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Particles and Patterns in Cellular Automata

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Abstract

This is the final report of a three-year, Laboratory Directed Research and Development (LDRD) project at Los Alamos National Laboratory (LANL). Our objective has been to develop tools for studying particle interactions in a class of dynamical systems characterized by discreteness, determinism, local interaction, and an inherently parallel form of evolution. These systems can be described by cellular automata (CA) and the behavior we studied has improved our understanding of the nature of patterns generated by CAs, their ability to perform global computations, and their relationship to continuous dynamical systems. We have also developed a rule-table mathematics that enables one to custom-design CA rule tables to generate patterns of specified types, or to perform specified computational tasks.

Background and Research Objectives

Cellular automata (CA) are a class of dynamical systems characterized by discreteness, determinism, local interaction, and an inherently parallel form of evolution. Specifically, CA are defined as lattices of processors whose site values are determined at any time step by the values of their neighboring processors at the previous time step. Although simple in construction, cellular automata generate a diverse range of highly complicated spatio-temporal behavior. Cellular automata are now widely recognized to be powerful simulation tools for the modeling of certain types of complex behavior.

The complexity and diversity of behavior generated by CA have motivated widespread use of these systems in the modeling of complex phenomena and in the design of parallel-processing algorithms. The application of CA models to these discrete systems has been hampered by the lack of a theoretical foundation. The goal of this research has been to develop analytical techniques for the understanding of phenomena such as transient behavior, stability of orbits, and pattern formation.

Newly obtained results in the mathematics of one-dimensional cellular automata promise to provide for the first time a method for analyzing the relationship between an

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automaton's microscopic level specifications through its rule table, and certain macroscopic features including pattern formation and computational capabilities that characterize its evolution. The central point is that these automata are characterized by their generation of mesoscopic structures that can be viewed as particles and backgrounds, and the global behavior of the automata may be described through a reformulation of the automation in terms of collision rules on this mesoscopic scale.

Our research has extended these results to develop a “rule-table mathematics” (i.e., a mathematical formulation for functions specified in rule-table, or lookup, form) for cellular automata. This mathematical approach helps to address questions fundamental to the use of cellular automata in simulations, such as the effects of perturbations in rule tables or in initial conditions. We have generated an “atlas” summarizing the range of possible backgrounds and particles for one-dimensional CA. We have shown that, in general, the background is generated by the “linear” component of the CA's interaction rule table, while the particles are generated by the “nonlinear” component of the rule table. Depending on the details of the linear component, the background may be either orderly—resembling a simple tiling—or stochastic. We derived mesoscopic rules describing the interactions among backgrounds and particles; in particular, the method permits the rigorous description of interactions resulting in the annihilation of particles by backgrounds, the annihilation of backgrounds by particles, and the spatial translation of particles induced by collision with backgrounds.

Importance to LANL's Science and Technology Base and National R&D Needs

Systems that exhibit CA behavior include percolation through porous media, dendritic growth of crystals, and patterns of electrical activity in simple neural networks generated by systems consisting of a large number of simple, identical, locally connected components. Cellular automata are important theoretically as prototypical, spatially extended, discrete dynamical systems—other examples of these systems include lattice gases, coupled lattice maps, neural nets, and sand pile models. Practically, CA are useful in the modeling of the type of complex behavior generated by the interaction of many simple, local connected components—such as multiphase flows and percolation through porous media—and in the design of parallel-processing algorithms for problems such as pattern recognition.

Scientific Approach and Accomplishments

Cellular automata are characterized by discreteness (in space, time, and state values), local interaction, and parallel evolution. The general form of a cellular automaton in one spatial dimension is given by

$$x_i^{t+1} = f(x_{i-r}^t, \dots, x_i^t, \dots, x_{i+r}^t),$$

$$f : \{0,1,\dots, k-1\}^{2r+1} \rightarrow \{0,1,\dots, k-1\}$$

where x_i^t denotes the value of site i at time t , f represents the update rule, and $r \geq 0$ is the radius of the rule. An analogous definition holds for CA in more than one space dimension.

Thus a CA consists of a lattice of sites, or processors, that evolve according to a local interaction rule in the form of a discrete lookup table. The value of each site at any time step is determined as a function of its neighboring site values at the previous time step, and is restricted to a finite set of integers. The combination of features distinctive to CA has attracted substantial attention from researchers interested in cellular automata as mathematical models for complex physical and biological phenomena, and as computational tools for parallel-processing applications.

In the context of simulations, cellular automata can be viewed as prototypical models for systems consisting of a large number of simple, identical, and locally connected components. Phenomena that are conventionally studied using continuous models based on partial differential equations, but that have been successfully modeled using cellular automata, include:

- turbulent flow resulting from the collisions of fluid molecules,
- dendritic growth of crystals resulting from aggregation of atoms, and
- patterns of electrical activity in simple neural networks resulting from neuronal interaction.

For these cases, cellular automata techniques yield models that differ completely in detail from, but provide results quantitatively consistent with, those obtained from a continuous approach, and the simulations based on cellular automata often provide a significant increase in computational efficiency.

In addition, cellular automata techniques have increasingly been used to simulate phenomena—ranging from algae bloom and related ecological features in estuaries, through the emergence of trade relations and other economic structures, to the development of

automobile traffic congestion—for which spatial inhomogeneities are known to be significant but for which partial differential equations are not necessarily appropriate. In these cases, CA provide a novel means of handling spatial components in the modeling of complex temporal phenomena.

In the context of computation, cellular automata have been useful in the design of parallel-processing algorithms with increased computational efficiency. Methods based on CA, for example, provide considerable improvement over conventional serial algorithms for pattern-recognition problems such as searching for a particular string of values in an arbitrary data stream. CA have also been proposed as efficient pseudo-random number generators.

The growing recognition of the importance of cellular automata as simulation and computational tools motivates this project's focus on analyzing the relationship between the discrete update rules that define cellular automata, and the features—including pattern formation and computational capabilities—that characterize their global behavior and that are fundamental to understanding both their mathematical structure and their use in applications.

Although the usefulness of cellular automata in modeling and parallel-processing applications is now generally recognized, there exist few theoretical tools for understanding their behavior. In particular, an outstanding obstacle in the application of these systems has been the lack of a “rule-table mathematics” (i.e., a mathematics for functions specified in lookup or rule-table form) to enable the analysis of their inherently discrete dynamics. In the absence of such mathematics, it is not possible to investigate features of CA—such as stability, metrics, perturbations—that are routinely used to characterize continuous systems but whose interpretation for discrete systems is not yet clear. The result is that fundamental questions essential to the use of CA in simulations—the effects of a perturbation such as a bit flip in the rule table, say, or in an initial condition—are not understood.

This project's research has represented some of the first steps toward development of a rule-table mathematics and its application to the analysis of the global behavior generated by CA. In particular, we have developed tools for studying particle interactions in cellular automata, and to use these tools to analyze the fundamental features that characterize CA behavior—including the nature of patterns generated by the automata, their ability to perform global computations, and their relationship to continuous dynamical systems. Our long-term goal has been to develop a rule-table mathematics that would enable one to custom-design CA rule tables to generate patterns of specified types, or to perform specified computational tasks.

Specifically, our research addressed the following questions:

- What is the range of particle types (gliders, bound states, self-replicators, counters, etc.) and particle interaction mechanisms associated with one-dimensional CA?
- How do the characteristics of the particle type generated by specific CA relate to the microscopic structure of their rule table?
- How do perturbations in a CA rule table affect the particles it produces?
- What is an appropriate metric for CA rules?
- How robust is the self-organizing behavior of CA?
- What are implications of different particle interaction mechanisms for computational capabilities—for example, the ability to solve a network synchronization problem—of CA?
- How does the particle interaction mechanism of a CA evolve as the system undergoes “learning” or optimization for a specific computational task?

We have developed a new approach, based on particle interactions in cellular automata, that promises to lead to a breakthrough in understanding the macroscopic behavior of cellular automata. This approach promises to provide for the first time a means for analyzing the relationship between an automaton's microscopic-level specification through its rule table, and certain macroscopic features—including pattern formation and computational capabilities—that characterize its evolution. The central point is that these features can be captured through a reformulation of the automaton in terms of mesoscopic collision rules for information-bearing “particles” generated by the system, as well as for the interaction of the particles with the backgrounds against which they propagate (see Fig. 1).

In what follows, “particles” may equivalently be interpreted as “dislocations” or “domain walls,” and “backgrounds” may equivalently be interpreted as “domains”. The major components of the approach have been to:

- 1) identify backgrounds; i.e., alphabets of site values that constitute backgrounds, together with the grammar that governs their construction;
- 2) identify particles; i.e., clusters of site values that constitute mesoscopic-level spatio-temporally coherent structures (gliders, bound states, self-replicators, etc.);
- 3) describe the mechanism by which backgrounds and particles emerge (“self-organize”) from random initial conditions;
- 4) derive collision laws that describe interactions among particles;
- 5) relate microscopic rule table to mesoscopic particle interactions; and

- 6) relate mesoscopic particle interactions to macroscopic features such as exact solvability.

On the basis of the preliminary results outlined above, two distinct prototypical mechanisms have been discovered for the interaction of particles with backgrounds. These are the motion of randomly diffusing annihilating particles propagating against a stochastic but highly structured background; and the collision of particles traveling with constant speed against a background that has self-organized itself into a simple, regular tiling. The two distinct particle interaction mechanisms differ in their implications for the global behavior of the associated cellular automata, in particular features such as:

- self-organization, such as the emergence of ordered structures from random initial conditions,
- exact solvability,
- information propagation in systems, and
- asymptotic characteristics of the systems.

A primary focus of the research has been on understanding the means by which differing particle interaction mechanisms lead to differing global features as outlined above. We conjecture, for example, that the capability to perform a wide variety of computational tasks requires the generation of particles of at least two distinct speeds. Establishing the validity of this, and other conjectures relating to the complexity of patterns generated by CA, involves detailed study of the algebra of particle interactions.

Under evolution of certain nonlinear automata, typically termed “chaotic”, it has been found that the lattice of processors typically “organizes” itself into multiple contiguous domains within which behavior is ordered and highly correlated. The domains are separated by domain walls that may be interpreted physically as dislocations or, equivalently, as propagators of information.

The analysis of domain structure and dislocation behavior has led to the development of a solution method for a wide class of nonlinear automata. The method maps the nonlinear data onto an exactly solvable linear “template” automaton with a closely-related evolution function.

A critical component of the mapping between nonlinear and linear systems is the characterization of the behavior of the dislocation processors. For the prototypical cellular automaton Rule 18 studied in the first stage of this project, we showed that the dislocations are analogous to classical, annihilating diffusive particles. The propagation of these particles provides a realization of quasi-periodicity in these discrete dynamical systems. Using the solution method developed for cellular automaton Rule 18, we have obtained results for the expected number of dislocations in an arbitrary finite lattice; the expected

transience time for collision and annihilation of dislocations; and maximal bounds on transience time for this cellular automaton. The results represent one of the very few analytical characterizations of nonequilibrium behavior for nonlinear, spatially extended dynamical systems.

In addition, a technique has been developed to characterize the “grammar” that generates the set of spatial sequences that appear in domain structures stable under evolution Rule 54. It has moreover been shown that all sequences recognized by this grammar are attracted to limit cycles of identical period. A corollary of the result provides a proof of the fact that the rule acts upon a certain well-defined subset of spatial sequences so as to induce a spatial reflection of those sequences. The result represents the first demonstration of a global reflection operator induced by a local interaction rule.

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Figure 1: The mesoscopic reformulation of cellular automata in terms of particles is also equivalent to viewing the system as evolving through the interaction of “dislocations” or domain walls separating spatio-temporal domains of coherent behavior. The dislocations undergo collisions that may result in annihilation, coalescence, or spontaneous generation of new types of dislocations.