

## Event-by-event fluctuations and the QGP

V. Koch<sup>a</sup>, M. Bleicher<sup>b</sup>, and S. Jeon<sup>c</sup>

<sup>a</sup>Nuclear Science Division, Lawrence Berkeley National Laboratory,  
1 Cyclotron Road, Berkeley, CA 94720, USA

<sup>b</sup>Institut für Theoretische Physik, Universität Frankfurt  
60054 Frankfurt am Main, Germany

<sup>c</sup>Physics Department, McGill University  
Montreal, QC H3A-2T8 Canada

We discuss the physics underlying event-by-event fluctuations in relativistic heavy ion collisions. We will emphasize how the fluctuations of particle ratios can be utilized to explore the properties of the matter created in these collisions. In particular, we will argue that the fluctuations of the ratio of positively over negatively charged particles may serve as a unique signature for the Quark Gluon Plasma.

### 1. INTRODUCTION

Any physical quantity measured in experiment is subject to fluctuations. In general, these fluctuations depend on the properties of the system under study (in the case at hand, on the properties of a fireball created in a heavy ion collision) and may contain important information about the system.

The original motivation for event-by-event (E-by-E) studies in ultra relativistic heavy ion collisions has been to find indications for distinct event classes. In particular it was hoped that one would find events which would carry the signature of the Quark Gluon Plasma. First pioneering experiments in this direction have been carried out by the NA49 collaboration [1]. They analysed the E-by-E fluctuations of the mean transverse momentum as well as the kaon to pion ratio (see Fig. (1)). Both observables, however, did not show any indication for two or more distinct event classes. Moreover, the observed fluctuations in both cases were consistent with pure statistical fluctuations.

On the theoretical side, the subject of E-by-E fluctuations has recently gained considerable interest. Several methods to distinguish between statistical and dynamic fluctuations have been devised [2,3]. Furthermore the influence of hadronic resonances and possible phase transitions has been investigated [48]. All these theoretical considerations assume that the observed fluctuations will be Gaussian and thus the physics information will be in the width of the Gaussian. In [9] one of us has shown, that in this case, the information contained in E-by-E fluctuations is that of a two particle density, which can alternatively be measured using a two arm spectrometer. This observation implies that possibly interesting fluctuations can be observed and verified by a whole array of detectors not only large acceptance ones.

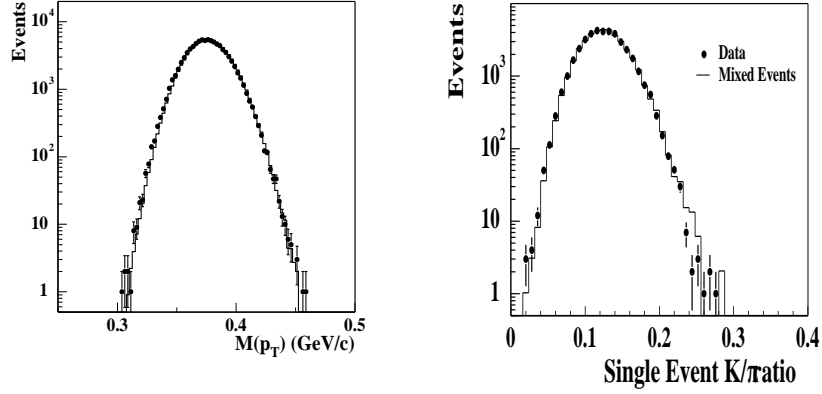


Figure 1. Results from the NA49 collaboration for the E-by-E fluctuations of the mean transverse momentum as well as the kaon to pion ratio [1]

## 2. Event-by-event fluctuations

Fluctuations have contributions of different nature. First there are the ‘trivial’ statistical fluctuations due to a finite number of particles used to define a particular observable in a given event. Second there are dynamical fluctuations. Those carry the information about the properties of the system. In the special case of heavy ion collisions, there is another source of ‘trivial’ fluctuations, namely the fluctuations of the volume. All these fluctuations contribute to E-by-E fluctuations. The obvious challenge, therefore, is to separate the ‘trivial’ from the interesting components.

### 2.1. Fluctuations in thermal systems

In a thermal system, the fluctuations of a quantity are proportional to its susceptibility, which is the second derivative of the appropriate free energy with respect to the conjugate variable. For example the fluctuations of the charge are given by

$$\langle \delta Q \rangle = -T \frac{\partial^2 F}{\partial \mu_Q^2} = VT \chi_Q \quad (1)$$

where  $\mu_Q$  is the charge chemical potential,  $T$  the temperature and  $V$  the volume of the system.  $\chi_Q$  is the charge susceptibility. Here we will mostly concentrate on the charge fluctuations. Therefore, let us discuss the charge susceptibility in some more detail. First, as shown in [10], the charge susceptibility is directly related to the electric screening mass in a system. Furthermore, it is given by the zero momentum limit of the static current-current correlation function [10]

$$\chi_Q = \Pi_{00}(\omega = 0, k \rightarrow 0) \quad (2)$$

which can be, and has been, evaluated in Lattice QCD [11,12] as well as in effective hadronic models [10,13,14].

## 2.2. Fluctuations in Heavy Ion Collisions

The systems created in a heavy ion collisions carry another important, but rather uninteresting source of fluctuations namely the fluctuations of the volume. Even for the tightest centrality selection the impact parameter and thus the volume of the system still fluctuates considerably. These volume fluctuations may even dominate. For example let us consider the fluctuation of the number of particles of a certain species. In a given event the number of particles is given by

$$N = \rho V \quad (3)$$

where  $\rho$  is the particle density and  $V$  the volume of the system created in this event. Thus the fluctuations have contributions from both the density fluctuations *and* the fluctuations of the volume

$$\langle (\delta N)^2 \rangle = \langle (\delta \rho)^2 \rangle \langle V^2 \rangle + \langle \rho^2 \rangle \langle (\delta V)^2 \rangle. \quad (4)$$

The interesting physics of course resides in the fluctuations of the density. Consequently, one needs to consider so called ‘intensive’ quantities, which do not explicitly depend on the volume. One such observable is the ratio of particle abundances, and this is what we will concentrate on here.

## 2.3. Fluctuations of particle ratios

As discussed in the previous section, the fluctuations of particle ratios should be independent of volume fluctuations. This is certainly true if one looks at similar particles such as  $\pi^+$  and  $\pi^-$ , where the freeze out volumes are expected to be the same. Some residual volume fluctuations may be present if one considers ratios of particles with different quantum numbers such as the  $K/\pi$  ratio, but they still should be small. Let us define the particle ratio  $R_{12}$  of two particle species  $N_1$  and  $N_2$

$$R_{12} = \frac{N_1}{N_2} \quad (5)$$

The fluctuations of this ratio are then given by [2,5,6]

$$\frac{\langle (\delta R_{12})^2 \rangle}{\langle R_{12} \rangle^2} = \left( \frac{\langle (\delta N_1)^2 \rangle}{\langle N_1 \rangle^2} + \frac{\langle (\delta N_2)^2 \rangle}{\langle N_2 \rangle^2} - 2 \frac{\langle \delta N_1 \delta N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \right). \quad (6)$$

The last term in Eq. (6) takes into account correlations between the particles of type 1 and type 2. This term will be important if both particle types originate from the decay of one and the same resonance. For example, in case of the  $\pi^+/\pi^-$  ratio, the  $\rho_0$ ,  $\omega$  etc. contribute to these correlations. Also this term is responsible to cancel out all volume fluctuations [5].

Let us note that the effect of the correlations introduced by the resonances should be most visible when  $\langle N_1 \rangle \simeq \langle N_2 \rangle$ . On the other hand, when  $\langle N_2 \rangle \gg \langle N_1 \rangle$ , as in the  $K$  to  $\pi$  ratio, the fluctuation is dominated by the less abundant particle type and the resonances feeding into it. The correlations are then very hard to extract. In [5] it was shown that in case of the  $K/\pi$ -ratio resonances and quantum statistics give rise to deviations from the statistical value of at most 2 %, in agreement with experiment [1].

As pointed out in [5] the measurement of particle ratio fluctuations can provide important information about the abundance of resonances at chemical freeze out, and thus provides a

crucial test for the picture emerging from the systematics of single particle yields [15]. In particular the fluctuations of the  $\pi^+/\pi^-$ -ratio should be reduced by about 30 % as compared to pure statistics due to the presence of hadronic resonances with decay channels into a  $\pi^+\pi^-$ -pair at chemical freeze out. About 50 % of the correlations originate from the decay of the  $\rho_0$  and the  $\omega$  mesons. Thus the fluctuations provide a complementary measurement to the dileptons.

### 3. Charge fluctuations

Measuring the charge fluctuations or more precisely the charge fluctuations per unit degree of freedom of the system created in a heavy ion collision would tell us immediately if we have created a system of quarks and gluons [16] (see also [17]). The point is that in the QGP phase, the unit of charge is  $1/3$  while in the hadronic phase, the unit of charge is 1. The net charge, of course does not depend on such subtleties, but the fluctuation in the net charge depends on the *squares* of the charges and hence are strongly dependent on which phase it originates from. However, as discussed in the previous section, measuring the charge fluctuation itself is plagued by systematic uncertainties such as volume fluctuations, which can be avoided if one considers ratio fluctuations. The task is then to find a suitable ratio whose fluctuation is easy to measure and simply related to the net charge fluctuation.

The obvious candidate is the ratio  $F = Q/N_{\text{ch}}$  where  $Q = N_+ - N_-$  is the net charge and  $N_{\text{ch}} = N_+ + N_-$  is the total charge multiplicity. Instead of using  $F$ , however, it is simpler to consider the charge ratio  $R = N_+/N_-$ . If  $\langle N_{\text{ch}} \rangle \gg \langle Q \rangle$ , i.e.  $|F| \ll 1$  the fluctuations of  $F$  are related to those of  $R$  by

$$\langle \delta R^2 \rangle \approx 4 \langle \delta F^2 \rangle \quad (7)$$

Furthermore in the limit of  $\langle Q \rangle \ll \langle N_{\text{ch}} \rangle$  the fluctuations are dominated by the small term and we find [16]

$$\langle \delta F^2 \rangle \approx \frac{\langle \delta Q^2 \rangle}{\langle N_{\text{ch}} \rangle^2} \quad (8)$$

Usually, the number of charged particles is directly related to the entropy generated in these collision. Thus the observable

$$D \equiv \langle N_{\text{ch}} \rangle \langle \delta R^2 \rangle = 4 \langle N_{\text{ch}} \rangle \langle \delta F^2 \rangle = 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{\text{ch}} \rangle} \quad (9)$$

provides a measure of the charge fluctuations per unit entropy.

In order to see how this observable differs between a hadronic system and a QGP let us compare the value for  $D$  in a pion gas and in a simple model of free quarks and gluons.

In a pion gas, the fundamental degrees of freedom are of course pions. Hence,  $N_{\text{ch}} = N_{\pi^+} + N_{\pi^-}$  and using thermal distributions and disregarding correlations the charge fluctuations are given by

$$\langle \delta Q^2 \rangle = \langle \delta N_+^2 \rangle + \langle \delta N_-^2 \rangle = w_\pi \langle N_{\text{ch}} \rangle \quad (10)$$

where  $w_\pi \equiv \langle \delta N_\pi^2 \rangle / \langle N_\pi \rangle$  is slightly bigger than 1 [5,6]. Hence for a pion gas,

$$D_{\pi\text{-gas}} \approx 4. \quad (11)$$

In the presence of resonances, this value gets reduced by about 30 % due to the correlations introduced by the resonances, as discussed in section (2.3).

For a thermal system of free quarks and gluons we have in the absence correlation

$$\langle \delta Q^2 \rangle = Q_u^2 w_u \langle N_u \rangle + Q_d^2 w_d \langle N_d \rangle \quad (12)$$

where  $Q_q$  is the charges of the quarks and  $N_q$  denotes the number of quarks *and* anti-quarks. The constant  $w_q \equiv \langle \delta N_q^2 \rangle / \langle N_q \rangle$  is slightly smaller than 1 due to the fermionic nature of quarks.

Relating the final charged particle multiplicity  $N_{\text{ch}}$  to the number of primordial quarks and gluons is not as simple. Using entropy conservation one finds [16]

$$N_{\text{ch}} \simeq \frac{2}{3} (N_g + 1.2N_u + 1.2N_d) \quad (13)$$

leading to

$$D_{QGP} \simeq 3/4 \quad (14)$$

Actually the charge fluctuations  $\langle (\delta Q)^2 \rangle$  have been evaluated in lattice QCD along with the entropy density [11]. Using these results one finds

$$D_{\text{Lattice-QCD}} \simeq 1 - 1.5 \quad (15)$$

where the uncertainty results from the uncertainty of relating the entropy to the number of charged particles in the final state. Actually the most recent lattice result [12] for the charge fluctuations, which was obtained in the quenched approximation, is somewhat lower than the result of [11].

But even using the larger value of  $D = 1.5$  for the Quark Gluon Plasma, there is still a factor of 2 difference between a hadronic gas and the QGP, which should be measurable in experiment.

#### 4. Observational issues and model calculations

The key question of course is, how can these reduced fluctuation be observed in the final state which consists of hadrons. Should one not expect that the fluctuations will be those of the hadron gas? The reason, why it should be possible to see the charge fluctuations of the initial QGP has to do with the fact that charge is a conserved quantity. Imagine one measures in each event the net charge in a given rapidity interval  $\Delta y$  such that

$$1 \ll \Delta y \ll \Delta y_{\text{max}} \quad (16)$$

where  $\Delta y_{\text{max}}$  is the width of the total charge distribution. If, as it is expected, strong longitudinal flow develops already in the QGP-phase, the number of charges inside the rapidity window  $\Delta y$  for a given event is essentially frozen in. And if  $\Delta y \gg 1$  neither hadronization nor the subsequent collisions in the hadronic phase will be very effective to transport charges in and out of this rapidity window. Thus, the E-by-E charge-fluctuations measured at the end reflect those of the initial state, when the longitudinal flow is developed. Ref. [18] arrives at the same conclusion on the basis of a Fokker-Planck type equation describing the relaxation of the charge fluctuation in a thermal environment. In this approach the relaxation time for the charge fluctuations is determined by the average rapidity shift of the charges in a collision and by the number of

collisions in the hadronic phase. These values can be easily extracted from a transport model and we find that for  $\Delta y > 3$  the signal should survive the hadronic phase. The smearing of the signal due to hadronization is more difficult to estimate, however. But it is rather unlikely that hadronization, which is a soft process, should give rise to large rapidity shifts.

Thus, in the limit of very large  $\Delta y$  our proposed signal should be robust. However, one should not make  $\Delta y$  too large, as eventually charge conservation becomes relevant. Actually in the limit that  $\Delta y \simeq \Delta y_{max}$  charge conservation requires that the fluctuations vanish. For moderate values of  $\Delta y$ , one can very successfully correct for the effect of charge conservation, and this needs to be done for a meaningful interpretation of any experimental data. This correction factor is given by [19]

$$C_y = 1 - \frac{\langle N_{ch} \rangle_{\Delta y}}{\langle N_{ch} \rangle_{total}} \quad (17)$$

Also in case of a finite net charge, which may be due to non negligible baryon stopping, some additional, though small, correction needs to be applied [19].

$$C_\mu = \frac{\langle N_{\Delta y}^+ \rangle^2}{\langle N_{\Delta y}^- \rangle^2} \quad (18)$$

Thus the actual observable turns out to be

$$\tilde{D} = \frac{D}{C_y C_\mu} \quad (19)$$

In Fig. (2) the importance of these corrections, in particular the charge conservation correction  $C_y$  is demonstrated. There we compare the uncorrected observable  $D$  with the corrected observable  $\tilde{D}$  as a function of the width of the rapidity window based on a URQMD [20] simulation. The effect of the charge conservation is clearly visible. With increasing rapidity window the charge fluctuation are suppressed. Once the charge-conservation corrections are applied, the value for  $\tilde{D}$  remains constant at the value for a hadron gas of  $\tilde{D} \simeq 3$ . This is to be expected for the URQMD model, which is of hadronic nature and does not have quark-gluon degrees of freedom.

Also for small  $\Delta y$  the value is  $\tilde{D} \simeq 4$  before it drops to  $\tilde{D} \simeq 3$  for  $\Delta y > 1.5$ . This effect, which was already predicted in [5], is simply due to the fact that the correlation introduced by the resonances gets lost if the acceptance window becomes too small.

In Fig. (2), we also show the predictions of the URQMD model as a function of impact parameter for SPS as well as RHIC energies [19]. For both energies hardly any centrality dependence is visible and the results agree within errors with the prediction for the hadron gas. Thus a measurement of  $\tilde{D} \simeq 1$  would clearly indicate the existence of a QGP in the system created in these collisions.

## 5. Some additional remarks

A similar argument as for the charge fluctuations also holds for the fluctuations of the baryon number, since in the QGP the quarks carry fractional baryon number [17]. The observation of reduced baryon number fluctuations, however, is much more difficult since one has to measure

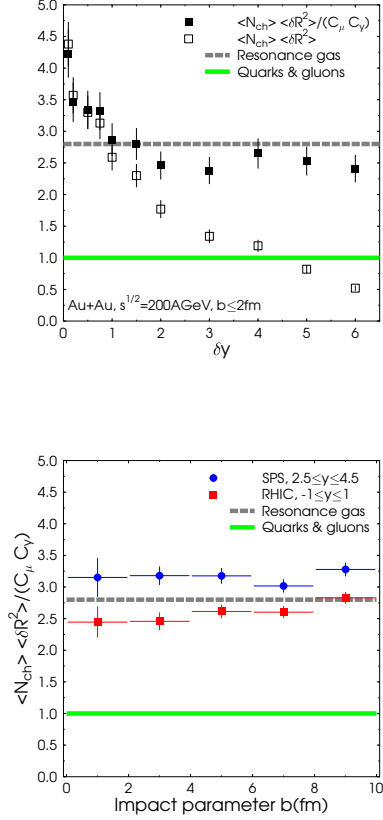


Figure 2. Charge fluctuations as a function of the size of the rapidity window(left) and centrality (right).

neutrons on an event by event basis. Also, so far, no obvious *intensive* observable has been proposed which would be sensitive to baryon number fluctuations.

Certainly, a measurement of  $D$  in proton-proton collisions will be necessary in order to rule out that possible small fluctuations are simply due to the nature of the proton wavefunction.

In order to be able to see the reduced charge fluctuations, they have to be imprinted into the hadronic phase. How can this be possible? As we have shown above, the charge fluctuations of a hadronic gas are three times larger than that of a QGP. The only way to reduce the fluctuations in a hadron gas is to introduce additional correlations (see eq. (6)). As we have argued above, neutral hadronic resonances are the obvious candidates, which would lead us to predict an enhancement of neutral resonances. However, if, in addition, we require that on average isospin should be conserved, increasing the number of neutral rho mesons would also imply an increase of the charged rho mesons and thus large charge fluctuations. This leaves us with the iso-scalar resonances, such as eta, omega, phi etc. We thus are lead to predict an enhancement of eta, omega and phi production, which can be readily observed via electromagnetic decay channels. An actual calculation based on this conjecture gives enhancement factors of the order

of 5.

## 6. Conclusions

We have discussed event-by-event fluctuations in heavy ion collisions. These fluctuations may provide useful information about the properties of the matter created in these collisions, as long as the ‘trivial’ volume fluctuations, inherent to heavy ion collisions, can be removed. We have argued that the fluctuations of particle ratios is not affected by volume fluctuations.

In particular the fluctuations of the ratio of positively over negatively charged particles measures the charge fluctuation per degree of freedom. Due to the fractional charge of the quarks, these are smaller in a QGP than in a hadronic system.

A measurement of our observable  $\tilde{D} \simeq 1$  would provide strong evidence for the existence of a QGP in these collisions. A measurement of  $\tilde{D} \simeq 3$  on the other hand does not rule out the creation of a QGP. There are a number of caveats (see [16]), which could destroy the signal, such as unexpected large rapidity shifts during hadronization.

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## REFERENCES

1. H. Appelshauser et al. (NA49 collaboration) Phys.Lett.**B459** (1999) 679-686; S.V. Afanasev et al (NA49 collaboration), hep-ex/0009053
2. S. Mrowczynski, Phys. Lett., **B430** (1998) 9; Phys. Lett. **B459**, (1999) 13; M. Gazdzicki and S. Mrowczynski, Z.Phys., C54 (1992) 127.
3. S.A. Voloshin et al, Phys. Rev. **C60** (1999) 204901.
4. K. Stephanov, K. Rajagopal, and E. Shuryak, Phys.Rev. **D60** (1999) 114028.
5. S. Jeon and V. Koch, Phys. Rev. Lett. **83** (1999) 5435.
6. G. Baym and H. Heiselberg, Phys. Lett. **B469**, (1999) 7.
7. H. Heiselberg and A.D. Jackson, nucl-th/0006021.
8. Henning Heiselberg , nucl-th/0003046.
9. A. Bialas and V. Koch, Phys. Lett. **B456** (1999) 1.
10. J.I. Kapusta , Phys. Rev. **D46** (1992) 4749.
11. S. Gottlieb et al., Phys. Rev. **D55**, (1997) 6852.
12. R.V. Gavai and S. Gupta, hep-lat/0103013
13. C. Song, Phys. Rev. **D48** (1993) 1375.
14. V.L. Eletsky, J.I. Kapusta, and R. Venugopalan, Phys. Rev. **D48** (1993) 4398.
15. M. Becattini, M. Gazdzicki and J. Sollfrank, Eur. Phys. Jour. C **5**, (1998) 143; P. Braun-Munzinger, I. Heppe and J. Stachel, Phys.Lett. **B465** (1999) 15.
16. S. Jeon and V. Koch, Phys. Rev. Lett. **85** (2000) 2076.
17. M. Asakawa, U. Heinz, and B. Müller, Phys.Rev.Lett. **85** (2000) 2072.



18. E.V. Shuryak and M.A. Stephanov, hep-ph/0010100
19. M. Bleicher, S. Jeon, and V. Koch, hep-ph-0006201, Phys. Rev. C, in press.
20. S. A. Bass et al., Prog. Part. Nucl. Phys. **41**,(1998) 225; M. Bleicher et al, J. Phys. **G 25** (1999) 1895.