Electromagnetic induction by a tilted magnetic dipole in an electrically anisotropic formation

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Abstract

Based on a previously developed analytic solution for induction in anisotropic media, we compute the eddy current paths and their associated magnetic fields for both dipping and horizontal sondes. The complex current paths are visualized using 3D rendering techniques in order to better develop a qualitative understanding of the physics involved. The results indicate that EM induction in a moderately anisotropic formation is characterized by eddy current paths with significant components in bedding plane, even when the source magnetic dipole is horizontal. The associated magnetic fields have a strong component in the direction perpendicular to the bedding planes—a finding that supports the utility of multi-component induction sondes to determine formation anisotropy and therefore minimize uncertainty in estimates of hydrocarbon potential.

Introduction

The electrical anisotropy of geologic formations is a topic that has attracted the attention of exploration geophysicists for much of the 20th century (Maillet and Doll, 1932; Kunz and Moran, 1958; Moran and Gianzero, 1979; Anderson et al., 1998). The primary objective of studies such as these is to reduce uncertainty in estimates of hydrocarbon reserves inferred from induction logs. In recent years, with the popularity of horizontal drilling and the challenge to log interpretation therein, there has been a renewed interest in studies of electromagnetic induction in anisotropic media as evidenced by the recent Special Topics Workshop on...
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Anisotropy, hosted by the Society for Professional Well Log Analysts (SPWLA), in Corpus Christi TX (2000).

Electrical anisotropy is a material property which specifies the directional dependence on the ability of a material to conduct electric current. Aside from materials that are intrinsically anisotropic (e.g. olivine crystals, Constable et al., 1992) electrical anisotropy can manifest itself as a macroscopic effect due to small scale geologic features. What is meant by "small" is a function of the resolving power of the instrument used for the investigation. The typical induction sonde samples the electrical properties of the surrounding formation within a few meters of the borehole. For the exploration geophysicist, geologic structures whose length scale is on the order of a few centimeters or less may give rise to a macroscopic electrical anisotropy. Examples include interbedded sand/shale sequences, crossbedded sandstones, and shales (figure 1). In the first example, the alternating sequence of conductive (shale) and resistive (sandstone) layers is the mechanism responsible for the anisotropic character of the formation. In the second, grain size variability, and therefore pore geometry, results in a sequence of layers which desaturate at different rates, and therefore give rise to the anisotropic electrical character of the formation. The third example occurs when shale platelets generate a preferred orientation to the geometrical arrangement of the interstices, and therefore, the mobility of free charge carriers within the rock. For reservoir rocks like those just listed, the ratio between the conductivity within the bedding plane and the conductivity perpendicular to the bedding plane can be quite substantial, reaching values as large as 100 (Klein et al., 1995).

The term transverse anisotropy refers to the situation where the electrical conductivity in the bed–parallel direction is different from that in the bed–perpendicular direction. By simple analogy with analogue circuits, the eddy currents perpendicular to bedding planes can be modeled as a current through a sequence of resistors wired in series. Likewise, the eddy currents within a bedding plane can be modeled as a current through a sequence of resistors wired in parallel. As a result of the rules governing simple resistor networks, the conductivity in the bed–perpendicular direction is always less than or equal to that in the bed–parallel direction. (Note: Transverse anistropic media where this inequality doesn’t hold represent conducting “rods” embedded in a resistive background—a conceptual model which may have implications for controlled–source marine electromagnetics. See Everett and Constable (1999) for more details.)
Historically, exploration wells were drilled vertically and mostly perpendicular to formation bedding planes. For a traditional induction sonde geometry consisting of coaxial source and receiver coils, eddy currents induced by the sonde circulate horizontally in the formation, in planes perpendicular to the borehole axis. Thus, the induction log records the effects of the macroscopic horizontal conductivity of the formation. The hydrocarbon potential of electrically resistive sand units can easily be underestimated if the sands are interbedded with conductive shales, resulting in a high horizontal conductivity values. Knowledge of the macroscopic vertical conductivity of the formation can help identify these “low-resistivity pay” zones by quantifying the degree of electrical anisotropy present.

For the case of a horizontal well in this transversely anisotropic sand/shale sequence, the eddy currents follow a more complicated path around the borehole axis, as we demonstrate later. When a moderate degree of anisotropy is present they are sensitive to both the (relatively low) vertical and (relatively high) horizontal conductivities of the formation. As a result, standard well log interpretation underestimates the water saturation and therefore overestimates the hydrocarbon potential of the formation. Through superposition, eddy currents induced by a sonde in a dipping well can be represented as a weighted average of components due to both a vertical and horizontal transmitter. Owing to the complex nature of the eddy current paths for a horizontal magnetic dipole source and the popularity of non-vertical well bores, analysis of induction by a horizontal dipole is an inescapable element of modern well log interpretation.

The problem of induction by a magnetic dipole source in anisotropic media has been solved analytically by many researchers (e.g. Sinha, 1968; Kong, 1972, 1975; Tabarovskii and Epov, 1977; Tang, 1979; Moran and Gianzero, 1979). However, even though the solutions have been available for some time now, a discussion of how anisotropy distorts the eddy current paths has not been presented as far as we know. Motivated by discussions at the recent SPWLA workshop, we present here an examination of eddy current paths and their associated magnetic fields generated by dipping and horizontal magnetic dipole sources in an transversely anisotropic medium. For simplicity, the effects of the borehole and invasion zones are not considered. The analysis is based on the analytic solution of Moran and Gianzero (1979) which is briefly outlined in the following section.

Methodology
Assuming an $e^{i\omega t}$ time-dependence and a point magnetic dipole source term, Maxwell's equations in the quasi-static limit are written as

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + i\omega \mathbf{M}_d,$$

(Ampere's Law)

and

$$\nabla \times \mathbf{E} = -i\omega \mu (\mathbf{H} + \mathbf{M}_d),$$

(Faraday's Law)

where the following definitions apply: $\mathbf{H}$ and $\mathbf{E}$ are the magnetic and electric fields, respectively; $\sigma$ is the electrical conductivity tensor; $\varepsilon$ is the dielectric constant; $\mu$ is the magnetic permeability; and $\mathbf{M}_d$ represents the magnetic dipole source. For the treatment that follows, the magnetic permeability $\mu$ is set to the value of that for free space, $4\pi \times 10^{-7}$. The form of the conductivity tensor considered here is given by,

$$\sigma = \text{diag}(\sigma_h, \sigma_h, \sigma_v),$$

(1)

where the $x$, $y$ and $z$ axes conform to the principal axes of the tensor (see figure 2). The value of $\sigma_h$ represents the electrical conductivity in the horizontal $x$ and $y$ directions while the value of $\sigma_v$ represents the conductivity in the vertical $z$ direction. At frequencies $f = \omega/2\pi$ less than approximately 1 MHz, the electrical conductivity of most rocks is sufficiently large that Ohmic conduction currents, $\sigma \mathbf{E}$, dominate over the displacement currents, $i\omega \mathbf{E}$, which are responsible for the propagation of electromagnetic waves. Instead, electromagnetic energy is propagated by a diffusion mechanism of Ohmic currents in this low-frequency regime.

For a constant value of $\sigma$, the vector Hertz potential $\mathbf{\pi} = \hat{x} \pi_x + \hat{y} \pi_y + \hat{z} \pi_z$ is introduced to satisfy the requirement imposed by Ampere’s law that the Ohmic currents are represented by a solenoidal vector field,

$$\sigma \mathbf{E} = i\omega \mu \sigma_h \nabla \times \mathbf{\pi},$$

(2)

where the constant $i\omega \mu \sigma_h$ is chosen for convenience. Equation 2, along with Faraday’s law, implies that the magnetic field can be written in terms of the Hertz vector and a curl-free term $\nabla \Phi$,

$$\mathbf{H} = i\omega \mu \sigma_h \mathbf{\pi} + \nabla \Phi.$$

(3)

Since the choice of $\Phi$ is arbitrary, a little algebra suggests that the relation $\sigma_v \Phi = \nabla \cdot (\sigma \mathbf{\pi})$ provides a set of three equations in terms of the components of the Hertz potential,

$$\frac{\partial^2 \pi_x}{\partial x^2} + \frac{\partial^2 \pi_x}{\partial y^2} + \frac{1}{\lambda^2} \frac{\partial^2 \pi_x}{\partial z^2} + i\omega \mu \sigma_v \pi_x = -\frac{M_x}{\lambda^2},$$

(4)
where the magnetic dipole source term is given by \( i'vf,\pi = 2MZ + ijMY + 331 = \) and the coefficient of anisotropy is defined as \( \lambda^2 = \sigma_h/\sigma_v. \) Note that equations 4 and 5 are completely decoupled by this choice of \( \Phi. \) Once solutions to equations 4 and 5 are obtained, their derivatives appear as known source terms in equation 6.

Each of the equations 4-6 can be solved using well-known Fourier transform methods. For brevity we state the solutions below (Moran and Gianzero, 1979).

In the case of an \( z-\)directed point magnetic dipole located at the origin \( (MZ = m_z \delta(0) \) and \( 31V = M= \) \( = (1 + \delta) \)) where \( \delta \) is the Dirac delta function) the Hertz vector and accompanying scalar potential \( \Phi \) are given by,

\[
\pi_x = \frac{m_z e^{iku_s}}{4\pi \lambda s}, \quad \pi_y = 0 \quad \text{and} \quad \pi_z = \frac{m_z}{4\pi \rho^2} \left[ \frac{\lambda z e^{iku_s} - z e^{iku_r}}{s} \right]
\]  

(7)

and

\[
\Phi = \frac{m_z i kh z}{4\pi \rho^2} \left[ e^{iku_s} - e^{iku_r} + \frac{\rho^2}{r^2} \left( 1 - \frac{1}{ik_h r} \right) e^{iku_r} \right],
\]  

(8)

where

\[
k_u^2 = i \omega \mu \sigma_u, \quad k_h^2 = i \omega \mu \sigma_h = k_u^2 \lambda^2,
\]  

(9)

\[
\rho^2 = x^2 + y^2, \quad r^2 = x^2 + y^2 + z^2 \quad \text{and} \quad s^2 = x^2 + y^2 + \lambda^2 z^2.
\]  

(10)

The solution for the case of a \( y-\)directed dipole follows analogously by substituting \( x \)'s for \( y \)'s and vice versa in equations 7 and 8.

In the case of a \( z-\)directed point magnetic dipole located at the origin \( (M_Z = m_z \delta(0) \) and \( M_y = M_z = 0 \)

where \( \delta \) is the Dirac delta function) the Hertz and scalar potential \( \Phi \) are given by,

\[
\pi_x = \pi_y = 0, \quad \pi_z = \frac{m_z e^{iku_r}}{4\pi r},
\]  

(11)

and

\[
\Phi = \frac{m_z i kh z}{4\pi \rho^2} \left( 1 - \frac{1}{ik_h r} e^{iku_r} \right)
\]  

(12)
Electric current density and magnetic fields for an arbitrarily oriented magnetic dipole source \((m_x, m_y, m_z ≠ 0)\) are obtained by superposition after substituting expressions for the potentials \(\pi\) and \(\Phi\) into equations 2 and 3. Note, for a vertical magnetic dipole source, the Hertz potential is a function of the horizontal conductivity, \(\sigma_h\), and not the vertical conductivity, \(\sigma_v\). Therefore, as a result of equation 2, the eddy currents are also strictly a function of the horizontal conductivity.

**Examples**

The first set of examples, using the analytic solutions developed in the previous section, demonstrate the effect of anisotropy on eddy currents induced by a horizontal magnetic dipole source. Shown in figure 3 are streamlines and the 0.01 A/m² isosurface for the quadrature component of the eddy current density. The streamlines of current density represent the path of the eddy currents in the conducting formation. From Faraday's law, we see that the quadrature component of the magnetic field generated by the eddy currents is responsible for the in-phase signal recorded in the receiver coil of the induction sonde (figure 2). Ampere’s law further dictates that it is the quadrature component of the current density which gives rise to the quadrature component of the signal-inducing magnetic field.

As expected for the isotropic case, the eddy currents form a set of set of circular paths centered coaxially on the dipole axis. However, as the coefficient of anisotropy is increased (vertical conductivity decreased for a fixed 1.0 S/m horizontal conductivity), the circular paths are distorted, acquiring a horizontal component in the same direction as the dipole source. For coefficients of anisotropy ~ 100 (not shown), the current paths are almost entirely horizontal.

The second set of examples demonstrate the anisotropy effect on eddy currents induced by a dipping dipole source. Streamlines and the 0.02 A/m² isosurface of the quadrature component of eddy current density are plotted in figure 4. Again, in the isotropic case the currents circulate coaxially around the dipole source, regardless of its orientation. However, as the anisotropy is increased, the eddy currents assume a closed path that is increasingly constrained to lie within the conductive horizontal plane. For large coefficients of anisotropy \((\lambda > 5)\) the magnetic field is nearly vertical, perpendicular to the closed eddy current loops (this effect is demonstrated in the last set of examples). Therefore, the decrease in the magnetic field intensity
along the borehole axis in the direction parallel to the dipole source is scaled by a geometrical factor, \( \cos(\theta) \). This fact is implicit in the apparent conductivity formula (eq. 44) given by Moran and Gianzero (1979).

Lastly, notice that the volume bounded by the current density isosurface increases and extends laterally as the vertical conductivity decreases. This behavior illustrates the increase in the effective skin depth of the electromagnetic fields as the formation conductivity is decreased.

The last set of examples (figure 5) shows the anisotropy effect on the magnetic fields in the vicinity of the receiver coil in a 2C–4CI induction sonde when the borehole is horizontal (figure 2). Note that the scale of the region plotted here shows the field variations within the region occupied by the borehole and drilling mud. Since the apparent conductivity is proportional to the quadrature component of the magnetic field in the vicinity of the point \( y = z = 0 \) (Moran and Kunz, 1962), decreasing the vertical conductivity of the formation results in a decrease in its apparent conductivity (top row). Note however, that the contour lines of the magnetic field components parallel to the borehole axis (top row) are not nearly as distorted as those components perpendicular (bottom row) to the borehole axis. Even for the small amount of anisotropy considered here (1 \( \leq \lambda^2 \leq 3 \)), these plots show that the magnetic fields perpendicular to the borehole axis are preferentially distorted toward the vertical direction, perpendicular to bed boundaries. This latter observation is consistent with the results shown in figure 3 where eddy current paths are constrained to the bedding planes as the formation anisotropy is increased.

Discussion and Conclusions

The computations shown here are presented as a guide to the underlying physics of induction log response of dipping wells in anisotropic formations. For simplicity, borehole, invasion zone, bed–boundary and “dipping” anisotropy effects have been omitted. Simulations of these more realistic logging scenarios have recently been addressed by other researchers (e.g. Anderson et al. (1998); Avdeev et al. (2000); Weiss and Newman, (2000)). Most of the attention, however, has been on instrument response, an analysis which is several steps removed from the fundamental processes of eddy current induction.

The eddy current paths shown here, while sometimes bizarre in appearance, reinforce the principle of conservation of electric current given by Ampere’s law. In the limiting case where the vertical conductivity
is zero. the current paths clearly must form a closed horizontal path (consider the case of thin interbeds of conducting and perfectly resistive sheets) since the medium is incapable of vertical current flow. For the 30°–60° dipping dipole source in the isotropic limit, the eddy current paths already contained a nontrivial amount of horizontal flow. Little additional anisotropy was required to deflect the paths almost entirely into the horizontal plane. For horizontal dipole source in the isotropic case, there was no component of the current in the direction of the dipole moment. However, the 10:1 anisotropy ratio (much less than the 100:1 ratio reported by Klein et al., 1995) was sufficient to generate a strong component in the direction of the dipole moment. The end effect of these bedding–plane constrained eddy currents is a stronger magnetic field component in the direction perpendicular to the bedding planes. This suggests that a multi–component induction sonde may be useful in determining to what degree, if any, anisotropy is present in a geologic formation.

Moran and Gianzero (1979) pointed out the that an anisotropy “dip–meter” (measuring the orientation of the borehole axis to the principal axes of the conductivity tensor) could be constructed based on a multi–component induction sonde. They speculated, however, that due to borehole effects, the instrument would meet with limited success. With the advent of fully 3D simulation codes such as those mentioned above, a more thorough investigation the multi–component sonde is now possible (see Alumbaugh and Wilt (1999) for a treatment of isotropic media). Building upon the results presented here, 3D analyses are an important next step in developing the necessary methods for determination of formation anisotropy from both conventional and experimental induction sonde designs.

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References


Figure Captions

Figure 1. Road cut along State Highway 536 2 mi W of intersection with Highway 14, east side of the Sandia Mountains, central New Mexico. Electrical anisotropy can manifest in layered elastics, such as this black shale from the Pennsylvanian Sandia Formation, as a result of preferred grain orientation during deposition and geometrical arrangement of the interstitial spaces.

Figure 2. Reference frame used for anisotropic model computations. Electrical conductivity of the formation is characterized by a diagonal conductivity tensor with azimuthal symmetry about the z-axis, diag(\(\sigma_h, \sigma_v\)). Also shown are the transmitter, Tx, and receiver, Rx, coils for a 2C-40 induction tool in an 8in diameter horizontal well oriented along the x-axis.

Figure 3. Quadrature components of the electric current density induced by a 25 kHz horizontal magnetic dipole source (orange arrow) oriented along the x-axis. Plot axes are labeled in meters. Shown here, for a fixed value \(\sigma_h = 1.0\) S/m, are current streamlines (yellow) and 0.01 A/m² isosurfaces (blue) for four cases of formation anisotropy (\(\lambda^2 = \sigma_h/\sigma_v\)): (a) \(\sigma_v = 1.0\) S/m, (b) \(\sigma_v = 0.333\) S/m, (c) \(\sigma_v = 0.1429\) S/m, and (d) \(\sigma_v = 0.10\) S/m. Streamlines illustrate different eddy current paths emanating from five neighboring points (units in meters) in the +z halfspace: (0.0, 0.0, 0.4), (0.1, 0.0, 0.4) (0.2, 0.0, 0.4), (0.3, 0.0, 0.4) and (0.4, 0.0, 0.4).

Figure 4. Quadrature components of the electric current density induced by a 25 kHz magnetic dipole source (orange arrow) tilted by 30°, 45° and 60° degrees in the y–z plane. Plot axes are labeled in meters. Shown here, for a fixed value \(\sigma_h = 1.0\) S/m, are current streamlines (yellow) and 0.02 A/m² isosurfaces (blue) for four cases of formation anisotropy: (a–c) \(\sigma_v = 1.0\) S/m, (d–f) \(\sigma_v = 0.5\) S/m, (g–i) \(\sigma_v = 0.333\) S/m, and (j–l) \(\sigma_v = 0.20\) S/m. Streamlines illustrate different eddy current paths emanating from five neighboring points, \((x_i, y_i, z_i)\) \((i = 1, \ldots , 5)\), positioned relative to the dip of the dipole source (angle \(\theta\) with respect to vertical in the y–z plane) where \(x_i = x_1 + 0.1(i - 1)\sin \theta\), \(y_i = y_1\) and \(z_i = z_1 + 0.1(i - 1)\cos \theta\) with \(x_1 = -0.4\cos \theta\) and \(z_1 = 0.4\sin \theta\).

Figure 5. Quadrature component of the magnetic field in the y–z plane at \(x = 40\) in. due to eddy currents
induced by a 25 kHz $x$-directed horizontal magnetic dipole source located at the origin. Note that the region plotted extends ±4 in. in the $y$ and $z$ directions. Keeping a fixed value $\sigma_h = 1.0$ S/m for the horizontal conductivity, field values for four different values of formation anisotropy shown: (a–b) $\sigma_a = 1.0$ S/m, (c–d) $\sigma_a = 0.7143$ S/m, (e–f) $\sigma_a = 0.5$ S/m, and (g–h) $\sigma_a = 0.3333$ S/m. The top row illustrates the contours of the field strength ($H_{\text{axial}}$) for the component oriented in the $x$-direction, parallel to the axis of the dipole source. The bottom row illustrates the contours of field strength ($H_\rho$) for the field component oriented radially outward from the $x$-axis.
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 5.