IMPACT OF PROFILE VARIATION ON THE EQUILIBRIUM AND STABILITY OF NSTX

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ABSTRACT. MHD equilibrium and stability analyses have been performed in support of the design and initial experimental operation of the National Spherical Torus Experiment (NSTX). Free-boundary equilibria have been generated to determine several aspects of the anticipated plasma configurations including the stability and shape domain. Boundary shape determination has been studied by considering random variations of modeled signals from the NSTX magnetic diagnostic set. The impact of equilibrium profile variations on the ideal low-\(n\) kink and ballooning stability of high-\(\beta\) configurations was also determined to assess the robustness of the stable operating space of NSTX. The profiles used included local perturbations from previously reported optimization studies and experimental pressure profiles from the START spherical torus, the DIII-D tokamak, and TRANSP modeling. The largest \(\beta\)-limits occur in plasmas with broader pressure profiles and are set by the \(n = 1\) kink/ballooning mode. Values of \(\beta = 38\%\) are achieved in the presence of a conformal conducting wall at \(b/a = 0.25\). The corresponding calculated no-wall \(\beta\)-limit is \(22\%\). The lowest limits (\(\beta = 19\%\)) were generated in plasmas with greater pressure peaking, and are set by high-\(n\) ballooning modes.

I. INTRODUCTION

MHD equilibrium and stability studies of plasma operational limits are standard procedure for the design and operation of a magnetically confined plasma device. Previous studies have included exploration of the accessible operating space of NSTX plasma configurations¹. In the present study, equilibrium solutions were calculated for both fixed and free-boundary conditions using the EFIT² and EGGAC³ equilibrium codes.

A major step in the identification of the machine's operating limits is the quantitative evaluation of the plasma response to equilibrium perturbation⁴. In this context, the study of the effectiveness of the magnetic diagnostic measurements in determining the plasma boundary position is important for the success of the equilibrium reconstruction process and its possible application to active plasma position control⁵. In Section II of the paper, we present results from a sensitivity analysis of the plasma boundary position performed by simulating a 3% relative random error in the magnetic signals. Both the full magnetic diagnostic set planned for routine NSTX operation, as well as a reduced set used for the initial plasma operation of NSTX⁶ were considered.

The other important component of the present investigation is the robustness of the stability properties of NSTX high-\(\beta\) configurations. In Section III of the paper, we present results from a study aimed at quantifying the stability response of NSTX plasmas to local perturbations of the pressure, \(p\), and the safety factor, \(q\), profiles. A unique aspect of this study is the use of experimental pressure profiles from other tokamak devices (the spherical torus START⁷ and the DIII-D⁸ tokamak) as a realistic specification of this free function, and
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to compare to previous studies using generic pressure profiles. In addition, simulations of pressure profiles obtained using the transport analysis code TRANSP with realistic NSTX neutral-beam deposition were considered. Variation of the \( q \) profile uncovered possible scenarios where direct access to second stability could be achieved.

**II. SENSITIVITY OF NSTX EQUILIBRIA**

A statistical analysis of the robustness of the plasma boundary position to variations of modeled magnetic diagnostic data has been carried out by introducing a gaussian distributed random error of 3% to previously calculated flux-loop (FL) and magnetic-probe (MP) data.

In this study, a baseline equilibrium configuration is modeled with \( (I_p = 1.0\,\text{MA}, B_0 = 0.3\,\text{T}, \beta = 23\% ) \) and the response of the FL and MP diagnostics is modeled. Here, \( \beta = 2\mu_0 \int pdV/B_0^2 \) where \( B_0 \) is the vacuum field at the midplane half width. The random error was then added to the modeled diagnostic data and the equilibrium was reconstructed to fit the perturbed data set. Since the functional form (polynomial in \( \psi \)) of the basis functions \( (p'(\psi) \text{ and } f'(\psi) \text{ where } f(\psi) = R B_0 \) used for the equilibrium profiles is arbitrary, we vary over the polynomial order (up to 5th order) of both \( p' \) and \( f' \) to help eliminate artificial constraints introduced by this choice of basis functions. The variation of the boundary position was analyzed for this ensemble and shown in Figure 1 for the case of “full magnetic coverage” (51 FL and 76 MP). For this ensemble, both \( p' \) and \( f' \) vanish at the edge of the plasma. The maximum uncertainty of the X-point positions (3\( \sigma \)) is approximately 0.3 cm while the outer gap radial position is known to within an error of 0.9 cm.

![Figure 1. Overlay of 100 reconstructed plasma boundaries (left) for a D-null NSTX plasma as a 3\% random error is introduced in the magnetic data. The “full set” of flux loops are shown as squares while the x represent the pickup coils. The sets of \( q \) and \( p \) profiles used are shown on the right.](image)

The \( q \) and \( p \) profiles for the equilibria used in this analysis are also shown in Figure 1. The variations of the profiles illustrate that the boundary position is insensitive to changes in the central \( q \) profile as well as changes to the \( p \) profile in the central 50\% of the plasma. A similar analysis was performed for a reduced magnetic diagnostic set expected for the initial plasma operation of NSTX \( (I_p = 0.1\,\text{MA}, B_0 = 0.1\,\text{T}, \beta = 4\% ) \). The set includes 21 flux loops of which 17 are placed along the center stack of the machine and 4 in the outer vessel side of the torus, and no pickup loops. In this case, the maximum uncertainty of the X-point position increases to about 19 cm, while the outer gap radius was known to within a maximum error of 7.5 cm. While these uncertainties are large, they were considered acceptable for the survey of the first plasma operation of NSTX.

**III. ROBUSTNESS OF STABILITY IN HIGH-\( \beta \) CONFIGURATIONS**

Studies to determine optimized \( \beta \)-limits for low aspect ratio plasmas have been performed for generic equilibrium profiles. In this study, we consider how optimized ideal MHD stability limits change due to local variations of safety factor by incorporating both modeled NSTX pressure profiles and those taken from
existing experiments. High-\(n\) stability calculations were performed using the STBAL\textsuperscript{11} code and low-\(n\) kink/ballooning instability was analyzed using the PEST\textsuperscript{12} code.

The baseline marginally stable, optimized reference scenario we used in this study is a \(\beta = 40.4\%\) (\(\beta_N = 8.1\) with \(\beta_N = 10^6 \beta a(m) B_0(T) / I_p(A)\)) case that was constrained to have a high bootstrap current fraction (~80\%) that was well-aligned with the total current profile. The main plasma parameters of these equilibrium are: \(R = 0.86\) m; \(a = 0.67\) m; \(k = 2.0\); \(\delta = 0.45\); \(I_p = 1\) MA; \(B_0 = 0.3\) T. The pressure profile in this case is quite broad (pressure peaking factor, \(F_P = p(0)/\langle p \rangle = 1.7\)).

We applied localized variations to the reference monotonic \(q\) profile (\(q_0 \sim 2.8\)), keeping the pressure profile constant, and we studied the impact on the calculated \(\beta\)-limits. The \(q\) profile was independently varied near the magnetic axis (+/- 30\%), the middle of the minor radius (+/- 40\%), and the plasma edge region (+/- 30\%). Ballooning calculations indicate that the plasma is robustly stable for variations of \(q_0\) in the entire range considered. Increasing \(q\) in the mid-region and decreasing it near the plasma edge also illustrate robust stability to high-\(n\) ballooning modes. Decreasing \(q\) in the mid-region and increasing it near the plasma edge each independently led to high-\(n\) instability in the respective regions. Combining these two variations produces a reversed shear profile, which is marginally stable on the second stability region boundary to high-\(n\) modes at \(\beta = 32.2\%\). Unlike higher aspect ratio tokamak, as plasma \(\beta\) is reduced in this configuration, the high-\(n\) unstable region vanishes. In this way, the plasma can directly access the second stability region to ballooning modes. This characteristic is a consequence of the combination of high \(q_0\) operation and low aspect ratio geometry.\textsuperscript{13}

Pressure profile variations were generated by considering both TRANSP\textsuperscript{9} modeling performed using the NSTX NBI geometry, and experimental profiles from the DIII-D\textsuperscript{8} tokamak and the START\textsuperscript{7} spherical torus.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Comparison between \(\rho\) profiles normalized to the peak value. Note that the DIII-D H-mode and the START profiles are remarkably similar.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Marginal \(\beta\) vs. toroidal mode number \(n\) when using START \(\rho\) profile in NSTX equilibrium.}
\end{figure}

In all of these cases, the contribution of the hot-ions to the pressure was included. These variations were considered as they may be more representative of what would be expected in an NBI heated experiment. In Fig. 2, we show a comparison between the shapes of these profiles. Both the TRANSP calculated and experimental \(\rho\) profile tend to be more...
peaked than the reference \( p \) profile (START, \( F_p \sim 3.5 \); DIII-D H-mode, \( F_p = 3.6 \); TRANSP-calculated, \( F_p = 4.2 \); DIII-D L-mode, \( F_p = 4.7 \)). As shown in the figure, the DIII-D H-mode and the START pressure profiles are nearly identical in shape.

Previous studies show that the stability of these plasmas degrades as \( F_p \) increases\(^\text{10}\). The \( \beta \)-limits computed for the \( p \) profiles with increasing pressure peakedness in this study are reduced but remain in the 20% range regardless of the \( q \) profile variations considered.

First, we examine the results for the reference \( q \) profile. In Fig. 3 the calculated marginal \( \beta \) values for different toroidal mode numbers are shown for the START pressure profile. In this case the no-wall maximum \( \beta \) is limited by an \( n = 1 \) kink/ballooning instability to a value of 22.3%. The TRANSP pressure profile, has greater total peakedness, but does not have the strong edge pressure gradient of the DIII-D H-mode or START profile. It is also limited by the \( n = 1 \) mode with a larger \( \beta = 25% \) which is due to the absence of the large edge pressure gradient driving the edge kink mode.

The stabilizing effect due to the presence of the passive plate structure on NSTX was considered and modeled as an ideal conformal conducting wall placed at a distance \( b/a = 0.25 \) from the edge of the plasma. For the START \( p \) profile, the effect is substantial, as the stability limit for the \( n = 1 \) kink/ballooning mode increases to \( \beta = 38.1\% \). The equilibrium using the TRANSP \( p \) profile equilibrium in the presence the conducting wall yields an \( n = 1 \) \( \beta \)-limit of 35.9%. The large difference between the wall and no-wall \( \beta \)-limits will allow NSTX experiments to clearly differentiate between these limits.

When the variations in the \( q \) profile leading to instability are combined with the START and TRANSP pressure profiles, the \( \beta \)-limits fall to 22.5% and 19.4% respectively. The limiting instability in these cases is the high-\( n \) ballooning mode, which is unaffected by the presence of nearby conducting structure. These plasmas are marginally stable on the first stability region boundary. Therefore, operation of NSTX significantly above \( \beta \sim 20\% \) might require favorable \( q \) profile shapes to avoid the high-\( n \) first stability region boundary, and conducting wall stabilization for \( n = 1 \) kink/ballooning stability.

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8. Reference for DIII-D shots