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Relativistic Foundations of Pseudospin Symmetry in Nuclear Structure and Scattering

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Pseudospin doublets were discovered about thirty years ago but their origins had been a mystery. We show that pseudospin doublets originate from an SU(2) symmetry of the Dirac Hamiltonian which occurs when the scalar and vector potentials are opposite in sign but equal in magnitude. Furthermore, we survey the evidence that pseudospin symmetry is approximately conserved for realistic nuclear scalar and vector potentials. We demonstrate that pseudospin symmetry is partially conserved in medium energy nucleon-nucleus scattering as well. We briefly discuss the relationship of pseudospin symmetry with chiral symmetry.

1 Pseudospin Symmetry

The spherical shell model orbitals that were observed to be quasi-degenerate have non-relativistic quantum numbers \( (n_r, \ell, j = \ell + 1/2) \) and \( (n_r - 1, \ell + 2, j = \ell + 3/2) \) where \( n_r, \ell, \) and \( j \) are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively \(^{1,2}\). This doublet structure is expressed in terms of a "pseudo" orbital angular momentum \( \tilde{\ell} = \ell + 1 \), the average of the orbital angular momentum of the two states in doublet and "pseudo" spin, \( \tilde{s} = 1/2 \). For example, \( (n_r s_{1/2}, (n_r - 1)d_{3/2}) \) will have \( \tilde{\ell} = 1 \), \( (n_r p_{3/2}, (n_r - 1)f_{5/2}) \) will have \( \tilde{\ell} = 2 \), etc. These doublets are almost degenerate with respect to pseudospin, since \( j = \tilde{\ell} \pm \tilde{s} \) for the two states in the doublet. Pseudospin "symmetry" was shown to exist in deformed nuclei as well \(^3\) and has been used to explain features of deformed nuclei, including superdeformation and identical bands \(^4,5,6\). However, the origin of pseudospin symmetry remained a mystery and "no deeper understanding of the origin of these (approximate) degeneracies" existed \(^7\). In this paper we shall review more recent developments that show that pseudospin symmetry is a relativistic symmetry \(^8,9,10\).

2 Symmetries of the Dirac Hamiltonian

The success of the shell model implies that nucleons move in a mean field produced by the interactions between the nucleons. Normally, it suffices to use the Schrödinger equation to describe the motion of the nucleons in this mean field. However, in order to understand the origin of pseudospin symmetry, we
need to take into account the motion of the nucleons in a relativistic mean field and thus use the Dirac equation. The Dirac Hamiltonian, $H$, with an external scalar, $V_S$, and vector, $V_V$, potentials is given by:

$$H = \bar{\psi} \alpha \cdot \vec{p} + \beta (m + V_S) + V_V,$$

where we have set $\hbar = c = 1$, $\alpha$, $\beta$ are the usual Dirac matrices, $m$ is the nucleon mass, and $\vec{p}$ is the three momentum. The Dirac Hamiltonian is invariant under an SU(2) algebra for two limits: $V_S - V_V =$ constant and $V_S + V_V =$ constant. The first condition leads to a spin symmetry which is relevant for mesons while the second leads to pseudospin symmetry. The generators for the SU(2) pseudospin symmetry, $\hat{S}_i$, which commute with the Dirac Hamiltonian, $[H, \hat{S}_i] = 0$, are given by $\hat{S}_i = \left( \begin{array}{cc} \hat{s}_i & 0 \\ 0 & -\hat{s}_i \end{array} \right)$ where $\hat{s}_i$ is the usual spin operator, $\hat{s}_i = U_p \hat{s}_i U_p$ and $U_p = 2^{1/2} \vec{p} / p$ is a unitary operator. These pseudospin generators have the spin operator $\hat{s}_i$ operating on the lower component of the Dirac wave function which has the consequence that the spatial wavefunctions for the two states in the pseudospin doublet are identical to within an overall phase.

This symmetry for $V_S + V_V =$ constant is general and applies to deformed nuclei as well as spherical nuclei. In the case for which the potentials are spherically symmetric, the Dirac Hamiltonian conserves the pseudo-orbital angular momentum, the generators of which are, $\hat{\mathbb{L}}_i = \left( \begin{array}{cc} \hat{\ell}_i & 0 \\ 0 & -\hat{\ell}_i \end{array} \right)$, where $\hat{\ell}_i = U_p \hat{\ell}_i U_p$, $\hat{\ell}_i = \vec{\mathbb{L}} \times \vec{p}$. Since the lower component is $\hat{\ell}_i$, and since $U_p$ conserves the total angular momentum but $\vec{p}$ changes the orbital angular momentum by one unit because of parity conservation, if the lower component of the Dirac wave function orbital angular momentum is $\hat{\ell}$, the upper component also has total angular momentum $j$, but orbital angular momentum $\ell = \ell \pm 1$. If $j = \ell + 1/2$, then it follows that $\ell = \ell + 1$, whereas if $j = \ell - 1/2$, then $\ell = \ell - 1$. This agrees with the pseudospin doublets originally observed and discussed at the beginning of this paper. This relativistic interpretation also gives the physical significance of the pseudo-orbital angular momentum $\hat{\ell}$ as the orbital angular momentum of the lower component. For axially symmetric deformed nuclei, there is a $U(1)$ generator corresponding to the pseudo-orbital angular momentum projection along the symmetry axis which is conserved in addition to the pseudospin, $\hat{\lambda} = \left( \begin{array}{cc} \hat{\lambda} & 0 \\ 0 & -\hat{\lambda} \end{array} \right)$, where $\hat{\lambda} = U_p \hat{\lambda} U_p$.

However, the exact symmetry limit can not be realized in nuclei, because, if $V_S + V_V =$ constant, there are no Dirac bound valence states and hence nuclei can not exist. 
3 Realistic mean fields

A near equality in the magnitude of mean fields, $V_S \approx -V_V$, is a universal feature of the relativistic mean field approximation (RMA) of relativistic field theories with interacting nucleons and mesons$^{14}$ and relativistic theories with nucleons interacting via zero range interactions$^{15}$, as well as a consequence of QCD sum rules$^{16}$. Recently realistic relativistic mean fields were shown to exhibit approximate pseudospin symmetry in both the energy spectra and wave functions$^{10,18,19}$. As mentioned in the last section pseudospin symmetry implies that the spatial wavefunction for the lower component of the Dirac wavefunctions will be equal in shape and magnitude for the two states in the doublet. In Figure 1, for example, the lower components of the $(2s_{1/2}, 1d_{3/2})$ Dirac wavefunctions are plotted and we see that they are approximately identical$^{10}$.

Figure 1: $^{208}$Pb lower component wavefunctions $f(r)$ for the $2s_{1/2}$ Dirac wavefunction (dash line) and $1d_{3/2}$ Dirac wavefunction (dot-dash line) as a function of the radius $r$. 

4 QCD Sum Rules

Applying QCD sum rules in nuclear matter\(^1\), the ratio of the scalar and vector self-energies were determined to be \( \frac{\sigma_s}{\sigma_v} \approx -\frac{\sigma_N}{2\pi^2} \) where \( \sigma_N \) is the sigma term which arises from the spontaneous breaking of chiral symmetry\(^2\). For reasonable values of \( \sigma_N \) and quark masses, this ratio is close to -1. The implication of these results is that chiral symmetry breaking is responsible for the scalar field being approximately equal in magnitude to the vector field, thereby producing pseudospin symmetry.

5 Nucleon - Nucleus Scattering

The relativistic optical model scalar and vector potentials determined from nucleon-nucleus scattering are almost equal and opposite in sign\(^2\). Since pseudosymmetry doesn’t care if the potentials are complex, this symmetry may arise in nucleon-nucleus scattering\(^2\). The pseudospin and spin symmetry breaking can be determined empirically if the polarization and spin rotation function are both measured as a function of the scattering angle\(^2\). In Figure 2 the square of the ratio of the pseudospin dependent amplitude to the pseudospin independent amplitude, \((B/A)^2\), and the square of the ratio of the spin dependent amplitude to the spin independent amplitude, \((B/A)^2\), are plotted for 800 MeV proton scattering\(^2\) on \(^{208}\)Pb. The pseudospin spin breaking is at most of the order of 10%, a factor of three lower than the spin breaking. On the other hand preliminary results for low energy proton scattering\(^2\) indicates that pseudospin may be badly broken, as predicted\(^2\).

6 Summary

We have shown that pseudospin symmetry is a broken SU(2) symmetry of the Dirac Hamiltonian which describes the motion of nucleons in realistic scalar and vector mean field potentials, \(V_S \approx -V_V\). This symmetry predicts that the spatial wavefunctions of the lower components for states in the doublet will be very similar in shape and size and this has been substantiated by relativistic mean field approximations of relativistic nuclear field theories and relativistic nuclear Lagrangians with zero range interactions. This symmetry has been linked via QCD sum rules to chiral symmetry breaking in nuclei. Finally, pseudospin symmetry has been shown to be approximately conserved in medium energy nucleon scattering from nuclei.

Although not covered in this brief survey, pseudospin symmetry implies a relationship between magnetic dipole transitions between the two states in the
Figure 2: The square of the ratio of the pseudospin dependent amplitude to the pseudospin independent amplitude, \((B/A)^2\), and the square of the ratio of the spin dependent amplitude to the spin independent amplitude, \((B/A)^2\), as a function of the scattering angle \(\theta\) for 800 MeV proton scattering on \(^{208}\text{Pb}\).

doublet and their magnetic moments and also similar relationships for Gamow-Teller transitions \(^{28,29}\). Future applications shall be made to deformed nuclei as well.

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