The parity partner of the nucleon in quenched QCD with domain wall fermions

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We present preliminary results for the mass spectrum of the nucleon and its low-lying excited states from quenched lattice QCD using the domain wall fermion method which preserves the chiral symmetry at finite lattice cutoff. Definite mass splitting is observed between the nucleon and its parity partner. This splitting grows with decreasing valence quark mass. We also present preliminary data regarding the first positive-parity excited state.

This work focuses on a notable feature in the mass spectrum of the nucleon and its excited states: the mass splitting between the nucleon $N(939)$ and its parity partner $N^*(1535)$ is remarkably large\textsuperscript{[1]}. As is well known, this splitting must be absent if chiral symmetry were preserved. Yet models with explicit chiral symmetry breaking such as non-relativistic quark models or bag models fail to reproduce this splitting. In a typical non-relativistic quark model with harmonic-oscillator quark wave function [2], the lowest negative-parity state is obtained by adding one oscillator quantum to the ground state. The known proton charge radius and magnetic moment lead to a few hundred MeV oscillator quantum, far underestimating the mass difference. It also gives the wrong ordering of positive- and negative-parity excited states: while the positive-parity $N^*(1440)$ lies below $N^*(1535)$ in nature, the model needs two oscillator quanta for $N^*$. A similar problem arises in bag models where the excitation energy is linked to the inverse of the bag radius which in turn is determined by the proton charge radius [3].

Thus it is an interesting question whether lattice QCD, which appears so successful in describing spontaneous breaking of chiral symmetry, can reproduce this mass splitting. Most conventional lattice fermion schemes are inadequate for this interesting challenge: they break chiral symmetry explicitly at finite lattice cutoff [4] and thus are prone to failure in explaining the splitting. Fortunately, however, the domain wall fermion (DWF) method seems capable of going around this pathology [5]. Here we report preliminary results of the first quenched calculation of this issue using DWF. In this paper, 24 well separated quenched gauge configurations on a $16^3 	imes 32$ lattice at $6/g^2=6.0$ are used. We use a fifth (DWF) dimension of $N_t=16$ sites and domain-wall height of $M=1.8$ [6].

We focus on the spin-half isodoublet baryons. Then there are only two possible choices for positive-parity baryons if we restrict them to contain no derivatives: $B_1^+ = \varepsilon_{abc}(u^T_5 C \gamma_5 d_b)u_c$ and $B_2^+ = \varepsilon_{abc}(u^T_5 C d_b) \gamma_5 u_c$, where $abc$, $ud$, $C$ and $\gamma_5$ have usual meanings as color, flavor, charge conjugation and Dirac matrix. In previous lattice calculations of ground-state hadrons, the operator $B_1^+$ was used for the nucleon ground state. Since the operator $B_2^+$ vanishes in the non-relativistic limit, it was considered ineffective. Indeed, nobody succeeded in extracting the nucleon mass using it [7]. We will come back to this point later.

The negative-parity baryon interpolating operators are defined with an extra $\gamma_5$ [8]: $B_1^- = \varepsilon_{abc}(u^T_5 C \gamma_5 d_b) \gamma_5 u_c$ and $B_2^- = \varepsilon_{abc}(u^T_5 d_b) \gamma_5 u_c$. As a result of the definition $B_{1,2}^+ = B_{1,2}^- B_{1,2}^+$, each two-point baryon correlator constructed from any one of them actually contains both positive- and
negative-parity contributions [9]. This means that there is contamination from the opposite parity state propagating backwards in time. Thus, to extract parity-eigenstate signals we use a linear combination of quark propagators, one obtained with periodic and another with anti-periodic boundary conditions in the time direction.

We use seven values for the valence quark mass $m$ in the range of $0.02 < m < 0.125$, corresponding to the $\pi\rho$ meson mass ratios $m_\pi/m_\rho \approx 0.59 - 0.90$. Quark propagators are calculated with wall source and point sink, and two different source positions are used for each gauge configuration.

Definite plateaus are seen in the effective mass plots for $B^+_1$, $B^-_1$, and $B^-_2$ operators. In Figure 1, we present our estimates of the nucleon ($N$) and its parity partner ($N^*$) mass values obtained by taking a weighted average of the effective mass in appropriate time ranges. The nucleon mass is extracted from the $B^+_1$ operator. $N^*$ mass estimates from $B^-_1$ and $B^-_2$ operators agree within errors in the whole quark mass range, as expected from their common quantum numbers. An important feature is that the $N-N^*$ mass splitting is observed in the whole range and even for light valence quark mass values. Another is that the splitting grows as the valence quark mass decreases, suggesting that the large splitting observed in nature indeed comes from the spontaneous breaking of chiral symmetry. Linear extrapolation in valence quark mass gives us $m_N = 0.56(2)$ and $m_{N^*} = 0.77(2)$ in lattice units for values in the chiral limit which are consistent with the experimental value ($a^{-1} \approx 1.9$ GeV from the $\rho$-meson mass [6]).

In Figure 2, we compare two mass ratios, one from the baryon parity partners $m_{N^*}/m_N$ and the other from pseudo-scalar and vector mesons $m_\pi/m_\rho$. Experimental points are marked with stars, corresponding to non-strange (left) and strange (right) sectors. In the strange sector we use $\Sigma$ and $\Sigma(1750)$ as baryon parity partners and $K$ and $K^*$ for mesons [1]. We find the baryon mass ratio grows with decreasing meson mass ratio, toward reproducing the experi-
mental values.

In contrast to our naive expectation that the operators $B^+_1$ and $B^+_2$ should give the same mass estimate, we find different plateaus in effective mass plots from these two operators. In Figure 3, shows that two masses extracted from $B^+_1$ and $B^+_2$ are quite different. For heavy quarks ($m \geq 0.04$), we identify $B^+_2$ with the first positive-parity excited state of nucleon ($N'$) for the following reasons: The operator $B^+_2$ is expected to couple weakly to the ground state of the nucleon as we mentioned earlier [7]. We suspect the reason why we see a clear $B^+_2$ signal for the first time in this study while previous studies failed to do so is related to mixing induced by explicit chiral symmetry breaking at finite lattice cutoff which is absent in the former but severe in the latter. Although $B^+_1$ and $B^+_2$ do not mix in the continuum because of different chiral structures, it is known that unwanted mixing between them comes about through the breaking of chiral symmetry by conventional lattice fermions [10]. On the other hand, the DWF exponentially suppresses this breaking and thus significantly reduce the unwanted mixing [11]. As a result, we are able to numerically confirm an expected feature of $\langle 0|B^+_2|N' \rangle \approx 0$ at a valence quark mass of $m=0.04$. So for this valence quark mass we believe the $B^+_2$ operator gives an $N'$ mass signal. For heavier quark mass values, the mass splitting between $N'$ and $N^*$ approaches the splitting between $N^*$ and $N$ just like in the naive quark or bag models. Unfortunately, however, we have yet to perform the $\langle 0|B^+_2|N \rangle$ calculation for lighter quark mass values and hence have not ruled out the possibility that $B^+_2$ couples to the ground-state nucleon.

In conclusion, we have studied the spectrum of the nucleon and its excited states by using DWF. We found the large mass splitting between $N$ and $N^*$ for light quark by using two distinct interpolating operators. Our $N^*$ mass $m_{N^*}=0.77(2)$ in the chiral limit is closer to the experimental value than any other study using other fermion schemes [8]. We also observed that the unconventional nucleon operator gives a clear signal for the first excited nucleon, at least for heavy quarks.

REFERENCES

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