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Supersymmetric radiative corrections at large $\tan\beta$ [*]

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In the minimal supersymmetric extension of the Standard Model (MSSM), fermion masses and Yukawa couplings receive radiative corrections at one loop from diagrams involving the supersymmetric particles. The corrections to the relation between down-type fermion masses and Yukawa couplings are enhanced by $\tan\beta$, which makes them potentially very significant at large $\tan\beta$. These corrections affect a wide range of processes in the MSSM, including neutral and charged Higgs phenomenology, rare B meson decays, and renormalization of the CKM matrix. We give a pedagogical review of the sources and phenomenological effects of these corrections.

1. INTRODUCTION

In the minimal supersymmetric extension of the Standard Model (MSSM) [1], radiative corrections involving supersymmetric (SUSY) particles modify the tree level relation between fermion masses and their Yukawa couplings [2]. In this paper we review the sources and behavior of these SUSY Yukawa corrections and describe their phenomenological effects.

The behavior of the SUSY Yukawa corrections is most easily derived in the context of an effective field theory (EFT), in which we take the low energy effective theory of the MSSM below the SUSY scale to be a two Higgs doublet model (2HDM) and absorb the effects of SUSY radiative corrections into the parameters of the EFT. At tree level, the fermion Yukawa couplings and masses arise from the Lagrangian,[2]

$$-\mathcal{L}^{\text{tree}} = \epsilon_{ij}\left[y_b \bar{b}_R H_1^i Q_L^j + y_t \bar{t}_R Q_L^i H_2^j\right] + \text{h.c.} \quad (1)$$

where we use third generation quark notation and the Higgs doublets are

$$H_1 = \begin{pmatrix} (v_1 + \phi_1^{0,r} - i\phi_1^{0,i})/\sqrt{2} \\ -\phi_1^- \end{pmatrix},$$
$$H_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^{0,r} + i\phi_2^{0,i})/\sqrt{2} \end{pmatrix}. \quad (2)$$

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[*]Talk given at *30 Years of Supersymmetry*, Minneapolis, Minnesota, USA, October 13-27, 2000.
[2]For a review of the MSSM Higgs sector, see Ref. [3].

The Higgs vacuum expectation values (vevs) are constrained by $v_1^2 + v_2^2 = 4M_W^2/g^2$ and their ratio is parameterized by $v_2/v_1 \equiv \tan\beta$. The fermion masses arise from replacing the Higgs fields with their vevs:

$$
\begin{aligned}
-\mathcal{L}^{\text{tree}} &= y_b \frac{v_1}{\sqrt{2}}\bar{b}_R b_L + y_t \frac{v_2}{\sqrt{2}}\bar{t}_R t_L + \text{h.c.} \\
&= m_b \bar{b}_R b_L + m_t \bar{t}_R t_L + \text{h.c.} \quad (3)
\end{aligned}
$$

so that $m_b = \sqrt{2}y_b M_W \cos\beta/g$ and $m_t = \sqrt{2}y_t M_W \sin\beta/g$. At tree level, down-type fermions receive their masses from H_1 while up-type fermions receive their masses from H_2. This is a consequence of the holomorphicity of the superpotential in unbroken SUSY; if SUSY were unbroken, it would be true to all orders in the EFT.

In a model with more than one Higgs doublet, tree-level flavor-changing neutral Higgs couplings can be eliminated if the right-handed fermion singlets with each value of hypercharge are allowed to couple to only one Higgs doublet [4]. Under this requirement, called "natural flavor conservation", there are two different configurations possible for the couplings of the two Higgs doublets H_1 and H_2 of a 2HDM to quarks: in the Type I 2HDM all the quarks couple to H_1 and none to H_2, while in the Type II 2HDM the down-type right-handed quark singlets couple to H_1 while the up-type right-handed quark singlets couple to H_2. The most general 2HDM without natural flavor conservation, in which the right-handed
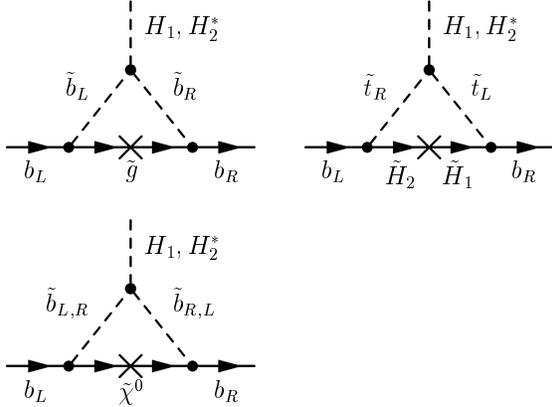
Figure 1. One-loop diagrams that contribute to Higgs-fermion couplings in broken SUSY.

fermion singlets with each value of hypercharge couple to both Higgs doublets, is called the Type III 2HDM. In unbroken SUSY, the Higgs couplings take the form of a Type II 2HDM.

However, SUSY must be broken in nature. When SUSY is broken, non-holomorphic Higgs couplings to fermions arise at one loop in the EFT. We first consider down-type fermions. At one loop, diagrams such as those in Fig. 1 give rise to a correction to the coupling of b quarks to H_1 and a non-holomorphic coupling of b quarks to H_2:

$$
\begin{aligned}
-\mathcal{L}^{1\,\text{loop}} &= \left(y_b + \Delta y_b^{(1)}\right) \bar{b}_R b_L H_1^0 \\
&+ \left(\Delta y_b^{(2)}\right) \bar{b}_R b_L H_2^{0*} + \text{h.c.}
\end{aligned} \tag{4}
$$

Examining Eq. 4, we see that in the EFT, the right-handed down-type quarks couple to both H_1 and H_2. Thus, *the low energy effective theory of the MSSM is a Type III 2HDM.*[3]

The b quark mass at one loop is obtained by inserting the Higgs vevs into Eq. 4:

$$
-\mathcal{L}^{1\,\text{loop}} = \left(y_b + \Delta y_b^{(1)}\right) \frac{v_1}{\sqrt{2}} \bar{b}_R b_L
$$

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[3] A review of the phenomenology of a general Type III 2HDM is given in Ref. [5].

$$
+ \left(\Delta y_b^{(2)}\right) \frac{v_2}{\sqrt{2}} \bar{b}_R b_L + \text{h.c.}
$$
$$
= m_b \bar{b}_R b_L + \text{h.c.} \tag{5}
$$

Solving for m_b we find,

$$
\begin{aligned}
m_b &= y_b \frac{\sqrt{2} M_W \cos\beta}{g} \\
&\times \left(1 + \frac{\Delta y_b^{(1)}}{y_b} + \frac{\Delta y_b^{(2)}}{y_b} \tan\beta\right). \tag{6}
\end{aligned}
$$

Here $\Delta y_b^{(1,2)}/y_b \ll 1$ because of the loop suppression. However, the third term in Eq. 6 is enhanced by a factor of $\tan\beta$, and thus can be quite large at large $\tan\beta$ [2,6]. This $\tan\beta$ enhanced correction to m_b has significant phenomenological consequences at large $\tan\beta$ because it modifies the relation between m_b and y_b.[4] At tree level, $y_b = g m_b / \sqrt{2} M_W \cos\beta$. At one loop, y_b can be significantly smaller or larger than the tree-level value, depending on the sign of $\Delta y_b^{(2)}/y_b$.

In the up-type fermion sector, an analogous calculation gives,

$$
\begin{aligned}
m_t &= y_t \frac{\sqrt{2} M_W \sin\beta}{g} \\
&\times \left(1 + \frac{\Delta y_t^{(1)}}{y_t} \cot\beta + \frac{\Delta y_t^{(2)}}{y_t}\right). \tag{7}
\end{aligned}
$$

In particular, there is no $\tan\beta$ enhancement and the corrections to the relation between m_t and y_t are small. These corrections can still be important; for example, they enter the MSSM Higgs mass calculation at the two loop level and lead to a few GeV change in the upper bound on M_{h^0} in some regions of parameter space [8].

We now examine the behavior of $\Delta y_b^{(1,2)}$ as the SUSY mass scale is varied. It is easy to see from, e.g., the first diagram in Fig. 1 that $\Delta y_b^{(1,2)}$ are independent of M_{SUSY}. This diagram scales like $\mu M_{\tilde{g}}/M_{SUSY}^2$, where the factor of $M_{\tilde{g}}$ comes from a helicity flip in the gluino line, the factor of μ comes from the dimensionful Higgs-squark-squark coupling, and the factor

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[4] In Ref. [7] it was shown that it is even possible to have $y_{b,s,d} = 0$ at tree level (so that $m_{b,s,d} = 0$) and generate the down-type quark masses at one loop through SUSY breaking terms.

of $1/M_{SUSY}^2$ comes from the three-point loop integral (here M_{SUSY} is the largest of the masses in the loop and we neglect external momenta; see Eq. 17). Thus if all the soft SUSY breaking parameters and the μ parameter are scaled by a common factor, $\Delta y_b^{(1,2)}$ are unchanged; i.e., they do not decouple with M_{SUSY} when the ratios between SUSY parameters are held fixed. These SUSY Yukawa corrections are also universal; i.e., they are independent of the external momenta and thus process-independent. In general, there are additional contributions to physical processes at one loop that depend on the external momenta; however, these are suppressed by powers of p^2/M_{SUSY}^2.

Although $\Delta y_b^{(1,2)}$ do not decouple with M_{SUSY}, they do not represent a violation of the decoupling theorem [9]: in the limit of large pseudoscalar Higgs mass M_A, the effects of $\Delta y_b^{(1,2)}$ appear only in the couplings of the heavy Higgs bosons, as we will now illustrate. Because in the Type III 2HDM the right-handed down-type fermions couple to both Higgs doublets, there is no longer a special Yukawa basis for the Higgs fields. We are then free to write them in a new basis chosen such that one of the doublets has zero vev (the "vev basis"):

$$
\begin{aligned}
\Phi_1 &= \begin{pmatrix} G^+ \\ (v_{SM} + \Phi_1^{0,r} + iG^0)/\sqrt{2} \end{pmatrix}, \\
\Phi_2 &= \begin{pmatrix} H^+ \\ (\Phi_2^{0,r} + iA^0)/\sqrt{2} \end{pmatrix},
\end{aligned} \tag{8}
$$

where $v_{SM} = 2M_W/g$ is the Standard Model (SM) Higgs vev and we have performed the rotation ($H_1^c = i\tau_2 H_1^*$)

$$
\begin{aligned}
H_1^c &= \cos\beta\,\Phi_1 - \sin\beta\,\Phi_2, \\
H_2 &= \sin\beta\,\Phi_1 + \cos\beta\,\Phi_2.
\end{aligned} \tag{9}
$$

Note that in the vev basis Φ_1 contains the Goldstone bosons while Φ_2 contains H^+ and A^0. The two CP-even neutral Higgs mass eigenstates h^0 and H^0 are given in terms of $\Phi_1^{0,r}$ and $\Phi_2^{0,r}$ by

$$
\begin{aligned}
h^0 &= \sin(\beta-\alpha)\Phi_1^{0,r} + \cos(\beta-\alpha)\Phi_2^{0,r} \\
H^0 &= \cos(\beta-\alpha)\Phi_1^{0,r} - \sin(\beta-\alpha)\Phi_2^{0,r}.
\end{aligned} \tag{10}
$$

In the limit of large M_A, $\cos(\beta - \alpha)$ goes to zero:

$$
\cos(\beta - \alpha) \simeq \frac{M_Z^2 \sin 4\beta}{2M_A^2}, \tag{11}
$$

so $h^0 \to \Phi_1^{0,r}$. Thus in the limit of large M_A, Φ_1 contains the Goldstone bosons, the SM Higgs vev, and the light Higgs boson h^0, while Φ_2 contains all the heavy Higgs states, $H^\pm$, A^0 and H^0.

The effective Lagrangian in the vev basis can be written as,

$$
-\mathcal{L}^{\rm eff} = \lambda_1^b \Phi_1^{0*}\bar{b}_R b_L + \lambda_2^b \Phi_2^{0*}\bar{b}_R b_L + {\rm h.c.}, \tag{12}
$$

where λ_1^b and λ_2^b are the Yukawa couplings in the vev basis. Inserting the Higgs vevs we obtain the b quark mass:

$$
\begin{aligned}
-\mathcal{L}^{\rm eff} &= \lambda_1^b \frac{v_{SM}}{\sqrt{2}} \bar{b}_R b_L + {\rm h.c.} \tag{13} \\
&= m_b \bar{b}_R b_L + {\rm h.c.} \tag{14}
\end{aligned}
$$

In particular, $\lambda_1^b = \sqrt{2}m_b/v_{SM} = gm_b/\sqrt{2}M_W$ is fixed by the b quark mass, while λ_2^b is unconstrained. Thus the coupling of Φ_1 to quarks must be SM-like while that of Φ_2 is unconstrained and can contain the effects of $\Delta y_b^{(1,2)}$. In the decoupling limit [10] of large M_A, $h^0 \to \Phi_1^{0,r}$ so the $h^0 b\bar{b}$ coupling approaches its SM value, while the effects of $\Delta y_b^{(1,2)}$ are confined to the heavy Higgs bosons in Φ_2.

We now write down the SUSY radiative corrections to the down-type mass-Yukawa relation (keeping only the $\tan\beta$ enhanced contributions) [2,11]:

$$
\begin{aligned}
m_b &\simeq y_b \frac{\sqrt{2}M_W \cos\beta}{g} \left(1 + \frac{\Delta y_b^{(2)}}{y_b} \tan\beta \right) \\
&\equiv y_b \frac{\sqrt{2}M_W \cos\beta}{g} \left(1 + \Delta_b \right), \tag{15}
\end{aligned}
$$

where the correction Δ_b is given by [5]

$$
\begin{aligned}
\Delta_b &\simeq \frac{2\alpha_s}{3\pi} M_{\tilde{g}}\mu \tan\beta\, I(M_{\tilde{b}_1}, M_{\tilde{b}_2}, M_{\tilde{g}}) \\
&\quad + \frac{\alpha_t}{4\pi} A_t \mu \tan\beta\, I(M_{\tilde{t}_1}, M_{\tilde{t}_2}, \mu). \tag{16}
\end{aligned}
$$

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[5]Because the SUSY-QCD contribution to Δ_b is proportional to the product $\mu M_{\tilde{g}}$, it has been suggested in Ref. [12] to use the sign of Δ_b and the sign of μ (measured in some other process) to determine the sign of $M_{\tilde{g}}$ and test the anomaly-mediated SUSY breaking scenario [13], which predicts a negative $M_{\tilde{g}}$.

Here $\alpha_t = y_t^2/4\pi$ and $I(a, b, c)$ is a loop integral,

$$
\begin{aligned}
I(a, b, c) = \ & [a^2 b^2 \log(a^2/b^2) + b^2 c^2 \log(b^2/c^2) \\
& + c^2 a^2 \log(c^2/a^2)]/ \\
& [(a^2 - b^2)(b^2 - c^2)(a^2 - c^2)], \quad (17)
\end{aligned}
$$

which is positive for any a, b, c and goes like $1/\max(a^2, b^2, c^2)$.

Solving Eq. 15 for y_b [6], we find

$$
y_b = \frac{g m_b}{\sqrt{2} M_W \cos\beta} \frac{1}{1 + \Delta_b}, \qquad (18)
$$

where we have not expanded the denominator to one loop order. It was proven in Ref. [14] that Eq. 18 includes a resummation of terms of order $(\alpha_s \mu \tan\beta/M_{SUSY})^n$ to all orders of perturbation theory. In particular, Δ_b in Eq. 18 gets no $\tan\beta$ enhanced corrections at higher orders of the form $(\alpha_s \mu \tan\beta/M_{SUSY})^n$. The remaining higher order corrections to Δ_b are not $\tan\beta$ enhanced and are thus insignificant compared to the one loop piece (Eq. 16). The proof in Ref. [14] that Eq. 18 receives no higher order $\tan\beta$ enhanced contributions relies on the facts (1) that $\tan\beta$ enters the calculation at higher orders only multiplied by m_b, e.g., from the $\mu m_b \tan\beta$ term in the bottom squark mass matrix; and (2) that no factors of $1/m_b$ can arise from the loop diagrams to cancel the m_b factor multiplying $\tan\beta$, because the Yukawa coupling operator is dimension four.

Expanding Eq. 18 to one loop order, we have

$$
y_b = \frac{g m_b}{\sqrt{2} M_W \cos\beta} (1 - \Delta_b). \qquad (19)
$$

An expression of this form arises in on-shell diagrammatic one-loop calculations, in which Δ_b enters through the b quark mass counterterm. Although Eqs. 18 and 19 are equivalent at one loop, they differ at higher orders: Eq. 19 does not contain a resummation of terms of order $(\alpha_s \mu \tan\beta/M_{SUSY})^n$ to all orders of perturbation theory.[6] In our EFT analyses in the remainder of this paper we will use Eq. 18 in order to take advantage of the resummation of higher order terms.

The remainder of this paper is organized as follows. We examine the effect of the SUSY Yukawa

corrections on the process $h^0 \to b\bar{b}$ in Sec. 2 and on the $H^+ \bar{t} b$ coupling in Sec. 3. In Sec. 4 we describe how flavor-changing neutral Higgs interactions arise in the EFT from the SUSY Yukawa corrections, and in Sec. 5 we summarize recent results for specific flavor changing processes. In Sec. 6 we discuss the renormalization of the CKM matrix. In Sec. 7 we briefly discuss resummation of the SUSY Yukawa corrections to flavor changing processes. Finally in Sec. 8 we give a summary and outlook.

2. SUSY CORRECTIONS TO $h^0 \to b\bar{b}$

Accurate knowledge of the $h^0 \to b\bar{b}$ branching ratio is important for determining the reach of the upcoming Higgs searches at the Tevatron and LHC [15]. Also, once a light Higgs boson is discovered, precision measurements of its branching ratios (e.g., at a future $e^+ e^-$ linear collider) can be used to distinguish a SM Higgs boson from a MSSM Higgs boson in some regions of parameter space [16]. The SUSY Yukawa corrections can have a significant effect on these branching ratios [6,15–17]. For example, the ratio $BR(h^0 \to b\bar{b})/BR(h^0 \to \tau^+\tau^-)$ is sensitive to the SUSY Yukawa corrections to $h^0 \to b\bar{b}$.[7] At tree level, the ratio $BR(h^0 \to b\bar{b})/BR(h^0 \to \tau^+\tau^-)$ is the same in the SM and the MSSM ($\propto m_b^2/m_\tau^2$). Further, in the MSSM the $h^0 b\bar{b}$ and $h^0 \tau^+\tau^-$ couplings have the same dependence on the CP-even Higgs mixing angle α, so that $BR(h^0 \to b\bar{b})/BR(h^0 \to \tau^+\tau^-)$ is also insensitive to the radiative corrections to α. Thus in the context of the MSSM, a deviation of $BR(h^0 \to b\bar{b})/BR(h^0 \to \tau^+\tau^-)$ from its SM value provides a direct window onto the SUSY Yukawa corrections [6,15–17].

2.1. EFT calculation

We begin by computing the SUSY corrections to $h^0 \to b\bar{b}$ in the EFT approach; that is, we assume $M_{SUSY} > M_A$ and neglect the external momenta.[8] Using Eqs. 4 and 18, the $h^0 b\bar{b}$ coupling

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[6] A proof as in Ref. [14] does not hold for Eq. 19, in which the b mass operator is renormalized, because the b mass operator is not dimension four. See Ref. [14] for details.

[7] The $h^0 \tau^+\tau^-$ coupling also receives $\tan\beta$ enhanced SUSY Yukawa corrections, but they are proportional to electroweak gauge couplings and are expected to be much smaller than the corrections to the $h^0 b\bar{b}$ coupling.

[8] We will consider the effects of the external momenta and lower M_{SUSY} in the next section.

is given by [6]

$$g_{hbb}^{\text{EFT}} \simeq \frac{gm_b \sin\alpha}{2M_W \cos\beta}$$
$$\times \left[1 - \frac{\Delta_b}{1+\Delta_b}\left(1 + \frac{1}{\tan\alpha\tan\beta}\right)\right] \quad (20)$$

where the tree level coupling is $g_{hbb}^{\text{tree}} = gm_b\sin\alpha/2M_W\cos\beta$ and $\mathcal{L} = g_{hbb}h^0\bar{b}b + \cdots$. As shown in Sec. 1, the SUSY radiative corrections to g_{hbb} should decouple in the limit of large M_A. Indeed, we have,

$$\left(1 + \frac{1}{\tan\alpha\tan\beta}\right) \simeq -\frac{2M_Z^2}{M_A^2}\cos 2\beta, \quad (21)$$

so that g_{hbb} approaches its SM value as expected in the decoupling limit [10,18].

2.2. Diagrammatic calculation

Up to this point we have neglected the effects of the external momenta on $h^0 \to b\bar{b}$. These give contributions of order $(p^2\tan\beta/M_{SUSY}^2)$, where $p^2 = M_{h^0}^2$. These contributions can be sizeable because of the $\tan\beta$ enhancement, especially when the SUSY scale is relatively low. For example, for $\tan\beta \sim 50$, $(M_{h^0}^2\tan\beta/M_{SUSY}^2)$ is of order one even for $M_{SUSY} \sim 1$ TeV. In order to treat the external momenta correctly, we must do an on-shell diagrammatic calculation. We focus here on the SUSY-QCD contributions to g_{hbb},[9] for which diagrammatic calculations are available in Refs. [18–20].[10] For compactness we give here an approximate formula[11] expanded in powers of M_{EW}^2/M_{SUSY}^2 and M_{EW}^2/M_A^2, where we have taken $M_{SUSY} \equiv M_S = \mu = M_{\tilde{g}} = A_b$ and $M_S^2 = (M_{\tilde{b}_1}^2 + M_{\tilde{b}_2}^2)/2$. Defining $g_{hbb}^{\text{diag}} = g_{hbb}[1 + \Delta_{SQCD}]$, we have [18]

$$\Delta_{SQCD} = \frac{\alpha_s}{3\pi}\left\{\frac{2M_Z^2}{M_A^2}\cos 2\beta\tan\beta\right.$$
$$+ \frac{M_{h^0}^2}{12M_{SUSY}^2}(\tan\beta - 1)$$

---

[9]The SUSY-electroweak contributions will be considered in the next section.

[10]Note that, unlike the EFT calculations, the one-loop diagrammatic calculations do not incorporate a resummation of higher order $\tan\beta$ enhanced contributions.

[11]The full one loop result may be found in Refs. [18,19].

$$+ \frac{M_Z^2}{3M_{SUSY}^2}\cos 2\beta(\tan\beta - 2)$$
$$\left. + \frac{m_b^2\tan^2\beta}{2M_{SUSY}^2}(\tan\beta - 4)\right\}. \quad (22)$$

The first term in Eq. 22 was found before in the EFT calculation; it remains constant as $M_{SUSY} \to \infty$ at fixed M_A. The remaining terms are neglected in the EFT approach; they decouple as $M_{SUSY} \to \infty$ but remain constant as $M_A \to \infty$ at fixed M_{SUSY}. Because of their $1/M_{SUSY}^2$ dependence they can be identified as coming from higher dimensional operators that were neglected in our EFT calculation. The correction Δ_{SQCD} decouples in the limit that both M_A and $M_{SUSY} \to \infty$.

2.3. SUSY-Electroweak corrections

The SUSY-electroweak corrections to $h^0 \to b\bar{b}$ are of order α_t instead of order α_s and arise from the second diagram of Fig. 1. The SUSY-electroweak Yukawa corrections are easily calculated in the EFT approach (see Eq. 16). There are also large corrections to h^0 decays at order α_t from the radiative corrections to the CP-even Higgs mixing angle α [6,21]. These corrections are usually taken into account by inserting the radiatively corrected value of α into the tree level Higgs couplings to fermions.

At tree level, the off-diagonal elements of the CP-even Higgs mass matrix are $(\mathcal{M}_{12}^2)_{\text{tree}} = -(M_A^2 + M_Z^2)\sin\beta\cos\beta$ [3]. When M_A is small and $\tan\beta$ is large, $(\mathcal{M}_{12}^2)_{\text{tree}}$ can be quite small, of the same order as its one loop radiative corrections. Then it is possible to tune the radiatively corrected $\mathcal{M}_{12}^2$ to zero by a careful choice of the MSSM parameters, thereby driving the mixing angle α to zero. If $\sin\alpha = 0$, the $h^0b\bar{b}$ and $h^0\tau^+\tau^-$ couplings both vanish at tree level. However, the SUSY-QCD Yukawa corrections to the $h^0b\bar{b}$ coupling are nonzero, because they depend on the h^0 couplings to bottom squarks which remain finite at $\sin\alpha = 0$. Thus at $\sin\alpha = 0$, the $h^0\tau^+\tau^-$ coupling vanishes while the $h^0b\bar{b}$ coupling is nonzero due to the SUSY-QCD Yukawa corrections. If we then vary the supersymmetric parameters, we can tune α so that the tree level value of the $h^0b\bar{b}$ coupling cancels its SUSY-QCD Yukawa corrections.

Then the corrected $h^0 b \bar{b}$ coupling vanishes while the $h^0 \tau^+ \tau^-$ coupling is again nonzero. Thus the $h^0 b \bar{b}$ and $h^0 \tau^+ \tau^-$ couplings vanish at different values of α, or equivalently, at different points in SUSY parameter space. This behavior was first pointed out in Ref. [15].

As for the SUSY-QCD corrections, an on-shell diagrammatic calculation is needed to take into account terms of order $(p^2 \tan\beta/M_{SUSY}^2)$ from the external momenta. In Ref. [22] a partial on-shell diagrammatic calculation was performed, which includes an on-shell calculation of the corrections to α and the on-shell SUSY-QCD corrections to $h^0 \to b \bar{b}$, but does not include the $\mathcal{O}(\alpha_t)$ SUSY-electroweak Yukawa corrections to $h^0 \to b \bar{b}$. A full one-loop diagrammatic calculation of the SUSY-electroweak corrections to $h^0 \to b \bar{b}$ is in progress [23].

3. SUSY CORRECTIONS TO THE $\mathbf{H^+ \bar{t} b}$ COUPLING

The $\tan\beta$ enhanced SUSY Yukawa corrections modify the $H^+ \bar{t} b$ coupling through, e.g., the first diagram of Fig. 1 with b_L ($\tilde{b}_L$) replaced by t_L ($\tilde{t}_L$) and taking the charged Higgs states in H_1, H_2^*. These corrections have important implications for the charged Higgs boson search through $t \to H^+ b$ at the Tevatron [24,14] because the reach in M_{H^+} and $\tan\beta$ depends on the effective t and b Yukawa couplings.

The one loop SUSY Yukawa corrections to $t \to H^+ b$ can be very large, of order the tree level amplitude, so that reliable calculations require a resummation or improvement. Using the EFT approach, we have at large $\tan\beta$ [14]

$$g_{H^+ tb} \simeq \frac{g m_b}{\sqrt{2} M_W} \frac{1}{1 + \Delta_b} \tan\beta, \quad (23)$$

with Δ_b as in Eq. 16. As described in Sec. 1, in this result Δ_b gets no $\tan\beta$ enhanced corrections at higher orders of the form $(\alpha_s \tan\beta \mu/M_{SUSY})^n$ [14]. The remaining higher order corrections to Δ_b are not $\tan\beta$ enhanced.

The SUSY Yukawa corrections to the $H^+ \bar{t} b$ coupling have also been computed diagrammatically in Refs. [25,26], giving results of the form

$$g_{H^+ tb} = \frac{g m_b}{\sqrt{2} M_W}(1 - \Delta_b) \tan\beta. \quad (24)$$

This is equivalent to Eq. 23 at one loop level, but in Eq. 24 Δ_b receives $\tan\beta$ enhanced radiative corrections at higher orders. In Ref. [26] the tree-level input parameters were carefully chosen to keep the one-loop corrections in Eq. 24 small. This is analogous to using running quark masses to absorb QCD corrections; however it does not take into account the higher order $\tan\beta$ enhanced SUSY corrections.

4. FLAVOR CHANGING NEUTRAL HIGGS COUPLINGS

The $\tan\beta$ enhanced SUSY Yukawa corrections lead to flavor changing neutral Higgs interactions in the EFT. This was first pointed out in Ref. [27] for B-$\bar{B}$ mixing and expanded upon in Ref. [28] for leptonic B meson decays. Similar results have been found in diagrammatic calculations for a number of processes; these will be discussed in Sec. 5. In this section we describe how flavor changing neutral Higgs interactions arise in the EFT.

As shown in Sec. 1, for $M_A < M_{SUSY}$ the low energy effective theory of the MSSM is the Type III 2HDM. In the Type III 2HDM, there is no natural flavor conservation and flavor changing neutral Higgs couplings arise in general [4]. In three generation notation, Eq. 12 becomes

$$-\mathcal{L}^{\text{eff}} = \lambda_1^{ij} \Phi_1^{0*} \bar{d}_R^i d_L^j + \lambda_2^{ij} \Phi_2^{0*} \bar{d}_R^i d_L^j + \text{h.c.}, \quad (25)$$

where we again use the vev basis, Eq. 8. Here i, j are fermion generation indices and $\lambda_{1,2}^{ij}$ are general Yukawa matrices, not necessarily diagonal. The down-type quark mass matrix is obtained by replacing the Higgs fields with their vevs:

$$-\mathcal{L}^{\text{eff}} = \lambda_1^{ij} \frac{v_{SM}}{\sqrt{2}} \bar{d}_R^i d_L^j + \text{h.c.} \quad (26)$$

Note that λ_2 does not appear in Eq. 26 because Φ_2 has zero vev. Diagonalizing the mass matrix in Eq. 26 diagonalizes the Yukawa matrix λ_1^{ij}, which parameterizes the down-type fermion couplings to Φ_1. The Yukawa matrix λ_2^{ij}, which parameterizes the down-type fermion couplings to Φ_2, is not in general diagonal in the fermion mass basis. Rewriting Eq. 25 in the fermion mass basis, we

have

$$-\mathcal{L}^{\text{eff}} = \frac{gm_i}{\sqrt{2}M_W} \Phi_1^{0*} \bar{d}_R^i d_L^i + \lambda_2^{ij} \Phi_2^{0*} \bar{d}_R^i d_L^j + \text{h.c.} \quad (27)$$

Clearly, the couplings of Φ_1 to down-type fermions are flavor diagonal, while those of Φ_2 are not. The off-diagonal elements in λ_2^{ij} give flavor changing couplings of the states in Φ_2^0 (A^0 and $\Phi_2^{0,r}$) to down-type fermions. Note that the flavor changing effects of the SUSY Yukawa corrections in physical processes should decouple at large M_A, for which $\Phi_2^{0,r} \simeq H^0$ and $M_{H^0} \simeq M_A$.

There are two potential sources of flavor changing neutral Higgs couplings in the MSSM: minimal and non-minimal flavor violation. In the MSSM with minimal flavor violation, the sfermion mass matrices are flavor-diagonal in the same basis as the quark and lepton mass matrices. Then the only source of flavor changing is the CKM matrix, as in the SM. In this case the flavor changing neutral Higgs couplings arise from one loop diagrams that contain a generation-changing $W^\pm$, $H^\pm$, or chargino coupling and an associated CKM matrix element.

Non-minimal flavor violation occurs when the sfermion mass matrices are not flavor-diagonal in the same basis as the quark and lepton mass matrices. When the sfermion mass matrices are diagonalized, flavor changing gluino-squark-quark and neutralino-sfermion-fermion couplings arise. Non-minimal flavor violation is present in the most general MSSM, and can lead to large flavor-changing effects in contradiction to experiment in, e.g., K-$\bar{K}$ mixing. This is the SUSY flavor problem, which must be solved in any realistic MSSM scenario. Attempts to solve the SUSY flavor problem include flavor-blind SUSY breaking scenarios (e.g., minimal supergravity), in which the sfermion mass matrices are flavor diagonal in the same basis as the quark and lepton mass matrices at the SUSY-breaking scale. However, a small amount of non-minimal flavor violation is typically regenerated as the sfermion and fermion mass matrices are run down to the electroweak scale. If non-minimal flavor violation is present in the MSSM, then the flavor changing neutral Higgs couplings receive contributions from one loop diagrams that contain a flavor-changing gluino or

neutralino coupling, in addition to the contributions present in the minimal flavor violation scenario.

5. SOME SPECIFIC FLAVOR CHANGING PROCESSES

5.1. $b \to s\gamma$

The SUSY contributions to $b \to s\gamma$ are well known [29]. In particular, the chargino-top squark contribution to the amplitude is proportional to $\tan\beta$. It is possible to obey the experimental constraint on $b \to s\gamma$ and still have large flavor changing effects in other processes from the chargino-top squark loop by choosing heavy enough chargino and top squark masses [30].

Recently in Ref. [31] the higher order $\tan\beta$ enhanced contributions to $b \to s\gamma$ were resummed, following the formalism of Ref. [14]. These higher order contributions can be large and can modify the regions of MSSM parameter space excluded by $b \to s\gamma$ [31].

5.2. $B_{s,d} \to \ell^+\ell^-$

Because the SM amplitude for $B \to \ell^+\ell^-$ is helicity suppressed, contributions from neutral Higgs boson exchange in the 2HDM or MSSM can compete with the SM amplitude at large $\tan\beta$. In the MSSM with large $\tan\beta$, the dominant contribution to the amplitude comes from h^0, H^0 and A^0 exchange with a flavor changing SUSY bubble in the external s quark line, and grows with $\tan^3\beta$ [28,30,32]. In the MSSM with minimal flavor violation the flavor changing SUSY bubble is a chargino-top squark loop; if non-minimal flavor violation is present, additional contributions come from a gluino-bottom squark and neutralino-bottom squark loop. All other one-loop SUSY contributions have at most a $\tan^2\beta$ enhancement. In the Type II 2HDM with large $\tan\beta$, penguin and box diagrams give a contribution to the amplitude that grows with $\tan^2\beta$ and is of the same order as the SM contribution [32–35].

In the MSSM, $B_{s,d} \to \ell^+\ell^-$ can be enhanced by three orders of magnitude over the SM rate because of the $\tan^3\beta$ enhancement in the amplitude. This has been demonstrated in an EFT

calculation in Ref. [28] and in diagrammatic calculations in Refs. [30,32]. The experimental sensitivity is best in the channel $B_s \to \mu^+\mu^-$; here the SM predicts $BR(B_s \to \mu^+\mu^-) = 4.3^{+0.9}_{-0.8} \times 10^{-9}$ [36], while the current experimental bound is $BR(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}$ (95% CL) [37]. At large $\tan\beta$ and low M_A, there are regions of SUSY parameter space in which $BR(B_s \to \mu^+\mu^-)$ can exceed its current upper bound [30]; thus SUSY parameter space is already being probed by this decay. Future experiments at the Tevatron Run II should improve the experimental bound by a factor of 40 with 2 fb^{-1} of data [38], and further at an extended Run II. At the LHC this decay is expected to be observed at the SM rate after three years of running at low luminosity [39].

5.3. B-$\bar{B}$ mixing

B-$\bar{B}$ mixing has been analyzed in the context of the effective Type III 2HDM in Refs. [27,28]. In the EFT, a tree-level neutral Higgs boson exchange diagram with flavor changing couplings at each vertex contributes to B-$\bar{B}$ mixing. If the tree-level relations for MSSM Higgs boson masses and couplings are used, then the three tree-level EFT diagrams involving h^0, H^0 and A^0 exchange sum to zero [28]. However, the tree-level relations for the MSSM Higgs boson masses and couplings are in general a poor approximation, and when radiative corrections to these masses and couplings are included the neutral Higgs mediated contributions to B-$\bar{B}$ mixing no longer cancel.

6. RENORMALIZATION OF THE CKM MATRIX

In the SM, the radiative corrections to the CKM matrix are very small electroweak effects. In the MSSM, however, there are $\tan\beta$ enhanced flavor changing loop diagrams that may yield significant corrections to the CKM matrix. In Ref. [40] the down-type quark mass matrix was run down from a high scale and the $\tan\beta$ enhanced SUSY corrections were included at the scale where the SUSY particles were integrated out; diagonalizing the quark mass matrices then yielded the radiatively corrected CKM matrix.

Here we describe the source of the SUSY corrections to the CKM matrix in the EFT.

When we diagonalized the down-type quark mass matrix in Eq. 26, the one loop SUSY corrections were implicitly included. Writing the one loop corrections explicitly, we have

$$
\begin{aligned}
-\mathcal{L}^{\text{eff}} &= \left(y_{ij} + \Delta y_{ij}^{(1)} \right) H_1^0 \bar{d}_R^i d_L^j \\
&\quad + \left(\Delta y_{ij}^{(2)} \right) H_2^{0*} \bar{d}_R^i d_L^j + \text{h.c.}
\end{aligned}
\tag{28}
$$

In the vev basis this becomes,

$$
\begin{aligned}
-\mathcal{L}^{\text{eff}} \\
= y_{ij}\cos\beta \left(1 + \frac{\Delta y_{ij}^{(1)}}{y_{ij}} + \frac{\Delta y_{ij}^{(2)}}{y_{ij}}\tan\beta \right) \Phi_1^{0*} \bar{d}_R^i d_L^j \\
- y_{ij}\sin\beta \left(1 + \frac{\Delta y_{ij}^{(1)}}{y_{ij}} - \frac{\Delta y_{ij}^{(2)}}{y_{ij}}\cot\beta \right) \Phi_2^{0*} \bar{d}_R^i d_L^j \\
+ \text{h.c.}
\end{aligned}
\tag{29}
$$

Comparing this to Eq. 25, λ_1^{ij} is given by

$$
\lambda_1^{ij} = y_{ij}\cos\beta \left(1 + \frac{\Delta y_{ij}^{(1)}}{y_{ij}} + \frac{\Delta y_{ij}^{(2)}}{y_{ij}}\tan\beta \right).
\tag{30}
$$

Diagonalizing the down-type quark mass matrix diagonalizes λ_1^{ij}. Since $\Delta y_{ij}^{(1,2)}$ are not in general diagonal in the same basis as the tree-level Yukawa matrix y_{ij}, they yield an additional rotation of the mass matrix. This additional rotation is a correction to the CKM matrix.

At present, the CKM elements are not predicted by theory and must be input from experiment. If the CKM elements are measured solely through W couplings to quarks, the SUSY corrections can simply be absorbed into the bare elements and are undetectable. However, if the CKM elements derived from W-quark-quark, $H^{\pm}$-quark-quark, and especially chargino-squark-quark couplings can be compared, the SUSY corrections may become apparent. We expect the chargino-squark-quark couplings to receive large corrections relative to the W-quark-quark couplings because SUSY-breaking effects will lead to different results for the flavor changing SUSY bubble diagrams in the external down-type quark and down-type squark lines.

7. RESUMMATION REVISITED

The SUSY radiative corrections that lead to flavor changing neutral Higgs couplings are enhanced by $\tan\beta$ and can be quite large, as we have seen. As in the flavor conserving case, these $\tan\beta$ enhanced corrections should be resummed. Such a resummation has been performed recently in Ref. [31] for higher-order corrections to $b \to s\gamma$, following the results in Ref. [14]. However, it is unclear whether this procedure will work in general because it is unclear whether the proof in Ref. [14] that all higher order $\tan\beta$ enhanced terms are automatically resummed if the relation in Eq. 18 is used is applicable to the flavor changing case. For example, factors of m_b/m_s or m_s/m_b can in principle arise at higher orders. This issue should be clarified before the resummation procedure is applied to additional flavor changing processes.

8. SUMMARY AND OUTLOOK

It is clear that the $\tan\beta$ enhanced SUSY corrections to the relation between down-type fermion masses and their Yukawa couplings has the potential to be a good handle on the MSSM at large $\tan\beta$. These corrections lead to large effects in both Higgs physics and B physics. Because these effects can be quite large, reliable calculations are needed. In Higgs boson decays, the external momenta are on the order of the electroweak scale and their effects should be included through diagrammatic calculations; these are available for the SUSY-QCD corrections to $h^0 \to b\bar{b}$ and calculations are in progress for the SUSY-electroweak corrections to both $h^0 \to b\bar{b}$ and $h^0 \to \tau^+\tau^-$. In addition, resummation of higher order $\tan\beta$ enhanced contributions beyond the one loop level is very important in some regions of parameter space. This resummation is well known and straightforward in the EFT approach but should also be incorporated into the diagrammatic calculations. In flavor changing processes such as rare B meson decays and CKM matrix renormalization, it is not entirely clear how the resummation should be done. Finally, the SUSY corrections to the CKM matrix may have a significant effect on the $H^\pm$ and chargino couplings to (s)fermions relative to the $W^\pm$ couplings. In short, the $\tan\beta$ enhanced SUSY corrections to the relation between down-type fermion masses and their Yukawa couplings offer a wide range of challenges and opportunities for MSSM phenomenology.

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