A Post Processing Algorithm to Add Damping to Undamped Model Responses

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ABSTRACT

In linear finite element models, proportional damping is often used. In general this does not produce results that match experimental measurements. Modal damping is a much better option, but sometimes is inconvenient. It may be cumbersome to calculate all the modes and keep track of what damping should be applied to each mode. If an explicit code is used, the modes are not available directly, so modal damping cannot be applied. A new approximate algorithm is demonstrated which allows the damping to be applied to undamped model response time histories. The damping is applied in user chosen frequency bands to as high a frequency as desired. Different damping may be applied to each response location. The method is demonstrated to be virtually equivalent to applying modal damping in bands. Examples are shown for a two degree of freedom spring-mass-damper system and a finite element model with 100 modes in the bandwidth.

NOMENCLATURE

DOF: degree of freedom
FRF: frequency response function
$\omega$: natural frequency
$\zeta$: viscous damping ratio

MOTIVATION

There is an increasing desire in industry to be able to predict absolute structural dynamic responses in environments that are difficult to test. Stiffness and mass of the model can be verified utilizing modal test data. However, damping cannot be modeled for many complex systems a priori. It is often inferred directly from test data. In many complex systems, the damping from the modal test is not appropriate for at-use levels, that is, the damping changes with higher level force inputs (usually increasing due to micro-slip of joints [2]). Hardware may be available to provide ground or laboratory test data at higher than modal levels from which damping may be inferred. In these data, actual forces or input accelerations are measured, and these provide the input loads for the model. Test responses observed in the frequency domain give some amplitude vs. frequency data to attempt to emulate with the model. With this data, it would be desirable to be able to optimize damping as appropriate in different frequency bandwidths, for each important sensor location on the structure. This can be done with modal damping. However, this may require significant bookkeeping efforts and computational time if the number of modes becomes large (e.g. hundreds of modes for a high frequency shock response). An explicit code does not even provide a modal solution. Utilizing material damping in an explicit model causes the solution time to increase dramatically, perhaps beyond feasible limits. The technique that follows provides the capability to add equivalent modal damping for a linear model by post processing the undamped impulse response.

THEORY OF DAMPING POST PROCESSING ALGORITHM

The algorithm is an approximate method that begins with the undamped impulse response time history from a model. The model may be of any type that can provide impulse response data such as a spring-mass or finite element model. This model has to be representative of the stiffness and mass of the system that is being modeled, so ideally it should be validated with modal test frequency and mode shape data. The goal is to add damping to the model so that its response matches that of some laboratory test data performed at operating levels. For the development of the theory, we will assume that the analyst desires to apply some known damping values in discrete frequency bandwidths. The first step is for the analyst to decide what these bandwidths will be. This is done by observing the Fourier Transform of the impulse response function at a certain response location in the model. The analyst identifies as many bands as

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necessary to apply the specific damping values. A key here is to make sure that each end of each frequency band is at a low point in the response, not at a resonance. As will be observed later, this keeps discontinuities in the frequency domain of the approximate response to a minimum. Suppose the analyst has chosen n bands, each of which will have a different damping ratio and dominant resonant frequency. It is assumed that all the response data is represented as discrete vectors that have even time steps. Now for each band, the analyst calculates a time decay vector of the well known damped exponential form

\[ \text{decay} = e^{-\zeta \omega t} \]  

(1)

where \( \omega \) is the natural circular frequency of the dominant mode in the bandwidth, \( \zeta \) is the modal damping ratio of that mode and \( t \) is time. Then each decay vector is multiplied, element by element, by the undamped time domain impulse response vector at the response location of interest. Now there are n damped time response vectors. The discrete Fourier Transform of each of these vectors is calculated. Then a composite discrete Fourier Transform vector is assembled from the n Fourier Transforms by using each individual Fourier Transform only in the bandwidth for which it was intended, in the step where the bandwidths were identified. This composite function is the approximate Frequency Response Function (FRF) for that response location [1]. There will be discontinuities where the individual bands are connected, but if the bands are chosen such that the value of the response is low at each band edge, the discontinuities are negligible. The FRF can be multiplied by the frequency domain input to calculate the output response. To prove the viability and illustrate the process let us apply it to a two degree of freedom spring-mass model.

**TWO DEGREE OF FREEDOM EXAMPLE**

In this example, the modal damping for the two modes of the spring-mass model are arbitrarily specified. If the approximate method is viable, the same damping ratio should be able to be applied in post processing the undamped impulse response and reproduce the damped response. Figure 1 shows the result. In this plot the solid line is the actual damped modal response and the dashed line is the response by the approximate method. The first step to accomplish the result is to look at the frequency domain response and see where the modes are. We can see the two resonances in Figure 1. In this case the analyst arbitrarily divided the response into two bandwidths from 0 to 500 Hz and from 500 Hz to 1024 Hz. The key here is that the 500 Hz transition point falls at a place of low response. From Figure 1, the dominant resonant frequencies can be picked off. These are about 100 Hz for the lower bandwidth and 900 Hz for the upper bandwidth. In Figure 2, a portion of the undamped impulse response from the model is displayed. The decay vectors from equation (1) are displayed in Figure 3, where the top plot represents the decay with the damping ratio and frequency for the first resonance and the bottom plot is the decay with the damping ratio and frequency for the second mode. In Figure 4, the decay vectors have been multiplied by the undamped impulse response vector to provide two damped time responses. These are discrete Fourier Transformed in Figure 5. The composite function is formed for 0-500 Hz from the top plot of Figure 5 and from 500-1024 Hz from the bottom plot. This gives the result which is compared with the actual damped response in Figure 1. Figure 6 shows the inverse Fourier Transform of Figure 1 showing the comparison in the time domain. The results are encouraging, but what about a case from a complex system which has many modes, some closely spaced?
To check the validity for a complex multi-mode system, the response was calculated from a finite element model of a shell/payload structure which had 100 modes in the bandwidth of interest (6000 Hz). The modal solution was obtained with an undamped and damped response. The damped solution applied a different damping ratio in every 1000 Hz bandwidth, i.e. 0-1000 Hz, 1000-2000 Hz, etc. The comparison of the actual damped response and the approximate damped response is shown in Figure 7 for a response point on the shell. In this case the analyst used 10 frequency bands. The first five bands corresponded with the first five resonances. The last five included multiple modes in each frequency band, and the most dominant frequency in the band was chosen as the frequency for the decay calculation. Figure 8 shows the time domain comparison. Figure 9 shows the frequency domain comparison for a point on the payload of the structure. In this case the analyst only used 5 bands, because there are large bands below 3000 Hz where few modes are active. Figure 10 shows the time domain comparison at the payload response location. The validity of the post processing algorithm is demonstrated with these comparisons.
APPLICATION

For useful applications the author plans to use this algorithm with an optimization package which will adjust the damping to cause the model response to give a best fit match to laboratory test data more representative of the operating levels than modal test data. Then the model will be used to predict responses in operating environments which cannot be tested.

It has been shown for certain systems that the damping may change in a complex structure from the modal test to the operating environments. This means that the damping is actually some other form than viscous or structural damping, i.e. it is nonlinear. Some have asked if this approach can be used on a system which has nonlinear stiffness as well. In a general sense, the author believes the answer is no. However, the approximate method may work adequately for a nonlinear system in the following special case. If the load is applied quickly and all the nonlinear response occurs early, the decay could be applied after the early time nonlinear response with fairly accurate results. Basically, this is just saying the technique can be applied to the late time linear ringdown.

UNCERTAINTY

The error at any particular resonance is based on the difference in the true product of $\zeta$ and $\omega$ and the $\zeta$ and $\omega$ the analyst chose for the bandwidth. The narrower the bandwidth, the less error there will be in $\omega$. Of course, if there is only one dominant mode in the bandwidth, and the analyst selects that natural frequency accurately, there will be almost no error due to frequency, even if it is a wide bandwidth. Theoretically the frequency error can be decreased to zero by having a bandwidth for each mode.
practice, the multi-mode example showed that very accurate response for a 100 mode system can be achieved with only five to ten bandwidths. The error in $\xi$ is not considered to be an issue here because it is assumed to be known or determined from an optimization process.

ADVANTAGES OF THE TECHNIQUE OVER MODAL DAMPING

It has been shown that one can achieve nearly equivalent results using the approximate technique and modal damping. So what advantage does the approximate technique have? One advantage is that there is no requirement with the approximate technique to do an eigenvalue problem to calculate the modes. An implicit direct transient solution can be used to get the undamped response. The author is aware of one high frequency application where the implicit direct transient solution is the only one feasible because the computation time to extract all the modes up to the frequency of interest is too great. If an explicit code is used, the calculation is much faster if there is no damping added to the model. Damping in an explicit code can make the solution time unfeasible because the critical time step becomes too small. The undamped explicit code impulse response can be damped by the approximate method, as long as the stiffness and mass are constant with time. If modal damping is applied in bands, the damping is the same at all response locations. This method is applied location by location, and therefore different functions of damping with frequency can be applied at different locations without recalculating the entire model response. This may be necessary because a mode with high damping may dominate the frequency response at one location while another mode with low damping may dominate the frequency response in that same band in another location.

CONCLUSIONS

An approximate method has been developed which can simulate the response of a system with modal damping by post processing the undamped impulse response of a validated model. The method was shown to be effective for a complex 100 mode system model with modal damping applied in six 1000 Hz bands. The method can be made to converge to any degree of accuracy by increasing the number of bandwidths chosen to apply the damping ratios. The approach is limited in general to linear models, although a restricted application was proposed for a nonlinear model. The major advantage of the approach is that it can be applied to undamped transient response calculations, which in many instances may be computationally feasible where damped modal response calculations may not be feasible.

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REFERENCES