This paper describes the development of a TMF model by extending the damage model proposed earlier by the authors [9, 10] to capture fatigue damage in solder. Evolution equations for fatigue damage are developed. The evolution laws are then used to calculate the damage accumulation per cycle by integrating the equations along a loading path. As the accumulated fatigue damage per cycle is dependent upon a loading path, or the type of cyclic wave form, a failure criterion based on the accumulated damage in a solder material is proposed. The effects of damage on mechanical responses of 63Sn-37Pb solder alloy under cyclic loading are also discussed in the paper.

DAMAGE MODEL
A damage model with two scalar damage variables, D and μ, was proposed earlier by Chow and Wei [8]. The model considers the damage accumulation due to microstructure changes that are associated with the stiffness degradation of a material. The resulting material degradation is characterized quantitatively with two state variables. According to the theory of damage mechanics [11], the effective stress tensor \( \bar{\sigma} \) considering the effects of damage is introduced to replace the true stress tensor (Cauchy stress) \( \sigma \) as

\[
\bar{\sigma} = M : \sigma
\]

where \( M \) is the damage effect tensor expressed as [8]

\[
M = \frac{1}{1-D} \begin{bmatrix}
1 & \mu & \mu & 0 & 0 & 0 \\
\mu & 1 & \mu & 0 & 0 & 0 \\
\mu & \mu & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1-\mu & 0 & 0 \\
0 & 0 & 0 & 0 & 1-\mu & 0 \\
0 & 0 & 0 & 0 & 0 & 1-\mu
\end{bmatrix}
\]

D and μ are two scalar damage variables. The equivalent damage \( w \) is introduced to quantify damage accumulation in materials and is taken as a driving force for material failure. Accordingly, a failure criterion is proposed postulating that: a material element is said to have ruptured when the total...
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
equivalent damage \( w \) reaches the critical value \( w_c \) of the material.

In order to derive damage evolution equations within the framework of thermodynamics, a dissipation potential function \( \phi \) is introduced consisting of two independent processes, i.e. a deformation process \( \phi^m \) and a damage process \( \phi^d \). Accordingly, the potential is expressed as

\[
\phi = \phi^m + \phi^d
\]  

(3)

The damage part of the dissipation potential is formulated with the equivalent damage energy release rate \( Y_{ind} \) as

\[
\phi^d = Y_{h_0} \left( \frac{Y_{ind}}{B_i + 1} \right)^{B_i+1}
\]  

(4)

where \( B_i \) is the damage-related material constant, \( Y_h \) is the damage hardening variable. The equivalent damage energy release rate \( Y_{ind} \) is defined as a function of thermodynamic conjugate forces of the damage variables \( D \) and \( \mu \)

\[
Y_{ind} = \left[ \frac{1}{2} (Y_D^2 + \gamma Y_{\mu}^2) \right]^{1/2}
\]  

(5)

where \( \gamma \) is the damage-related material constant associated with the change of Poisson’s ratio, and \( Y_D, Y_{\mu} \) are the thermodynamic conjugate forces of the damage variables \( [8, 9] \).

\[
Y_D = \frac{1}{1-D} \sigma^T : \mathbf{C}^{-1} : \sigma
\]

\[
Y_{\mu} = \frac{1}{1-D} \sigma^T : \mathbf{Z} : \sigma
\]  

(6)

\( \mathbf{C} \) is the effective elastic tensor for a damaged material and can be derived with the form of the damage effect tensor \( \mathbf{M} \) in Equation (2) as

\[
\mathbf{C}^{-1} = \mathbf{M}^T : \mathbf{C}_0^{-1} : \mathbf{M}
\]  

(7)

\( \mathbf{C}_0 \) is the fourth order elastic tensor without damage.

\[
\mathbf{C}^{-1} = \begin{vmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1+\nu) \end{vmatrix}
\]

(8)

\[
E = \frac{E_n(1-D)}{1-4
\nu \mu + 2(1+\nu)\mu^2}
\]

\[
Y_{ind} = \frac{1}{E_n(1-D)} \begin{vmatrix} z_1 & z_2 & z_3 & 0 & 0 & 0 \\ z_2 & z_1 & z_3 & 0 & 0 & 0 \\ z_3 & z_1 & z_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & (z_1 - z_3) & 0 & 0 \\ 0 & 0 & 0 & 0 & (z_1 - z_3) & 0 \end{vmatrix}
\]

(9)

\[
z_1 = 2(1 - \nu_0) \mu - 2\nu_0
\]

\[
z_2 = (1 + \mu)(1 - \nu_0) - 2\mu\nu_0
\]

\( E_0 \) and \( \nu_0 \) are the Young’s modulus and the Poisson’s ratio for undamaged or as-received material. Then the damage evolution equations can be derived as

\[
\dot{D} = -\dot{\lambda}_m \frac{\partial \phi^m}{\partial Y_D} = -\dot{\lambda}_m \frac{Y_D}{2Y_{ind}}
\]

\[
\dot{\mu} = -\dot{\lambda}_m \frac{\partial \phi^d}{\partial Y_{\mu}} = -\dot{\lambda}_m \frac{\gamma Y_{\mu}}{2Y_{ind}}
\]

(10)

where \( \dot{\lambda}_m \) is a multiplier which is equal to the equivalent inelastic strain \( p^{in} \) [10]. Therefore, Equation (10) can be alternatively written as

\[
\dot{D} = -p^{in} \left( \frac{Y_{ind}}{Y_h} \right) \frac{Y_D}{2Y_{ind}} = -\dot{\lambda} \frac{Y_D}{2Y_{ind}}
\]

\[
\dot{\mu} = -p^{in} \left( \frac{Y_{ind}}{Y_h} \right) \frac{\gamma Y_{\mu}}{2Y_{ind}} = -\dot{\lambda} \frac{\gamma Y_{\mu}}{2Y_{ind}}
\]

(11)
where the equivalent damage rate \( \dot{w} \) is expressed as
\[
\dot{w} = \dot{p}_{in} \left( \frac{Y_{ind}}{Y_h} \right)^\frac{B_1}{B_2}
\]
(12)
The damage hardening variable \( Y_h \) may be expressed in terms of the equivalent damage \( w \), the absolute temperature \( T \) and the equivalent inelastic strain rate \( \dot{p}_{in} \) as
\[
Y_h(w, T) = Y_0 \left( \frac{\dot{p}_{in}^{B_3}}{B_4} e^{\frac{B_2}{T}} \right)
\]
(13)
where \( Y_0, B_2, B_3 \) and \( B_4 \) are damage-related material parameters.

In general, a damage accumulation process in solder materials can be considered as the viscoplastic damage induced from the monotonic loading and the fatigue damage resulted from cyclic loading. It is postulated that both damage evolution laws follow Equations (11)-(13), except that the material constants are defined differently. Then the fatigue damage evolution equations are:
\[
\begin{align*}
\dot{D}_f &= -\dot{w}_f \frac{Y_0}{2Y_{ind}} \\
\dot{\mu}_f &= -\dot{w}_f \frac{Y_\mu}{2Y_{ind}} \\
\dot{w}_f &= \dot{p}_{in} \frac{Y_{ind}}{Y_h} \\
Y_{uf} &= Y_0 \left( \frac{\dot{p}_{in}^{B_3}}{B_4} e^{\frac{B_2}{T}} \right)
\end{align*}
\]
(14)
The viscoplastic damage evolution equations are:
\[
\begin{align*}
\dot{D}_m &= -\dot{w}_m \frac{Y_0}{2Y_{ind}} \\
\dot{\mu}_m &= -\dot{w}_m \frac{Y_\mu}{2Y_{ind}} \\
\dot{w}_m &= \dot{p}_{in} \frac{Y_{ind}}{Y_h} \\
Y_h &= Y_0 \left( \frac{\dot{p}_{in}^{B_3}}{B_4} e^{\frac{B_2}{T}} \right)
\end{align*}
\]
(15)
The total damage is the sum of fatigue damage and viscoplastic damage as
\[
D = D_f + D_m, \quad \mu = \mu_f + \mu_m, \quad w = w_f + w_m
\]
(16)
where \( D_f, \mu_f \) and \( w_f \) are fatigue damage variables, \( D_m, \mu_m \) and \( w_m \) are viscoplastic damage variables. Both damage accumulations can be calculated by integrating Equations (14) and (15) over a loading history.

**DAMAGE-COUPL ED CONSTITUTIVE EQUATION**

The proposed viscoplastic model considers the change of grain/phase size and the damage as internal state variables [9,10]. The total strain \( c \) consists of the elastic strain \( \varepsilon_e \) and the inelastic strain \( \varepsilon_{in} \).
\[
\varepsilon = \varepsilon_e + \varepsilon_{in}
\]
(17)
The damage-coupled elastic equation is:
\[
\varepsilon_e = C^{-1} : \sigma
\]
\[
\sigma = C : \varepsilon_e
\]
(18)
The damage-coupled viscoplastic equations are [10]:
\[
\dot{\varepsilon}_{in} = \dot{p}_{in} \frac{3S - X}{2J_2}
\]
(19)
\[
\dot{p}_{in} = \frac{1 - \mu}{1 - D} \left[ \frac{f \exp \left( -\frac{Q}{RT} \right) \lambda_0}{\lambda} \right] \sinh \left( \frac{1 - \mu}{1 - D} \alpha(c + \bar{c}) \right)
\]
(20)
where \( \dot{p}_{in} \) is the equivalent inelastic strain, \( S \) is the deviatoric stress tensor, \( X \) is the back stress tensor, \( J_2 \) is a second invariant of the stress difference, \( f, \mu, p, m \) and \( Q \) are material parameters, \( R \) is the gas constant, \( T \) is the absolute temperature, \( \lambda \) is the current grain diameter, \( \lambda_0 \) is the initial grain diameter, \( \alpha \) is a scalar function of the absolute temperature, \( c \) and \( \bar{c} \) are state variables. Evolution equations for these internal state variables have been described by the authors [9,10].

**APPLICATIONS**

Material constants for 63Sn-37Pb solder alloy with the viscoplastic constitutive model were reported in the reference [10]. The damage-related material constants for proposed damage evolution equations are given in Table 1.

<table>
<thead>
<tr>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_3 )</th>
<th>( B_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>5.0</td>
<td>3.34E+3</td>
<td>0</td>
</tr>
<tr>
<td>( Y_0 ) (MPa)</td>
<td>( Y_{0f} ) (MPa)</td>
<td>( \gamma )</td>
<td>( w_c )</td>
</tr>
<tr>
<td>3.77E-8</td>
<td>3.40E-5</td>
<td>-0.2</td>
<td>0.58</td>
</tr>
</tbody>
</table>

The TMF damage model has been implemented into ABAQUS (version 5.8) with its user subroutine UMAT for FEM analysis. The proposed model was applied to predict
the hysteresis loops of 63Sn-37Pb solder material at temperature 80°C. Three different strain rates, $10^{-3}$/s, $10^{4}$/s and $10^{-5}$/s, were chosen for the numerical simulations. The corresponding experimental tests were carried out with a MTS machine under strain-control. The calculated hysteresis loops for the first few cycles, two strain ranges, ±0.5% and ±1.0%, and three strain rates are compared with testing data in Figs. 1, 2 and 3. It can be observed from the figures that predictions for the maximum stress and stress range agree well with experimental results. However, the figures also exhibit a degree of discrepancy during the transition from the region of elastic dominant strain to inelastic dominant strain. It is worth noting that all material constants used for the simulation were determined under monotonic loading, as reported in the reference [10]. The observed discrepancies may be attributed to the difference in mechanical behavior under monotonic and cyclic loading. More accurate predictions may be achieved if all the material constants are determined from cyclic tests. This will be the subject of future investigations.

Softening behavior was observed from the beginning of cyclic loading. Conventionally, the load-drop ratio has been adopted to determine low cycle fatigue life. Accordingly, the proposed model is employed to simulate hysteresis loops at different percentages of fatigue life. The results are summarized in Fig. 4.

The model is also applied to predict fatigue life under strain-controlled cyclic loading with 120-second ramp time at temperature 80°C. The fatigue damage accumulation in the material is calculated using Equation (14). The fatigue life under a certain strain range can be determined when the accumulated fatigue damage reaches a critical value $w_c$. A value of 0.58 was used for $w_c$. The predicted results are compared with the test data reported by Guo, et al.[1] as shown in Fig. 5 where the minimum strain used was 0%. The fatigue life determined from the tests is based on the criterion of a sudden acceleration of load drop.

CONCLUSIONS

The fatigue damage evolution equations are proposed to characterize fatigue damage accumulation in 63Sn-37Pb solder material under TMF loading. A failure criterion is formulated with the concept of equivalent damage accumulation. The proposed TMF model is applied successfully to predict fatigue life of 63Sn-37Pb solder material at temperature 80°C by comparing with Guo’s data.

The damage model can be used to evaluate quantitatively damage accumulation and its effects on mechanical properties of the material under cyclic loading, including cyclic softening behavior. It may also be applied to evaluate the effects of hold time, strain rate and type of load wave form on fatigue life.

REFERENCES


Fig. 1 Hysteresis loops for strain rate $10^{-3}$/s at temperature 80°C

Fig. 2 Hysteresis loops for strain rate $10^{-4}$/s at temperature 80°C

Fig. 3 Hysteresis loops for strain rate $10^{-5}$/s at temperature 80°C

Fig. 4 Simulated softening behavior for strain range ±1% with strain rate $10^{-4}$/s at temperature 80°C
Fig. 5 Comparison of computed and experimentally measured fatigue life as a function of strain range (minimum strain 0%) at a temperature of 80°C.