - J_0(k_{12}R) N_1(k_{12}a)}{\epsilon_1 k_{12} J_0(k_{12}a) N_0(k_{12}R) - J_0(k_{12}R) N_0(k_{12}a)}, \quad (2.1)$$

where the functions of $J_m(x)$ and $N_m(x)$ are m th-order Bessel functions of the first and second kinds and $I_m(x)$ is the modified Bessel function; a and R are radii of the central vacuum hole and the outer waveguide wall, respectively, and ϵ_1 and ϵ_2 are the dielectric constants of the inner and outer regions. Obviously, for the central vacuum hole $\epsilon_1 = 1$. Note the k_{11} and k_{12} are the transverse wave numbers in region 1 and 2, and in Eq. (2.1), we have substituted $k_{11}^2 = -k_{\perp 1}^2$ for the slow wave, so that

$$k_{11}^2 = k_z^2 - \epsilon_1 k_0^2, \quad (2.2)$$

$$k_{12}^2 = \epsilon_2 k_0^2 - k_z^2, \quad (2.3)$$

FIG. 3. Normalized phase velocity v_{ph}/c vs normalized outer radius R/λ_0 for the TM_{01} mode waveguide with an alumina liner when the radius of the vacuum hole varies. From left to right, $a/R=0$ (completely filled), 0.3, 0.4, 0.5, and 1.0 (vacuum waveguide). Note that for $a/R=0.3$, $v_{ph} \leq c$ for $R \geq 0.15\lambda_0$.

where k_z is the axial wave number of the waveguide TM_{0n} mode and $k_0 = \omega/c$ is the wave number in free space for a wave of radian frequency ω . Combining Eqs. (2.2) and (2.3), we have

$$k_{12}^2 = (\epsilon_2 - \epsilon_1)k_0^2 - k_{11}^2. \quad (2.4)$$

By eliminating k_{12} from Eq. (2.1) using Eq. (2.4), we can solve for the multiple eigenvalues k_{1i} for the loaded waveguide, corresponding to distinct numbers n of radial maxima in the rf electric fields of the TM_{0n} modes. Accordingly, the normalized phase velocity v_{ph} and guide wavelength λ_g of each mode can be obtained from the respective eigenvalue. It then follows that

$$\frac{v_{ph}}{c} = \frac{\lambda_g}{\lambda_0} = \left(1 + \frac{k_{1i}^2}{k_0^2}\right)^{-1/2}, \quad (2.5)$$

where λ_0 is the free space wavelength of the field. As an example, Fig. 3 shows the normalized phase velocity of the TM_{01} mode as a function of the ratio R/λ_0 . For this example, the dielectric constant ϵ_2 is taken to be 9.4 and the ratio of hole radius to outer dielectric radius a/R is taken to be 0 (completely filled), 0.3, 0.4, 0.5, and 1.0 (no dielectric at all). For the specific configuration of $a/R=0.3$, one finds from Fig. 3 (second curve from the left) that a phase velocity of c and below obtain when $R/\lambda_0 > 0.15$, from which one finds the design values of outer waveguide radius and hole radius in terms of the rf vacuum wavelength for a given desired phase velocity.

For operation at 2.856 GHz, the frequency of the rf source available for the MICA, one has $\lambda_0 = 10.504$ cm. Since the injected 6-MeV electrons have velocities of about $0.997c$, the desired phase velocity is very close to c . From the curve in Fig. 3 with $a/R=0.30$, one selects $R=1.59$ cm and $a=0.48$ cm for $v_{ph}/c \approx 1.0$. The cutoff frequency of the TM_{01} mode is 2.69 GHz, and for the TM_{02} mode it is 6.72 GHz. By neglecting the hole, one can estimate cutoff frequencies for the TE_{11} -like and TE_{01} modes to be 1.80 and 3.76 GHz, respectively. The dimensions selected for operation at 2.856 GHz thus appear to guarantee freedom from

interference by other nearby modes, since only the TE_{11} -mode with nearly orthogonal polarization is not cut off at 2.856 GHz.

B. rf field distributions

The electromagnetic fields that are distributed in the two regions are joined at the interior boundary between vacuum and the dielectric and satisfy conditions appropriate for a conducting metallic wall at the outer boundary. For the TM_{0n} modes, only three components exist (in cylindrical coordinates r, θ, z), namely, E_z , E_r , and B_θ . They have the following forms. When $0 < r < a$ (region 1), one has

$$E_{z1}(r, z, t) = E_0 J_0(k_{11}r) \cos(\omega t - k_z z), \quad (2.6)$$

$$E_{r1}(r, z, t) = -E_0 \frac{k_z}{k_{11}} I_1(k_{11}r) \sin(\omega t - k_z z), \quad (2.7)$$

$$B_{\theta1}(r, z, t) = -E_0 \frac{\epsilon_1 k_0}{k_{11}} I_1(k_{11}r) \sin(\omega t - k_z z). \quad (2.8)$$

When $a < r \leq R$ (region 2), one has

$$E_{z2}(r, z, t) = E_0 G_0(k_{12}r) \cos(\omega t - k_z z), \quad (2.9)$$

$$E_{r2}(r, z, t) = -E_0 \frac{k_z}{k_{12}} G_1(k_{12}r) \sin(\omega t - k_z z), \quad (2.10)$$

$$B_{\theta2}(r, z, t) = -E_0 \frac{\epsilon_2 k_0}{k_{12}} G_1(k_{12}r) \sin(\omega t - k_z z), \quad (2.11)$$

where

$$g_i = \frac{I_0(k_{1i}a)}{J_0(k_{12}a)N_0(k_{12}R) - J_0(k_{12}R)N_0(k_{12}a)}, \quad (2.12)$$

$$G_0(k_{12}r) = g_i [J_0(k_{12}r)N_0(k_{12}R) - J_0(k_{12}R)N_0(k_{12}r)], \quad (2.13)$$

$$G_1(k_{12}r) = g_i [J_1(k_{12}r)N_0(k_{12}R) - J_0(k_{12}R)N_1(k_{12}r)]. \quad (2.14)$$

The amplitude E_0 is the peak axial field strength on axis. Its value in the absence of a beam and with negligible power losses in the dielectric and the walls can be determined from the total microwave power transmitted through the waveguide after integrating the Poynting vector over the guide cross section. Figure 4 is a snapshot of relative amplitudes of the field components vs radial coordinates r/R . This is a field profile seen by a specific electron whose relative phase with respect to the field is very close to π and therefore it is accelerated by the maximum axial field while suffering little influence from the transverse field components. This result thus establishes that is indeed possible to design a dielectrically loaded waveguide where the axial electric field in the beam hole is nearly uniform with radius, having its strongest magnitude on axis, where the beam is to be located.

As the acceleration of electrons proceeds, the microwave field will gradually lose part of its energy to the electrons and therefore E_0 will be a decreasing function of axial distance. However, the rf energy stored in the accelerating structure will be much larger than the energy to be imparted to the

FIG. 4. Profile of field component amplitudes vs r/R for the TM_{01} mode with an alumina liner $a/R = 0.3$. This is the field "seen" by an electron whose relative phase to the field is close to π ; the electron experiences an intense and uniform axial field but much smaller transverse fields.

microbunches that pass through in one fill time. As a result, rf power depletion along the MICA will be small, and it is a good approximation to neglect the small axial decrease of E_0 along the beam path. Were the MICA to be operated with strong rf power depletion successive microbunches could encounter diminishing accelerating fields, the final energy of late-arriving microbunches could be significantly lower than that of early microbunches, and the overall beam energy spread could be substantial; this would defeat one goal of the research, namely, low-energy spread. A rf beam chopper would allow only one in ten (or even fewer) high-current microbunches to enter the MICA, thus ensuring negligible energy spread due to beam loading

III. RF POWER FLOW, ENERGY DENSITY, AND ENERGY LOSS

The flow of energy in the loaded waveguide is described by the complex Poynting vector

$$\mathbf{S} = \frac{c}{4\pi} \frac{1}{2} (\mathbf{E} \times \mathbf{H}^*). \quad (3.1)$$

Accordingly, the energy density is

$$w = \frac{1}{8\pi} \frac{1}{2} (\mathbf{E} \cdot \mathbf{D}^* + \mathbf{B} \cdot \mathbf{H}^*). \quad (3.2)$$

Substituting Eqs. (2.6)–(2.11) in (3.1) and (3.2), we can represent $\mathbf{S}$ and w in terms of the axial electric field in both regions. To evaluate the total power P and the field energy per unit length U of the waveguide, we integrate the axial component of $\mathbf{S}$ and w over the cross-sectional areas, i.e., $P = \int \mathbf{S} \cdot d\mathbf{A}$ and $U = \int w dA$. The integration requires the use of Green's first identity

$$\int_A |\nabla_{\perp} E_z|^2 dA = \oint_A c E_z^* \frac{\partial E_z}{\partial n} dl - \int_A E_z^* \nabla_{\perp}^2 E_z dA$$

on the boundary surfaces of each region [12]. The first integral vanishes on the outer surface of region 2 (perfect conductor condition). However, on the interface between the two regions, care must be taken to consider

TABLE I. Phase velocity and group velocity of the TM_{01} mode in the dielectric-loaded waveguide for different ϵ_r .

ϵ_r	v_{ph}/c	v_g/c
9.35	0.9964	0.1076
9.40	0.9732	0.1096
9.45	0.9516	0.1115
9.50	0.9314	0.1133
9.55	0.9123	0.1150
9.60	0.8944	0.1168

$$\frac{\partial E_{z1}}{\partial n_1} = \nabla E_{z1} \cdot \mathbf{n}_1 = \frac{\partial E_{z1}}{\partial r}, \quad \frac{\partial E_{z2}}{\partial n_2} = \nabla E_{z2} \cdot \mathbf{n}_2 = -\frac{\partial E_{z2}}{\partial r},$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the unit normal vectors of regions 1 and 2 on the interface, respectively. By means of the eigenmode equations, the second integral may be reduced to the normalization integral for E_z ; then we have

$$\int_{A1} |\nabla_{\perp} E_{z1}|^2 dA = 2\pi a \left[E_{z1}^* \frac{\partial E_{z1}}{\partial r} \right]_{r=a} + k_{\perp 1}^2 \int_{A1} |E_{z1}|^2 dA, \quad (3.3)$$

$$\int_{A2} |\nabla_{\perp} E_{z2}|^2 dA = -2\pi a \left[E_{z2}^* \frac{\partial E_{z2}}{\partial r} \right]_{r=a} + k_{\perp 2}^2 \int_{A2} |E_{z2}|^2 dA. \quad (3.4)$$

Substituting Eqs. (3.3) and (3.4) into our integrations, consequently, we get the expressions for transmitted power and energy per unit length

$$P = \frac{\omega k_z}{8\pi} \left\{ \frac{\epsilon_1}{k_{\perp 1}^2} \int_{A1} |E_{z1}|^2 dA + \frac{\epsilon_2}{k_{\perp 2}^2} \int_{A2} |E_{z2}|^2 dA + 2\pi a \left[E_{z1}^* \frac{\partial E_{z1}}{\partial r} \right]_{r=a} \frac{\epsilon_1}{k_{\perp 1}^2} \left[\frac{1}{k_{\perp 1}^2} - \frac{1}{k_{\perp 2}^2} \right] \right\}, \quad (3.5)$$

$$U = \frac{k_0^2}{8\pi} \left\{ \frac{\epsilon_1^2}{k_{\perp 1}^2} \int_{A1} |E_{z1}|^2 dA + \frac{\epsilon_2^2}{k_{\perp 2}^2} \int_{A2} |E_{z2}|^2 dA + 2\pi a \left[E_{z1}^* \frac{\partial E_{z1}}{\partial r} \right]_{r=a} \frac{\epsilon_1}{k_{\perp 1}^2} \left[\frac{\epsilon_1}{k_{\perp 1}^2} - \frac{\epsilon_2}{k_{\perp 2}^2} \right] \right\}. \quad (3.6)$$

The last terms on the right-hand side of the above equations indicate power flow and energy in a surface wave that can exist only on the interface between two differing regions. Given the level of input rf power P , Eq. (3.5) can be used to determine the amplitude of the axial electric field E_0 .

A comparison of the energy per unit length U with the power flow P shows that P and U are exactly proportional to one another. The constant of proportionality has the dimensions of velocity (velocity of energy flow) and can be identified with the group velocity $v_g = P/U$. Values of v_g/c and v_{ph}/c have been computed for the TM_{01} -mode MICA structure with $a/R = 0.30$ and $R/\lambda_0 = 0.15$ for a variety of values of ϵ_r ranging from 9.35 to 9.60, representing the spread cited for commercially produced alumina, depending upon purity. Results are listed in Table I. It is seen that the group velocity

of the TM_{01} mode in the loaded waveguide is much slower than the phase velocity. Checking these results, one may find they are very close to the relation $v_{ph}v_g = c^2/\epsilon$, the result for completely filled waveguide. The close agreement arises since the unfilled volume only occupies about 9% of the waveguide and the stored energy per unit length there is calculated to be only about 3% of the total.

MICA operation with $v_g/c \approx 0.10$ would be similar to that of conventional rf linear accelerator operation, where low group velocities are also employed. In the MICA case, the bulk of the energy is stored in the high-dielectric-constant material, while in the conventional linear accelerator the bulk of the energy is stored in periodic structures that act similarly to cavities. Low group velocity implies that the energy fill-time is much longer than the microbunch transit time along an accelerator section, so that significant energy depletion would cause late-following bunches to experience less acceleration than early-leading bunches. After several fill times, a steady state can be reached, but beam loading will reduce the field amplitudes, bringing about less net acceleration than in the absence of beam loading. This situation is undesirable when the accelerator is designed for high-energy gain and high-energy resolution. Therefore one must ensure that the energy carried away by the beam during a sequence of microbunches (i.e., during one fill time $\tau = L/v_g$) is much less than the stored energy. The available rf power is about 15 MW. For a group velocity $v_g/c \approx 0.10$ and a length $L = 150$ cm, the total stored rf energy $UL = 0.75$ J and the fill time is 50 nsec. The estimated energy gain is 10 MeV. If one is to accelerate 10^9 electrons per microbunch (0.16 nC) then the energy gain per microbunch is 1.6 mJ, much smaller than the total stored energy. However, if one microbunch is injected into each rf cycle (i.e., each 350 psec), the number of microbunches per fill time is 143 and the energy carried away by the beam in one fill time would be 0.23 J, neglecting beam loading. Since this value is not negligible compared to the total stored energy of 0.75 J, significant beam loading would be expected and the estimate for energy gain is too large. This undesirable situation can be avoided in the MICA by use of a beam chopper that will reduce the number of microbunches per fill time by a factor of 10; details of the proposed beam chopper will be discussed elsewhere. It should be noted that beam loading as described here is independent of the particular accelerator structure and depends only upon the group velocity and fill time; usually low group velocity is chosen in linear accelerators to afford a higher acceleration gradient for the same available rf power, as compared with high group velocity.

Power flow considerations discussed so far have applied to waveguides with perfectly conducting walls and ideal lossless dielectric. The axial wave number k_z above cutoff is purely real in that situation. However, in reality, the waveguide walls will have finite conductivity and the dielectric permittivity of the alumina medium will be complex. Both factors contribute to circuit dissipation, which might influence MICA operation. The above analysis has been extended to include both Ohmic wall losses and complex dielectric permittivity. We here summarize the results of these effects.

To consider the finite conductivity wall we employ the Poynting vector (3.1) to find the power dissipated in Ohmic losses per unit length of the guide, viz.,

$$\frac{dP_w}{dz} = \frac{c^2}{32\pi^2\sigma\delta} \oint_C \frac{\epsilon^2 k_0^2}{k_1^4} \left| \frac{\partial E_z}{\partial n} \right|^2 dl, \quad (3.7)$$

where the integral is taken around the boundary of the waveguide, $\delta = (c/\sqrt{4\pi})(2/\mu_c\omega\sigma)^{1/2}$ is the skin depth, and σ is the electrical conductivity of the conductor. The above equation will allow us to calculate approximately the resistive losses for the dielectric-loaded waveguide and cavity, using the fields we have found for the idealized problem of infinite conductivity in Sec. II. For pure copper walls, $\delta = 1.22 \times 10^{-4}$ cm at 2.856 GHz, one finds the attenuation along the MICA due to Ohmic wall losses to be 0.465 dB over its 150 cm length.

Turning to the dielectric losses, we express the complex dielectric constant as $\epsilon_2 = \epsilon_r - i\epsilon_i$. According to Eq. (2.3), we have

$$k_z^2 = k_{z0}^2 - i\epsilon_i k_0^2, \quad (3.8)$$

where $k_{z0}^2 = \epsilon_r k_0^2 - k_{12}^2$ and k_{z0} is the value of the axial wave number for the perfect dielectric. We express the axial wave number for the dielectric as $k_z = k_{zr} - ik_{zi}$, and from Eq. (3.8), we solve approximately

$$k_{zr} \approx k_{z0} + \frac{1}{8} \epsilon_i^2 k_0^2 \left(\frac{k_0}{k_{z0}} \right)^3, \quad (3.9)$$

$$k_{zi} \approx \frac{1}{2} \epsilon_i k_0 \left(\frac{k_0}{k_{z0}} \right). \quad (3.10)$$

For alumina with $\epsilon_r = 9.4$ and (typically) $\epsilon_i/\epsilon_r = 9.4 \times 10^{-5}$, $k_0 = 0.598$ (for $f = 2.856$ GHz), and $k_0/k_{z0} \approx 1$, we get $k_{zi} \approx 2.64 \times 10^{-4}$. In the total length of the waveguide $z = 150$ cm, the field amplitude will drop from unity to $\exp(-k_{zi}z) \approx 0.96$ due to the dielectric losses, corresponding to an attenuation of 0.355 dB. The total attenuation due to dielectric and wall losses is 0.82 dB, corresponding to a diminution in field strength along the 150-cm accelerating section of 9.0%. The actual diminution might be expected to exceed this value, since the copper skin on the alumina pipe could have a conductivity lower than the ideal value. Furthermore, poor-quality alumina, with a loss tangent greater than 10^{-4} , could also give rise to higher losses. These losses in both the copper skin and the dielectric will reduce the acceleration energy below that calculated for zero loss. This phenomenon is examined further in the following section.

IV. PARTICLE MOTION AND ACCELERATION IN THE WAVEGUIDE FIELDS

Substituting Eqs. (2.6)–(2.11) in the Lorentz force equation gives the change in relativistic momentum $\mathbf{P} = m\gamma\mathbf{v}$ of the beam particles in the MICA, namely,

$$\frac{d\mathbf{P}}{dt} = -e \left[\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right], \quad (4.1)$$

where γ is the Lorentz energy factor of a beam electron and m is the rest electron mass. In what follows, the fields are not self-consistent, in that the mutual effect of the beam particles upon one another is neglected. However, self-field effects

TABLE II. Simulation parameters of the MICA.

Electron beam parameters	
Initial electron energy	$\gamma_0 = 13$
Maximum initial transverse velocity	$\beta_{\perp} = 2.60 \times 10^{-3}$
Initial axial velocity (6 MeV)	$\beta_z = 0.9970$
Beam radius (cm)	$r_b = 0.05$
	$r_b/R = 0.032$
Waveguide parameters	
Waveguide radius (cm)	$R = 1.59$
Radius of vacuum hole (cm)	$a = 0.48$
	$a/R = 0.30$
Dielectric constant (alumina)	$\epsilon = 9.4$
Waveguide length (cm)	$z = 150$
Waveguide mode	TM ₀₁
Radiofrequency wave	
Field power (MW)	$P = 15$
Maximum field strength (MV/m)	$E_{z,max} = 6.29$
Frequency (GHz)	$f_0 = 2.856$
Normalized phase velocity	$V_{ph}/c = 0.9943$
Free space wavelength (cm)	$\lambda_0 = 10.50$
Waveguide wavelength (cm)	$\lambda_g = 10.46$

have been taken into account in simulation studies using PARMELA as described below. As will be shown, self-fields are of minor consequence for the beam currents considered for the MICA. The three components of the force Eq. (4.1) in cylindrical form are

$$\dot{\beta}_r - \mu\beta_\theta = -\frac{e}{mc^2\gamma}[(1-\beta_r^2)E_r - \beta_r\beta_z E_z - \beta_z B_\theta], \quad (4.2)$$

$$\dot{\beta}_\theta + \mu\beta_r = -\frac{e}{mc^2\gamma}[-\beta_\theta\beta_r E_r - \beta_\theta\beta_z E_z], \quad (4.3)$$

and

$$\dot{\beta}_z = -\frac{e}{mc^2\gamma}[-\beta_z\beta_r E_r + (1-\beta_z^2)E_z - \beta_r B_\theta], \quad (4.4)$$

where $\mu = \dot{\theta}$ is the normalized angular velocity of electron and the overdot represents the time derivative of the quantities $d/d\tau$, $\tau = ct$.

The numerical simulation is based on the force Eqs. (4.2)–(4.4) with the field components given by Eqs. (2.6)–(2.11). The parameters used are listed in Table II. The input 6-MeV electrons from the rf gun are taken to be monoenergetic but with small transverse velocities that vary randomly from particle to particle. At the entrance of the waveguide they are randomly distributed inside the beam cross section as shown in Fig. 5(a1). Their distribution in β_x - β_y space is shown in Fig. 5(b1).

The acceleration of electrons in the MICA is mainly axial because the intense axial field E_z dominates the other field components near the axis. In Fig. 6(a) we show the result for 15-MW traveling wave power. The electron energy increases almost linearly as the particles move down the waveguide. An analytical derivation from Eqs. (4.2)–(4.4) leads to

FIG. 5. Beam cross section in x - y space and β_x - β_y space (a1) and (b1) at the entrance of waveguide, (a2) and (b2) at the end of waveguide with the TM₀₁ mode inside, and (a3) and (b3) at the end of waveguide with no rf fields. The vacuum hole radius $a/R = 0.30$.

$$\frac{d\gamma}{d\tau} = -\frac{e}{mc^2\gamma}[\beta_r E_r + \beta_z E_z], \quad (4.5)$$

which explicitly gives an approximate energy expression as

$$\gamma^2 = 1 + \left[\sqrt{\gamma_0^2 - 1} - \frac{eE_0\tau}{mc^2} \right]^2, \quad (4.6)$$

where γ_0 is the initial electron energy. When γ_0 is large ($\gamma_0 = 13$, for example, in this case), the above equation can be further simplified as $\gamma = \gamma_0 + e|E_0|\tau/mc^2$. When $|E_0| = 210$ statvolt/cm ($= 63$ kV/cm), $\tau = 150$ cm, it gives $\gamma = 31.5$, in close agreement with the simulation result. The validity of Eq. (4.6) requires that the relative phase of the electrons with respect to the rf field is equal to π ; checking the field distribution in Fig. 2, one sees that this corresponds to an initial distribution of particles at the position $\lambda_g/2$. Particles in this position will experience the maximum axial field, as we show in Fig. 6(b). Also shown in Fig. 6 (as dotted lines) is the result when 10% of power depletion is taken into account, which does not drop the particle energy too much. Due to the small difference between the electron

FIG. 6. (a) Electron energy as a function of the axial distance and (b) the axial accelerating field seen by one particle as it moves down the waveguide. The dotted lines are the case when a 10% power depletion is taken into account.

velocity and the wave phase velocity, one may expect that the electron will gradually slip from the maximum acceleration position, forward or backward depending on whether the beam is going faster or slower. In the current simulation, we find a phase slippage of $\Delta\phi \approx 24^\circ$ in 1.5 m with the electrons moving ahead of the rf field, corresponding to a slippage interval of $\Delta\tau_0 \approx 23$ ps. For a rf gun with a beam bunch length of only $\Delta\tau_0 = 5$ ps, we can expect excellent trapping and acceleration of electrons during the entire propagation along the waveguide, without a taper of the dielectric element. In our calculation, the electron energy increases to about 16 MeV in 150 cm. If the dielectric surface breakdown strength is adequate, the electron energy can increase more.

When the electrons are located in a small "phase window" of acceleration, the radial component of the field E_r will prevent the electrons from spreading out even though the particles have an initial transverse velocity distribution. Shown in Figs. 5(a2) and 5(b2) are the cross sections of the electrons in the beam at the end of the MICA in x - y and β_x - β_y space separately. Electrons remain well confined inside the hole in the dielectric and the transverse velocity spread shrinks. An algebraic analysis from Eqs. (4.2)–(4.4) gives

$$\frac{1}{2} \frac{d\beta_\perp^2}{d\tau} = -\frac{e}{mc^2\gamma} [(1-\beta_\perp^2)\beta_r E_r - \beta_\perp^2 \beta_z E_z - \beta_r \beta_z B_\theta], \quad (4.7)$$

where $\beta_\perp^2 = \beta_r^2 + \beta_\theta^2$. An approximate solution of Eq. (4.7) can be obtained when $\beta_r \rightarrow 1$ and $\beta_z^2 \ll \beta_\perp^2$, namely,

$$\beta_\perp = \beta_{\perp 0} \exp \left[\frac{eE_0\tau}{mc^2\gamma} \left(\frac{1}{2} \Delta k r \sin\phi_0 + \cos\phi_0 \right) \right], \quad (4.8)$$

where ϕ_0 is the initial phase of electrons, $\Delta k = k_z - k_0 = 3.5 \times 10^{-3}$, and when $\phi_0 = \pi$, $\bar{\gamma} = 22$, we get $\beta_\perp = 1.1 \times 10^{-3}$, which is consistent with the maximum $\beta_\perp$ in Fig. 5(b2). This calculation shows indeed that radial motions in the MICA are sufficiently constrained by the rf fields to maintain a small beam cross section and that particles are confined close to the axis so long as $\phi_0 \sim \pi$ and $\Delta\phi \ll 1$. By contrast, shown in Figs. 5(a3) and 5(b3) is the beam cross section with no rf field in waveguide: in this case the beam cross section in β_x - β_y space does not change, whereas in x - y space is spreads uniformly. Beam spreading becomes serious when electrons are out of acceleration phase. For instance, when particles are initially injected somewhere near $z_0 = 0, \lambda_g$ or $\phi_0 = 0, 2\pi$, our simulation found that the beam can neither be accelerated nor focused; instead the electrons will spread radially and collide with the dielectric wall.

In order to further examine radial stability of the MICA acceleration process, simulations for an injected off-axis beam have been carried out. For example, when the initial off-axis radial displacement was taken to be 0.5 mm, the off-axis displacement at the end of the waveguide remained at 0.5 mm and the beam cross section remains symmetrical. This is not too surprising since, by reference to the field distribution shown in Fig. 4, one observes that a small offset of the beam from the axis does not change by much the fields experienced by the beam. Furthermore, since $E_{r1} \approx -E_0(k_z r) \sin\phi$ and $\beta_{\theta 1} \approx -E_0(k_0 r) \sin\phi$, the radial forces on an off-axis z -directed relativistic particle nearly cancel when the phase velocity of wave is close to the speed of light.

V. SELF-FIELD EFFECT AND EMITTANCE EVOLUTION

Let us now include the self-field effect of electron current in beam dynamics. The space charge field of a bunch can be approximately determined by the formula for the terminal space charge field of a cylindrical beam [13]

$$E_s = \frac{0.18q}{a_0^2} \left[1 + \frac{a_0}{\gamma l} - \sqrt{1 + \left(\frac{a_0}{\gamma l} \right)^2} \right], \quad (5.1)$$

where E_s is the longitudinal space charge field in MV/m, a_0 is the beam radius in cm, l is the bunch length in cm, and q is the total charge of the bunch in nC. For a bunch of peak current 10 A with bunch length $\tau = 5$ psec one has $q = 0.05$ nC, $l = 0.15$ cm, and if the beam radius $a_0 = 0.05$ cm, we have $E_s = 0.094$ MV/m, a value that is far below the anticipated applied axial rf field of $E_z = 6.3$ MV/m. Although this suggests that one can neglect the space charge for this level of q , we wish to determine if there are more subtle changes in the bunch quality; for this purpose we have conducted simulations of the MICA using the PARMELA accelerator code [14].

PARMELA is a versatile multiparticle electron linear accelerator code that is widely used in accelerator community [15]. In PARMELA, the electron beam, represented by a collection of macroparticles, may be transformed through a lin

TABLE III. PARMELA results about beam emittance and energy spread at the entrance and exit of the waveguide.

z (cm)	a_0 (mm)	E_{beam} (MeV)	$4\sigma_\phi$ (deg)	$4\sigma_E$ (keV)	$\epsilon_{nx\text{rms}}$ (π mm mrad)	$\epsilon_{ny\text{rms}}$ (π mm mrad)	$\epsilon_{nz\text{rms}}$ (π deg keV)
0	0.5	6.0	5.78	68.9	3.05	2.90	24.9
150	3.0	15.6	5.83	80.2	3.05	2.91	25.8

ear accelerator and/or transport system. The self-field effects (both electric and magnetic) are automatically taken into account in the simulation. Since the code usually applies to a periodic loading structure, it was necessary to modify the code so that it has the capability of modeling traveling-wave acceleration in the smooth-bore MICA structure, based on the field distribution given by Eqs. (2.6)–(2.11). In Table III we summarize some parameters of main interest for a bunch at the entrance ($z=0$) and the exit ($z=150$ cm) of the waveguide. Entrance conditions are taken from the rf gun and beam line computation, while the output is determined by our simulation results. The input parameters for the simulation are displayed in Fig. 7. The initial parameters in our PARMELA simulations are the same as we used in the single-particle dynamics run shown in Table II, except a bunch charge of $q=0.05$ nC is now included.

Comparing the PARMELA output results (Fig. 8) with the single-particle results (Fig. 5), we find that in both simulations the acceleration gradients, the final particle energy, the beam cross section, and the particle velocity evolution are all in excellent agreement. The 1000 particles used in PARMELA simulation are all "good" particles, meaning that there is no particle loss in the MICA. PARMELA shows very clearly that

the transverse emittance $\epsilon_{nx\text{rms}}$ and $\epsilon_{ny\text{rms}}$ are constant throughout the acceleration, even though the longitudinal emittance $\epsilon_{nz\text{rms}}$ has a very slight change because of the minute longitudinal bunching that makes the particle energy spectrum more narrow. This PARMELA simulation is also compared to a test run where the net charge is set to zero: we observe only trivial differences. This shows that the self-field effects are not significant in any meaningful detail. However, when the beam current is increased, the self-field effects do affect the ultimate beam quality. For PARMELA runs with 20-A peak micropulse current a noticeable growth in normalized transverse and longitudinal emittance is found, while for 200 A the growth is substantial. These results suggest that achievement of the goal of a normalized transverse emittance of 5π mm mrad for 0.16 -nC (10^9) particles, 5-psec bunch is realistic.

VI. MEASUREMENT OF THE DIELECTRIC BREAKDOWN LIMIT

A. Analysis of loaded cavity resonator

The MICA, as described thus far, is a traveling-wave accelerator; however, an alternative arrangement would em-

FIG. 7. Initial emittance, energy spectrum, and beam cross section of the electron bunch when the space charge effect is taken into account, $q=0.05$ nC.

FIG. 8. Beam distribution at the exit of the lined waveguide. The transverse normalized emittance is constant, the longitudinal emittance has a very slight change, the energy spread becomes narrower, and the cross section of the bunch expands as if there is no space charge effect.

ploy a reflector at one end to allow standing waves to build up, so that a higher accelerating gradient could be obtained for a given rf power. Also, in order to determine the breakdown limits at 2.856 GHz, we have designed and constructed a cavity resonator with an alumina liner. Thus it is worthwhile to derive formulas for the resonance frequency and quality factor Q for a TM resonator constructed with a section of dielectric-lined waveguide with conducting plates closing the ends. Since the exact value of the alumina dielectric constant of the sample we used was not accurately known, we began with a low-power test of the resonant modes of a simple cavity incorporating an alumina annulus with metallic surfaces, which is coupled to a signal generator and a detector as shown in Fig. 9.

For a cylindrical cavity of axial length d , the rf fields are a superposition of forward and backward traveling waves of

the forms given by Eqs. (2.6)–(2.11). For the component E_z , superposition of equal amplitude waves gives, in regions 1 (hole) and 2 (dielectric), the form

$$E_{z1}(r, z, t) = E_{c0} I_0(k_{11}r) \cos(k_{zp}z) \cos(\omega t) \quad (0 \leq r \leq a) \quad (6.1)$$

and

$$E_{z2}(r, z, t) = E_{c0} G_0(k_{12}r) \cos(k_{zp}r) \cos(\omega t) \quad (a < r \leq R), \quad (6.2)$$

where E_{c0} is the amplitude of the standing wave, and to satisfy the boundary conditions of zero tangential electric field at the end planes, the axial wave number takes on discrete values $k_{zp} = p\pi/d$, $p = 0, 1, 2, \dots$. Correspondingly, the cavity resonance frequency is

$$\omega_{0np} = \frac{c}{\sqrt{\epsilon_1}} \left[-k_{11n}^2 + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2} = \frac{c}{\sqrt{\epsilon_2}} \left[k_{12n}^2 + \left(\frac{p\pi}{d} \right)^2 \right]^{1/2}. \quad (6.3)$$

For a given microwave frequency ω (e.g., $\omega = 2\pi \times 2.856$ GHz) and resonance mode TM_{0np} and using the eigenvalue of the lined waveguide we solved in Sec. II, Eq. (6.3) will give the longitudinal dimension d . Conversely, if the resonance frequency is measured for resonator of given length, Eq. (6.3) can be used to find the value of the dielectric permittivity ϵ_2 .

The unloaded quality factor Q of the cavity can be expressed as

FIG. 9. Sketch of the test resonator fabricated from a short section of alumina pipe coated on its exterior with silver paint. A low-power rf input was used.

$$\frac{1}{Q} = \frac{1}{Q_w} + \frac{1}{Q_d}, \quad (6.4)$$

with contributions from wall losses and dielectric losses expressed as $Q_w = \omega(U_s/P_w)$ and $Q_d = \omega(U_s/P_d)$, where U_s is the time-average energy stored in the cavity, P_w is the Ohmic losses on the wall, and P_d is the dielectric losses. To determine the Q of a cavity, we need to calculate the time-average energy stored in the cavity and then determine the power loss in the walls and in the dielectric. The integral of (3.6) over the cavity length yields the energy stored in the cavity

$$\begin{aligned} U_s = \int_0^d U dz = \frac{k_0^2}{8\pi} \left\{ \frac{\epsilon_1^2}{k_{11}^2} \int_0^d dz \int_{A_1} |E_{z1}|^2 dA \right. \\ \left. + \frac{\epsilon_2^2}{k_{12}^2} \int_0^d dz \int_{A_2} |E_{z2}|^2 dA \right. \\ \left. + 2\pi a \int_0^d \left[E_{z1}^* \frac{\partial E_{z1}}{\partial r} \right]_{r=a} dz \frac{\epsilon_1}{k_{11}^2} \left[\frac{\epsilon_1}{k_{11}^2} - \frac{\epsilon_2}{k_{12}^2} \right] \right\}. \end{aligned} \quad (6.5)$$

Considering the Ohmic losses on both the sidewall and the two ends of the cavity, the power loss at the walls is

$$\begin{aligned} P_w = \frac{c^2}{32\pi^2 \sigma \delta} \left\{ \frac{\epsilon_2^2 k_0^2}{k_{12}^4} \int_0^d dz \oint_C \left| \frac{\partial E_{z2}}{\partial n} \right|_{r=R} dl \right. \\ \left. + 2 \frac{\epsilon_1^2 k_0^2}{k_{11}^4} \int_{A_1} |\nabla_{\perp} E_{z1}|_{z=0}^2 dA + 2 \frac{\epsilon_2^2 k_0^2}{k_{12}^4} \int_{A_2} |\nabla_{\perp} E_{z2}|_{z=d}^2 dA \right\}. \end{aligned} \quad (6.6)$$

The dielectric losses can be determined from the electrical conductivity $\sigma_d = (\omega/4\pi)\epsilon_i$, leading to the result

$$P_d = \frac{1}{2} \frac{\omega}{4\pi} \epsilon_i \int_0^d dz \int_{A_2} \left[\frac{k_z^2}{k_{12}^4} \left| \frac{\partial E_{z2}}{\partial r} \right|^2 + |E_{z2}|^2 \right] dA. \quad (6.7)$$

The design parameters of an ideal cavity resonator, based on the sample available, are listed in Table IV. The cavity operates in the TM_{012} mode; that is, the length of the cavity is one guide wavelength of the TM_{01} waveguide mode. The cavity has a moderate quality factor $Q = 4620$. The relation between the maximum axial field in the cavity and the power coupled in is also given in Table IV in terms of the parameter $E_{z\max}/P^{1/2}$, where P is the total power lost in both walls and dielectric.

B. Measurements of a loaded cavity resonator and breakdown results

Measurements were conducted with a resonator fabricated from a short section of alumina pipe coated on its exterior with silver (Fig. 9). The alumina samples, supplied by LSP Ceramics, Inc., had inner and outer radii of 1.429 and 0.508 cm, respectively. There are some differences in dimensions between the proposed waveguide and the test cavity (the outer radius is 10% smaller than the required value of 1.5875

TABLE IV. Simulation parameters of dielectric-loaded cavity resonator.

Cavity parameters	
Radius of empty hole (cm)	$a = 0.508$
Radius of cylindrical cavity (cm)	$R = 1.429$
	$a/R = 0.356$
Length of cavity (cm)	$d = 12.11$
Real part of the dielectric constant	$\epsilon_r = 9.62$
Imaginary part of the dielectric constant	$\epsilon_i/\epsilon_r = 9.4 \times 10^{-5}$
rf wave parameters	
Cutoff frequency (GHz)	$f_c = 3.118$
Resonance frequency (GHz)	$f_0 = 3.210$
Transverse wave number in the hole (cm^{-1})	$k_{1r} = 0.4281$
Transverse wave number in the dielectric (cm^{-1})	$k_2 = 2.021$
Wavelength in the cavity resonator (cm)	$\lambda_g = 12.11$
Cavity mode	TM_{012}
Quality factor of the cavity	
Q for the conducting walls	$Q_w = 7929$
Q for the dielectric	$Q_d = 11070$
Q of the cavity	$Q = 4620$
Power loss ratio $(\text{MV/m})/(\text{MW})^{1/2}$	$E_{\max}/P^{1/2} = 20.32$

cm, while the inner radius is 7% larger). Nevertheless, measurements on the samples available still provide a good test of theory. Raw data for the observed rf transmission by the cavity is shown in Fig. 10(a), over a frequency range between 3 and 6 GHz. Figure 10(b) shows a plot of the square of the 12 observed resonance frequencies in Fig. 10(a) versus the square of the resonance index. From Eq. (6.3), one sees that the slope of this line should be the reciprocal of the relative dielectric constant; for the data in Fig. 10(b), this reciprocal slope is 9.62. This differs from 9.4, the canonical value taken in the analysis given above, but 9.62 is well within the range quoted for good purity alumina. It is highly significant that no other resonances for this structure between 3 and 6 GHz were found that did not fit on the line shown in Fig. 10(b), despite attempts having been made to excite non-axisymmetric modes using a nonaxisymmetric antenna. One can conclude from this observation that potentially disruptive nonaxisymmetric modes of the dielectric pipe were not excited. Calculation of the properties of non-axisymmetric modes would be a formidable task, one that these experimental tests appear to render unnecessary.

The vertical intercept for the line in Fig. 10(b) should be the square of the TM_{01} -mode waveguide cutoff frequency, which in this case is observed to be 3.216 GHz. For $\epsilon_2 = 9.62$, $R = 1.429$ cm, and $a = 0.508$ cm, the calculated value is 3.118 GHz, a value of 3.05% lower than the measurement. This discrepancy is not unreasonable, considering the added circuit reactance of the coupling antennas and the incomplete closure of the end walls. Typical Q values for the observed cavity resonances were in the range of 400–500, much lower than the calculated unloaded value of 4620. This is also not too surprising, considering the open ends of the beam hole and the strong external loading that was required to make accurate resonance measurements on all 12 modes. However, this exercise emphasizes the need to carefully test alumina samples prior to acceptance and prior to selection of

FIG. 10. (a) Mode spectrum of the resonator obtained in measurement-cavity transmission in arbitrary units and (b) square of the mode resonance frequency versus the square of the axial mode number from measurements on the resonator shown in (a). The reciprocal of the slope of the line is 9.62, which can be inferred to be the dielectric permittivity ϵ_2 , from Eq. (6.3).

the final parameters for the 150-cm accelerating sections. In particular, an accurate advance measurement of dielectric constant, phase velocity, and loss tangent must be made from samples taken from the alumina batch to be used for the final accelerating sections.

Measurements using high-power microwaves applied to the alumina samples were also carried out to determine rf breakdown limits. Since the cavity described above has resonances above 3.2 GHz, an alternative experimental arrangement was devised to subject the alumina surfaces to high tangential rf electric fields at 2.856 GHz (obtained from a SLAC klystron). A diagram of the arrangement used is shown in Fig. 11. A standing-wave resonance was established in WR-284 rectangular waveguide using inductive irises. Measurements with the alumina sample in place showed this arrangement to give an effective gain of over 11 dB, as deduced from signals on the sample probe with and without the irises. Under these conditions, the peak tangential rf electric field at the inner alumina surface is calculated

FIG. 11. Sketch of the experimental arrangement used to measure the maximum tangential rf electric field that can be sustained at an alumina surface.

to be $33.6P^{1/2}$ V/cm, where P is the incident power in watts. This indicates that a field of 63 kV/cm would be applied when $P = 3.52$ MW. In the experiments, the rf power level was increased over an ~ 12 -h period to provide gradual rf processing of the structure, without allowing the background pressure to exceed 2×10^{-6} Torr. It was found that this procedure could be continued up to a power level of 6.25 MW, without evidence of arcing at the alumina surface. This corresponds to a tangential field of 84 kV/cm. These observations suggest that acceleration gradients of at least 8.4 MV/m should be achievable in the MICA, where a design with superior vacuum integrity and coating of the alumina is planned.

VII. CONCLUSIONS

We have studied a microwave inverse Čerenkov accelerator, which has an acceleration mechanism similar to that of a conventional rf linear accelerator. However, the accelerating structure, which comprises a continuous coated ceramic pipe, should be less expensive to fabricate than that of the linear accelerator. In the absence of any periodic loading structures in the waveguide, wake-field generation that can lead to emittance growth and beam breakup should be minimized. Thus MICA's advantages of a relatively compact structure, smooth-bore design, and no need of magnetic focusing make it a very competitive facility as a simple, low-cost electron accelerator.

In this paper, we have studied numerically the eigenmode, field profile, energy flow, particle dynamics, and space charge effects; experimentally, we measured the dielectric breakdown limit of alumina. We find that a thick liner with a high dielectric constant is very helpful not only to store high rf energy but also to maintain an intense and uniform axial accelerating field in the central hole. The particle motion in the waveguide is nearly one dimensional with all input particles being accelerated and no interception by the dielectric. There is no beam breakup and the beam bunches have good stability even if they are slightly off axis. For the beam current under consideration, the space charge effect is not an issue and the initial low normalized emittance, within 3π

mm mrad, is constant throughout the acceleration. The acceleration gradient in the simulation is 6.3 MeV/m, in which case the electron energy increases from 6 to 16 MeV in 150 cm. However, without exceeding the breakdown limit measured by experiment (greater than 8.4 MeV/m) and using higher microwave power and/or a higher- Q structure, the electron energy could increase even more, perhaps in the range of 10–15 MeV/m if techniques for improving the dielectric breakdown [9] on the surface using Ti or TiN evaporated coatings can be used successfully.

We identify some challenging technical issues such as the finish machining of the waveguide and liner, since the phase velocity of a rf wave in the vicinity of the speed of light is

very sensitive to the radius of the vacuum hole and tube. Also, one must pay attention to the matching of the power feeding system and the accelerator waveguide because of the substantial difference of the wave group velocity (or impedance) in these two sections.

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A microwave inverse Cherenkov accelerator (MICA)

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Abstract

By “inverting” the stimulated Cherenkov effect to stimulated Cherenkov absorption, it is possible to build an electron accelerator device driven by high power microwaves that propagate in a slow-wave TM mode (axial E -field). In this paper, we have solved for the wave dispersion in the structure, found the field distributions, and then used the Lorentz force equations to obtain the motion of a group of electrons distributed in radius and velocity. We find the radial forces are focusing. Electrons in a well-defined filament ($r < 0.5$ mm) remain collimated and do not strike the dielectric. By using the 15 MW of rf power available at 2.865 GHz, we can accelerate an electron beam (~ 6 MeV, few ps pulses) to energy ~ 16 MeV. This results in a relatively compact structure that has the advantage of a smooth-bore design and no need of magnetic focusing. The techniques for improving the dielectric breakdown the surface should permit axial fields in the range of 100–200 kV/cm.

1. Introduction

The stimulated Cherenkov effect is a well-understood mechanism for generating coherent radiation from an energetic electron beam [1–3]. The radiating electrons move at speed greater than that of the velocity of light in the structure (hence the name “Cherenkov”); although here are several ways to slow light waves, as a general rule the term is used when the slowing is caused by a dielectric element. When one does a linearized treatment of the fields and the self-consistent motion of the particles, a dispersion relation is obtained for growth or decay of radiation in the system. One of the three roots obtained corresponds to a damped wave; this we identify with the mechanism of stimulated absorption, whereby an electron will gain energy at the expense of the rf field. In the discussion which follows, we consider the application of stimulated absorption in the nonlinear regime of particle trapping, which applies to an electron accelerator device. Thus we refer to as a microwave inverse Cherenkov accelerator (“MICA”).

Acceleration of the electron is done by appropriate timing of an electron bunch which is emitted from an rf linac, so that a continuous accelerating force is applied to all the electrons, which move synchronously with the slow wave. Variation of the wave speed is possible by a small perturbation in the filling factor of the dielectric element. Thus the device resembles an rf linac, but without the periodic

loading structures in the waveguide. As the MICA is smooth-bore and the motion of the particles is rather one-dimensional, we expect that the emittance of the electron beam produced will be attractive. The MICA under consideration will use a SLAC source of microwave power at 2.85 GHz, and with a bunch length of only 5 ps compared with the rf period of 350 ps, we can expect excellent trapping and acceleration of a monoenergetic bunch of electrons. Another approach [4–7], the ICA (Inverse Cherenkov Accelerator) experiment at Brookhaven National Laboratory, uses a CO_2 laser and an axicon to accelerate an electron beam at 40 MeV energy; the light wave is slowed by introducing hydrogen gas into the beamline. The gas contributes to some electron scattering, and the main disadvantage of the short laser wavelength is that electrons interact with the wave over the full range (2π) of phase; that is, the bunch length is long compared with the rf wavelength. In the MICA, the electrons move down a 1 cm diameter hole in an alumina dielectric liner as a filamentary beam of under 1 mm diameter. The main limitation here is that of the maximum axial field gradient (120–160 kV/cm [8]) along the dielectric surface.

In this paper, we describe first the analysis of a wave inside of a dielectric annular cylinder fitted into a cylindrical waveguide. We find dispersion relations [9] for the axisymmetric TM_{01} -like mode of the system, that is, the mode that gives maximum electric field along the axis. This calculation provides both the slowing factor and the distribution of fields. We then model the acceleration and motion of electrons in the vacuum fields of this device.

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The objective of this effort is to determine whether a compact high quality accelerator of this type is feasible.

2. Dispersion relation, field distribution and particle dynamics in MICA

We take a waveguide of radius R , lined with a dielectric sleeve, with the central vacuum hole of radius a as shown in Fig. 1. Under the appropriate boundary conditions, we solve the Maxwell equations by a standard procedure [9], and arrive at a dispersion relation of the system for eigenmode TM_{0n} :

$$\frac{I_1(k_{11}a)}{I_0(k_{11}a)} = \epsilon \frac{k_{11} J_1(k_2 a) N_0(k_2 R) - J_0(k_2 R) N_1(k_2 a)}{k_2 J_0(k_2 a) N_0(k_2 R) - J_0(k_2 R) N_0(k_2 a)}, \quad (1)$$

where, the functions J and N are Bessel functions and I is the modified Bessel function; $k_{11} = -ik_1$, k_1 and k_2 are the transverse wave numbers in the vacuum hole and the dielectric element separately, $k_1^2 = k_0^2 - k_z^2$, $k_2^2 = \epsilon k_0^2 - k_z^2$. k_z is the axial wave number of the waveguide TM_{0n} mode, $k_0 = \omega/c$ is the wave number in free space and ϵ is the dielectric constant of the material. The normalized phase velocity of the mode can be obtained from the eigenvalues of Eq. (1). As an example, Fig. 2 shows the normalized phase velocity of TM_{01} mode as a function of the ratio R/λ_0 , where λ_0 the free space wavelength of field, the dielectric constant $\epsilon = 9.4$ for alumina, and the ratio of hole radius to outer dielectric radius is $a/R = 0.30$. One sees from Fig. 2 that phase velocity of c and below obtain when $R/\lambda_0 > 0.15$, which gives a relation between the waveguide radius and the rf wavelength for a slow wave structure.

Fig. 2. Normalized phase velocity v_{ph}/c vs. normalized outer radius for TM_{01} mode waveguide with alumina liner. Note: $v_{ph} \leq c$ for $R \geq 0.15\lambda_0$.

With the eigenvalue of the waveguide mode solved from the dispersion relation Eq. (1), we can calculate the field distribution in this specific MICA configuration. The electromagnetic field distributions in the loaded waveguide include two separate parts:

When $0 \leq r \leq a$, in the hole:

$$E_{z1}(r, z, t) = E_0 I_0(k_{11}r) \cos(\omega t - k_z z), \quad (2a)$$

$$E_{r1}(r, z, t) = -E_0 \frac{k_z}{k_{11}} I_1(k_{11}r) \sin(\omega t - k_z z), \quad (2b)$$

$$B_{\theta 1}(r, z, t) = -E_0 \frac{k_0}{k_{11}} I_1(k_{11}r) \sin(\omega t - k_z z). \quad (2c)$$

When $a < r \leq R$, in the dielectric:

$$E_{z2}(r, z, t) = E_0 g_1 [J_0(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_0(k_2 r)] \cos(\omega t - k_z z), \quad (3a)$$

Fig. 1. Longitudinal cross-section of loaded waveguide showing axial electric field distribution as well as electron phase positions along waveguide.

$$E_{r2}(r, z, t) = -E_0 g_1 \frac{k_z}{k_2} [J_1(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_1(k_2 r)] \sin(\omega t - k_z z), \quad (3b)$$

$$B_{\theta 2}(r, z, t) = -E_0 g_1 \frac{ck_0}{k_2} [J_1(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_1(k_2 r)] \sin(\omega t - k_z z). \quad (3c)$$

The coefficient g_i in Eqs. (3a)–(3c) is a constant relating the field amplitude in the dielectric to that in the hole and can be derived by the boundary condition. $E_0 = E_0(z)$ is the axial field strength on the axis; it can be determined in terms of the total microwave power by integrating the Poynting vector in the cross-section of waveguide.

Substituting Eqs. (2a)–(3c) in the Lorentz force equation for the relativistic beam particles, we are now in a position to study the motion of electrons in the MICA. The three components of the force equation in cylindrical form are:

$$\dot{\beta}_r - \mu \beta_\theta = -\frac{e}{mc^2 \gamma} [(1 - \beta_z^2) E_r - \beta_r \beta_z E_z - \beta_z B_\theta], \quad (4a)$$

$$\dot{\beta}_\theta + \mu \beta_r = -\frac{e}{mc^2 \gamma} [-\beta_\theta \beta_r E_r - \beta_\theta \beta_z E_z], \quad (4b)$$

$$\dot{\beta}_z = -\frac{e}{mc^2 \gamma} [-\beta_z \beta_r E_r + (1 - \beta_z^2) E_z - \beta_r B_\theta], \quad (4c)$$

where, $\mu = \dot{\theta}$ is the normalized angular velocity of electron, and the overdot represents the time derivative of the quantities $d/d\tau$, $\tau = ct$.

3. Numerical results and discussion

The MICA configuration is a circular waveguide loaded by high ϵ dielectric material, but before selecting this configuration we also explored another dielectric structure, a rectangular waveguide loaded by a dielectric slab or slabs. For the latter, the maximum axial field strength occurs inside the vacuum-dielectric interface [10] and so this configuration is not appropriate for electron acceleration. However, the use of a high ϵ annulus inside circular waveguide maintains a large uniform E_z field inside the hole which is of particular interest for acceleration purpose.

The numerical simulation is based on the force Eqs. (4a)–(4c) with the field components given by Eqs. (2a)–(3c). The parameters used are listed in Table 1. The electrons are taken to be monoenergetic but with small transverse velocities which vary randomly from particle to particle. At the entrance of the waveguide they are randomly distributed inside the beam cross-section as shown in Fig. 4a (1). Their distribution in β_r – β_θ space is shown in Fig. 4b (1).

The acceleration of electrons in the MICA configuration is straightforward because of the intense axial field E_z . In

Table 1
Simulation parameters of MICA

Electron beam parameters	
Initial electron energy	$\gamma_0 = 13$
Maximum initial transverse velocity	$\beta_{\perp} = 0.4670 \times 10^{-3}$
Initial axial velocity	$\beta_z = 0.9970$
Beam radius	$r_b = 0.05$ cm
	$r_b/R = 0.032$
Waveguide parameters	
Waveguide radius	$R = 1.59$ cm
Radius of vacuum hole	$a = 0.48$ cm
Dielectric constant (alumina)	$\epsilon = 9.4$
Waveguide length	$z = 150$ cm
Waveguide mode	TM_{01}
Radiofrequency wave	
Field power	$P = 15$ MW
Maximum field strength	$E_{z,max} = 6.29$ MV/m
Frequency	$f_0 = 2.85$ GHz
Normalized phase velocity	$V_{ph}/c = 0.9943$
Free space wavelength	$\lambda_0 = 10.52$ cm
Waveguide wavelength	$\lambda_g = 10.46$ cm

Fig. 3 we show the result for 15 MW traveling wave power. The electron energy increases almost linearly as the particles move down the waveguide. An analytical derivation from Eq. (4c) gives an approximate energy expression as

$$\gamma^2 = 1 + \left[\sqrt{\gamma_0^2 - 1} - \frac{e E_0 \tau}{mc^2} \right]^2, \quad (5)$$

where γ_0 is the initial electron energy. When γ_0 is large ($\gamma_0 = 13$, for example in this case), the above equation can be further simplified as $\gamma = \gamma_0 + e|E_0|\tau/mc^2$. When $|E_0| = 210$ (CGS), $\tau = 150$ cm, it gives $\gamma \approx 31.5$, in close agreement with the simulation result. The validity of Eq. (5) requires that the relative phase of the electrons with respect to the rf field is π ; checking the field distribution in Fig. 1, one sees that this corresponds to an initial distribution of particles at the position $\lambda_g/2$. Particles in this position will experience the maximum axial field. Due to the small difference between the electron velocity and the wave phase velocity, one may expect that the electron will

Fig. 3. Electron energy as a function of the axial distance.

Fig. 4. Beam cross-section in x - y space (a) and β_x - β_y space (b): (1) at the entrance of waveguide; (2) at the end of waveguide with TM_{01} mode inside; (3) at the end of waveguide with no rf field. The vacuum hole radius $a/R = 0.30$.

gradually slip from the maximum acceleration position. In the current simulation, we find a phase slippage of $\Delta\phi \approx 24^\circ$ in one and a half meters with the electrons moving ahead of the rf field, corresponding to a slippage interval of $\Delta\tau_0 \approx 23$ ps. For a rf gun with a beam bunch length of only $\Delta\tau_0 = 5$ ps, we can expect excellent trapping and acceleration of electrons during the entire propagation along the waveguide, without a taper of the dielectric element. In our calculation, the electron energy increases to about 16 MeV in 150 cm. If the dielectric surface strength is adequate, and with a higher Q structure, the electron energy can increase more.

When the electrons are located in the small "phase window" of acceleration, the radial component of rf field E_r will prevent the electrons from spreading out even though the particles have an initial transverse velocity

distribution. Shown in Figs. 4a (2) and 4b (2) are the cross sections of the electrons in the beam at the end of the MICA in x - y and β_x - β_y space separately. Electrons remain well confined inside the hole in the dielectric and the transverse velocity spread shrinks. Algebraic analysis from Eqs. (4a) and (4b) gives an approximate transverse velocity as $\beta_{\perp} \approx 0.20 \times 10^{-3}$ which is consistent with the maximum $\beta_{\perp}$ in Fig. 4b (2). Shown in Figs. 4a (3) and 4b (3) are the beam cross-section with no rf field in waveguide: in this case beam cross-section in β_x - β_y space does not change, whereas in x - y space it continues to spread. Beam spreading can also occur when electrons are out of acceleration phase. For instance, when particles are initially injected somewhere near $z_0 = 0$, λ_e or $\phi_0 = 0, 2\pi$, our simulation found the beam can neither be accelerated nor focused; instead the electrons will spread radially and collide with the dielectric wall.

In this paper we do not deal with the effects of electron beam loading (negligible for low beam current) or space charge. As to the latter, our estimates show that space charge will result in some radial expansion and bunch lengthening for electron beam peak pulse current >10 – 30 A, but we leave this matter for future calculations which will make use of the detailed accelerator code PARMELA.

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A Survey of Microwave Inverse FEL and Inverse Cerenkov Accelerators

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Abstract. A Microwave Inverse FEL Accelerator (MIFELA) and a Microwave Inverse Cerenkov Accelerator (MICA) are currently under construction at the Yale Beam Physics Laboratory. MIFELA and MICA will share the same injector, a thermionic cathode rf gun that should furnish 5psec, 6MeV, 0.2nC electron pulses spaced by 350psec, using microwave power of many MW provided from a 2.85GHz klystron. MIFELA is to operate with ~4MW of 11.4GHz microwave power in the TE₁₁ mode, with beam injection into each fourth of cycle; a variable pitch and field undulator together with a guide magnetic field are present as well. MICA will operate at 2.85GHz using an alumina-lined waveguide driven in the TM₀₁ mode; the phase velocity is just below c , with no guide field. MIFELA produces a beam of spiralling electrons, while MICA makes an axially-directed beam. This is a survey of the operating principles of these smooth-bore "tabletop" accelerators (~ 15MeV) as they are understood prior to operation.

1. Introduction to the MICA

The stimulated Cerenkov effect is a well-understood mechanism for generating coherent radiation from an energetic electron beam [1-3]. The radiating electrons move at speed greater than that of the velocity of light in the structure (hence the name "Cerenkov"); although there are several ways to slow light waves, as a general rule the term is used when the slowing is caused by a dielectric element. When one does a linearized treatment of the fields and the self-consistent motion of the particles, a dispersion relation is obtained for growth or decay of radiation in the system. One of the three roots obtained corresponds to a damped wave; this we identify with the mechanism of stimulated absorption, whereby an electron will gain energy at the expense of the rf field. In the discussion which follows, we consider the application of stimulated absorption in the nonlinear regime of particle trapping, which applies to an electron accelerator device. This we refer to as a microwave inverse Cerenkov accelerator ("MICA").

Acceleration of the electron is done by appropriate phasing of an electron bunch which is emitted from an rf gun, so that a continuous accelerating force is applied to all the electrons, which move synchronously with the slow rf wave. Variation of the wave speed is possible by a small taper in the filling factor of the dielectric element. Thus the device resembles an rf linac, but without the periodic loading structures in the waveguide. As the MICA is smooth-bore and the motion of the particles is rather one-dimensional, we expect that the emittance of the electron beam produced will be attractive. The MICA under consideration will use a SLAC source of microwave power at 2.85GHz, and with a bunch length of only

5psec compared with the rf period of 350psec, we can expect excellent trapping and acceleration of a monoenergetic bunch of electrons. Another approach[4-7], the ICA(Inverse Cerenkov Accelerator) experiment at Brookhaven National Laboratory, uses a CO₂ laser and an axicon to accelerate an electron beam at 40MeV energy; the light wave is slowed by introducing hydrogen gas into the beamline. The gas contributes to some electron scattering, and the main disadvantage of the short laser wavelength is that electrons interact with the wave over the full range (2π) of phase; that is, the bunch length is long compared with the rf wavelength. In the MICA, the electrons move down a 1cm diameter hole in an alumina dielectric liner as a filamentary beam of under 1mm diameter. The main limitation here is that of the maximum axial field gradient (120-160kV/cm [8]) along the dielectric surface.

In this paper, we describe first the analysis of a wave inside of a dielectric annular cylinder fitted into a cylindrical waveguide. We find dispersion relations[9] for the axisymmetric TM₀₁-like mode of the system, that is, the mode that gives maximum electric field along the axis. This calculation provides both the slowing factor and the distribution of fields. We then model the acceleration and motion of electrons in the vacuum fields of this device. In this paper we shall not deal with the effects of electron beam loading (negligible for low beam current) or space charge; the reader is referred to a more comprehensive publication that includes experimental tests[10] of the breakdown fields. The objective of this effort is to determine whether a compact high quality accelerator of this type is feasible.

2. Dispersion relation and field distribution of the dielectric-loaded waveguide

We take a waveguide of radius R, lined with a dielectric sleeve, with the central vacuum hole of radius a as shown in Figure 1. Under the appropriate boundary conditions, we solve the Maxwell equations by a standard procedure[9], and arrive at a dispersion relation of the system for eigenmodes TM_{0n}:

$$\frac{I_1(k_{1i}a)}{I_0(k_{1i}a)} = \epsilon \frac{k_{1i} J_1(k_2a) N_0(k_2R) - J_0(k_2R) N_1(k_2a)}{k_2 J_0(k_2a) N_0(k_2R) - J_0(k_2R) N_0(k_2a)} \quad (1)$$

where, the functions J and N are Bessel functions and I is the modified Bessel function; $k_{1i} = -ik_1$, k_1 and k_2 are the transverse wave numbers in the vacuum hole and the dielectric element separately.

$$k_1^2 = k_0^2 - k_z^2 \quad (2)$$

$$k_2^2 = \epsilon k_0^2 - k_z^2 \quad (3)$$

k_z is the axial wave number of the waveguide TM_{0n} mode, $k_0 = \frac{\omega}{c}$ is the wave number in free space and ϵ is the dielectric constant of the material. The normalized phase velocity of the mode can be obtained from the eigenvalue of Eq. (1):

Figure 1. Longitudinal cross-section of loaded waveguide showing axial electric field distribution as well as electron phase positions along waveguide.

Figure 2. Normalized phase velocity v_{ph}/c vs normalized outer radius for TM_{01} mode waveguide with alumina liner. Note: $v_{ph} \leq c$ for $R \geq 0.15 \lambda_0$.

$$\frac{v_{ph}}{c} = \frac{\omega}{k_z c} = \frac{1}{\sqrt{1 + \frac{k_{1i}^2}{k_0^2}}} \quad (4)$$

As an example, Figure 2 shows the normalized phase velocity of TM_{01} mode as a function of the ratio R/λ_0 , where λ_0 is the free space wavelength of field, the dielectric constant $\epsilon = 9.4$ for alumina, and the ratio of hole radius to outer dielectric radius is $a/R=0.30$. One sees from Figure 2 that phase velocity of c and below obtain when $R/\lambda_0 > 0.15$, which gives a relation between the waveguide radius and the rf wavelength for a slow wave structure.

With the eigenvalue of the waveguide mode solved from the dispersion relation Eq.(1), we can calculate the field distribution in this specific MICA configuration. The electromagnetic field distributions in the loaded waveguide include two separate parts: one part in the vacuum hole and the other in the dielectric, connected by the boundary conditions. For the TM_{0n} mode, only three components E_z , E_r and B_θ exist. In cylindrical coordinates, they have the following form:

when $0 \leq r \leq a$, in the hole:

$$E_{z1}(r, z, t) = E_0 I_0(k_{1i} r) \cos(\omega t - k_z z) \quad (5.1)$$

$$E_{r1}(r, z, t) = -E_0 \frac{k_z}{k_{1i}} I_1(k_{1i} r) \sin(\omega t - k_z z) \quad (5.2)$$

$$B_{\theta 1}(r, z, t) = -E_0 \frac{k_0}{k_{1i}} I_1(k_{1i} r) \sin(\omega t - k_z z) \quad (5.3)$$

when $a < r \leq R$, in the dielectric:

$$E_{z2}(r, z, t) = E_0 g_i \left[J_0(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_0(k_2 r) \right] \cos(\omega t - k_z z) \quad (6.1)$$

$$E_{r2}(r, z, t) = -E_0 g_i \frac{k_z}{k_2} \left[J_1(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_1(k_2 r) \right] \sin(\omega t - k_z z) \quad (6.2)$$

$$B_{\theta 2}(r, z, t) = -E_0 g_i \frac{ek_0}{k_2} \left[J_1(k_2 r) N_0(k_2 R) - J_0(k_2 R) N_1(k_2 r) \right] \sin(\omega t - k_z z) \quad (6.3)$$

The coefficient g_i in Eqs(6.1-6.3) is a constant relating the field amplitude in the dielectric to that in the hole:

$$g_i = \frac{I_0(k_1 a)}{J_0(k_2 a) N_0(k_2 R) - J_0(k_2 R) N_0(k_2 a)} \quad (7)$$

$E_0 = E_0(z)$ is axial field strength on the axis; it can be determined in terms of the total microwave power by integrating the Poynting vector in the cross-section of waveguide.

3. Single-Electron Motion and Acceleration in the Waveguide Fields

Substituting Eqs.(5.1-6.3) in the Lorentz force equation for the relativistic beam particles,

$$\frac{d\mathbf{P}}{dt} = -e \left[\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right] \quad (8)$$

we are now in a position to study the motion of electrons in the MICA. In Eq.(8) the symbols have their conventional meaning; $\mathbf{P} = m\gamma\mathbf{v}$, γ is the Lorentz factor of beam particle. Since the field is not self-consistent, we account for single particle effects only; but, for a low beam current and intense driving field, this a reasonable approximation. The three components of the force equation (8) in cylindrical form are:

$$\dot{\beta}_r - \mu\dot{\beta}_\theta = -\frac{e}{mc^2\gamma} \left[(1 - \beta_r^2)E_r - \beta_r\beta_z E_z - \beta_z B_\theta \right] \quad (9.1)$$

$$\dot{\beta}_\theta + \mu\dot{\beta}_r = -\frac{e}{mc^2\gamma} \left[-\beta_\theta\beta_r E_r - \beta_\theta\beta_z E_z \right] \quad (9.2)$$

$$\dot{\beta}_z = -\frac{e}{mc^2\gamma} \left[-\beta_z\beta_r E_r + (1 - \beta_z^2)E_z - \beta_r B_\theta \right] \quad (9.3)$$

where, $\mu = \dot{\theta}$ is the normalized angular velocity of electron, and the overdot represents the time derivative of the quantities d/dt , $\tau = ct$.

4. Numerical results and discussion

The MICA configuration is a circular waveguide loaded by high ϵ dielectric material, but before selecting this configuration we also explored another dielectric structure, a rectangular waveguide loaded by a dielectric slab or slabs. No conventional TM or TE modes exist in the latter, but rather LSE(Longitudinal-Section Electric mode) or LSM(Longitudinal-Section Magnetic mode) with either zero electric or zero magnetic field normal to the dielectric surface[11]. The maximum axial field strength occurs inside the vacuum-dielectric interface and so this configuration is not appropriate for electron acceleration. However, the use of

TABLE I. Simulation parameters of MICA

<u>Electron beam parameters</u>		
Initial electron energy	$\gamma_0 = 13$	
Maximum initial transverse velocity	$\beta_{\perp} = 0.4670 \times 10^{-3}$	
Initial axial velocity	$\beta_z = 0.9970$	
Beam radius	$r_0 = 0.05$ (cm)	
	$r_0/R = 0.032$	
<u>Waveguide parameters</u>		
Waveguide radius	$R = 1.59$ (cm)	
Radius of vacuum hole	$a = 0.48$ (cm)	
	$a/R = 0.30$	
Dielectric constant(alumina)	$\epsilon = 9.4$	
Waveguide length	$z = 150$ (cm)	
Waveguide mode	TM ₀₁	
<u>Radiofrequency wave</u>		
Field power	$P = 15$ (MW)	
Maximum field strength	$E_{z\max} = 6.29$ (MV/m)	
Frequency	$f_0 = 2.85$ (GHz)	
Normalized phase velocity	$V_{pb}/c = 0.9943$	
Free space wavelength	$\lambda_0 = 10.52$ (cm)	
Waveguide wavelength	$\lambda_g = 10.46$ (cm)

Figure 3. Electron energy as a function of the axial distance.

a high ϵ annulus inside circular waveguide maintains a large uniform E_z field inside the hole which is of particular interest for acceleration purposes.

The numerical simulation is based on the force Eqs.(9.1-9.3) with the field components given by Eqs.(5.1-6.3). The parameters used are listed in Table I. The electrons are taken to be monoenergetic but with small transverse velocities which vary randomly from particle to particle. At the entrance of the waveguide they are randomly distributed inside the beam cross-section as shown in Figure 4(a1). Their distribution in β_x - β_y space is shown in Figure 4(b1).

The acceleration of electrons in the MICA configuration is straightforward because of the intense axial field E_z . In Figure 3 we show the result for 15MW travelling wave power. The electron energy increases almost linearly as the particles move down the waveguide. An analytical derivation from Eqs(9.3) gives an approximate energy expression as

$$\gamma^2 = 1 + \left[\sqrt{\gamma_0^2 - 1} - \frac{eE_0\tau}{mc^2} \right]^2 \quad (10)$$

where γ_0 is the initial electron energy. When γ_0 is large ($\gamma_0=13$, for example in this case), the above equation can be further simplified as $\gamma = \gamma_0 + eE_0\tau/mc^2$. When $|E_0|=210$ (CGS) , $\tau=150$ cm, it gives $\gamma = 31.5$, in close agreement with the simulation result. The validity of equation (10) requires that the relative phase of the electrons with respect to the rf field is π ; checking the field distribution in Figure 1, one sees that this corresponds to an initial distribution of particles at the position $\lambda_g/2$. Particles in this position will experience the maximum axial field. Due to the small difference between the electron velocity and the wave phase velocity, one may expect that the electron will gradually slip from the maximum acceleration position. In the current simulation, we find a phase slippage of $\Delta\phi \approx 24^\circ$ in one and a half meters with the electrons moving ahead of the rf field, corresponding to a slippage interval of $\Delta t_0 \approx 23$ ps. For a rf gun with a beam bunch length of only $\Delta t_0 = 5$ ps, we can expect excellent trapping and acceleration of electrons during the entire propagation along the waveguide, without a taper of the dielectric element. In our calculation, the electron energy increases to about 16 MeV in 150 cm. If the dielectric surface strength is adequate, and with a higher Q structure, the electron energy can increase more.

When the electrons are located in the small "phase window" of acceleration, "momentum compaction" will retard the radial spreading out of the electrons even though the particles have an initial transverse velocity distribution. Shown in Figure 4(a2,b2) are the cross sections of the electrons in the beam at the end of the MICA in x-y and β_x - β_y space separately. Electrons remain well confined inside the hole in the dielectric and the transverse velocity spread shrinks. An algebraic analysis from Equation(9.1.9.2) gives an approximate transverse velocity as

$$\beta_{\perp} = \beta_{\perp 0} \exp \left[\frac{eE_0\tau}{mc^2\gamma} \left(\frac{1}{2} \Delta k \tau \sin\phi_0 + \cos\phi_0 \right) \right] \quad (11)$$

Figure 4. Beam cross-section in x - y space and β_x - β_y space: (1) at the entrance of waveguide(a1,b1); (2) at the end of waveguide with TM_{01} mode inside(a2,b2); (3) at the end of waveguide with no rf field(a3,b3). The vacuum hole $a/R = 0.30$.

where ϕ_0 is the initial phase of electrons, $\Delta k = k_z - k_0 = 3.5 \times 10^{-3}$, and when $\phi_0 = \pi$, $\bar{\gamma} = 22$, we get $\beta_{\perp} = 0.20 \times 10^{-3}$ which is consistent with the maximum $\beta_{\perp}$ in Figure 4(b2). Shown in Figure 4(a3,b3) are the beam cross-section with no rf field in waveguide: in this case beam cross-section in β_x - β_y space does not change, whereas in x - y space it continues to spread. Beam spreading can also occur when electrons are out of acceleration phase. For instance, when particles are initially injected somewhere near $z_0 = 0$, λ_g or $\phi_0 = 0, 2\pi$, our simulation found the beam can neither be accelerated nor confined; instead the electrons will spread radially and collide with the dielectric wall.

In reference 10 is described a more detailed study of the MICA. We have found that the rf breakdown of the alumina is in excess of 8.4 MV/m, and simulations with the PARMELA code find that space charge effects do not appreciably modify the conclusions we reached with regard to Figure 4.

5. Introduction to the MIFELA

The free-electron-laser has proved to be a very efficient tunable radiation source. If we regard the FEL as an electron "decelerator" where the energy of electrons is transferred in the undulator to amplify electromagnetic radiation, then it is very natural to take advantage of the analogy between FEL devices and radio frequency accelerators in which a high power electromagnetic field is used to accelerate an electron beam from low energy to high energy. The FEL operated in this process is called the inverse free-electron-laser accelerator (IFELA). The principle of the IFELA was described many years ago [12], and has been re-examined in more detail both theoretically [13,14] and experimentally [15] in the last a few years. It would generate a spiralling beam of electrons at energy ~15 MeV.

The principle of acceleration is as follows. In the rest frame of the electron beam, the magnetostatic periodic field of the undulator (or wiggler) is transformed into an electromagnetic wave that beats with the microwave source; acceleration occurs by trapping a bunch of electrons into the resulting ponderomotive wave, and then increasing the velocity of this wave by tapering the undulator field and/or amplitude. We can identify some of the problems facing the IFELA by considering the acceleration mechanism:

$$\frac{d\gamma}{dz} = -a_s \frac{\omega_s}{c} \frac{a_w}{\gamma} \sin \psi \quad (12)$$

where γ is the relativistic factor of electron, $a_s = eA_s/mc^2$ is the normalized vector potential of the radiation field, ω_s is the radiation frequency, a_w/γ involves the normalized vector potential of the undulator field and amounts to the ratio of electron velocity perpendicular to the axis to the velocity parallel to the axis of the device, and is caused by the undulator interaction with the electrons. ψ is the relative phase of the electron with respect to the rf driving field.

It can be seen from (12) that the relative phase ψ plays an important part in the FEL. When $\psi > 0$, $d\gamma/dz < 0$ and this case represents stimulated emission, as in the FEL. When $\psi < 0$, then $d\gamma/dz > 0$ and this case represents stimulated absorption which is called the inverse FEL process. We can inject the electrons into a small "phase window" so that all the electrons are located in the accelerating buckets. This can be done in our example because the acceleration power is obtained from a microwave source that has a "long wavelength" (unlike some IFELA's that use a laser source of power). In our simulation the small "phase window" of phase angles is between $-\pi/8$ and $-3\pi/8$. This can be done in practice by gating the injected electron beam emerging from a rf gun and buncher cavity as it enters the IFELA; the $\pi/4$ spread corresponds to a pulse length of ~ 6 psec. In further consideration of (12), we find that a strong rf electric field (~ 0.5 MV/cm—below the vacuum breakdown limit) is required, as well as a strong transverse undulator magnetic field (~ 0.6 T), in order to achieve a sizable accelerating gradient.

The MIFELA is now under construction at Yale University. Before construction, numerical results were obtained from a nonlinear 3D FEL code "ARACHNE" [16,17]; these recent results "calibrate" reasonably well against the results of the 1D analysis presented here.

6. Theoretical Model of MIFELA

We present in this section the basic theoretical model used to describe the acceleration process in an IFELA. The system uses a helical undulator and circularly polarized driving field that propagates in a cylindrical waveguide. The fundamental equations used herein to describe the IFELA interaction are the well-known FEL equations [18,19] that define the energy and relative phase of a resonant electron in terms of the undulator magnetic field, the undulator period, and the driving field.

$$\frac{d\gamma_j}{dz} = -\frac{a_w a_s \frac{\omega_s}{c}}{\gamma_j} \sin \psi_j \left[1 - \frac{\mu^2 - 2a_w a_s \cos \psi_j}{\gamma_j^2} \right]^{-1/2} \quad (13)$$

$$\frac{d\psi_j}{dz} = k_w + k_s - \frac{\omega_s}{c} \left[1 - \frac{\mu^2 - 2a_w a_s \cos \psi_j}{\gamma_j^2} \right]^{-1/2} + \frac{\partial \phi}{\partial z} \quad (14)$$

Here $\mu^2 = 1 + a_w^2 + a_s^2$, z is the axial distance along the system and is the independent variable. γ_j is the relativistic factor of the j th electron, ψ_j is the phase of the j th electron with respect to the driving field, ϕ is the phase shift of the driving radiation field.

The driving field equation is a solution of the Maxwell's equations. It has the following form

$$\left[\nabla_{\perp}^2 + 2ik_s \frac{\partial}{\partial z} - \delta k_s^2 \right] u(r, z) = - \frac{\omega_p^2}{c^2} \left\langle \frac{e^{-i(\psi + \phi)}}{\gamma} \right\rangle \quad (15)$$

$$\delta k_s^2 = k_s^2 - \frac{\omega_s^2}{c^2} \quad (16)$$

The angular brackets in the right-hand side of equation (15) indicates an ensemble average over all electrons. Equation (15) can be solved with the cylindrical waveguide boundary conditions.

Equations (13)-(15) comprise a complete set of nonlinear coupled equations. The motion equations (13) and (14) are valid for every electron, and given the initial conditions $\psi(0)$ and $\chi(0)$ for every electron along with the parameters of the undulator, these equations can be integrated numerically to yield the value of ψ and γ as a function of the longitudinal position z . The motion of the electron in the undulator and a guiding magnetic field (used for beam transport) is represented by the term a_w/γ , and is obtained by separate solution of the orbit equation [20,21].

$$\beta_{\perp} = \frac{2\Omega_{\perp} \frac{I_1(\lambda)}{\lambda} \beta_{\parallel}}{\Omega_0 - \gamma k_w c \beta_{\parallel} \pm 2\Omega_{\perp} I_1(\lambda)} \quad (17)$$

where $\lambda = \pm \beta_{\perp}/\beta_{\parallel}$, $\beta_{\perp}$ and $\beta_{\parallel}$ are the transverse and axial velocities of electrons respectively; $\Omega_{\perp}$ and Ω_0 are the gyrofrequencies of electrons in the undulator and guiding fields; $I_1(\lambda)$ is the first-order Bessel function of imaginary argument.

The driving field, which is coupled with the equations of motion through $u(r, z) \equiv a_s e^{i\phi}$ in the right-hand of (13) and (14), is also a self-consistent solution to this set of equations. The procedure we used here is to solve the equations (13) to (15) for a large number of particles and obtain the final coordinates in phase space for every particle. The code we use is based on the equations above with a one-dimensional description for the motion of electrons, but it is two-dimensional for the driving field dynamics. This is a single-pass code corresponding to the Compton regime, for which the space charge effect is not considered: this approximation is satisfactory if the electron beam current is not higher than 100A.

7. Simulation Results and Discussion of MIFELA

Figure 5a shows the schematic of the accelerator and how the electrons are injected from an rf gun. The rf for the cavity injector is obtained from a 2.85 GHz high power klystron; this power will be converted [22] to the fourth harmonic at 11.4 GHz so as to drive the IFELA. We imagine that the waveguide will fill with microwaves for several nsec, interacting with a reflector in the waveguide so as to

Figure 5(a): Schematic of the MIFELA;

5(b): Rf wave forms, injecting a bunch into the MIFELA.

Figure 6. Electron energy as a function of the axial distance along the undulator. The dotted line is the resonant energy of trapped particles.

build up a high intensity field for electron acceleration. Then the subharmonic cavity will inject a short pulse of electrons into the IFELA, which will absorb the rf energy. The TE_{11} mode was chosen since it has a lowest cut-off frequency. In the numerical study, we increase the undulator period and the axial guiding field linearly in the accelerating section. The gradient of the axial field is determined so that the accelerator works with the stable Group I electron orbits (however, the original stability condition applies for constant energy, undulator period, and undulator field, which is not the case here).

To get the optimum accelerating gradient, we vary the undulator period $l_w(z)$ and the undulator parameter $a_w(z)$, so as to gradually increase the resonant energy of the trapped electrons and put most of the beam electrons into an accelerating bucket. Figure 6 shows the electron energy as a function of axial distance along the MIFELA. The electrons are injected monoenergetically from the gun at $\gamma = 13$ in a small initial "phase window" between $-\pi/8$ and $-3\pi/8$. The resonant energy of the design structure is shown as a dotted line. It can be seen from Fig. 6 that the beam energy and the resonant energy match well along the acceleration section. The idea of small initial "phase window" is significant in this application since once injected, all the electrons are located in the accelerating buckets. Experimentally, this can be done by gating the injected electron beam emerging from a rf gun and buncher cavity as it enters the MIFELA (Figure 5b); the $\pi/4$ spread corresponds to a pulse length of ~ 6 psec. Figure 7 shows the phase plot and the energy spectrum of the accelerated electrons at the end of the accelerating section. Because of this small "phase window", all the electrons we simulated are trapped in the accelerating bucket. In the phase plot there is no spread of the particles, and the electron energy distribution in this case is also narrow ($\Delta\gamma/\gamma \sim 1/2\%$).

At the end of the MIFELA, we can allow the guide and undulator fields to decrease gradually. With a sufficiently gentle gradient, one can show that the defocusing effect of the decreasing guide field can be overcome by the natural focusing of the helical undulator. If the rf field continues on in this end section of the MIFELA, we find there is still a small increase of electron energy since the electrons remain in the accelerating buckets. After that, with the decrease of guide field, a_w/γ drops, and the electrons fall out of resonance. Since the electrons have random phase with respect to the driving field, no net energy increase occurs in the end section. In this way, the output beam of the MIFELA can be extracted to zero guide field. The accelerating gradient we have obtained in this example is $\sim 7\text{MeV/m}$.

We have also designed an "entry" section of adiabatic undulator field increase for the MIFELA. A linear or sine-squared variation of the undulator field over about five periods will allow the electron beam to spiral up to a radius $\sim 8\text{mm}$, which remains approximately the same thereafter; by careful design, the electrons will enter the accelerator at the correct phase to undergo acceleration.

The device will use the power at the fourth harmonic of a 24MW, 2.856GHz XK-5 SLAC klystron as the rf pump. Calculations[22] indicate that a conversion efficiency of $\sim 70\%$ is to be expected for a cold beam, and $>50\%$ efficiency for a beam with velocity spread of 1%. We envision the use of a TE_{11m} cavity to increase the rf pump parameter in the MIFELA by approximately a factor of five compared with the value for a free travelling wave. The loaded Q would then be

Figure 7. The electron distribution in phase space(above) and energy spectrum(below).

>5000 with an iris coupler; however the real power flow to the beam is limited to the available 12MW. This will determine the amount of charge that can be accelerated as well as the pulse repetition rate. A recent numerical study also supports a MIFELA which uses power directly obtained from the 2.85GHz klystron; this requires a larger drift tube ($R = 3.1\text{cm}$) and undulator period $\sim 12\text{cm}$ together with a larger undulator winding radius, but otherwise its performance is quite similar to the higher frequency device we have just described.

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Microwave Inverse Cerenkov Accelerator

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A Microwave Inverse Cerenkov Accelerator (MICA) is currently under construction at the Yale Beam Physics Laboratory. The accelerating structure in MICA consists of an axisymmetric dielectrically lined waveguide. For the injection of 6 MeV microbunches from a 2.856 GHz RF gun, and subsequent acceleration by the TM_{01} fields, particle simulation studies predict that an acceleration gradient of 6.3 MV/m can be achieved with a traveling-wave power of 15 MW applied to the structure. Synchronous injection into a narrow phase window is shown to allow trapping of all injected particles. The RF fields of the accelerating structure are shown to provide radial focusing, so that longitudinal and transverse emittance growth during acceleration is small, and that no external magnetic fields are required for focusing. For 0.16 nC, 5 psec microbunches, the normalized emittance of the accelerated beam is predicted to be less than 5π mm-mrad. Experiments on sample alumina tubes have been conducted that verify the theoretical dispersion relation for the TM_{01} mode over a two-to-one range in frequency. No excitation of axisymmetric or non-axisymmetric competing waveguide modes was observed. High power tests showed that tangential electric fields at the inner surface of an uncoated sample of alumina pipe could be sustained up to at least 8.4 MV/m without breakdown. These considerations suggest that a MICA test accelerator can be built to examine these predictions using an available RF power source, 6 MeV RF gun and associated beam line.

I. INTRODUCTION

The stimulated Cerenkov effect is a well-understood mechanism for generating coherent radiation from an energetic electron beam [1-3]. The radiating electrons move at speeds greater than that of the velocity of light in the structure (hence the name "Cerenkov"). Although there are several ways to slow light waves, as a general rule the term is used when the slowing is caused by a dielectric element. When one does a linearized treatment of the fields and the self-consistent motion of the particles, a dispersion relation is obtained for growth or decay of radiation in the system. One of the three roots obtained corresponds to a damped wave; this we identify with the mechanism of stimulated absorption, whereby an electron will gain energy at the expense of the RF field. In the discussion which follows, we consider the application of stimulated absorption in the nonlinear regime of particle trapping, which applies to an electron accelerator. This we refer to as a Microwave Inverse Cerenkov Accelerator ("MICA") [4,5].

Acceleration of the electrons is achieved by appropriate phasing of a 6 MeV electron bunch which is emitted from a thermionic cathode RF gun, so that a continuous accelerating force is applied to all the electrons which move synchronously with the slow RF wave. Variation of the wave speed, if necessary, can be done by using a small taper in the filling factor of the dielectric element. Thus the device resembles an RF linac, but without the periodic loading structures in the waveguide. As the MICA is smooth-bore and the motion of the particles is essentially one-dimensional, the quality of the electron beam produced can be expected to be good. The MICA under consideration will use a SLAC klystron source of microwave power at 2.856 GHz. With a bunch length of only 5 psec compared with the RF period of 350 psec, we can expect excellent trapping and acceleration of a monoenergetic bunch of electrons. Another approach [6-7], the ICA (Inverse Cerenkov Accelerator) experiment at Brookhaven National Laboratory, uses a CO₂ laser and an axicon to accelerate a 40 MeV electron beam, and the light wave is slowed by introducing hydrogen gas into the beam line. The gas contributes to some electron scattering, and the main disadvantage

FIGURE 1. (a) Schematic diagram of MICA layout. A pre-accelerated short bunch of ~6 MeV electrons from an RF gun is injected in the appropriate accelerating phase into MICA, so that trapping and further acceleration of the entire bunch can occur. (b) MICA accelerating section.

of the short laser wavelength is that electrons interact with the wave over the full range (2π) of phase; that is, the bunch length is long compared with the RF wavelength. In MICA, the electrons move down a 1 cm diameter hole in an alumina dielectric liner as a filamentary beam of under 1 mm diameter. The main limitation here is that of the maximum axial field gradient (120-160 kV/cm [8]) along the dielectric surface. Shown in Fig. 1 is a schematic layout of the MICA.

II. EIGENMODES OF THE DIELECTRIC-LOADED WAVEGUIDE

The MICA configuration is a circular waveguide loaded by high ϵ dielectric material, with a small hole on axis for passage of the beam as shown in Fig. 1(b). The axisymmetric modes of this cylindrical dielectric-loaded system are either TE or TM, and we consider the TM_{0n} -like mode of this system, i.e. the mode with finite axial electric field on axis, no azimuthal variations and one radial maximum for the axial electric field. The use of a high ϵ annulus inside a circular waveguide maintains a large uniform E_z field inside the hole which is ideal for an accelerator. Using the appropriate boundary conditions at the interface of two differing media as well as at the outer metallic conducting wall, we solve Maxwell's equations using standard procedures [9], and arrive at a dispersion relation of the system for TM_{0n} eigenmodes. Accordingly, the normalized phase velocity v_{ph}/c can be obtained from the respective eigenvalue [5]. As an example,

FIGURE 2. Normalized phase velocity v_{ph}/c vs. normalized outer radius R/λ_0 for TM_{01} mode waveguide with an alumina liner when the radius of the vacuum hole $a/R = 0.3$. Note: $v_{ph} \leq c$ when $R \geq 0.15\lambda_0$.

Fig. 2 shows the normalized phase velocity of the TM_{01} mode as a function of the ratio R/λ_0 , the dielectric constant ϵ is taken to be 9.4, and the ratio of hole radius to outer dielectric radius a/R is taken to be 0.3. One finds from Fig. 2 that a phase velocity of c and below obtains when $R/\lambda_0 > 0.15$, from which one finds the

TABLE 1. Simulation parameters of MICA

<u>Electron beam parameters</u>		
Initial electron energy	$\gamma_0 = 13$	
Maximum initial transverse velocity	$\beta_{\perp} = 2.60 \times 10^{-3}$	
Initial axial velocity (6 MeV)	$\beta_z = 0.9970$	
Beam radius	$r_b = 0.05$	(cm)
	$r_b/R = 0.032$	
<u>Waveguide parameters</u>		
Waveguide radius	$R = 1.59$	(cm)
Radius of vacuum hole	$a = 0.48$	(cm)
Ratio of two radii	$a/R = 0.30$	
Dielectric constant (alumina)	$\epsilon = 9.4$	
Waveguide length	$z = 150$	(cm)
Waveguide mode	TM_{01}	
<u>Radiofrequency wave</u>		
Field power	$P = 15$	(MW)
Maximum field strength	$E_{zmax} = 6.29$	(MV/m)
Frequency	$f_0 = 2.856$	(GHz)
Normalized phase velocity	$V_{ph}/c = 0.9943$	
Free space wavelength	$\lambda_0 = 10.50$	(cm)
Waveguide wavelength	$\lambda_g = 10.46$	(cm)

along an accelerator section, so that significant energy depletion, if any, would cause late-following bunches to experience less acceleration than early-leading bunches. After several fill times, a steady-state can be reached, but beam loading will reduce the field amplitudes, bringing about less net acceleration than in the absence of beam loading. This situation is undesirable when the accelerator is designed for high energy gain and narrow energy resolution. Therefore one must

FIGURE 3. (a) Electron energy as a function of the axial distance. (b) The axial accelerating field seen by one particle as it moves down the waveguide. The dotted lines are the case when a 10 percent power depletion is taken into account.

insure that the energy carried away by the beam during a sequence of microbunches (i.e. during one fill time $\tau = L/v_g$) is much less than the stored energy. Our beam loading computation shows that for an average current of $I_0 = 0.143$ A, which corresponds to peak current of 10A with 5 ps bunch averaged over the 350 ps RF cycle established in the MICA, there is only a slight drop of 2.3 % from the maximum no-load energy gain 9.06 MV.

Due to the small difference between the electron velocity and the wave phase velocity, one may expect that the electron will gradually slip from the maximum acceleration position, forward or backward depending on whether the beam is going faster or slower. In the current simulation, we find a phase slippage of $\Delta\phi = 24^\circ$ in 1.5 m with the electrons moving ahead of the RF field, corresponding to a slippage interval of $\Delta\tau_0 \approx 23$ ps. For a RF gun with a beam bunch length of only $\Delta\tau_0 = 5$ ps, we can expect excellent trapping and acceleration of electrons during the entire propagation along the waveguide, without a taper of the dielectric element.

When the electrons are located in a small "phase window" of acceleration, the radial component of the field E_r will prevent the electrons from spreading out even though the particles have an initial transverse velocity distribution. Our simulation shows that electrons remain well confined inside the hole in the dielectric and the transverse velocity spread shrinks. Beam spreading becomes serious only when electrons are out of acceleration phase.

IV. SELF FIELD EFFECT AND EMITTANCE EVOLUTION

The self field effect due to finite electron current has been investigated by running the PARMELA accelerator code [10], a versatile multiparticle electron linac code widely used in accelerator community [11]. In PARMELA the electron beam, represented by a collection of macroparticles, may be transformed through a linac and/or transport system. The self field effects (both electric and magnetic) are automatically taken into account in the simulation. Since the code usually applies to a periodic loading structure, it was necessary to modify the code so that it has the capability of modeling traveling wave acceleration in the smooth-bore MICA structure. In Table 2, we summarize some parameters of main interest for a bunch at the entrance ($z=0$) and the exit ($z=150$ cm) of the waveguide. Entrance conditions are taken from the RF gun and beam line computation, while the output is determined by the simulation results. The initial parameters in the PARMELA simulation are the same as we used in the single particle dynamics run shown in Table 1, except a finite bunch charge of $q = 0.05$ nC is now included.

TABLE 2. PARMELA results for beam emittance and energy spread at the entrance and exit of waveguide

z	a_0	E_{beam}	$4\sigma_p$	$4\sigma_E$	ϵ_{axrms}	ϵ_{ayrms}	ϵ_{azrms}
(cm)	(mm)	(MeV)	(deg.)	(keV)	(π mm mrad)	(π mm mrad)	(π deg-keV)
0	0.5	6.0	5.78	68.9	3.05	2.90	24.9
150	3.0	15.6	5.83	80.2	3.05	2.91	25.8

Comparing the PARMELA output results with the single particle results, we find that in both simulations the acceleration gradients, the final particle energy, the beam cross-section and the particle velocity evolution are all in excellent agreement [5]. The 1000 particles used in PARMELA simulation are all "good" particles, meaning that there is no particle loss in the MICA. PARMELA shows very clearly that the transverse emittance ϵ_{axrms} and ϵ_{ayrms} are constant throughout the acceleration, even though the longitudinal emittance ϵ_{azrms} has a very slight change because of the minute longitudinal bunching which makes the particle energy spectrum more narrow. This PARMELA simulation is also compared to a test run where the net charge is set to zero: we observe only trivial differences. This shows that the self field effects are not significant for 10 A peak current. However, when the beam current is increased, the self field effects do affect the ultimate beam quality. For PARMELA runs with 20 A peak micropulse current a noticeable growth in normalized transverse and longitudinal emittance is found, while for 200 A the growth is substantial. These results suggest that achievement of the goal of a normalized transverse emittance of 5π mm-mrad for a 0.16 nC (10^{19} particles), 5 psec bunch is realistic.

V. ATTEMPT TO MEASURE THE DIELECTRIC BREAKDOWN LIMIT

In order to determine the breakdown limits at 2.856 GHz, we have designed and constructed a cavity resonator with an alumina liner. Thus it is necessary to determine the resonance frequency and quality factor Q for a TM_{01} resonator constructed with a section of dielectric-lined waveguide with conducting plates closing the ends. Since the exact value of the alumina dielectric constant of the sample we used was not accurately known, we began with a low power test to determine the resonant modes of a simple cavity incorporating an alumina annulus with outer metallic surfaces, which is coupled to a signal generator and a detector as shown in Figure 4.

FIGURE 4. Sketch of the test resonator fabricated from a short section of alumina pipe coated on its exterior with silver paint. A low power RF input was used.

The design parameters of an ideal cavity resonator, based on the sample available, are listed in Table 3. The cavity operates in the TM_{012} mode; that is, the length of the cavity is one guide wavelength of TM_{01} mode. The cavity has a moderate quality factor $Q = 4620$. The relation between the maximum axial field in the cavity and the power coupled in is also given in Table 3 in terms of the parameter $E_{zmax} / P^{1/2}$ where P is the total power lost in both walls and dielectric.

TABLE 3. Simulation Parameters of Dielectric-loaded Cavity Resonator

<u>Cavity parameters:</u>		
Radius of empty hole	$a = 0.508$	(cm)
Radius of cylindrical cavity	$R = 1.429$	(cm)
	$a/R = 0.356$	
Length of cavity	$d = 12.11$	(cm)
Real part of the dielectric constant	$\epsilon_r = 9.62$	
Imaginary part of the dielectric constant	$\epsilon_i/\epsilon_r = 9.4 \times 10^{-5}$	
<u>RF wave parameters:</u>		
Cut-off frequency	$f_c = 3.118$	(GHz)
Resonance frequency	$f_0 = 3.210$	(GHz)
Transverse wave number in the hole	$k_{11} = 0.4281$	(1/cm)
Transverse wave number in the dielectric	$k_2 = 2.021$	(1/cm)
Wavelength in the cavity resonator	$\lambda_g = 12.11$	(cm)
Cavity mode	TM_{012}	
<u>Quality factor of the cavity</u>		
Q for the conducting walls	$Q_w = 7929$	
Q for the dielectric	$Q_d = 11070$	
Q of the cavity	$Q = 4620$	
Power loss ratio (MV/m)/(MW) ^{1/2}	$E_{max}/P^{1/2} = 20.32$	

Measurements were conducted with a resonator fabricated from a short section of alumina pipe coated on its exterior with silver (Fig. 4). The alumina samples, supplied by LSP Ceramics, Inc., had inner and outer radii of 1.429 cm and 0.508 cm, respectively. There are some differences in dimension between the proposed waveguide and the test cavity (the outer radius is 10% smaller than the required value of 1.5875 cm, while the inner radius is 7% larger). Nevertheless, measurements on the samples available still provide a good test of theory. Raw data for the observed RF transmission by the cavity is shown in Fig. 5(a), over a frequency range between 3 and 6 GHz. Fig. 5(b) shows a plot of the square of the 12 observed resonance frequencies in Fig. 5(a) versus the square of the resonance index. From theoretical analysis we know that the slope of this line should be the reciprocal of the relative dielectric constant; for the data in Fig. 5(b), this reciprocal slope is 9.62. This differs from 9.4, the canonical value taken in the analysis given above, but 9.62 is well within the range quoted for good purity alumina. It is highly significant that no other resonance for this structure between 3 and 6 GHz were found that did not fit on the line shown in Fig. 5(b), despite attempts having been made to excite non-axisymmetric modes using a

FIGURE 5. (a) Mode spectrum of the resonator obtained in measurement cavity transmission is in arbitrary units; (b) Square of mode resonance frequency versus square of axial mode number from measurements on the resonator shown in (a). The reciprocal of the slope of the line is 9.62, which can be inferred to be the dielectric permittivity ϵ .

non-axisymmetric antenna. One can conclude from this observation that potentially disruptive non-axisymmetric modes of the dielectric pipe were not excited. Calculation of the properties of non-axisymmetric modes would be a formidable task, one which these experimental tests appear to render unnecessary.

The vertical intercept for the line in Fig. 5(b) should be the square of the TM_{01} -mode waveguide cutoff frequency, which in this case is observed to be 3.216 GHz. For $\epsilon = 9.62$, $R = 1.429$ cm and $a = 0.508$ cm, the calculated value is 3.118 GHz, a value 3.05% lower than the measurement. This discrepancy is not unreasonable, considering the added circuit reactance of the coupling antennas, and the incomplete closure of the end walls. Typical Q values for the observed

cavity resonance were in the range of 400-500, much lower than the calculated unloaded value of 4620. This is also not too surprising, considering the open ends of the beam hole, and the strong external loading that was required to make accurate resonance measurements on all 12 modes. But this exercise emphasizes the need to carefully test alumina samples prior to acceptance, and prior to selection of the final parameters for the 150 cm accelerating sections. In particular, an accurate advance measurement of dielectric constant, phase velocity and loss tangent must be made from samples taken from the alumina batch to be used for the final accelerating sections.

Measurements using high power microwaves applied to the alumina samples were also carried out to determine RF breakdown limits. Since the cavity described above has resonance above 3.2 GHz, an alternative experimental arrangement was devised to subject the alumina surfaces to high tangential RF electric fields at 2.856 GHz (obtained from a XK-5 klystron). A standing-wave resonance was established in WR-284 rectangular waveguide using inductive irises. Measurements with the alumina sample in place showed this arrangement to give an effective gain of over 11 dB, as deduced from signals on the sample probe with and without the irises. Under these conditions, the peak tangential RF electric field at the inner alumina surface is calculated to be $33.6 P^{1/2}$ V/cm, where P is the incident power in watts. This indicates that a field of 63 kV/cm would be applied when $P = 3.52$ MW. In the experiments, the RF power level was increased over a ~12 hour period to provide gradual RF processing of the structure, without allowing the background pressure to exceed 2×10^{-6} Torr. It was found that this procedure could be continued up to a power level of 6.25 MW, without evidence of arcing at the alumina surface. This corresponds to a tangential field of 84 kV/cm. These observations suggest that acceleration gradients of at least 8.4 MV/m should be achievable in MICA, where a design with superior vacuum integrity and coating of the alumina is planned.

V. CONCLUSIONS

We have studied a Microwave Inverse Cerenkov Accelerator, which has an acceleration mechanism similar to that of a conventional RF linac. However, the accelerating structure, which comprises a continuous coated ceramic pipe, should be less expensive to fabricate than that of the linac. In the absence of any periodic loading structures in the waveguide, wakefield generation that can lead to emittance growth and beam breakup should be minimized. Thus MICA's advantages of a relatively compact structure, smooth-bore design and no need of magnetic focusing make it a very competitive facility as a simple, low cost electron accelerator.

In this paper, we have discussed briefly the eigenmode, field profile, particle dynamics, beam loading and space charge effects. Experimentally, we failed to observe dielectric breakdown of alumina at fields up to 8.4 MV/m. We find that a thick liner with a high dielectric constant is very helpful not only to store high RF energy but also to maintain an intense and uniform axial

accelerating field in the central hole. The particle motion in the waveguide is nearly one dimensional with all input particles being accelerated and no interception by the dielectric. There is no beam breakup, and the beam bunches have good stability even if they are slightly off-axis. For the beam current under consideration, the initial low normalized emittance—less than $3 \times \text{mm mrad}$ —is constant throughout the acceleration. The acceleration gradient in the simulation is 6.3 MV/m in which case the electron energy increases from 6 to 16 MeV in 150 cm. However, without exceeding the breakdown limit measured by experiment ($>8.4 \text{ MV/m}$), and using higher microwave power and/or a higher Q structure, the electron energy could increase even more, perhaps in the range of 10-15 MeV/m if techniques for improving the dielectric breakdown[8] on the surface using Ti or TiN evaporated coatings can be used successfully.

Challenging technical issues must be overcome. These include precision grinding such as the finish of the waveguide liner, since the phase velocity of a RF wave in the vicinity of c is very sensitive to the radius of the vacuum hole and tube, and careful design of the matching for the power feeding system and the accelerator waveguide because of the substantial difference of the wave group velocity (or impedance) in these two sections.

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ENGINEERING DESIGN AND FABRICATION OF MICA PROTOTYPE

Included in this section are a sequence of diagrams and plots that show the evolution of the final engineering design for MICA. This engineering and fabrication was carried out by Titan-Beta, of Dublin, CA, under a sub-contract from Omega-P, Inc. The critical issues were the matching of the dielectric-loaded TM-01 mode cylindrical waveguide to standard WR-238 rectangular waveguides at input and output ports, and the insertion loss of the MICA structure itself.

Design simulations and cold tests were carried out to optimize the designs, as seen in the enclosed diagrams and plots. Fig. 1 shows the input matching structure. Figs. 2 and 3 show the return loss (match) for two locations of the short position. As seen in Fig. 2, the return loss for the cold-test model could be reduced to loss than -30 db at one select frequency. Fig. 4 shows the overall outline design for MICA, including the input and (identical) output coupler and three accelerating sections. (Five were actually fabricated by Titan-Beta.) Fig. 5 shows the details of the input (output) coupler design. Fig. 6 shows the details of construction of the ceramic (metalized on its exterior) and the manner in which it was brazed into the surrounding water jacket. Three pages of assembly steps are included, detailing the construction sequence followed by Titan-Beta. Measurements of the return loss, and overall insertion loss for the fully-assembled MICA prototype are shown in Fig. 7; as is seen these quantities are far below specifications, and much worse than the cold-test values. Shortage of additional funds to rebuild the structures, and shortage of time remaining in the 2-year grant period dictated that preliminary attempts be made to utilize these imperfect structures for acceleration, postponing a decision regards further modifications until acceleration results could be studied. Table I shows results of tests to optimize dimensions of the dielectric pipe, depending upon dielectric constant. Fig. 8 shows the influence of dielectric constant upon expected acceleration. Fig. 9 is a photograph of the input (output) coupler. Fig. 10 is a photograph of one of the MICA accelerator modules. Fig. 11 is a photograph of the assembled MICA structure on a strongback. Fig. 12 is a photograph of one MICA module and the input and output couplers installed on one leg of the Yale Beam Physics 6-MeV beamline. Fig. 13 is a photograph of some of the overall experimental set-up in the Yale Beam Physics accelerator tunnel. Visible are MIFELA (with green quadrupole magnets and antler waveguide input feeds and, to its right the MICA beamline. Not installed at the time of this photograph are the necessary quadrupoles for focussing the beam along MICA. Just visible beyond the MICA structure is a magnetic spectrometer for measuring the energy and energy distribution of the beam after passage through the MICA structure. At this writing (7/99) 6-MeV bunches have been successfully transported through the MICA structure, and imaged on screens inserted into the beamline ahead of, and beyond the MICA structure. Approval from Yale University radiation safety officials is awaited before acceleration beyond 6 MeV can be attempted. Moreover, transfer of the S-band input waveguide feed from MIFELA to MICA must await completion of experiments on MIFELA that are underway. Definitive tests of MICA are scheduled for Fall-Winter 1999-2000.

MATCH (RETURN LOSS)

SHORT POSITION; $L_s = 1.850$ inches

CH1: A/R - 29.42 dB
REF - .00 dB

Fig. 2. Measured return loss on MICA cold-test structure with short position at 1.85".

MICA TRANSITION MATCH (RETURN LOSS)

CERAMIC TUBE ($\epsilon' = 9$)
DIELECTRIC IZING ($\epsilon' = 9$)

Fig. 3. Measured return loss on MICA cold-test structure with short position at 2.375".

[Signature]
6/13/97

Fig. 4. Overall assembly drawing of MICA with three modules shown.

WAVEGUIDE TO CERAMIC TRANSITION

Fig. 5. Engineering assembly drawing of MICA input coupler.

END SECTION SEQUENCE

1. Assemble/braze waveguide sub-assembly. Machine as necessary, vacuum leak check.
2. Metallize ceramic O.D., except ends. No metallizing on larger diameter end ceramic.
3. Copper plate metallized areas.
4. Braze eyelets, (Kovar) to ceramic using ceramic tooling for precise location. Leak check.
5. Pre-braze coolant, connectors to water jacket.
6. Weld water jacket to ceramic assembly.
7. Pressure test water channel.
8. Titanium coat ceramic I.D. & ends.
9. Weld larger diameter end ceramic to waveguide assembly of step 1.
10. Weld cylindrical ceramic assembly to waveguide sub-assembly. Leak check.
11. Bag in N₂ to hold for next assembly.

CERAMIC TO CERAMIC JOINT

Fig. 6. Engineering assembly drawing of ceramic-to-ceramic joint at center of MICA module.

INTERNAL SECTION SEQUENCE

1. Metallize ceramic O.D. except ends.
2. Copper plate metallized areas.
3. Braze eyelets (Kovar) to ceramic using ceramic tooling for precise location.
4. Pre-braze coolant connectors to water jacket.
5. Weld water jacket to ceramic assembly.
6. Pressure test water channel.
7. Titanium coat I.D. & ends of ceramic.
8. Bag in N_2 to hold for next assembly.

FINAL ASSEMBLY SEQUENCE

- 1. Carefully align mating sections and clamp in place.**
- 2. Weld sections together.**
- 3. Vacuum leak check.**
- 4. Microwave tests; Measure match and insertion loss.**
- 5. Evacuate; Bakeout?; Seal off.**
- 6. Package for shipment.**
- 7. Ship.**

INPUT MATCH (RETURN LOSS) INSERTION LOSS
 CH1: A/R - 14.45 dB CH2: B/R - 3.94 dB
 2.0dB/ REF + .00 dB 1.0dB/ REF + .00 dB

Fig. 7. Measured return loss and insertion loss of assembled final MICA Structure, input-to-output.

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Fig. 8. Measured return loss and insertion loss of assembled final MICA Structure, output-to-input.

Table 1: ϵ fitting of mica tube

sample	n	ϵ	ω_{co} (fitting)	ω_{co} (theory)	$\Delta\omega/\omega(\%)$	χ	OD(in.)	$V_{ph}(C)$
2B	2	8.19	2.5820	2.7050	4.65	10.49		
	3	9.14	2.5418	2.5647	0.89	0.52	1.246	0.9973
	4	10.09	2.4992	2.4419	2.32	11.80		
3A1	2	8.10	2.5987	2.7201	4.57	14.54		
	3	9.04	2.5583	2.5792	0.81	0.37	1.252	0.9988
	4	9.98	2.5155	2.4558	2.40	8.21		
3A2	2	8.09	2.5846	2.7232	5.22	14.28		
	3	9.02	2.5439	2.5820	1.48	0.73	1.253	0.9970
	4	9.96	2.5008	2.4584	1.71	9.49		
3B1	2	8.13	2.5811	2.7158	5.09	11.04		
	3	9.07	2.5406	2.5749	1.34	0.34	1.250	0.9968
	4	10.02	2.4977	2.4516	1.86	11.30		
3B2	2	8.14	2.5848	2.7140	4.88	11.68		
	3	9.08	2.5444	2.5732	1.12	0.34	1.249	0.9988
	4	10.03	2.5017	2.4561	2.08	10.66

(a)

(b)

(c)

(d)

Figure 9: simulation results for various conditions. dielectric constant: 9.09, inner diameter: 0.35 in. and outer diameter: (a) 1.249 in., (b) 1.251 in., (c) 1.247 in., (d) 1.253 in. V_{ph}/C : (a) 0.9955, (b) 0.9805, (c) 1.011, (d) 0.9662

Fig. 10. Photograph of input (output) coupler.

Fig. 11. Photograph of accelerator module.

Fig. 12. Photograph of MICA module with three accelerator modules assembled on strong-back.

Fig. 13. Photograph of one MICA module with input and output couplers installed on the East leg of the Yale Beam Physics Laboratory beamline.

Fig. 14. View looking North along accelerator tunnel at Yale Beam Physics Laboratory, showing MIFELA (in line with green quadrupole magnets) and MICA. Just beyond MICA, the dipole spectrometer is visible.

WAKE FIELDS IN DIELECTRIC-LOADED WAVEGUIDES

Since the dielectric-loaded waveguide employed in MICA is a slow-wave structure, it will support wake fields that travel with injected charge bunches. These wake fields can influence the motion of the injected bunches, and thus modify the energy gain in MICA. There may even be regimes of operation in which wake fields cause the bunch motion to be unstable, causing break-up of the bunch and/or motion into the wall. These attributes are common to all slow-wave accelerating structures, including those in conventional rf linacs. It is thus of importance that the origin and structure of the wake fields that arise in MICA be understood. Accordingly, a study of wake fields in dielectric-lined waveguides was originated under this Phase II MICA research program, and subsequently continued under another SBIR Phase I grant from DoE. Results of this study have led to a new concept for a wake field accelerator, and for a novel source of picosecond rf pulses. These results are discussed in detail in the following publications, copies of which are enclosed.

"Stimulated dielectric wake field accelerator," by T.-B. Zhang, J. L. Hirshfield, T. C. Marshall, and B. Hafizi, *Phys. Rev. E* **56**, pp. 4647-4655 (1997).

"A Cerenkov source of high-power picosecond pulsed microwaves," by T.-B. Zhang, T. C. Marshall and J. L. Hirshfield, *IEEE Trans. Plasma Science* **26**, pp. 787-793 (1998).

"Transverse fields in dielectric wake field accelerators," by J. L. Hirshfield, S. Y. Park, and T.-B. Zhang, *Advanced Accelerator Concepts - 8th Workshop*, AIP Conf. Proc. **472**, pp. 668-675 (AIP Press, Woodbury, NY, 1999).

Stimulated dielectric wake-field accelerator

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A wake-field accelerator is described based on the use of a waveguide structure in which many modes can participate in wake-field formation, and in which the wake-field period equals the period of a train of drive bunches. A dielectric-lined waveguide is analyzed that is shown to support multimode propagation with all modes having nearly equal phase velocities, equal to the initial velocity of injected charge bunches that drive the wake fields. For this waveguide, the ratio of wake field to drag field for a bare drive bunch is 4.7, as compared to 2.0 for a single-mode waveguide. The composite TM_{0n} wake field of such a structure is shown to include highly peaked axial electric fields localized on each driving bunch in a periodic sequence of bunches. This allows stimulated emission of wake-field energy to occur at a rate that is larger than the coherent spontaneous emission from a single driving bunch of equal charge and energy. This mechanism can make possible the design of a stimulated dielectric wake-field accelerator that has the potential of providing an acceleration gradient for electrons or positrons in the range of 50–100 MV/m, taking a driving bunch charge of a few nC. We present calculations for such wake fields from a bunched sheet beam in a two-dimensional dielectric waveguide. Numerical examples are given, including the acceleration of a 30 MeV test bunch to 155 MeV in a structure 200 cm in length, using ten identical 2 nC/mm drive bunches. [S1063-651X(97)03610-6]

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I. INTRODUCTION

In the conventional dielectric wake-field accelerator, a dielectric-lined waveguide supports wake fields with longitudinal electric fields induced by the passage of an electron bunch of high charge number (the "driving bunch"). Phase velocities for the modes of dielectric-lined waveguide can be less than the speed of light [1], so that Cherenkov radiation occurs [2], manifesting itself as a wake field that fills the waveguide behind the driving bunch. If a "test" bunch of low charge number is injected at a suitable interval after the driving bunch, it can move in synchronism with the wake fields and experience net acceleration [3–5]. This approach for development of novel accelerators is appealing because no external source of rf power is required for acceleration, and because high-gradient longitudinal fields are predicted for achievable high intensity driving bunches. For example, Rosing and Gai [4] consider a 100 nC, 1.0 mm long driving bunch passing through a dielectric-lined waveguide with an inner radius of 2.0 mm and an outer radius 5.0 mm; they took the relative dielectric constant of the outer liner to be $\kappa = \epsilon/\epsilon_0 = 3.0$. For this they predict a peak wake-field accelerating gradient of $E_{z, \text{peak}} = 240$ MV/m, a value about 14 times that at the Stanford Linear Collider. Experimental confirmation of wake-field generation in a dielectric-lined waveguide has been obtained [5] using 21 MeV driving bunches of 2.0–2.6 nC and 15 MeV test bunches of much lower charge. Acceleration gradients of 0.3–0.5 MV/m were observed in the experiments, in agreement with supporting theory. Acceleration gradients in all dielectric-lined waveguides must be below the breakdown field of the dielectric [6]. This will limit achievable acceleration gradients in any dielectric-lined waveguide to a level that may not be as high as the 240 MV/m value predicted in Ref. [4].

The particular dielectric waveguide analyzed in this paper

for the Stimulated Dielectric Wake-field Accelerator (S-WAC) enjoys two uncommon virtues. The first arises because many waveguide modes can participate in wake-field formation, and these are designed to have phase velocities nearly equal to one another, and to the bunch velocity. This leads to a coherent superposition of many copropagating waveguide modes, so the net wake-field amplitude can be significantly larger than amplitudes of individual modes. The second virtue arises because the near-periodic character of the wake fields allows constructive interference of field amplitudes from successive bunches. Furthermore, the analysis given here is formulated to add to the *spontaneous* Cherenkov wake-field emission of one bunch, the *stimulated* Cherenkov emission from a train of succeeding bunches. The bunches are assumed to be identical, and each bunch is injected to move initially with near synchronism in the net wake field of prior bunches. It will be shown that stimulated emission from each trailing bunch can exceed spontaneous Cherenkov emission from a bunch moving alone. Consequently, the drag field that decelerates a "dressed" bunch can be much larger than the drag field acting on a "bare" bunch. (Terms in quotes refer to the presence or absence of decelerating wake fields from prior bunches.) Thus a dressed bunch leaves behind a stronger wake than a bare bunch, and so forth for succeeding dressed bunches. Of course, the successive wake maxima are found to be not exactly periodic, and decelerating particles can slip behind the wake-field maxima; so the cumulative superposition of wakes can be less than a sum of peak values. But the validity of building up a substantial wake-field amplitude by stimulated wake-field emission from a number of driving bunches of modest charge will be demonstrated here. It is this that is proposed as a new means for achieving high wake-field acceleration gradients, without need for bunches of exceptionally high charge.

Ruth *et al.* [7] have modeled the wake field from multiple driving bunches, assuming that successive bunches radiate wake-field energy identically. In their model, the composite wake field of a train of bunches would be a linear superposition of individual wake-field amplitudes. Synchronism of multiple driving bunches with the wake-field period has been discussed previously by Onishchenko *et al.* [8], who have experimentally observed intense radiation attributable to the progressive buildup of a strong wake field. Finally, Bane, Chen, and Wilson [9] have considered collinear wake-field acceleration generally, and show that a "transformer ratio" (i.e., the ratio of wake field to drag field for a bare bunch) greater than two is possible in multimode structures where the mode eigenfrequencies ω_n are equally spaced, with $\omega_n = \omega_0(2n+1)$; this signifies that particles can be accelerated to energies greater than twice the energy of particles in the driving bunch. In S-WAC, the multimode aspect of the dielectric waveguide chosen is a crucial factor since, as will be demonstrated below, the waveguide eigenfrequency spectrum can be designed to nearly fit the relationship $\omega_n = \omega_0(2n+1)$.

In this paper, we shall present the wake-field theory for a simplified slab geometry driven by a bunched sheet beam. Numerical examples are provided to show how a well-defined, spatiotemporally localized wake field may be produced using high dielectric constant low-loss material, providing the geometrical variables are correctly chosen. We discuss how wake-field amplitudes may be further enhanced by the superposition of contributions from successive drive bunches, together with stimulated emission. We provide a numerical example showing acceleration of test electrons from 30 to 155 MeV, in a 200 cm long two-dimensional dielectric waveguide, using 2-nC/mm drive bunches. For this example, the transformer ratio is 4.7. Further examples are given in which drive bunches are removed from the interaction once they have lost most of their energy, but before they can slip into accelerating phases and drain wake-field energy, and thereby reduce the available accelerating gradient for the test bunch.

II. WAVEGUIDE MODES AND WAKE-FIELD STRUCTURE

The model analyzed here is simplified to bring out the essential physics. Thus a two-dimensional waveguide is considered, in which two parallel slabs of dielectric are separated by a small vacuum gap, and in which the outer surfaces of the slabs are sheathed in a lossless conductor. The relative dielectric constant κ for the slabs is assumed to be independent of frequency. The geometry is depicted in Fig. 1, and all quantities are taken to be independent of y . The electrons are injected along the z axis in sheet bunches, with charge density given by $\rho(x, z, t) = -eN\delta(x)h(z - vt)$, where e is the magnitude of the electron charge, N is the charge number per unit length in the y direction along the sheet, $\delta(x)$ is the transverse charge distribution, assumed to be of infinitesimal width in x , and $h(z - vt)$ is the longitudinal charge distribution for bunch particles moving at axial speed v . Simplification is afforded when the geometry is two dimensional (2D), with orientation of the dielectric as shown in Fig. 1. In this case, orthonormal wave functions can be found for the electromagnetic fields that separate into TE^x and TM^x classes

FIG. 1. Geometry for the two-dimensional dielectric-lined waveguide.

with respect to the x axis; these are also known as LSE and LSM modes [10]. In 2D geometry, only the TM^x mode has an axial electric field; this is the mode considered here. For 3D geometry, when the waveguide shown in Fig. 1 is closed from above and below by conducting planes normal to the y axis, both TE^x and TM^x modes must be included. In cylindrical geometry, only axisymmetric excitations separate into TE and TM modes, and generally one must deal with hybrid modes [1]. For the geometry shown in Fig. 1, conditions can be found where all TM^x modes have phase velocities equal to v , corresponding to wake fields that move in synchronism with the electron bunches. The field components for the complete orthonormal TM^x mode set are given by

$$E_z(x, z, t) = \sum_{m=0}^{\infty} E_m \frac{f_m(x)}{\alpha_m} e^{-i\omega_m z_0/v}, \quad (1)$$

where

$$f_m(x) = \frac{1}{\sin p_m(b-a)} \times \begin{cases} \cosh k_m a \sin p_m(b+x), & -b \leq x \leq -a \\ \cosh k_m x \sin p_m(b-a), & -a \leq x \leq a \\ \cosh k_m a \sin p_m(b-x), & a \leq x \leq b, \end{cases} \quad (2)$$

and $z_0 = z - vt$;

$$E_x(x, z, t) = i \sum_{m=0}^{\infty} E_m \frac{g_m(x)}{\alpha_m} e^{-i\omega_m z_0/v}, \quad (3)$$

where

$$g_m(x) = \frac{\gamma}{\kappa \cos p_m(b-a)} \times \begin{cases} -\sinh k_m a \cos p_m(b+x), & -b \leq x \leq -a \\ \kappa \sinh k_m x \cos p_m(b-a), & -a \leq x \leq a \\ \sinh k_m a \cos p_m(b-x), & a \leq x \leq b, \end{cases} \quad (4)$$

and

$$H_y(x, z, t) = c\beta E_x(x, z, t) \begin{cases} \kappa, & -b \leq x \leq -a \\ 1, & -a \leq x \leq a \\ \kappa, & a \leq x \leq b. \end{cases} \quad (5)$$

The normalizing constant is

$$\alpha_m = 1 + \frac{\sinh(2k_m a)}{(2k_m a)} + \cosh^2(k_m a) \left[\frac{\kappa(b-a)}{a \sin^2 p_m(b-a)} - \frac{\tanh(k_m a)}{(k_m a)} \right];$$

$\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, the eigenfrequencies are $\omega_m = c\beta p_m / \sqrt{\kappa\beta^2 - 1}$, and $k_m = \omega_m / c\beta\gamma = p_m / \gamma \sqrt{\kappa\beta^2 - 1}$. The (evanescent) transverse wave number in the vacuum is k_m , and the (real) transverse wave number in the dielectric is p_m , and $\omega_m = c\beta k_m$. For the fields given by Eqs. (1)–(5), orthonormalization is obtained in the form

$$\int_{-b}^b dx E_{z,n} D_{z,n}^* = \delta_{mn} \frac{a}{\sqrt{\alpha_m \alpha_n}} \epsilon_0 E_m E_n \times \exp[-iz_0(\omega_m - \omega_n)/v], \quad (6)$$

where $D_{z,n}^* = \epsilon E_{z,n}^* = \kappa \epsilon_0 E_{z,n}^*$ in the dielectric slabs and $D_{z,n}^* = \epsilon_0 E_{z,n}^*$ in the vacuum gap. The dispersion relation is found to be

$$p_m \tanh(k_m a) = \kappa k_m \cot[p_m(b-a)]. \quad (7)$$

It is noted that one can have eigenfrequencies with nearly equal spacing, since $p_m(b-a) \rightarrow (n+1/2)\pi$ as $\kappa \rightarrow \infty$. As $m \rightarrow \infty$ the asymptotic eigenfrequency spacing approaches $\Delta\omega = \pi c\beta[(b-a)\sqrt{\kappa\beta^2 - 1}]^{-1}$. The wake field is more strongly peaked and more closely periodic in z_0 as the eigenfrequencies become more nearly periodic, i.e., as a higher value of κ is used.

To find wake fields induced by an electron bunch, one expands in orthonormal modes the solution of the inhomogeneous wave equation,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\kappa(x)}{c^2} \frac{\partial^2}{\partial t^2} \right] E_z(x, z, t) = S_z(x, z, t), \quad (8)$$

with the source function

$$S_z(x, z, t) = \mu_0 \frac{\partial j_z}{\partial t} + \frac{1}{\epsilon_0} \frac{\partial \rho}{\partial z},$$

where the z component of the current density is $j_z(x, z, t) = v\rho(x, z, t)$, and where $S_z(x, z, t) = 0$ for $|x| \geq a$. A complete solution can be constructed from fields as given in Eq. (1), since these are solutions of Eq. (8) with $S_z(x, z, t) = 0$ everywhere. We expand the solution of Eq. (8) in the interval $-b \leq x \leq b$ in a Fourier series:

$$E_z(x, z, t) = \sum_{m=0}^{\infty} \frac{1}{\alpha_m} f_m(x) \int_{-\infty}^{\infty} dk A_m(k) e^{-ikz_0}. \quad (9)$$

Inserting Eq. (9) into Eq. (8), and multiplying both sides by $w(x)D_{z,n}^*(x, z, t)$ gives

$$A_m(k) = \frac{1}{2\pi\alpha_m(k^2 - \omega_m^2/v^2)} \int_{-b}^b dx' \int_{-\infty}^{\infty} dz'_0 S_z(x', z'_0) \times \kappa(x') f_m(x') w(x') e^{ikz'_0}. \quad (10)$$

Then, integrating over k , with due regard for the choice of the contour of integration consistent with causality, and invoking Eq. (6) yields

$$E_z(x, z, t) = \sum_{m=0}^{\infty} \int_{-b}^b dx' \int_{-\infty}^{\infty} dz'_0 S_z(x', z'_0) G_m(x, z_0; x', z'_0), \quad (11)$$

where the Green's function $G_m(x, z_0; x', z'_0)$ is

$$G_m(x, z_0; x', z'_0) = \frac{-i\nu\kappa(x')}{2\omega_m\alpha_m a} w(x') f_m(x') f_m(x) \times e^{-i\omega_m|z_0 - z'_0|/v}, \quad (12)$$

and the weighting factor is $w(x) = [(\kappa\beta^2 - 1)^{-1} - \gamma^2, (\kappa\beta^2 - 1)^{-1}]$ in the intervals $[(-b \leq x \leq -a), (-a \leq x \leq a), (a \leq x \leq b)]$, respectively.

For a rectangular bunch $\rho(x, z, t) = -Ne\delta(x)/\Delta z$ in the interval $z_0 - \Delta z/2 \leq z \leq z_0 + \Delta z/2$, and $\rho(x, z, t) = 0$ otherwise, one finds for the coherent spontaneous wake field from a bunch containing N electrons the result

$$E_z(x, z, t) = -E_0 \sum_{m=0}^{\infty} \frac{f_m(x)}{\alpha_m} \frac{\sin(\omega_m \Delta z/2\nu)}{(\omega_m \Delta z/2\nu)} e^{-i\omega_m z_0/v}. \quad (13)$$

While for a Gaussian bunch of N electrons with $\rho(x, z, t) = [Ne\delta(x)/\Delta z] \exp[-(z_0/\Delta z)^2]$, one finds the result

$$E_z(x, z, t) = -E_0 \sum_{m=0}^{\infty} \frac{f_m(x)}{\alpha_m} e^{-(\omega_m \Delta z/2\nu)^2} e^{-i\omega_m z_0/v}. \quad (14)$$

In Eqs. (13) and (14), $E_0 = -Ne/2\epsilon_0 a$ is a measure of the Coulomb field of the bunch, and causality dictates that the results are valid only behind the bunch, i.e., for $z_0 \leq 0$; ahead of the bunch the fields are, of course, zero.

The fields for the Gaussian bunch case have been evaluated for a waveguide with $a = 0.30$ cm, $b = 1.147$ cm, $\kappa = 10.0$, $\Delta z = 3.0$ mm, $-Ne = Q = -2$ nC/mm, and $\gamma = 60$. A relative dielectric constant of $\kappa = 10.0$ is close to the value of 9.6 for alumina, the material that could be used to construct a proof-of-principle device. It is assumed in this analysis that κ is independent of frequency. For these parameters, $E_0 = -37.7$ MV/m. The wake field is computed by including modes up to $m = 50$ in the sum in Eq. (14), although beyond the 12th mode the relative amplitudes are less than 1% of that for $m = 1$. For this case, the first eigenfrequency interval $(\omega_2 - \omega_1)/2\pi$ is 5.70 GHz, while the asymptotic interval $\Delta\omega/2\pi$ is 5.88 GHz; eigenfrequency intervals differ by at most 3.1%. The computed coherent spontaneous wake-field pattern for $E_z(x=0, z_0)$ of a single bunch is shown in Fig. 2

FIG. 2. Spontaneous wake field from a -2 nC/mm, 30 MeV, 3.0 mm long Gaussian sheet bunch after it has traveled 100 cm left-to-right in the 2D waveguide.

for $0 \leq z \leq 100$ cm. The wake-field peaks are seen generally to alternate in sign; each is relatively concentrated in z and has a period of 10.5 cm, corresponding to the vacuum wavelength at 2.856 GHz, i.e., half the asymptotic frequency interval $\Delta\omega/2\pi$. The peak values of E_z for the first wake are -5.57 and $+5.54$ MV/m, and later wakes develop oscillatory precursors. Figure 3(a) shows the mode frequencies, and Fig. 3(b) shows the mode amplitudes A_m . It is clear that consideration of only the first few modes would give an incomplete picture of the wake-field structure. If the bunch length is decreased, the spectral width increases, and addi-

FIG. 3. (a) Wake-field mode frequencies; and (b) wake-field mode amplitudes.

FIG. 4. Peak amplitude of the composite axial electric field for the wake-field example shown in Figs. 2 and 3.

tional high-frequency structure develops; this is undesirable in connection with the superposition of several bunches. Also, the wider the bandwidth, the greater the influence of dielectric dispersion (neglected here). Below, we consider multiple injected bunches with parameters identical to this first bunch.

The transverse structure of the waveguide fields has also been computed for the example discussed in the preceding paragraph. Figure 4 shows the peak axial electric field $E_z(x, z_0=0)$. It is seen to be essentially independent of x in the vacuum gap, but it falls rapidly to zero in the dielectric regions. The peak transverse fields $E_x(x, z_0=z_m)$ and $H_y(x, z_0=z_m)$ have been computed from Eqs. (3) and (5), where $z_m = (2m+1)\pi\nu/2\omega_m$; these fields are depicted in Figs. 5 and 6. It is seen that the transverse fields are antisymmetric in x , essentially linear with x in the vacuum gap, but not insignificant in the dielectric regions. Figure 7 shows the Poynting vector $S_z = E_x \times H_y$, which is seen to vanish at $x=0$, to be discontinuous at the dielectric-vacuum interface, and to be largest near the conducting walls. Most of the power flow is seen to be within the dielectric, so that losses must be minimized to avoid undue heating. The wake-field power per unit height can also be found, as $P_{\text{wake}} = \int_{-b}^b dx S_z(x)$; for the example discussed, one finds $P_{\text{wake}} = 80.0$ kW/mm. The wake-field power at any point in the waveguide is maintained at this level for the full 3.3 nsec

FIG. 5. Peak amplitude of the composite transverse electric field for the wake-field example shown in Figs. 2-4.

FIG. 6. Peak amplitude of the composite transverse magnetic field for the wake-field example shown in Figs. 2-5.

transit time of the bunch. In contrast, the peak power in the 10.0 psec bunch, defined as $P_b = (mc^2/e)(\gamma-1)(Q\nu/\Delta z)$, is 6.0 GW/mm.

III. STIMULATED EMISSION OF WAKE FIELDS BY A TRAIN OF BUNCHES

The choice of the particular dielectric-lined waveguide parameters in Sec. II is seen to result in a spacing of 10.5 cm between the first few sharply peaked positive polarity wake-field features. This spacing is equal to the interbunch spacing from a typical 2.856 GHz rf injector gun, or for that matter, a rf linac driven at this frequency. Therefore, if successive bunches were injected into the dielectric waveguide, the second bunch will find itself riding just on the crest of the first decelerating wake feature generated by the first bunch. Instead of generating only a coherent spontaneous Cherenkov wake as did the first bunch, the second bunch will be decelerated in the field of the first wake, and its energy will be radiated as additional stimulated Cherenkov energy, which builds up its own wake. Successive bunches will interact similarly. To make this quantitative, one calculates the incremental energy ΔW radiated into the waveguide by a bunch in advancing a distance Δz , and equates $\Delta W/\Delta z$ to the energy loss rate of the bunch. This loss rate is identified with a drag field E_{drag} acting on the bunch. Thus,

$$QE_{\text{drag}} = \Delta W/\Delta z. \quad (15)$$

For a bare bunch, $E_{\text{drag}} = E_{\text{spon}}$, the drag field corresponding only to coherent spontaneous emission. But for a dressed bunch that follows behind N prior bunches, the drag field consists of the spontaneous drag field added to the combined wake fields of the preceding bunches. The total wake field is incremented by equating the sum of energies lost by all bunches to the change in wake-field energy, the latter being proportional to the square of the sum of wake-field amplitudes. It is also noted that perfect synchronism is assumed in the above simplified discussion, so that wake amplitudes (and not energies) are added constructively.

The rate of energy accumulation in wake fields behind any bunch (per unit height along the sheet bunch) is given by

FIG. 7. Poynting vector for the wake-field example shown in Figs. 2-6.

$$\begin{aligned} \frac{dW}{dz} &= \frac{1}{4} \sum_{m=0}^{\infty} \int_{-b}^b dx \{ \epsilon(x) [E_{x,m}^2 + E_{z,m}^2] + \mu_0 H_{y,m}^2 \} \\ &= \frac{\epsilon_0 E_0^2 b}{2} F \sum_{m=0}^{\infty} \frac{h^2(\xi_m)}{\alpha_m^2} \left\{ \frac{a}{b} \gamma^2 \left[\frac{\sinh(2k_m a)}{2k_m a} - \beta^2 \right] \right. \\ &\quad \left. + \frac{b-a}{b} \frac{\kappa}{\kappa\beta^2 - 1} \left[\frac{\cosh(k_m a)}{\sin[p_m(b-a)]} \right]^2 \right. \\ &\quad \left. \times \left[\frac{\sin[2p_m(b-a)]}{2p_m(b-a)} + \kappa\beta^2 \right] \right\}. \end{aligned} \quad (16)$$

For a rectangular bunch, the structure factor is given by $h(\xi_m) = (\sin \xi_m / \xi_m)^2$; and for the Gaussian bunch, $h(\xi_m) = \exp(-\xi_m^2)$, with $\xi_m = \omega_m \Delta z / 2\nu$. In Eq. (16), F is a scale factor fixed by balancing the energy loss rate between drag on the bunch, and increase in the wake-field energy, as described in the prior paragraph. For a bare bunch $F=1$. This scaling procedure assumes that the source current and charge distributions in Eq. (8) remain constant during the interaction. If not, the individual mode amplitudes must be adjusted iteratively; this amounts to introduction of a z -dependent structure factor $h(\xi_m)$ in equations such as Eqs. (13) and (14).

For the parameters chosen, Eq. (16) with $F=1$ gives $dW/dz = 23.6 \times 10^{-4}$ J/m mm. (Note: the mm^{-1} comes from a 1 mm height up the sheet beam.) Equating dW/dz to the energy loss rate QE_{drag} gives $E_{\text{drag}} = E_{\text{spon}} = 1.18$ MV/m for a bare bunch. The wake field induced by a bare bunch can be as high as 5.54 MV/m, as seen in Fig. 2, namely, 4.7 times the drag field. This factor, commonly referred to as the "transformer ratio," is larger than the customary factor of 2 because of the multimode nature of wake fields that can participate in this case [9].

Now, when a second bunch is introduced into the waveguide at the peak of the first bunch wake field, it can be decelerated by up to $1.18 + 5.54 = 6.72$ MV/m, the sum of its own bare drag field associated with spontaneous emission, plus the wake field of the first bunch; this produces additional stimulated emission. Second bunch deceleration at a rate of 6.72 MV/m, i.e., 5.7 times that of a bare bunch, can clearly not proceed as far as the point $z = 30/6.7 = 4.5$ m before which synchronism fails, due to slippage between the

TABLE I. Constructive superposition of wake fields for ten successive bunches in a 100 cm long dielectric waveguide, as described in the text. Initial $\gamma = 60$. The simulation result shown in the last column is at the time the first bunch has reached $z = 100$ cm.

Bunch number	$E_{\text{drag},i}$ (MV/m)	$E_{w,i}$ (MV/m)	$\Sigma_i E_{w,i}$ (MV/m)	$\Sigma E_{w,i}$ (simulation) (MV/m)
1	1.18	5.54	5.54	5.82
2	6.72	8.79	14.33	13.69
3	15.51	10.34	24.67	23.05
4	25.85	11.12	35.79	32.80
5	36.97	11.56	47.35	42.66
6	48.53	11.84	59.19	52.49
7	60.37	12.04	71.23	62.20
8	72.41	12.17	83.40	71.80
9	84.58	12.28	95.68	81.25
10	96.36	12.36	108.04	90.61

particles and the wake fields as the bunch energy is depleted. The wake field of the second bunch adds its energy to that of the first bunch to give a combined wake field of 14.33 MV/m total. This addition of amplitudes only continues as long as synchronism is maintained between bunches and peak wake fields. Highly relativistic bunches can maintain synchronism while losing a larger fraction of their initial energy, as compared with bunches of lesser energy, since for the former velocity slip is lower, i.e., $\Delta\beta \approx \Delta\gamma/\gamma^3$.

In the following simple model for the buildup of a cumulative wake field from a driving bunch train, the bunches are taken as point charges that remain perfectly synchronized with the wake fields. The energy radiated into the wake field of the n th bunch can be written in terms of the net drag on the charge Q as

$$\begin{aligned} \frac{dW_n}{dz} &= R \left[\left(\sum_{i=1}^n E_i \right)^2 - \left(\sum_{i=1}^{n-1} E_i \right)^2 \right] \\ &= Q \left[E_{\text{spont}} + \sum_{i=1}^{n-1} E_i \right] = Q E_{\text{drag}}, \end{aligned} \quad (17)$$

from which the combined wake field is found to be

$$\sum_{i=1}^n E_i = \left[\left(\sum_{i=1}^{n-1} E_i \right)^2 + \frac{Q}{R} \left(E_{\text{spont}} + \sum_{i=1}^{n-1} E_i \right) \right]^{1/2}. \quad (18)$$

The factor R is obtained from the coefficient of the electric field squared in Eq. (16); $R = 76.9$ nC/MV for the example cited. The individual wake field from the i th bunch is E_i .

Table I (except for the last column) shows a compilation from this (admittedly crude) estimate of the drag fields, individual wake fields and combined wake fields, as they would build up for ten injected bunches. The fifth column in the table lists, for purposes of comparison with the fourth column, the wake field obtained from the numerical study described in the next section. For comparison, the drag field that results when the transformer ratio is two is, for each bunch, an odd integer multiple of 1.18 MV/m, i.e., 1.18, 3.54, 5.90, 8.26 MV/m, etc. [7]. One can appreciate from

Table I that participation of many copropagating modes, and stimulated emission from a periodic sequence of driving bunches, can increase dramatically the extraction of energy from the bunches, which in turn promotes the buildup of an intense overall wake field after passage of a relatively few moderate-charge driving bunches. These results suggest that stability problems can be avoided that may attend the propagation of a single drive bunch of very high charge that is needed to produce a strong wake field on its own.

IV. NUMERICAL SIMULATIONS

The conceptual model discussed in the prior section assumes perfect synchronism between driving bunches and peak wake fields, and assumes the wake-field amplitude to be uniform over the finite spatial extent of each bunch. The problem has been examined with greater accuracy in a numerical simulation, using 100 particles per bunch, and taking slip and actual wake-field amplitude variations into account. Particles in each 3.0 mm long bunch are injected each 350 psec (10.5 cm) around the peak of the wake field from prior bunches. The energy loss rate from each bunch is given by Eq. (16), with successive values of F found from the drag field on that bunch, i.e., from the sum of its spontaneous drag field E_{spont} and the net wake field from prior bunches. Particles in a given bunch obey the one-dimensional equation of motion $d\gamma/dz = (e/mc^2)E_{\text{drag}}$, evaluated at each particle's instantaneous location, where E_{drag} is obtained from Eq. (17). The initial energy of each bunch is chosen with $\gamma_{\text{initial}} = 60$, or about 30 MeV. In this model, the bunches—now distributed spatially—will lose energy and thus can slip with respect to the wake fields. Motion in the x - y transverse plane is neglected.

In Figs. 8 and 9 are shown the results of injecting a test electron bunch of small charge into the accelerating phase of the wake field set up by the passage of ten prior driving bunches in a structure 100 cm in length, taking the initial energy of all test electrons to be about 30 MeV (i.e., $\gamma_{\text{initial}} = 60.0$). Beam loading by the test bunch is neglected, so it is assumed that the test bunch charge is very small. The parameters of this simulation are identical to those pertaining to Fig. 2. Figure 8(a) shows the buildup of the wake field from the ten driving bunches, and Fig. 8(b) shows the location of the finite-length test electron bunch to be accelerated. The test bunch (No. 11) enters behind the tenth drive bunch at the peak accelerating field, after the first drive bunch has traveled $\nu_0 t = 100$ cm. Bunch velocities are initially close to c (i.e., $0.99972c$ for $\gamma_{\text{initial}} = 60.0$). The location of the drive bunches at this instant is indicated on Fig. 8(a). As the drive bunches proceed through the 100 cm long device, the energy of the test electrons increases. In Fig. 9 we plot the energy of every other drive bunch and the energy of the accelerated test bunch (No. 11). After $\nu_0 t = 200$ cm, the 94.5 cm train of drive bunches has moved out of the structure, and the accelerated electrons are just departing. Their energy has been increased to 100 MeV, with an average acceleration gradient of about 70 MV/m. The value 70 MV/m is greater than ten times the 5.6 MV/m peak wake field of a single bunch. This shows that a stronger wake field can be produced by a multibunch train, than by a single bunch containing the total charge of all the bunches. The peak wake field in the struc-

FIG. 8. (a) Cumulative wake-field set up by ten identical successive bunches, at the time the first bunch has moved 100 cm along the waveguide. The positions of the bunches are indicated by the black dots in the figure. (b) Location of the test bunch (No. 11) in the accelerating wake field near the entrance of the waveguide, when the first bunch has moved 100 cm.

ture, 90.6 MV/m, does not exceed known theoretical [6] or experimental breakdown limits [11]. Table I shows, for comparison, that the simplified model predicts a total wake field of 108 MV/m after the 10th driving bunch. The later drive bunches show energy depletion, and indeed by $v_0 t = 120$ cm they decrease in energy so that there is first a slippage off the wake-field maximum, followed by a further slippage into the following accelerating phase, after which the drive bunch energy begins to increase again. In this way, the maximum wake-field amplitude is eroded downstream. In Fig. 10 is shown the energy spread of the accelerated electrons, about 14% over the bunch length of 3 mm, due to the variation of the accelerating wake field over the finite-extent of the test bunch; forward slippage of the test bunch is insignificant.

The wake-field power at any point in the waveguide builds up in a stepwise fashion, over a 3.3 nsec interval, to a level of 19.4 MW/mm. [This value is found by multiplying the single-bunch value of 80.0 kW/mm by the square of the ratio of peak wake-field amplitudes for ten bunches and one bunch, namely, $(90.61/5.82)^2 = 242.4$.] The 19.4 MW/mm level is maintained for a further interval of 3.3 nsec, after which it diminishes in stepwise fashion back to zero.

One can find an approximate upper limit for the ultimate energy to which a test bunch can be accelerated when it follows a train of driving bunches. This can be determined by calculating the wake-field energy available for absorption by the test bunch. For the example discussed in the preceding

FIG. 9. History of the energy of various drive bunches and the test bunch, up to the time the test bunch just leaves the waveguide.

paragraphs, the wake-field energy is 174 mJ behind the test bunch at $v_0 t = 200$ cm, i.e., when the first drive bunch is leaving the waveguide and when the test bunch energy is 101.7 MeV. If, for example, the test bunch charge were 0.5 nC, it would have absorbed 36 mJ in acquiring 72 MeV up to $v_0 t = 200$ cm, so about 138 mJ is still available. Thus the test bunch could in principal acquire another 272 MeV, for a maximum ultimate energy of 374 MeV. However, as the test bunch approaches this energy, beam loading will have depleted the wake-field energy, and the acceleration gradient will have fallen below the 56 MV/m value it experienced at $v_0 t = 200$ cm. Thus the distance necessary to reach, say, 350 MeV will be greater than 6 m.

From the results shown in Figs. 9 and 10, it is apparent that one should endeavor to eliminate driving bunches from the system after their energy has decreased so much that they begin to slip into an accelerating field. The consequence of eliminating spent driving bunches has been examined by numerically decoupling a driving bunch from the computation before reacceleration can take place. In Fig. 11, we show a

FIG. 10. Energy spread of electrons in the test bunch at $v_0 t = 200$ cm.

FIG. 11. Energy of the test electrons vs $v_0 t$ for two cases. The dotted line is for the case described in Figs. 8 and 9, but carried out for a 200 cm long waveguide, where the bunches are left in the system the entire length. The solid line is for the case where the drive bunches are eliminated after moving 100 cm along the 200 cm long waveguide.

result obtained using a 200 cm long dielectric structure, where the ten drive bunches are abruptly deflected out after traveling 100 cm. We find that the energy of the test particles increases steadily to 155 MeV after traversing the system. This compares favorably with what happens when one leaves the drive bunches in for the full 200 cm, where the test particles only reach an energy of 135 MeV. When drive bunches decrease in energy so that particle speeds are significantly reduced below c , their wake fields will not remain synchronized with the fields of the more energetic drive bunches; the net wake field could lose its spatiotemporal coherence. But, since stimulated wake-field emission is proportional to the local net field of prior bunches, this will fall rapidly once a driving bunch loses synchronism. Nevertheless, the reacceleration of drive bunches will drain wake-field energy and diminish the accelerating field available for the test bunch, as seen in Figs. 9 and 11. One cure for this is to deflect away a drive bunch when its energy falls below a certain limit, e.g., using a transverse magnetic field that is too small to deflect the orbit of the test electrons appreciably, and then using a second magnet to correct the orbit of the test bunch. We have run another case to show this, as depicted in Fig. 12, where six bunches are deflected out at $z = 65$ cm. The energy of the (initially) 30 MeV drive bunches does not fall below 3 MeV, and the test electrons reach 126 MeV at the end of the 200 cm long structure, for a mean accelerating gradient of 48 MV/m. As in the prior example, this value is seen to be more than six times the wake-field amplitude 5.6 MV/m of a single bunch, showing again that a stronger wake field can be generated by a multibunch train, than by a single bunch whose charge is equal to the total charge of all bunches in the train.

Deflection of drive bunches when their energy falls, with-

FIG. 12. History of the energy of drive bunches in a six bunch train, when the bunches are eliminated at $z = 65$ cm. The total waveguide length is 200 cm, and the average accelerating wake field produced is 42.5 MV/m.

out seriously affecting test bunches, will doubtless require careful design and control of the deflecting field, and will require slots in the dielectric waveguide for egress of spent drive bunches. An alternative is to allow drive bunches to move rectilinearly, but to deflect the wake fields into a second waveguide where the test bunch also moves rectilinearly, a scheme that resembles one suggested in Ref. [12]. Either of these approaches lends itself to multistage acceleration, in which achievement of energies of interest for high-energy particle physics experiments is possible. In any case, it is clear that collinear transport of drive bunches and test bunches will impose limitations on the net acceleration one can achieve with a wake-field accelerator, as others have already concluded [9].

V. DISCUSSION

The model presented in this paper is admittedly a simplified one, in that it is a two-dimensional waveguide and a bunched sheet beam that were discussed and analyzed. Clearly a practical arrangement will be three-dimensional, utilizing either a dielectric-lined cylindrical waveguide or a rectangular waveguide containing one or two dielectric slabs. The analysis should be extended to these cases to examine their potential for producing wake fields similar to those described in this paper. Preliminary study [13] of dispersion in a cylindrical dielectric-lined waveguide has shown that eigenmodes with phase velocities close to c will have minimum deviations in eigenfrequency spacing that are somewhat more than double the deviations for planar waveguide [cf. Fig. 3(a)]. But cylindrical waveguides may not allow transformer ratios as high as those found in this paper, since radiated wake field energy must fill a larger volume proportionally than for the two-dimensional case. The possibility of employing advanced materials with dielectric constant κ

greater than 10 should also be examined since, for short driving bunch lengths, higher κ results in sharper wake-field peaks. It also may be that a multilayer dielectric liner could be used to introduce compensation for the dispersion of a single-layered liner, but preliminary study of this question has so far yielded inconclusive results.

Significant wake-field power flow has been found for the train of ten driving bunches, namely, 19.4 MW/mm. The corresponding value for a cylindrical waveguide could exceed 100 MW, in about a dozen TM_{0m} modes ranging in frequency from 2.856 to perhaps 50 GHz, and in a pulse about 10 nsec long. Comparable power and pulse width with such a spectrum cannot be obtained from a conventional rf source, without a complex system for pulse compression and harmonic multiplication. But a limitation exists in peak and average power capability for a dielectric waveguide, on account of dielectric breakdown and bulk heating from volumetric losses.

Transverse wake fields have not been addressed in the analysis presented here. But recently, it was shown that these vanish in ideal planar geometry as $\gamma \rightarrow \infty$ [14]. For dielectric-lined cylindrical waveguides, transverse fields from a single driving bunch have been examined [3,4]; it was found that these can be weaker than in traditional disk-loaded waveguides. Nevertheless, transverse wake fields will need to be analyzed for S-WAC to determine how they build up in a multibunch train, and how they influence beam stability.

Another assumption underlying the calculation is that wake fields continue to travel with the speed of the 30 MeV particles, even when a drive bunch is significantly decelerated. In view of the highly relativistic motion and the finite

length (100 cm) along the first part of the system, this is a good assumption until the drive bunch energy has been reduced below a few MeV, as discussed above. Additionally, it would be necessary to include a variable structure factor $h(\xi_m)$ [see Eq. (16)] to account for distortions in bunch distribution when severe energy depletion sets in. On the other hand, accelerated particles always travel very close to c and, by the time 200 cm has been reached, the accelerated electrons have not slipped significantly ahead of an accelerating wake-field pulse. Extension of the theory to include beam loading by a test bunch is necessary to obtain accurate acceleration lengths. It will also be necessary to examine means to deflect away either the spent driving bunches, or to diffract away the wake field, once the driving bunches begin to slip. These improvements to the theory are needed to allow optimization of strong multibunch wake fields, with acceleration of one or more test bunches. But, the above limitations notwithstanding, it seems reasonable to expect that the simplified calculations presented in this paper can provide an adequate guide for further detailed calculations, and ultimately for the design of a convincing proof-of-principal "two-beam" stimulated dielectric wake-field accelerator experiment.

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A Cerenkov Source of High-Power Picosecond Pulsed Microwaves

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Abstract—One or more electron bunches passing along the axis of a dielectric-lined cylindrical waveguide are shown to emit picosecond pulses of high-power broad-band microwave radiation. The bunches can be generated by an S-band RF gun, and thus spaced from one another by 10.5 cm in a macropulse sequence, or a single more intense bunch can be generated using a laser-illuminated photocathode in the RF gun. Theory is developed for the excitation of TM_{0m} modes of this waveguide which propagate at the bunch velocity from Cerenkov radiation emitted by a single intense bunch. A train of picosecond coherent wakefield pulses is shown to follow the bunch, when the waveguide modes have nearly constant spacing in frequency. An example is shown for an alumina-lined waveguide with 10-nC 3–15-ps bunches having an initial energy of 6 MeV. Computations are presented of the mode spectrum of the radiation and its time structure. It is also shown that measurements of the mode spectrum, or of the energy loss of the bunch, can be used to infer the axial density profile of the bunch. Certain features of the theory are compared with the results of a preliminary experiment.

Index Terms—Cerenkov effect, dielectric-lined waveguide, electron beam, high-power wakefield.

I. INTRODUCTION

IF HIGH-ENERGY electrons pass along a channel traversing a dielectric medium, coherent millimeter and microwave radiation can be produced by the Cerenkov effect, if the electron speed is greater than the phase velocity of the waves [1]–[4]. When the electrons are in a continuous beam, linearized analysis for the fields and the self-consistent motion of the particles leads to a dispersion relation which predicts exponential growth or decay of the waves. For the nonlinear regime of particle trapping, the phase relationship between the waves and the particles will either allow the extraction of energy from the electrons and emission of radiation, or the acceleration of the particles [5], [8]. In previous analyses of this problem, the electron stream has been initially unbunched, and the radiation occurred at a discrete frequency given approximately by the intersection of the electron beam dispersion line ($\omega = \beta ck_z$) with the dielectric-loaded waveguide line. However, as we shall show in this paper, if the beam consists of a single short bunch of

electrons, or a succession of such bunches, then many modes of the structure can be excited simultaneously. By proper choice of waveguide dimensions and dielectric constant, we find that these modes can be phased so as to constructively interfere to produce a distinctive sharply pulsed Cerenkov wakefield which trails the particles as they move along the axis of the channel in the dielectric [6]. This Cerenkov wakefield, which here is embodied in the TM_{0m} modes of the structure, has a short-pulse (picosecond) time signature and megawatt-level peak power for available experimental conditions. The efficiency of this high-power microwave source depends on the waveguide geometry, length, bunch energy, and bunch size. In the example discussed below, 20% of the electron bunch energy is converted into radiation for the parameters chosen; this percentage increases as the length of the interaction increases. Broad-band picosecond microwave and millimeter-wave radiation may have applications in remote sensing and spectroscopy. Conversely, measurements of the radiated spectrum and/or bunch energy loss in a selected dielectric waveguide segment may be used to infer the axial bunch length.

In what follows, we analyze a system depicted in Fig. 1 which consists of the following elements. A thermionic or photoelectric cathode RF electron gun is taken as the source of the electron bunch or bunches. Such an RF gun is typically powered by a conventional microwave source (e.g., a klystron) which provides 5–10 MW of L- or S-band power. This gun will, at the crest of the RF field, emit pulses of electrons about 3–9 ps (1–3 mm) in length which are spaced apart by the RF period (10.5 cm for the S-band case). The energy of the electrons can be as high as 6 MeV for a 2-1/2 cell RF gun, as confirmed in experimental tests in the Yale Beam Physics Laboratory [7]. The S-band source can be pulsed at up to several hundred hertz, with a macropulse length of several microseconds. The stream of electron pulses provided by this gun (with suitable downstream focusing elements) is directed down the axis of a hollow dielectric cylinder, with outer radius R and inner radius a , coated on its outside with a conducting layer. The waveguide modes of this structure which readily interact with the on-axis electrons are of the TM_{0m} class. For simplicity in what follows, we shall assume the waves to move only along the device in the direction of the electron flow; e.g., there is no counter-traveling wave from reflections. Spent electrons can be deflected and collected at the end of the system. The axial phase velocity of the waves nearly equals the speed of light, so the radiation presumably can be coupled into free space using a gentle uptaper in the

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Fig. 1. Schematic for a Cerenkov wakefield wide-band radiation system. The dielectric-lined waveguide is used for wakefield generation. The radii of waveguide and empty hole are R and a , respectively.

inner and outer waveguide radii. However, discussion of this tapered waveguide and the resulting radiation pattern in free space is beyond the scope of this paper.

In Section II we present the wakefield theory in such a cylindrical waveguide lined with a thick shell of alumina. Alumina has excellent vacuum and mechanical properties, a high relative dielectric constant ($\sim 9-10$) and low loss. It is a ceramic which can be fabricated into the geometry we have chosen, coated on its interior to avoid charge buildup, and metalized on its exterior to provide a good outer conducting wall. In Section III, we present computational results under typical conditions for a single injected electron bunch having a charge of 10 nC that will be seen to excite fields with up to 3.3-MW peak power. Wakefield radial and axial electric field patterns, temporal power histories, and mode amplitudes are presented. It is pointed out that measurement of the mode spectrum can be used to infer the axial bunch profile, and that measurement of the energy loss of a bunch can be used to infer the effective bunch length. In Section IV, we describe a preliminary experiment in which a spectrum of modes was excited in a dielectric-lined cylindrical waveguide, using a 600-keV unbunched beam; the observed mode frequencies are compared with frequencies calculated using the theory given in Section II. In Section V, results of the paper are summarized.

II. WAKEFIELD THEORY AND RADIATION MODES

The model analyzed here is a cylindrical waveguide, consisting of dielectric material with an axisymmetric hole; the outer surface of the cylinder is coated with a low-loss conductor. The dielectric constant κ is assumed to be independent of frequency. The geometry is depicted in Fig. 1. Electrons are injected along the z axis in discrete axisymmetric bunches, with charge density given by $\rho(r, z, t) = -Ne(1/2\pi r)\delta(r)f(z - vt)$, where e is the magnitude of the electron charge, N is the total charge number in the bunch, $\delta(r)$ is the transverse

charge distribution, assumed to be of infinitesimal width in r , and $f(z - vt)$ is the longitudinal charge distribution for bunch particles moving at axial speed v . For this geometry, orthonormal wave functions can be found for the electromagnetic fields that separate into TE and TM classes. Only the TM modes have an axial electric field; these are the modes considered here. For the geometry shown in Fig. 1, conditions can be found where all TM modes have phase velocities equal to v , corresponding to wakefields that move in synchronism with the electron bunches. The field components for the complete orthonormal TM mode set are given by

$$E_z(r, z, t) = \sum_{m=0}^{\infty} E_m \frac{f_m(r)}{\alpha_m} e^{-i\omega_m z_0/v} \quad (1)$$

where

$$f_m(r) = \frac{1}{P_0(k_{2m}, R, a)} \times \begin{cases} P_0(k_{2m}, R, a)I_0(k_{1m}r), & 0 \leq r \leq a \\ P_0(k_{2m}, R, r)I_0(k_{1m}a), & a \leq r \leq R \end{cases} \quad (2)$$

and $z_0 = z - vt$

$$E_r(r, z, t) = i \sum_{m=0}^{\infty} E_m \frac{g_m(r)}{\alpha_m} e^{-i\omega_m z_0/v} \quad (3)$$

where

$$g_m(r) = \frac{1}{P_0(k_{2m}, R, a)} \times \begin{cases} \gamma P_0(k_{2m}, R, a)I_1(k_{1m}r), & 0 \leq r \leq a \\ \gamma_\kappa P_1(k_{2m}, R, r)I_0(k_{1m}a), & a \leq r \leq R \end{cases} \quad (4)$$

and

$$H_\theta(r, z, t) = c\beta E_r(r, z, t) \begin{cases} 1, & 0 \leq r \leq a \\ \kappa, & a \leq r \leq R \end{cases} \quad (5)$$

As we will see later, the field amplitude E_m is expressed by the product of the Coulomb field and a structure factor

that depends on the electron bunch size. In the above equations, $P_0(k, R, r) = J_0(kR)N_0(kr) - J_0(kr)N_0(kR)$ and $P_1(k, R, r) = J_0(kR)N_1(kr) - J_1(kr)N_0(kR)$; $J_m(x)$ and $N_m(x)$ are m th order Bessel functions of the first and second kinds, and $I_m(x)$ is the modified Bessel function; a and R are radii of the central vacuum hole and the outer waveguide wall, respectively. The normalizing constant is found to be

$$\alpha_m = \frac{1}{2} \left\{ \frac{1}{\kappa} \left(\frac{\gamma}{\gamma_\kappa} \right)^2 I_1^2(k_{1m}a) \times \left[\left(\frac{R}{a} \right)^2 \left(\frac{P_1(k_{2m}, R, R)}{P_1(k_{2m}, R, a)} \right)^2 - 1 \right] - (\kappa - 1) I_0^2(k_{1m}a) - I_1^2(k_{1m}a) \right\}$$

where $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, and $\gamma_\kappa = (\kappa\beta^2 - 1)^{-1/2}$. The (evanescent) transverse wavenumber in the vacuum is k_{1m} ; the (real) transverse wavenumber in the dielectric is k_{2m} , and $k_{1m} = \omega_m/c\beta\gamma = k_{2m}\gamma_\kappa/\gamma$. The eigenfrequencies are $\omega_m = c\beta\gamma k_{1m} = c\beta\gamma_\kappa k_{2m}$. Since all TM modes have phase velocities equal to v , we also have $\omega_m = c\beta k_{z,m}$. For the fields given by (1)–(5), orthonormalization obtains in the form

$$\int_0^R dr r E_{z,m} D_{z,n}^* = \delta_{mn} \frac{a^2}{\sqrt{\alpha_m \alpha_n}} \epsilon_0 E_m E_n \exp[-iz_0(\omega_m - \omega_n)/v] \quad (6)$$

where $D_{z,n}^* = \epsilon E_{z,n}^* = \kappa \epsilon_0 E_{z,n}^*$ in the dielectric and $D_{z,n}^* = \epsilon_0 E_{z,n}^*$ in the vacuum hole. The dispersion relation is found to be

$$\frac{I_1(k_{1a})}{I_0(k_{1a})} = \frac{\kappa k_1 J_0(k_2 R) N_1(k_2 a) - J_1(k_2 a) N_0(k_2 R)}{k_2 J_0(k_2 R) N_0(k_2 a) - J_0(k_2 a) N_0(k_2 R)} \quad (7)$$

It is noted that one can have eigenfrequencies with nearly periodic spacing, since $k_{2m}(R - a) \rightarrow (n + 1/2)\pi$ as $\kappa \rightarrow \infty$. As $m \rightarrow \infty$, the asymptotic eigenfrequency spacing approaches $\Delta\omega = \pi c\beta[(R - a)\sqrt{\kappa\beta^2 - 1}]^{-1}$. The wakefield is more strongly peaked and more closely periodic in z_0 as the eigenfrequencies become more nearly periodic, i.e., as a higher value of κ is used.

To find wakefields induced by an electron bunch, one expands in orthonormal modes the solution of the inhomogeneous wave equation

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} - \frac{\kappa(r)}{c^2} \frac{\partial^2}{\partial t^2} \right] E_z(r, z, t) = S_z(r, z, t) \quad (8)$$

with the source function $S_z(r, z, t) = \mu_0 \frac{\partial j_z}{\partial t} + \frac{1}{\epsilon_0} \frac{\partial \rho}{\partial z}$, where the z -component of the current density is $j_z(r, z, t) = v\rho(r, z, t)$, and where $S_z(r, z, t) = 0$ for $r \geq a$. A complete solution can be constructed from fields as given in (1)–(5), since these are solutions of (8) with $S_z(r, z, t) = 0$ everywhere. We expand the solution of (8) in the interval $0 \leq r \leq R$ in a Fourier integral

$$E_z(r, z, t) = \sum_{m=0}^{\infty} \frac{1}{\alpha_m} f_m(r) \int_{-\infty}^{\infty} dk A_m(k) e^{-ikz_0} \quad (9)$$

Inserting (9) into (8) and multiplying both sides by $w(r)D_{z,n}^*(r, z, t)$ gives

$$A_m(k) = \frac{1}{2\pi\alpha_m(k^2 - \omega_m^2/v^2)} \int_0^R r' dr' \times \int_{-\infty}^{\infty} dz'_0 S_z(r', z'_0) \kappa(r') f_m(r') w(r') e^{ikz'_0} \quad (10)$$

where the weighting factor is $w(r) = [-\gamma^2, \gamma_\kappa^2]$ in the intervals $[(0 \leq r \leq a), (a \leq r \leq R)]$, respectively. Then, integrating over k , with due regard for choice of the contour of integration consistent with causality, and invoking (6) yields

$$E_z(r, z, t) = \sum_{m=0}^{\infty} \int_0^R r' dr' \int_{-\infty}^{\infty} dz'_0 S_z(r', z'_0) G_m(r, z_0; r', z'_0) \quad (11)$$

where the Green's function $G_m(r, z_0; r', z'_0)$ is

$$G_m(r, z_0; r', z'_0) = \frac{-iv\kappa(r')}{2\omega_m\alpha_m a} w(r') f_m(r') f_m(r) e^{-i\omega_m|z_0 - z'_0|/v} \quad (12)$$

For a rectangular bunch $\rho(r, z, t) = -Ne(1/2\pi r)\delta(r)/\Delta z$ in the interval $z_0 - \Delta z/2 \leq z \leq z_0 + \Delta z/2$, and $\rho(r, z, t) = 0$ otherwise, one finds for the spontaneous wakefield the result

$$E_z(r, z, t) = -E_0 \sum_{m=0}^{\infty} \frac{f_m(r)}{\alpha_m} \frac{\sin(\omega_m \Delta z/2v)}{(\omega_m \Delta z/2v)} e^{-i\omega_m z_0/v} \quad (13)$$

While for a Gaussian bunch $\rho(r, z, t) = -\frac{Ne\delta(r)}{2\pi r \Delta z} \exp[-(z_0/\Delta z)^2]$, one finds

$$E_z(r, z, t) = -E_0 \sum_{m=0}^{\infty} \frac{f_m(r)}{\alpha_m} e^{-(\omega_m \Delta z/2v)^2} e^{-i\omega_m z_0/v} \quad (14)$$

In (13) and (14), $E_0 = -Ne/4\pi\epsilon_0 a^2$ is the Coulomb field of the bunch at the edge of the hole, and causality dictates that the results are valid only behind the bunch, i.e., for $z_0 \leq 0$; ahead of the bunch the fields are of course zero.

Equation (7) has been evaluated for a waveguide with $a = 0.128$ cm, $R = 1.0$ cm, and $\kappa = 10.0$. A relative dielectric constant of $\kappa = 10.0$ is close to the value of 9.6 for alumina. It is assumed that κ is independent of frequency. The phase velocity of all the TM modes corresponds to the speed of an electron bunch with $\gamma = 13$, i.e., 0.99704c. Fig. 2 shows the eigenmode frequencies for this choice of parameters. For this case, the first frequency interval $(\omega_2 - \omega_1)/2\pi = 5.32$ GHz, while the asymptotic interval $\Delta\omega/2\pi$ is 5.71 GHz. It is thus seen that conditions can be found where the eigenmodes all have equal phase velocities with nearly equal eigenfrequency spacings. The maximum deviation in frequency interval for the example shown in Fig. 2 is 6.8%, about double the value found for planar geometry [6].

Fig. 2. Wakefield mode frequencies.

Fig. 3. Axial electric field component of wakefield radiation from a 10-nC 6-MeV 1-mm-long Gaussian bunch at the instant the bunch has traveled 30 cm left to right in the cylindrical waveguide, plotted as a function of axial coordinate.

III. POWER OUTPUT AND FIELD STRUCTURE

The theoretical results of Section II were evaluated numerically for comparison with wakefield experiments conducted at Argonne National Laboratory [8]. Good agreement was obtained for the wakefield structure, intensity, and mode frequencies. In this case, only the first few modes make significant contributions, as also noted by the Argonne group.

The detailed computations presented in this section are mostly for a Gaussian bunch of 1.0-mm length, but other bunch sizes are also taken for evaluations of the mode spectrum and energy loss. The wakefield structure is examined for a total charge of 10 nC, although the results can be scaled in direct proportion to the total charge (so long as the bunch energy loss is not so great as to significantly reduce the bunch velocity). It is seen from Fig. 2 that, for this example, consideration of only the first few modes would give an incomplete picture of the wakefield structure. Therefore, the wakefield is computed here by including modes up to $m = 50$ in the sum in (14), even though modes with the higher indices will make smaller contributions. For $Q = 10$ nC, the Coulomb field $E_0 = 54.93$ MV/m. The computed spontaneous wakefield pattern of a single bunch is shown in Fig. 3 for $0 \leq z_0 \leq 30$ cm, i.e., at the instant that the bunch has traveled 30 cm into the waveguide. The wakefield peaks are seen generally to alternate in sign, to each be relatively concentrated in z_0 , and

Fig. 4. Radial distribution of axial electric field at several axial locations within the first half wakefield period behind the bunch, at the instant the bunch has traveled 30 cm left to right for the example in Fig. 3. It is seen that the field maximum at points behind the bunch advances outward from the axis toward the wall, where it is reflected with a reversal in polarity, and then advances back toward the axis. This "conical" field pattern for guided Cerenkov radiation is reminiscent of that for unguided Cerenkov radiation where of course no wall reflection occurs.

to have a period of 10.5 cm, corresponding to the vacuum wavelength at 2.856 GHz, i.e., half the asymptotic frequency interval $\Delta\omega/2\pi$. The peak values of E_z for the first wake at the bunch and at 5.2 cm behind the bunch are +20.51 and -19.40 MV/m, and later wakes develop secondary oscillations and are somewhat smaller. The waveguide length chosen is to allow a moderate 20% energy loss from the bunch, as will be discussed below. The radial distribution of axial electric field at different axial locations within the first half wake period is shown in Fig. 4. This figure shows that when the electron bunch passes through the waveguide, the emitted radiation travels in a pulse moving at an angle with the waveguide axis until it reflects from the wall and heads back to the axis, forming a radiation cone reminiscent of a Cerenkov cone in free space. This reflection of the pulse from the wall back to the axis repeats periodically and is responsible for the periodic sharp peaks in axial wakefield shown in Fig. 3.

Fig. 5 shows the instantaneous (time-dependent) power structure, computed as

$$P = 2\pi \int_0^R dr \bar{r} \left(\sum_m \vec{E}_m \right) \times \left(\sum_n \vec{H}_n \right) \cdot \hat{e}_z. \quad (15)$$

It is noted that terms with $m \neq n$ contribute to the instantaneous power and are responsible for the sharp spikes in Fig. 5. In the time-averaged power, only terms with $m = n$ contribute. For this example, the peak power is 3.3 MW and the average power is 1.23 MW. Fig. 5(a) shows the full instantaneous power pulse, which occupies a time duration of 1 ns. Fig. 5(b) shows an expanded plot of the sharp power spike near the center of the pulse, indicating the 3.3-MW peak with a width of 3 ps. Such unusual high-power picosecond pulses could find application in time-dependent spectroscopy or remote sensing.

The mode spectrum of the radiation is sensitive to the bunch length and profile. Shown in Fig. 6 is the wakefield mode spectrum for a Gaussian bunch with a total charge $-Ne = Q = -10$ nC, for several bunch lengths $\Delta z = 1, 2$, and 3 mm. If the bunch length is increased the mode spectrum

Fig. 5. (a) Wakefield power distribution from a 10-nC 6-MeV 1-mm-long Gaussian bunch at the instant it has traveled 30 cm left to right in the cylindrical waveguide. (b) An extended plot of the sharp power spike near the center of the pulse, i.e., about 16 cm behind the bunch, plotted as a function of time.

Fig. 6. Wakefield power spectrum for three axial bunch lengths, for 10-nC 6-MeV Gaussian bunches.

narrows, with diminished amplitudes for the higher frequency modes. Also, the narrower the bandwidth, the less dielectric dispersion (neglected here) will matter. The strong dependence of the width of the mode spectrum on bunch length could be the basis for a passive, nonintercepting diagnostic tool for monitoring bunch size for relativistic electron beams. Profiles with non-Gaussian shapes will have different mode spectra.

Now we turn to considerations of energy loss by the bunch. The rate of energy accumulation with distance in wakefields

behind the bunch dW/dz is given by integrating the field energy density over the waveguide cross section, thus leading to the following relation:

$$\begin{aligned} \frac{dW}{dz} = & \frac{1}{4} \sum_{m=0}^{\infty} \int_0^{2\pi} d\theta \int_0^R r dr \{ \epsilon(r) \\ & \times [E_{r,m}^2 + E_{z,m}^2] + \mu_0 H_{\theta,m}^2 \} \\ = & \frac{\epsilon_0 E_0^2 \pi R^2}{2} \sum_{m=0}^{\infty} \frac{h^2(\xi_m)}{\alpha_m^2} \left\{ \frac{1}{R^2} \int_0^a I_0^2(k_{1m}r) r dr \right. \\ & + (1 + \beta^2) \gamma^2 \frac{1}{R^2} \int_0^a I_1^2(k_{1m}r) r dr \\ & + \kappa \left(\frac{I_0(k_{1m}a)}{P_0(k_{2m}, R, a)} \right)^2 \left[\frac{1}{R^2} \int_a^R P_0^2(k_{2m}, R, r) r dr \right. \\ & \left. \left. + (1 + \kappa\beta^2) \frac{\gamma_k^2}{R^2} \int_a^R P_0^2(k_{2m}, R, r) r dr \right] \right\}. \quad (16) \end{aligned}$$

The source current and charge distributions in (8) are assumed to remain constant during the interaction. If not, the individual mode amplitudes must be adjusted iteratively; this amounts to introduction of a z -dependent structure factor $h(\xi_m)$ in equations such as (13) and (14). For the rectangular bunch, the structure factor $h(\xi_m) = (\sin \xi_m / \xi_m)^2$; and for the Gaussian bunch, $h(\xi_m) = \exp(-\xi_m^2)$, with $\xi_m = \omega_m \Delta z / 2v$. For the parameters chosen and $Q = 10$ nC, (16) gives $dW/dz = 4.01 \times 10^{-2}$ J/m. Equating dW/dz to the energy loss rate QE_{drag} gives $E_{\text{drag}} = 4.01$ MV/m for a single bunch. This drag field will slow down the 6-MeV bunch to 4.80 MeV within 30 cm. For this example, the efficiency for transfer of bunch kinetic energy into radiation energy is seen to be 20%. If the waveguide were to extend to 100 cm in length, this drag field would transfer 4 MeV of beam energy into wakefield radiation, giving an efficiency of 67%. Larger energy loss by the bunch will result in reduction in the phase velocities of the wakefield modes that are excited, with a corresponding change in the wakefield pattern.

Measurement of the energy loss of a bunch traversing such a dielectric waveguide can also be used as a bunch length diagnostic, if the total bunch charge is known. This is illustrated in Fig. 7, which for a 10-nC Gaussian bunch shows the energy loss in traversing a 30-cm length of dielectric waveguide as a function of the bunch width Δz . It is seen that energy loss is a strongly decreasing function of increasing bunch length. This diagnostic method could be more easily implemented than the measurement of radiation spectrum, since the latter requires careful calibration of the detectors over a wide frequency range, while the former requires knowledge of only the bunch energy before and after traversing the dielectric waveguide. Of course, the spectrum can in principal be inverted to give the actual axial bunch profile, rather than only the effective width.

IV. AN EXPERIMENTAL TEST OF CERTAIN FEATURES OF THIS THEORY

A preliminary experiment was carried out to excite Cerenkov radiation in a cylindrical tube made of alumina,

Fig. 7. Energy loss by Cerenkov radiation as a function of axial bunch length for a 6-MeV 10-pC Gaussian bunch traversing a 30-cm-long waveguide. It is seen that measurement of the energy loss can be a sensitive diagnostic for inferring the axial bunch length.

Fig. 8. Schematic of an experiment demonstrating wakefield generation in an alumina-loaded cylindrical waveguide. An initially unbunched beam was used in this experiment, but the observed intense radiation suggests that strong bunching of beam developed during its passage along the waveguide.

using an unbunched beam. The schematic of the experiment is shown in Fig. 8. The alumina tube has an outer diameter of 32 mm and fits into a copper pipe having an inner diameter of 33 mm; the axial hole through the alumina cylinder is 9 mm in diameter. The alumina cylinder is 110 cm long, and the electron beam is passed along a length of 85 cm before it is deflected to the wall by distorting the guide magnetic field using an iron "beam stop." An otherwise uniform guide magnetic field of approximately 0.9 T is provided for electron beam guiding and focusing. The electron beam has a diameter of about 3 mm and carries a current of about 100 A. The beam is extracted from a cold-cathode diode made of graphite, which field-emits when a 600-kV negative pulse is applied to the cathode. Only a small portion of the emitted current is passed through an aperture in the anode, a procedure which has been shown to improve the beam quality [9]. The diode is attached to a pulse-line accelerator, which applies a 150-ns pulse of high voltage to the cathode. A pressure below 10^{-6} torr is obtained in the diode and in the hole through which the beam propagates. Radiation exits the waveguide into air through a plexiglass window which is cemented onto the end of the alumina cylinder.

Radiation was initially picked up by several detectors connected to sections of waveguide used as high-pass filters.

Fig. 9. Power spectrum of the emitted wakefield radiation observed in the experiment shown in Fig. 8.

TABLE I
COMPARISON OF THEORETICAL AND EXPERIMENTAL MODE FREQUENCIES

Mode number	f_6	f_7	f_8	f_9
Theoretical (GHz)	23.96	28.32	32.68	37.06
Experimental (GHz)	23	27	31	35

The strongest signals were obtained at S-band, for which the TM_{01} mode should match the electron speed ($0.97c$) at 2.9 GHz. From the geometry of the system and the calibration of the detectors, it is estimated that megawatt levels of radiation were observed for pulses lasting <100 ns. However, it was found that the radiated power could be propagated in X- and K-band waveguides as well. This suggested that higher modes were being excited, so a quasi-optical grating spectrometer was installed to study the spectrum of the radiation at wavelengths shorter than 12 mm [10]. An example of this spectrum is shown in Fig. 9. The horizontal axis is labeled in frequency units, with the frequencies of the peaks indicated; the spectrometer was calibrated against a 24-GHz source. The data points reflect averages with a standard deviation of roughly 30%. In addition to the peak frequencies indicated on this figure, another peak of radiation was observed at 39 GHz, but very little beyond 40 GHz. The spacing of the modes is seen to be about 4 GHz.

In Table I we compare the observed peak frequencies radiated versus the mode frequencies predicted from the dispersion relation (7) of Section II using a dielectric coefficient of 10 for alumina. The frequency spacing predicted was about 4.3 GHz, and the mode number is also indicated. The theory appears to be in relatively good agreement with the experimental results for the mode frequencies.

It is not unreasonable to ask why this system should radiate high power at the discrete frequencies as predicted by theory, since the beam is initially unbunched whereas the theory is appropriate to either one bunch or to a sequence of bunches. We point out that the observed level of power radiated cannot be explained without the presence of considerable bunching. Evidently a linear beam Cerenkov instability is present that leads to rapid bunching. While the bunching is probably not

nearly so distinct as used in the theory, it is apparently enough to excite intense power in a mode spectrum up to $m \sim 9$. Indeed, a recently reported free-electron laser experiment has found direct evidence of tight spatial beam bunching when the emitted power reaches saturation [11]. Thus it seems reasonable to compare the results of this experiment with the theory of Section II, but we emphasize that this experiment does not check other features of this theory, such as the wakefield spatial structure or the time structure of the emitted power.

V. CONCLUSION

By exciting a dielectric-loaded waveguide system with short bunches of electrons, theory predicts that high-power microwave pulses of Cerenkov radiation are emitted with picosecond-scale time variations. This occurs because the short bunches excite a highly localized wakefield which trails the electron bunch in the waveguide. The wakefield is made possible by the coherent superposition of many discrete TM waveguide modes. We have studied the example of a 6-MeV 10-nC bunch, which can be obtained from a conventional RF gun driven by a high-power pulsed klystron at 2.856 GHz. The example showed that 20% of the kinetic energy in the pulse can be converted into radiation, with up to 3.3-MW peak power having width of a few picoseconds. This is achieved using a short section of waveguide, and only a single bunch is analyzed. When more than one bunch is present in the waveguide, much higher radiation power may be reached. To our knowledge, this is the first time that this approach to the generation of picosecond megawatt-level microwave power has been discussed. This radiation could find applications in material studies or remote sensing. Diagnostic methods to determine the axial bunch profile or the effective axial bunch width are also suggested by the theory. We have not dealt with the problem of extraction of the power pulse from the dielectric waveguide into free space, for actual application of the power.

A preliminary experiment has also been described, using an initially unbunched beam. The beam was found to be capable of exciting some of the waveguide modes with eigenfrequencies close to those calculated for the dielectric waveguide. The observation of high-power output suggests that the beam

becomes bunched, although probably not as highly bunched as in the case we analyze that uses picosecond bunches.

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T.-B. Zhang, photograph and biography not available at the time of publication.

T. C. Marshall, photograph and biography not available at the time of publication.

J. L. Hirshfield, photograph and biography not available at time of publication.

Transverse Fields in Dielectric Wake Field Accelerators

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Abstract. Theory is presented for excitation of hybrid electric/magnetic (*HEM*) wake-field modes by passage of an electron drive bunch in a dielectric-lined cylindrical waveguide. The drive bunch is moving parallel to the waveguide axis, but is displaced by a radial increment r_0 . Knowledge of the amplitudes of all *HEM* modes allows calculation of the transverse forces on a bunch or bunches that follow the displaced drive bunch. Approximate formulae for the transverse forces on a trailing bunch are given, valid in the limit of small r_0 . These transverse forces can lead to beam instability, if cumulative transverse motion is significant on the time-scale of passage of the bunch through the accelerator module. Constructive interference can be present amongst TM_{0m} monopole modes that produce highly-peaked spatiotemporally-localized axial wake-fields, with peak fields at the locations of trailing bunches in a multi-bunch train. In this case, the spectrum of dipole (and higher order) *HEM* modes may not enjoy the same degree of constructive interference, and may not have the same axial periodicity as the monopole modes. Further analytical and computational study is needed to determine the limitations that transverse wake fields may impose on a multi-bunch dielectric wake-field accelerator.

INTRODUCTION

Elsewhere in this volume, Marshall, Zhang and Hirshfield (1) present analysis for the axisymmetric wake fields induced by a train of drive bunches moving along the axis of a dielectric-lined cylindrical waveguide. Formulation of wake field theory for this structure is carried out for cases where many TM_{0m} waveguide modes can participate in wake-field formation. The waveguide is designed so that, with phase velocities equal to the bunch velocity and to one another, the excited modes have nearly equally-spaced eigenfrequencies. This leads to a strongly-peaked spatiotemporal superposition of many co-propagating waveguide modes, resulting in a net wake-field amplitude that can be much larger than the amplitudes of individual modes. When the nearly periodic highly-peaked wake-fields have the same period as the bunches, constructive interference occurs between fields from successive bunches, so that stronger wake-fields are stimulated than would be the case for a bunch not influenced by synchronous fields from bunches ahead of it. These considerations lead to a new approach to dielectric wake-field accelerators, in which a train of moderate-charge drive bunches is used to build up a strong cumulative wake field that can provide high-gradient acceleration for a following test bunch. In an example discussed in (1), an unloaded acceleration gradient of 133 MV/m is predicted using three 2 nC drive bunches in an appropriately-designed dielectric-lined waveguide.

But one concern, as with any accelerating structure, is orbit instability driven by transverse forces that arise from non-axisymmetric excitations. That concern is addressed in this paper, in which theory is developed for excitation of hybrid *HEM* modes excited by a non-axisymmetric bunch, or train of such bunches. Theories along this line have been developed heretofore, including those by Rosing and Gai (2), and by Ng (3), but these authors do not agree with one another on some important issues. These prior theories are not based on expansion of the wave equation in a sequence of orthonormal waveguide modes, so their treatment of multi-mode, multi-bunch effects may be more cumbersome than by use of the normal-mode expansion method that is expounded in this paper. Here we present an outline of the theory, leading to compact formulae for the transverse force on a test bunch that follows behind an non-axisymmetric drive bunch. This result can easily be generalized to a train of drive bunches, under conditions where the wake-fields interfere constructively. The purpose of this work is to provide the theoretical framework to allow scrutiny of beam instability driven by off-axis excursions of the drive bunches, especially for the case of a synchronous multi-bunch train.

TRANSVERSE WAKEFIELD THEORY

The model analyzed here is a cylindrical waveguide, consisting of a dielectric cylinder with an axisymmetric hole, whose outer surface is coated with a low-loss conductor. The dielectric constant κ is assumed to be independent of frequency. Radii of the central vacuum hole and the outer waveguide wall are a and R , respectively. For axisymmetric excitations from a bunch centered on the waveguide axis, orthonormal wave functions can be found for the electromagnetic wake fields that separate into *TE* and *TM* classes. However, the general solution with a non-axisymmetric source, such as an off-axis bunch, does not so separate, and the orthonormal wave functions are combination of *TE* and *TM* modes, often called hybrid electromagnetic (*HEM*) modes. The field solutions and dispersion relation (but not the orthogonality relation) were obtained by Chang and Dawson (4). The eigenfunctions for the complete set of orthonormal *HEM* modes in the hole and dielectric regions are given by

$$E_{z,l}(r, \theta, k_z) = P_l(k_2 a) I_l(k_1 r) \cos l\theta \quad (1)$$

$$E_{r,l}(r, \theta, k_z) = i \frac{k_z}{k_1} \left[P_l(k_2 a) I'_l(k_1 r) + \beta e_l \frac{l}{k_1 r} Q_l(k_2 a) I_l(k_1 r) \right] \cos l\theta \quad (2)$$

$$E_{\theta,l}(r, \theta, k_z) = -i \frac{k_z}{k_1} \left[\frac{l}{k_1 r} P_l(k_2 a) I_l(k_1 r) + \beta e_l Q_l(k_2 a) I'_l(k_1 r) \right] \sin l\theta \quad (3)$$

$$B_{z,l}(r, \theta, k_z) = e_l Q_l(k_2 a) I_l(k_1 r) \sin l\theta \quad (4)$$

$$B_{r,l}(r, \theta, k_z) = i \frac{k_z}{k_1} \left[\beta \frac{l}{k_1 r} P_l(k_2 a) I_l(k_1 r) + e_l Q_l(k_2 a) I'_l(k_1 r) \right] \sin l\theta \quad (5)$$

$$B_{\theta,l}(r, \theta, k_z) = i \frac{k_z}{k_1} \left[\beta P_l(k_2 a) I_l'(k_1 r) + e_l \frac{l}{k_1 r} Q_l(k_2 a) I_l(k_1 r) \right] \cos l\theta \quad (6)$$

for $0 \leq r \leq a$, and

$$E_{z,l}(r, \theta, k_z) = I_l(k_1 a) P_l(k_2 r) \cos l\theta \quad (7)$$

$$E_{r,l}(r, \theta, k_z) = -i \frac{k_z}{k_2} \left[I_l(k_1 a) P_l'(k_2 r) + \beta e_l \frac{l}{k_2 r} I_l(k_1 a) Q_l(k_2 r) \right] \cos l\theta \quad (8)$$

$$E_{\theta,l}(r, \theta, k_z) = i \frac{k_z}{k_2} \left[\frac{l}{k_2 r} I_l(k_1 a) P_l(k_2 r) + \beta e_l I_l(k_1 a) Q_l'(k_2 r) \right] \sin l\theta \quad (9)$$

$$B_{z,l}(r, \theta, k_z) = e_l I_l(k_1 a) Q_l(k_2 r) \sin l\theta \quad (10)$$

$$B_{r,l}(r, \theta, k_z) = -i \frac{k_z}{k_2} \left[\kappa \beta \frac{l}{k_2 r} I_l(k_1 a) P_l(k_2 r) + e_l I_l(k_1 a) Q_l'(k_2 r) \right] \sin l\theta \quad (11)$$

$$B_{\theta,l}(r, \theta, k_z) = -i \frac{k_z}{k_2} \left[\kappa \beta I_l(k_1 a) P_l'(k_2 r) + e_l \frac{l}{k_2 r} I_l(k_1 a) Q_l(k_2 r) \right] \cos l\theta \quad (12)$$

for $a \leq r \leq R$. All field components propagate as $e^{i(\omega t - k_z z)}$. The transverse wave number in the hole and the dielectric regions are $k_1^2 = k_z^2 - k_0^2$, and $k_2^2 = \kappa k_0^2 - k_z^2$, with $k_0 = \omega/c$; $P_l(k_2 r) = J_l(k_2 R) N_l(k_2 r) - N_l(k_2 R) J_l(k_2 r)$; $J_l(x)$ and $N_l(x)$ are l -th order Bessel

functions of the first and second kinds; $I_l(x)$ is the modified Bessel function; $Q_l(k_2 r) = N_l'(k_2 R)J_l(k_2 r) - J_l'(k_2 R)N_l(k_2 r)$; $\beta = k_o/k_z$; e_l is a constant which relates the field components E_z and B_z , i. e.,

$$e_l = -l\beta \left(\frac{1}{x_1^2} + \frac{\kappa}{x_2^2} \right) \frac{P_l(x_2)}{Q_l(x_2)} \left[\frac{I_l'(x_1)}{x_1 I_l(x_1)} + \frac{Q_l'(x_2)}{x_2 Q_l(x_2)} \right]^{-1} \quad (13)$$

where $x_1 = k_1 a$ and $x_2 = k_2 a$.

The dispersion relation is

$$\left[\frac{I_l'(x_1)}{x_1 I_l(x_1)} + \kappa \frac{P_l'(x_2)}{x_2 P_l(x_2)} \right] \left[\frac{I_l'(x_1)}{x_1 I_l(x_1)} + \frac{Q_l'(x_2)}{x_2 Q_l(x_2)} \right] = l^2 \beta^2 \left(\frac{1}{x_1^2} + \frac{\kappa}{x_2^2} \right)^2. \quad (14)$$

The first bracket represents *TM* modes, while the second represents *TE* modes. For the axial symmetric modes $l = 0$, and two decouple; otherwise, they are coupled hybrid *HEM* modes with, in general, six interdependent non-zero field components in each region. Eqs. 1-14 are all consistent with results obtained by Chang and Dawson (4).

We have confirmed by a detailed proof that a general orthogonality relation given by Collins (5) is applicable for any cylindrical waveguide with perfectly conducting walls and any number of annular dielectric layers, i. e.,

$$\int_0^R dr r \int_0^{2\pi} d\theta \left[E_{r,lm} B_{\theta,l'n}^* - E_{\theta,lm} B_{r,l'n}^* \right] = \delta_{ll'} \delta_{mn} \alpha_{lm}^2. \quad (15)$$

Eq. 15 states that power flow at each frequency occurs mode-by-mode, without mixing between field components of different modes. The normalization factor $\alpha_{lm}(\omega, k_{zm})$ is

$$\alpha_{lm}^2 = \pi a^2 \left\{ \left(\frac{k_{zm}}{k_{1m}} \right)^2 \left[\beta \left(P_l^2(x_{2m}) + e_l^2 Q_l^2(x_{2m}) \right) C_{I,lm} + (1 + \beta^2) e_l P_l(x_{2m}) Q_l(x_{2m}) D_{I,lm} \right] \right. \\ \left. + \left(\frac{k_{zm}}{k_{2m}} \right)^2 I_l^2(x_{1m}) \left[\beta \left(\kappa C_{P,lm} + e_l^2 C_{Q,lm} \right) + (1 + \kappa \beta^2) e_l D_{PQ,lm} \right] \right\} \quad (16)$$

where

$$C_{I,lm} = \frac{1}{x_{1m}^2} \int_0^{x_{1m}} dx x \left\{ I_l'^2(x) + \left[\frac{l}{x} I_l(x) \right]^2 \right\}; \quad D_{I,lm} = \frac{l}{x_{1m}^2} I_l^2(x_{1m})$$

$$C_{P,lm} = \frac{1}{x_{2m}^2} \int_{x_{2m}}^{y_{2m}} dx x \left\{ P_l'^2(x) + \left[\frac{l}{x} P_l(x) \right]^2 \right\};$$

$$C_{Q,lm} = \frac{1}{x_{2m}^2} \int_{x_{2m}}^{y_{2m}} dx x \left\{ Q_l'^2(x) + \left[\frac{l}{x} Q_l(x) \right]^2 \right\};$$

$$D_{PQ,lm} = \frac{1}{x_{2m}^2} [Q_l(y_{2m})P_l(y_{2m}) - Q_l(x_{2m})P_l(x_{2m})]; \quad x_{1m} = k_{1m}a, \quad x_{2m} = k_{2m}a \quad \text{and}$$

$y_{2m} = k_{2m}R$. The above integrals all reduce to standard forms, but writing out the full result would occupy more space than is justified. Prior analyses of dielectric-lined cylindrical waveguides do not give an orthogonality proof or a normalization constant.

To find wake fields induced by an electron bunch, one expands in orthonormal modes the solution of the inhomogeneous wave equations for E_r and E_θ , namely

$$\left[\nabla^2 - \frac{\kappa(r)}{c^2} \frac{\partial^2}{\partial t^2} \right] E_r - \frac{2}{r^2} \frac{\partial E_\theta}{\partial \theta} - \frac{1}{r^2} E_r = S_r, \quad (17)$$

$$\left[\nabla^2 - \frac{\kappa(r)}{c^2} \frac{\partial^2}{\partial t^2} \right] E_\theta + \frac{2}{r^2} \frac{\partial E_r}{\partial \theta} - \frac{1}{r^2} E_\theta = S_\theta \quad (18)$$

with the source functions $S_r(\mathbf{r}, t) = 4\pi \partial \rho(\mathbf{r}, t) / \partial r$ and $S_\theta(\mathbf{r}, t) = 4\pi \partial \rho(\mathbf{r}, t) / r \partial \theta$. These forms assume that the beam current is flowing in only in the z -direction; of course $S(\mathbf{r}, t) = 0$ for $r \geq a$. A complete solution can be constructed from fields as given in Eq. 1-12, since these are solutions of Eq. 17 and Eq. 18 with $S(\mathbf{r}, t) = 0$ everywhere. To proceed, we expand the solutions of Eq. 17 and Eq. 18 in the interval $0 \leq r \leq R$ in a Fourier integral.

$$\begin{Bmatrix} E_r(\mathbf{r}, t) \\ E_\theta(\mathbf{r}, t) \end{Bmatrix} = \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} d\omega A_{lm}(k, \omega) \begin{Bmatrix} E_{r,lm}(r, k, \omega) \cos l\theta \\ E_{\theta,lm}(r, k, \omega) \sin l\theta \end{Bmatrix} e^{-i(kz - \omega t)}. \quad (19)$$

Inserting Eq. 19 into Eqs. 17 and 18, multiplying Eq. 17 by $B_{\theta, l'n}^*(r, k', \omega') \cos l'\theta e^{i(k'z - \omega't)}$ and Eq. 18 by $B_{r, l'n}^*(r, k', \omega') \sin l'\theta e^{i(k'z - \omega't)}$, then combining the two equations, taking the integral over $d^3\mathbf{r} dt$ and invoking the orthogonality relation Eq. 15, yields for the Fourier amplitude the result

$$A_{lm}(k, \omega) = \frac{-1}{4\pi^2 \alpha_{lm}^2 (k^2 - k_{zm}^2)} \times \iiint d^3 \mathbf{r}' \int_{-\infty}^{\infty} dt' [S_r(\mathbf{r}', t') B_{\theta, lm}^*(r', k, \omega) \cos l\theta' - S_{\theta}(\mathbf{r}', t') B_{r, lm}^*(r', k, \omega) \sin l\theta'] e^{i(kz' - \omega t')} \quad (20)$$

For the balance of this paper, we specialize to the case of a point charge displaced from the axis by an increment r_0 , so that the charge density is given by $\rho(\mathbf{r}, t) = Q_0(1/r) \delta(r - r_0) \delta(\theta) \delta(z - vt)$. In this instance, we find for the radial electric field

$$E_r(\mathbf{r}, t) = \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} A_{lm} E_{r, lm}(r, k_{zm}) \cos l\theta e^{-ik_{zm}z_0} \quad (21)$$

where $z_0 = z - vt$. The amplitude of all the field components (see Eqs. 1-12) is given by

$$A_{lm} = \frac{2\pi Q_0}{a^2} \left(\frac{a^2}{\alpha_{lm}^2} \right) b_{lm}(r_0, k_{zm}) \quad (22)$$

where

$$b_{lm}(r_0, k_{zm}) = \frac{i}{k_{zm}} \left[\frac{\partial}{\partial r} B_{\theta, lm}^*(r, k_{zm}) \right]_{r=r_0} + \frac{1}{r_0} B_{\theta, lm}^*(r_0, k_{zm}) - \frac{l}{r_0} B_{r, lm}^*(r_0, k_{zm}) \quad (23)$$

This result, Eqs. 21-23, together with the other five field components, constitutes a complete formal solution for all the wake-field modes excited by a point charge moving with an axial velocity v , and displaced from the axis of the waveguide by a radial increment r_0 .

For judging beam stability, one begins by calculating the transverse forces on a bunch with charge q that follows a drive bunch whose wake-fields are given above. The results are

$$F_r(\mathbf{r}, t) = q[E_r(\mathbf{r}, t) - \beta B_{\theta}(\mathbf{r}, t)] \\ = \frac{q}{\gamma} \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} A_{lm} P_l(k_{2m}a) I_l'(k_{1m}r) \cos l\theta \sin k_{zm}z_0 \quad (24)$$

and

$$F_{\theta}(\mathbf{r}, t) = q[E_{\theta}(\mathbf{r}, t) + \beta B_r(\mathbf{r}, t)]$$

$$= -\frac{q}{\gamma} \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} A_{lm} P_l(k_{2m}a) \frac{l}{k_{1m}r} I_l(k_{1m}r) \sin l\theta \sin k_{zm}z_0 \quad (25)$$

The full significance of these results cannot be known without numerical computations using parameters for dielectric wake-field accelerators of interest. These computations are beyond the scope of this paper. However, some insight into the results can be inferred by considering the limit of a small radial displacement r_0 for the drive bunch, and small displacements r for a following test bunch. If one inserts Eqs. 5 and 6 for the eigenmodes $B_{r,lm}$ and $B_{\theta,lm}$ in Eq. 23, and expands the result for small beam displacement, one has

$$b_{lm}(r_0, k_{zm}) \approx \frac{1}{2} \frac{1}{(l+1)!} [(l+2)\beta P_l(k_{2m}a) - le_l Q_l(k_{2m}a)] (k_{1m}r_0/2)^l \quad (26)$$

This allows an estimate to be made of the forces associated with each azimuthal order of the HEM_{lm} eigenmodes. Thus, for $l=0$, where the eigenmode is actually TM_{0m} , one has to lowest order in r_0 the results

$$F_{r,0m}(\mathbf{r}, t) = F_0 \sum_{m=1}^{\infty} c_{0m} (k_{1m}r/2) \sin(k_{zm}z_0) \quad (27)$$

and
where,

$$F_{\theta,0m}(\mathbf{r}, t) = 0 \quad (28)$$

$$F_0 = \frac{2\pi q Q_0}{a^2}, \text{ and } c_{0m} = \frac{1}{\gamma} \left(\frac{a^2}{\alpha_{0m}^2} \right) \beta P_0^2(k_{2m}a).$$

One notes that Eq. 27 does not depend upon r_0 , and that, mode-by-mode, the radial force is a quarter-cycle in z_0 out of phase with the axial electric field, which varies as $\cos k_{zm}z_0$. However, the train of drive bunches in the multi-bunch dielectric wake-field accelerator are spaced by a full cycle in z_0 , and the bunch to be accelerated is spaced by a half-cycle from the last drive bunch. Thus, one can conclude that, to lowest order, the radial force from the TM_{0m} modes of a slightly displaced driving bunch will not be felt by following bunches.

The radial and azimuthal forces for $l \geq 1$, for small displacement r_0 of the drive bunch, and for small deviations r of the test bunch from the axis, are

$$F_r(\mathbf{r}, t) = F_0 \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} c_{lm} \left(\frac{k_{1m}r_0}{2} \right)^l \left(\frac{k_{1m}r}{2} \right)^{l-1} \cos l\theta \sin k_{zm}z_0, \quad (29)$$

and

$$F_{\theta}(\mathbf{r}, t) = -F_0 \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} c_{lm} \left(\frac{k_{1m} r_0}{2} \right)^l \left(\frac{k_{1m} r}{2} \right)^{l-1} \sin l\theta \sin k_{zm} z_0, \quad (30)$$

where

$$c_{lm} = \frac{1}{4\gamma} \left(\frac{a^2}{\alpha_{lm}^2} \right) \frac{P_l(k_{2m}a)}{(l-1)!(l+1)!} [(l+2)\beta P_l(k_{2m}a) - le_l Q_l(k_{2m}a)].$$

Clearly, the largest components are those for $l=1$, where both forces are independent of r and linearly proportional to r_0 . The degree to which the transverse dipole modes HEM_{1m} propagate synchronously with one another depends upon the wavenumber spectrum for the k_{zm} for $l=1$. The degree to which transverse dipole forces act at the locations of drive or test bunches depends upon the similarity of the wavenumber spectra of the k_{zm} for $l=1$ to that for $l=0$. It would be unusual for these two spectra to not be dissimilar. These preliminary observations provide motivation for performing further analysis and detailed numerical computations of the full transverse forces obtained by summing over all significant HEM_{lm} eigenmodes. This can lead to an accurate prediction of radial bunch trajectories and stability limits for the multi-bunch dielectric wake-field accelerator.

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DISCUSSION

As described in the third section of this report, a prototype MICA structure has been fabricated by Titan-Beta. One module of this structure, with rf input and output couplers, is installed along the East leg of the Yale Beam Physics Laboratory 6-MeV beamline, with a switching dipole that can direct the beam either into MICA or into MIFELA. A magnetic-dipole energy spectrometer is installed downstream of the MICA module to allow measurement of the beam energy and energy distribution. The microwave circuit directs up to 15 MW of 2.856 GHz rf power to the MICA module, and up to 7 MW to the rf gun that is used as the source of 6-MeV electron bunches; the circuit allows for a variable phase delay to be imposed to adjust the bunch arrival time to coincide with a given location near the crest of the rf wave. This is to allow achievement of maximum acceleration, and to study the influence on energy spread of off-crest operation. Successful transport through MICA of the 6-MeV beam has been observed, prior to application of rf power to the MICA structure. At this writing, Yale Radiation Safety officials have yet to give approval of the radiation shielding in place for beam energies higher than 6 MeV. Approval for operation to 8 MeV is anticipated before Fall, 1999. Furthermore, the XK-5 klystron used to drive rf gun and MICA accelerator module is currently in regular use for the MIFELA experiment; this experiment is scheduled to be completed also before Fall, 1999. It is therefore anticipated that operation and evaluation of MICA will take place during Winter 1999-2000.

However, there is concern that the measured poor rf matching of the input and output couplers and the strong insertion loss of the MICA module may prevent a full evaluation of the potential of MICA. These disappointing parameters for the components fabricated by Titan-Beta were not observed during the cold-testing, when performance was acceptable. Only after high-vacuum brazing and final assembly were the serious flaws in the structure evident. By then, Titan-Beta had exhausted all available funds under the purchase order from Omega-P, Inc. (It was stated that a significant investment by Titan-Beta was also made in the fabrication.) Repair of the structure by Titan-Beta would evidently have a low priority, and no guaranteed date for delivery of the specified structure was forthcoming. Accordingly, it was decided to attempt evaluation of this MICA prototype as it stands. Further steps are to await results of first trials during Winter 1999-2000. A report of results of these tests will be forwarded to DoE and, following normal publication procedures, with the accelerator community.