

A Deterministic Method for Transient, Three-Dimensional Neutron Transport

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Abstract

A deterministic method for solving the time-dependent, three-dimensional Boltzmann transport equation with explicit representation of delayed neutrons has been developed and evaluated. The methodology used in this study for the time variable of the neutron flux is known as the improved quasi-static (IQS) method. The position, energy, and angle-dependent neutron flux is computed deterministically by using the three-dimensional discrete ordinates code TORT. This paper briefly describes the methodology and selected results.

The code developed at the University of Tennessee based on this methodology is called TDTORT. TDTORT can be used to model transients involving voided and/or strongly absorbing regions that require transport theory for accuracy. This code can also be used to model either small high-leakage systems, such as space reactors, or asymmetric control rod movements. TDTORT can model step, ramp, step followed by another step, and step followed by ramp type perturbations. It can also model columnwise rod movement can also be modeled. A special case of columnwise rod movement in a three-dimensional model of a boiling water reactor (BWR) with simple adiabatic feedback is also included. TDTORT is verified through several transient one-dimensional, two-dimensional, and three-dimensional benchmark problems. The results show that the transport methodology and corresponding code developed in this work have sufficient accuracy and speed for computing the dynamic behavior of complex multi-dimensional neutronic systems.

1. INTRODUCTION

Multigroup, multi-dimensional, nodal diffusion theory has been widely used to model space-dependent neutron kinetics problems in transient reactor analysis and reactor safety.^{1,2} However, the validity of models based on diffusion theory is limited due to the assumptions made in deriving the diffusion theory. Therefore, kinetics methods based on transport theory³⁻⁶ are required to provide better accuracy and serve as a benchmarking tool for verifying more approximate methods⁷⁻¹³ based on diffusion theory.

The methodology used in this study for the time variable of the neutron flux is known as the improved quasi-static (IQS) method. The position, energy, and angle variables are computed deterministically by using the three-dimensional discrete ordinates code TORT¹⁴. This combination of methods represents a new approach for transient, three-dimensional deterministic transport theory with explicit treatment of delayed neutrons. Previous transient three-dimensional methods use Monte Carlo, nodal diffusion theory, even parity functions, or other simplifications and/or approximations.

2. IMPROVED QUASI-STATIC METHOD

The improved quasi-static method is described in detail in the literature.^{12,13} However, a brief description of the methodology is provided in this paper for completeness. The time-dependent flux is expressed as product of a space-, energy-, and angle-dependent shape function which is usually slowly varying in time and a purely time-dependent amplitude function which is usually fast varying in time. This factorization is made unique by imposing a normalization condition known as γ normalization in which γ is kept constant throughout the transient. Since the shape calculation is the most CPU-intensive part of the calculation, the resulting shape equation is solved only when shape needs to be updated. However, the much simpler amplitude function is solved over many very small time intervals.^{15,16} Reactivity, generation time, and effective delayed neutron fractions are computed by using the interpolated shapes at intermediate size time intervals.

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3. NUMERICAL SOLUTION

The shape equation and point kinetics equations are solved numerically by using three different integration time intervals as shown in Figure 1. The shape is assumed to vary linearly over each Δt_n time interval. The point kinetics parameters are calculated using the Δt_k time intervals with linearly interpolated flux shapes. The point kinetics equations are then solved over the smallest time intervals, Δt_i .

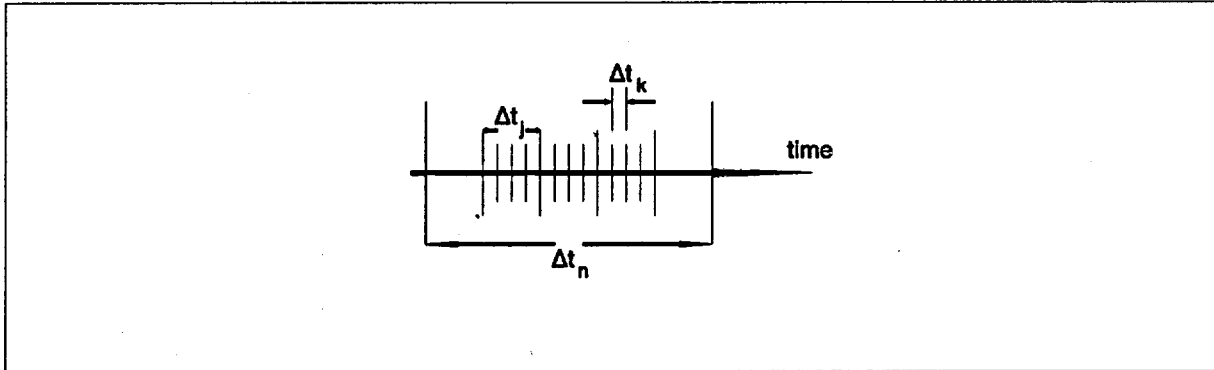


Fig. 1. The improved quasi-static method time intervals.

4. RESULTS

To verify that TDTORT accurately solves the time-dependent, three-dimensional Boltzmann transport equation with explicit representation of delayed neutrons, the code was used to model benchmark problems 16-A1, 16-A2, 8-A1, and 14-A2 from the Argonne National Laboratory Benchmark Problem Book.¹⁷ All TDTORT calculations presented in this paper were performed on a SUN Ultra 1 workstation with a 167 MHZ CPU and 192 MBytes of RAM located in the University of Tennessee Nuclear Engineering Department. Also, the fully symmetric quadrature set supplied by TORT is used for all problems. All of the problems considered in this study have isotropic scattering. Problems 16-A1 and 16-A2 are for a one-dimensional transport model of a liquid metal fast breeder reactor. Problem 8-A1 is a 2-D (r-z) diffusion model of a heavy water reactor. Problem 14-A2 is a three-dimensional (x,y,z) diffusion problem for a boiling water reactor (BWR). All diffusion theory based benchmark problems are converted to transport theory problems using the following relationship between the diffusion coefficient and the total scattering cross section:

$$D = \frac{1}{3\Sigma_s(1 - \cos\psi)} \quad (1)$$

where,

D = diffusion coefficient,

Σ_s = scattering cross section,

ψ = scattering angle in the laboratory system.

If scattering is isotropic in the laboratory system, the above equation reduces to:

$$D = \frac{1}{3\Sigma_s} \quad (2)$$

which is the case for the diffusion problems considered in this work. Using this scattering cross section along with the specified values of the downscattering and absorption cross sections, the within-group scattering cross section and the total cross section are calculated. For all the transients presented, the γ normalization mentioned above is constant to within 1% throughout the transients, except for problem 14-A2, which is constant within 5%.

Furthermore, the shape calculation time intervals are selected so that two iterations over the shape time interval are sufficient, and increasing the number of iterations beyond two does not significantly improve accuracy. In addition, since there is no pre-defined relationship between the time-step size and a particular transient problem, a time-step size selection algorithm has not been developed in this work. Therefore, a new TDTORT user should model several of the benchmark problems to gain experience selecting time steps.

Figures 2-5 show the results of power-versus-time for the four problems. The agreement between TDTORT and benchmark results for 16-A1 and 16-A2 is excellent. For 8-A1, the TDTORT results are in good agreement with benchmark results. Also, the results for 14-A2 exhibit satisfactory agreement with benchmark results. Deviations in 8-A1 and 14-A2 results are due to the fact that these benchmark problems are diffusion theory problems, and the benchmark solutions are calculated with codes using the diffusion theory.

5. CONCLUSIONS

A methodology for solving the time-dependent, three-dimensional Boltzmann transport equation with explicit representation of delayed neutrons has been developed and implemented in a new transient code, TDTORT. TDTORT has been compared with several computational benchmark problems. All comparisons show that the methodology and code developed have sufficient accuracy and speed to serve as a benchmarking tool for other less accurate models and codes. More importantly, a new computational tool now exists for analyzing the dynamic behavior of complex neutronic systems including voided reactors; small, high-leakage reactors, such as space reactors; and other systems for which the validity of diffusion theory is questionable.

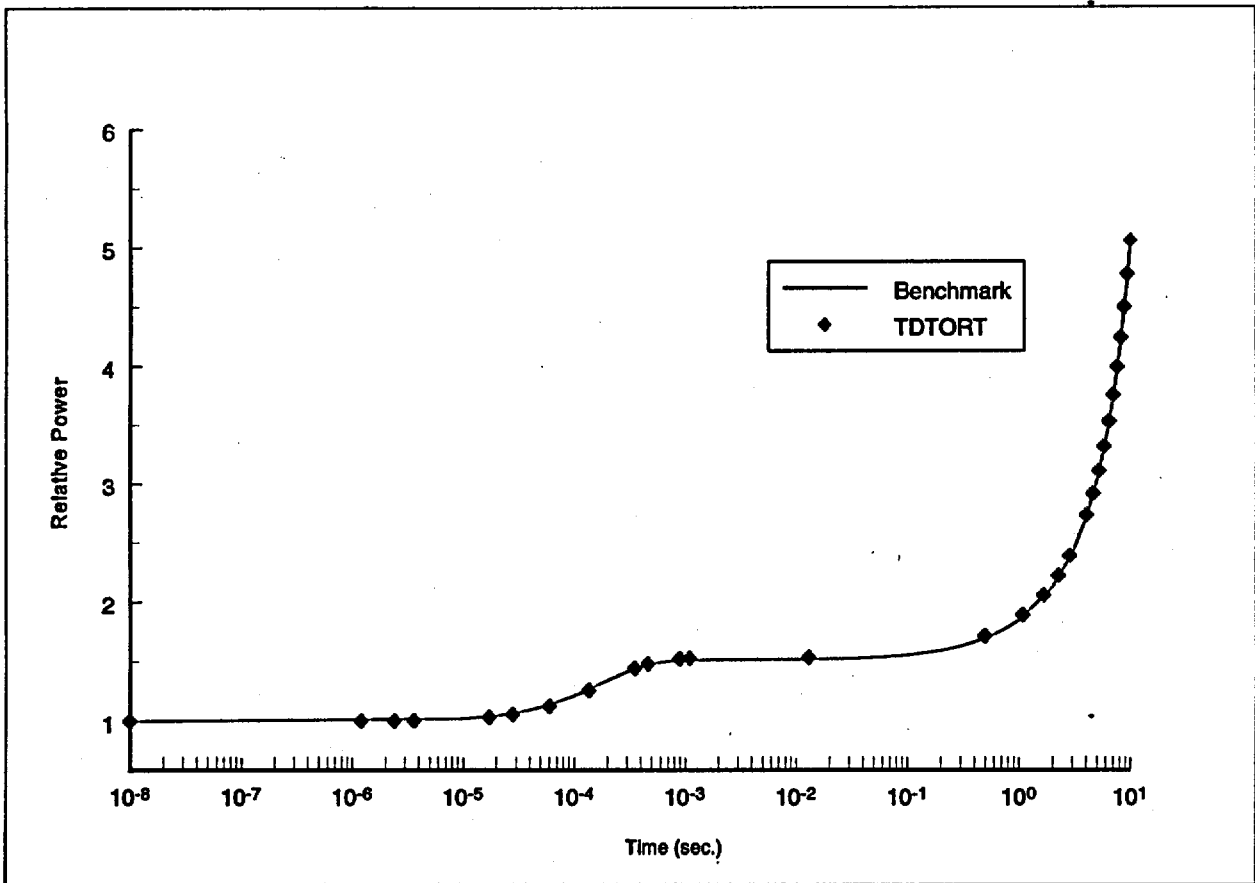


Fig.2. Relative power comparisons of TDTORT and TIMEX (Benchmark) for problem 16-A1.

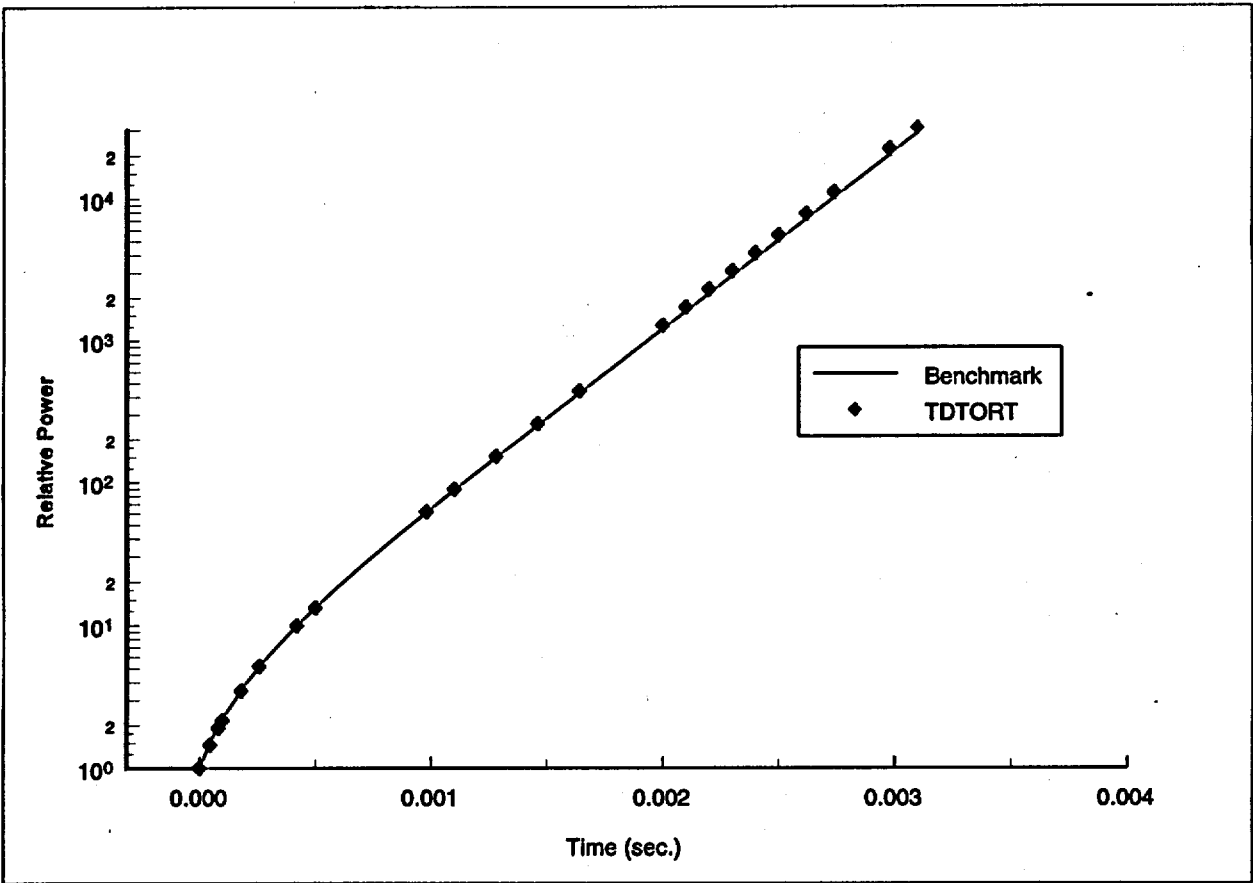


Fig. 3. Relative power comparisons of TDTORT and TIMEX (Benchmark) for problem 16-A2.

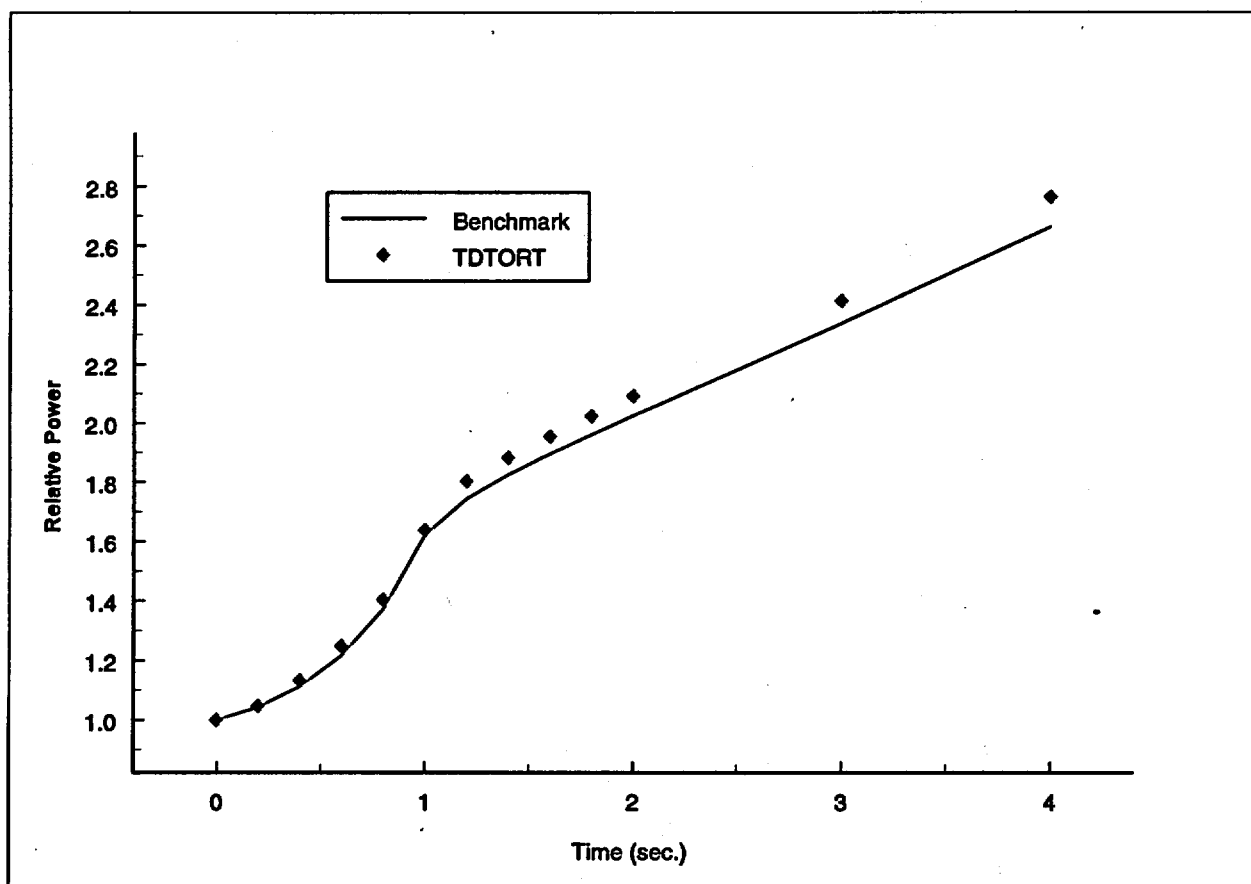


Fig. 4. Relative power comparisons of TDTORT and TW0DTA(Benchmark) for problem 8-A1.

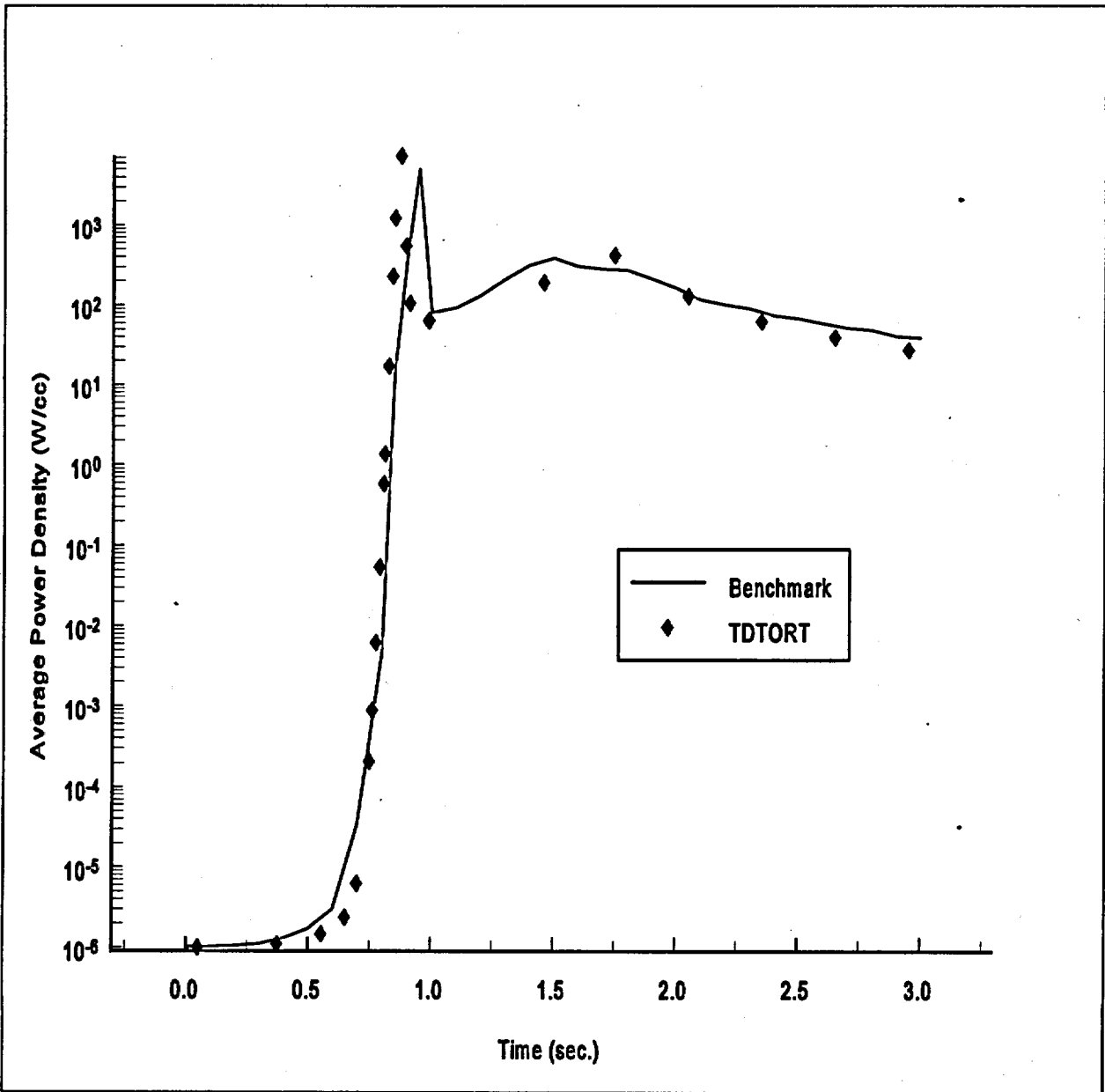


Fig. 5. Average power density vs. time comparisons of TDTORT and benchmark for problem14-A2.

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