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FOR AIR-WATER BUBBLY FLOW

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STUDY OF INTERFACIAL AREA TRANSPORT AND SENSITIVITY ANALYSIS FOR AIR-WATER BUBBLY FLOW

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ABSTRACT

The interfacial area transport equation applicable to the bubbly flow is presented. The model is evaluated against the data acquired by the state-of-the-art miniaturized double-sensor conductivity probe in an adiabatic air-water co-current vertical test loop under atmospheric pressure condition. In general, a good agreement, within the measurement error of ±10%, is observed for a wide range in the bubbly flow regime. The sensitivity analysis on the individual particle interaction mechanisms demonstrates the active interactions between the bubbles and highlights the mechanisms playing the dominant role in interfacial area transport. The analysis employing the drift flux model is also performed for the data acquired. Under the given flow conditions, the distribution parameter of 1.076 yields the best fit to the data.
NOMENCLATURE

English

$A_i$ average surface area of fluid particles
$a_i$ interfacial area concentration
$B_d$ volume of a typical dispersed fluid particle
$F_D$ standard drag force
$F_V$ virtual mass force
$f$ fluid particle number density distribution function per unit mixture and bubble volume
$h_{ki}$ interfacial heat transfer coefficient
$L_s$ length scale
$M_{ik}$ generalized interfacial drag
$m_k$ mean mass transfer rate
$n(x,t)$ total number of particles of all sizes per unit mixture volume
$q_{ki}$ interfacial heat flux
$S_j$ particle source/sink rates per unit mixture volume due to $j^{th}$ particle interactions
$T_i$ temperature at the interface
$T_k$ bulk temperature
$t$ time
$V$ fluid particle volume
$v$ fluid particle velocity
\( v_i \)  
interfacial velocity

\( v_{pm} \)  
average local particle velocity weighted by the particle number

**Greek Symbol**

\( \alpha \)  
void fraction

\( \eta_{ph} \)  
rate of volume generated by nucleation per unit mixture volume

\( \phi_j \)  
interfacial source or sink due to \( j^{th} \) interaction mechanism

\( \phi_k \)  
heat dissipation

\( \Gamma_k \)  
mass generation

\( \mu_m \)  
mixture viscosity

\( \tau_i \)  
interfacial shear stress

**Subscripts**

\( f \)  
liquid

\( g \)  
gas

\( i \)  
interface

\( ph \)  
phase change
1. INTRODUCTION

In performing a detailed assessment of the given two-phase system, many two-phase system analysis codes employ the formulation using the two-fluid model [1], which is based on the detailed treatment of the phase interactions at the interface. However, since the two-fluid model solves the conservation equations of mass, momentum and energy for each phase separately, the phase-interaction terms arise.

In view of practical applications, the two-fluid model was simplified by Ishii and Mishima [2, 3] as

\[ \frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k v_k) = \Gamma_k ; \text{Continuity} \] (1)

\[ \frac{\partial \alpha_k \rho_k v_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k v_k v_k) = -\alpha_k \nabla \rho_k + \nabla \cdot \alpha_k (\bar{\tau} + \tau_i') + \alpha_k \rho_k g + v_k \Gamma_k + M_k - \nabla \alpha_k \cdot \tau_i' \; ; \text{Momentum} \] (2)

\[ \frac{\partial \alpha_k \rho_k H_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k H_k v_k) = -\nabla \cdot \alpha_k (q_k + q_i') + \alpha_k \frac{D}{Dt} p_k + H_k \Gamma_k + \frac{q_{in}}{L_i} + \phi_k \; ; \text{Enthalpy Energy} \] (3)

where \( \Gamma_k, M_k, \tau_i, q_{in} \) and \( \phi_k \) are the mass generation, generalized interfacial drag, interfacial shear stress, interfacial heat flux and dissipation, respectively. Here, the subscript \( i \) denotes the value at the interface, and the \( L_i \) denotes the length scale at the interface, where \( 1/L_i \) has the physical meaning of the interfacial area per unit mixture volume [1], such that

\[ \frac{1}{L_i} = a_i \; ; \text{interfacial area per unit mixture volume} \] (4)

In equations (1) through (3), the interfacial transfer terms can be modeled [1-3] as

\[ \Gamma_k = a_i m_k \; ; \text{Generation of Mass} \] (5)

\[ M_{id} = \frac{\alpha_i F_k}{B_k} + \frac{\alpha_i F_k}{B_k} + \frac{9}{2} \frac{\alpha_d}{r_d} \frac{\rho \mu}{\pi} \int \frac{D_k}{D_k^2} (v_j - v_i) \frac{d \xi}{\sqrt{t - \xi}} \; ; \text{Generalized Drag} \] (6)
\[
\Gamma_i H_i + \frac{q_{\text{th}}}{L_i} = a_i \left[ m_k H_k + h_k (T_i - T_k) \right]; \text{Interfacial Energy Transfer} \tag{7}
\]

where \( m_k \), \( F_{Dk} \), \( B_d \), \( F_v \), and \( \mu_m \) are the mean mass transfer rate, standard drag force, volume of a typical dispersed fluid particle, virtual mass force and mixture viscosity, respectively. With the last term in the right hand side of equation (6) being the Basset force, the interfacial transfer term due to standard drag can be written in terms of \( a_i \) as

\[
\frac{a_i F_{Dk}}{B_d} = -a_i \left[ \frac{C_{Dk}}{4} \left( \frac{r_{sm}}{r_P} \right) \rho \nu \frac{v_A v_i}{2} \right] \tag{8}
\]

As shown in equations (5) through (8), the interfacial transfer terms are expressed in terms of \( a_i \) and the driving force such that

\[\text{(Interfacial Transfer Term)} \sim a_i \times \text{(Driving Force)} \tag{9}\]

Therefore, the closure relation for the \( a_i \) is necessary in the two-phase system analysis employing the two-fluid model. Nonetheless, the interfacial area concentration \( (a_i) \) and the void fraction \( (\alpha) \) are the key geometric parameters in the fluid particle transport and the heat transfer capability. Hence, accurate estimation of these parameters is indispensable in accurate assessment of the given system.

Despite the lack of accuracy, however, the flow-regime dependent correlations are being implemented for the \( a_i \) in most of the current thermal-hydraulic system analysis codes. This approach does not dynamically represent the changes in the interfacial structure of the system and may lead to the numerical oscillations in system behavior due to its static and flow regime dependent nature. Therefore, improvements in the evaluation of the interfacial area will not only enhance the capability of the system analysis codes, but also improve the assessment accuracy. In view of this, the use of the first order equation to characterize the interfacial area transport had been recommended[2,3].

In efforts of solving this closure problem for the \( a_i \), the foundation of the interfacial area transport equation has been established by Kojasoy and Ishii[4] based on the Boltzmann particle transport equation. It was followed by the formulation of the preliminary interfacial area transport equation for the bubbly flow by Wu et al.[5], where the source and sink terms were established through mechanistic modeling of bubble
interaction phenomena in the bubbly flow regime. The focus of the present study is to establish a reliable interfacial area transport equation applicable to bubbly flow regime in the given test section geometry.

2. DEVELOPMENT OF INTERFACIAL AREA TRANSPORT EQUATION

The interfacial area transport equation is based on the Boltzmann transport equation, where the equation was expressed as an integro-differential equation of the particle distribution function. Letting $f(V,x,v,t)$ be the particle number density distribution function per unit mixture and particle volume, which is assumed to be continuous and specifies the probable number density of fluid particles moving with particle velocity $v$, at a given time $t$, in the spatial range $\delta x$ about a position $x$, with particle volumes between $V$ and $V+\delta V$, one can write

$$f(V + \delta V, x + v\delta t, v + F\delta t, t + \delta t)\delta\mu - f(V, x, v, t)\delta\mu = \sum_j S_j + S_{ph} \delta\mu \delta t$$

(10)

where $\delta\mu$ is a volume element in $\mu$ space and $F(V,x,v,t)$ is the force per unit mass. The $S_j$ and $S_{ph}$ are the particle source/sink rates per unit mixture volume due to $j^{th}$ particle interactions such as break-up or coalescence and the source rate due to phase change, respectively. Then, if we assume that the change of particle velocity within the time interval $t$ to $t + \delta t$ is negligible, such that the distribution function can be approximated as a function of $f(V,x,t)$, then equation (10) reduces to

$$\frac{df}{dt} + \nabla \cdot (fv) + \frac{\partial}{\partial V} \left( f \frac{dV}{dt} \right) = \sum_j S_j + S_{ph}$$

(11)

2.1 Interfacial Area Concentration Transport Equation

In order to formulate the interfacial area transport equation, we multiply equation (11) by the average interfacial area of particles with volume $V$, which is independent of coordinate system. Furthermore, since the transport equation given by (11) is much too detailed to be employed in practice, it is integrated over the volume of all particle sizes. After applying the Leibnitz rule of integration we obtain
\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) - \left( \frac{\dot{V}}{V} \right) \int_{V_{\text{min}}}^{V_{\text{max}}} fVdA_i = \int_{V_{\text{min}}}^{V_{\text{max}}} \left[ \sum_j S_j + S_{ph} \right] A_i dV 
\]

where the average \( a_i \) of all fluid particles and the interfacial velocity are given respectively by

\[
a_i(x,t) = \int_{V_{\text{min}}}^{V_{\text{max}}} f(V,x,t)A_i(V)dV, \tag{13}
\]

\[
v_i(x,t) = \frac{\int_{V_{\text{min}}}^{V_{\text{max}}} f(V,x,t)A_i(V)\nu(V,x,t)dV}{\int_{V_{\text{min}}}^{V_{\text{max}}} f(V,x,t)A_i(V)dV}, \tag{14}
\]

In acquiring equation (12), it is assumed that the time rate of change in the particle volume is independent of its volume. Also, since the volume source can be expressed as

\[
\frac{1}{V} \frac{dV}{dt} = \frac{1}{\rho_g} \left[ \frac{\eta_{ph} \rho_g}{\alpha} - \frac{d \rho_g}{dt} \right] = \frac{1}{\alpha} \left[ \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \mathbf{v}_g - \eta_{ph} \right], \tag{15}
\]

and by defining the volume- and surface-equivalent diameters of fluid particles with volume \( V \) as

\[
V = \frac{\pi}{6} D_e^3 \quad \text{and} \quad A_i = \pi D_s^2, \tag{16}
\]

respectively, the interfacial area transport equation is given by

\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \mathbf{v}_i) - \frac{2}{3} \left( \frac{a_i}{\alpha} \right) \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \mathbf{v}_g - \eta_{ph} \right) = \int_{V_{\text{min}}}^{V_{\text{max}}} \left[ \sum_j S_j + S_{ph} \right] A_i dV \tag{17}
\]

In order to solve equation (17), it can be seen that the source and sink terms in the right-hand-side of the equation should be specified by the constitutive relations. In view of this, we define

\[
\int_{V_{\text{min}}}^{V_{\text{max}}} \sum_j S_j dV = \sum_j R_j; \quad \text{particle number source/sink, and}
\]

\[
\int_{V_{\text{min}}}^{V_{\text{max}}} \sum_j S_jA_i dV = \sum_j \phi_j; \quad \text{interfacial area concentration source/sink.} \tag{18, 19}
\]

where \( R_j \)'s should be mechanistically modeled for each interaction mechanism which serve as source or sink in particle number through disintegration or coalescence.
processes. Furthermore, noting that $\phi_j$ can be written in terms of change of surface area of the fluid particle after a given particle interaction process, we write

$$\phi_j = R_j \Delta A,$$

(20)

Here, $\Delta A_i$ depends on the interaction mechanism, namely whether it is a break-up process or a coalescence process. In view of specifying the $\Delta A$, we may consider the coalescence and break-up processes as illustrated in Figure 1. Then, since the total volume of the particles should be conserved, assigning the subscripts 1 and 2 for the bubbles of smaller and larger volume, respectively, we can write

$$2V_1 = V_2 \quad \text{or} \quad D_2 = 2^{1/3} D_1$$

(21)

Hence, the change of surface area for one particle after a given interaction processes is given by

$$\Delta A = -0.413 \Delta A_1; \text{ for a coalescence process}$$

(22)

and

$$\Delta A = -0.260 \Delta A_1; \text{ for a break-up process}$$

(23)

where $A_i$ is the total initial surface area of the particle(s) subject to the given particle interaction, and the minus sign accounts for the reduction of surface area compared to the initial surface area. Furthermore, since the particle number density can be specified through $a_i$ and $\alpha$ by

$$a_i = n A_i \quad \text{and} \quad \alpha = n V,$$

(24)

such that

$$n = \psi \frac{a_i^3}{\alpha^2},$$

(25)

with

$$\psi = \frac{1}{36\pi} \left( \frac{D_{sa}}{D_e} \right)^3,$$

(26)

where the Sauter mean diameter is defined by

$$D_{sa} = \frac{6\alpha}{a_i},$$

(27)

Thus, we can rewrite $\phi_j$ as
Similarly, the nucleation source \( \phi_{ph} \) can be given by
\[
\phi_{ph} = \pi D_{bc}^2 R_{ph} \tag{29}
\]
where \( D_{bc} \) is the critical bubble size, which should be determined depending on the given nucleation process, namely, critical cavity size for the bulk boiling or condensation process, and bubble departure size for the wall nucleation. The \( R_{ph} \) should be modeled independently based on the given phase-changing mechanism, such as nucleation or condensation. Thus, combining with the constitutive relations given above, we can rewrite the interfacial area transport equation as
\[
\frac{\partial a_i}{\partial t} + \nabla \cdot (a_i \nu_i) = \frac{2}{3} \left( \frac{a_i}{\alpha} \left( \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \nu_i - \eta_{ph} \right) + \frac{1}{3 \psi} \left( \frac{\alpha}{a_i} \right)^2 \right) \sum_j R_j + \pi D_{bc} R_{ph} \tag{30}
\]
where the total change of \( a_i \) is expressed in terms of changes due to pressure, particle interactions and the phase-change.

Figure 1. Illustration of the fluid particle coalescence and disintegration processes.
2.2 Constitutive Relations

In this section, the identities and the constitutive relations in developing the interfacial area transport equation are summarized. First, by obtaining the volume transport equation from equation (11) through similar approach as in the previous section, we can write

\[
\frac{I}{\rho_k} \left[ \frac{\partial \alpha_p}{\partial t} + \nabla \cdot (\alpha_p \mathbf{v}_g) - \Gamma_g \right] = \int_{v_m}^{v_m} \sum_j S_j VdV .
\]  

Then, since the total particle volume is conserved, the right hand side of equation (31) identically equals to zero, and the left-hand side corresponds to the gas-phase continuity equation. Hence, following identities can be deduced from the void fraction transport equation;

\[
\frac{\partial \alpha_p}{\partial t} + \nabla \cdot (\alpha_p \mathbf{v}_g) - \Gamma_g = 0 ; \text{ gas-phase mass conservation (32)}
\]

\[
\int_{v_m}^{v_m} \sum_j S_j VdV = 0 ; \text{ volume conservation in bubble coalescence or break-up (33)}
\]

In view of constitutive relations necessary for the interfacial area transport equation to be solved, we may summarize them as;

\[
\int_{v_m}^{v_m} \sum_j S_j dV = R_j ; \text{ particle number source/sink, (34)}
\]

\[
\int_{v_m}^{v_m} \sum_j S_j A dV = \phi_j = R \Delta A_i ; \text{ interfacial area concentration source/sink, (35)}
\]

\[
\phi_{ph} = \pi D_{ph} R_{ph} ; \text{ interfacial area source due to phase-change, (36)}
\]

\[
n = \psi \frac{a^i}{\alpha^i} \text{ where } \psi = \frac{1}{36\pi} \left( \frac{D_{sw}}{D_i} \right)^i,
\]

and

\[
D_{sw} = \frac{6\alpha}{a_i} .
\]  

Here, the number sources/sinks in equation (34) should be established through mechanistic modeling of the major particle interactions that contribute to the \(a_i\) change. Accounting for the wide range of two-phase flow, the major particle interaction
mechanisms, which may attribute to the particle coalescence or disintegration can be summarized as:

- **Random Collision** ($R_{RC}$): coalescence through collision driven by turbulent eddies,
- **Wake Entrainment** ($R_{WE}$): coalescence through collision due to acceleration of the following particle in the wake of the preceding particle,
- **Turbulent Impact** ($R_{TI}$): disintegration upon impact of turbulent eddies,
- **Shearing-Off** ($R_{SO}$): shearing off around the base rim of the cap bubble,
- **Surface Instability** ($R_{SI}$): break-up of large cap bubble due to surface instability at the interface,
- **Rise Velocity** ($R_{RV}$): collision due to the difference in the bubble rise velocity,
- **Laminar Shear** ($R_{LS}$): break-up due to the laminar shear in viscous fluid, and
- **Velocity Gradient** ($R_{VG}$): collision due to the velocity gradient

Among the given mechanisms above, the $R_{LS}$ can be important in a viscous fluid flow, and the $R_{VG}$ may not be neglected for the flow very close to the wall. In the present study, however, we only consider the mechanisms applicable to the current system of air-water, and the effect from the local phenomenon is assumed to be negligibly small in the overall contribution. Therefore, the $R_{LS}$ and $R_{VG}$ are neglected in the source and sink terms. Furthermore, the $R_{RV}$ can be considered as a part of $R_{WE}$ and can be dropped for simplicity of the model.

In view of the particle transport for dispersed bubbles, the major contribution in the $a_i$ transport can be attributed to three mechanisms. They are the turbulent impact ($TI$), the random collision ($RC$) and the wake entrainment ($WE$). In view of this, the preliminary models for the one-group particle interaction phenomena have been studied by Wu et al.[5]. In the scope of the present study, detailed modeling approaches on these models will not be discussed but are left to the readers to refer to the references given above. Accounting for the improvements made by Kim[6], the source and sinks attribute to the $a_i$ in small bubble transport can be summarized as

$$R_{TI} = C_T \left( \frac{nu}{D_b} \right) \exp \left( -\frac{We_{cr}}{We} \right) \left[ 1 - \frac{We}{We_{cr}} \right] \text{ where } We > We_{cr}; \ TI (source), \quad (39)$$
and

\[ R_{we} = C_{we} C_{D}^{1/3} (D_b/n^2 u_e (D_b)); \quad WE (sink) \]  

where \( D_b, We_{cr}, \alpha_{max}, \) and \( C_D \) are the bubble diameter, critical Weber number over which the bubble break-up occurs, maximum packing limit, and drag coefficient, respectively, and \( C_{Th}, C_{RC}, C, \) and \( C_{WE} \) are the coefficients to be determined through experiment. Here, \( C \) accounts for the effective range for the turbulent eddies to drive two particles to collide, whereas \( C_{Th}, C_{RC}, \) and \( C_{WE} \) are to account for the collision probability and break-up/coalescence efficiency for each mechanism.

### 2.3 One-Group Interfacial Area Transport Equation

When the fluid particles of interest remain similar in shape and size after the particle interactions (as can be found in bubbly flow regime), their transport phenomena can be described by one transport equation. However, when fluid particles of various shapes and size present simultaneously, the transport phenomena can be quite different due to the significant difference in the drag. In such case, multiple transport equations may be necessary to describe the particle transport. In the present study, we limit our scope of study within the two-phase flow system of dispersed bubbles, where all the present bubbles can be categorized as one group in view of transport phenomena. In one group transport, we assume that the bubbles are approximately spherical in shape, and the drag-based bubble transport is similar.

In bubbly two-phase flow condition, the Sauter mean diameter is approximately equal to the volume equivalent diameter. Therefore, \( \psi \) defined in equation (37) reduces to

\[ \psi = \frac{1}{36\pi} = 8.85 \times 10^{-3} ; \quad \text{for dispersed bubbles} \]  

Furthermore, since the phase change term can be replaced by

\[ \eta_{ph} = \int_{V_{mm}}^{V_{ph}} S_{ph} V dV = R_{ph} \frac{\pi}{6} D_{he}^3. \]
Also, noting that critical bubble size due to nucleation is much smaller compared to the average bubble Sauter mean diameter, we may assume

\[
\left( \frac{D_{bc}}{D_{sm}} \right) \approx 0.
\]

Combining these with equation (30), the interfacial area transport equation for dispersed bubbles, or one-group interfacial area transport equation is given by

\[
\frac{\partial a}{\partial t} + \nabla \cdot (a \vec{v}) = \frac{2}{3} \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \vec{v}_g + \frac{1}{3\gamma} \left( \frac{\alpha}{\alpha_r} \right)^{\frac{\gamma}{\gamma-1}} \left( R_{Rh} - R_{Rc} - R_{WE} \right) + \pi D_h^2 R_{ph} \tag{45}
\]

where the \( R_{Th}, R_{RC} \) and \( R_{WE} \) are given respectively by equations (39) through (41).

3. EXPERIMENTAL STUDIES

In order to validate the present model, the experiments are performed in an adiabatic vertical air-water two-phase flow loop and an extensive database is built. In the present experiment, two different heights of test section with hydraulic diameter of 2-cm is employed.

3.1 Experimental Conditions

The test section has a flow area of 1x20-cm² and is made of the acrylic Lucite for flow visualization. Use of this geometry permits unambiguous visualization of the two-phase flow patterns without the distortion caused by curved surfaces. Moreover, for bubbly flow conditions in the present flow path, the bubbles transport more likely in 2-dimension and can be simpler compared to a more conventional round tube geometry of the same hydraulic diameter.

The first set of experiments is performed in a test section of 188-cm in height, and the second set is performed in a test section of 295-cm in height. The instrumentation ports are furnished on the gap side of the flow path at six different axial locations at \( L/D_p=8.02, 34.76, 61.49, 88.22, 114.96 \) and 141. The local two-phase flow parameters are acquired by the state-of-the-art miniaturized double-sensor conductivity probe[7,8], which has a measurement error of less than \( \pm10\% \). At each axial location, the probe is traversed
along the flow cross-sectional area in both gap (1-cm) and width (20-cm) directions, such that the measurements are made at 36 local points across one quarter of the total flow area. The local parameters acquired by the probe are bubble number \((N_b)\), bubble velocity \((v_g)\), \(\alpha\) and \(a_i\). The bubble Sauter mean diameter is obtained by the relation given by equation (38).

In total, 9 flow conditions are selected mostly within the bubbly flow regime as shown in Figure 2, where the flow regime map is given by Mishima and Ishii[8]. Also summarized in Table 1 are the flow rates for each experimental run. In order to highlight the bubble transport phenomena, the experimental data are also obtained at flow conditions near the flow regime transition.

![Figure 2. Present flow conditions of interest shown in the flow regime transition boundary given by Mishima and Ishii[8].](image-url)
Table 1. Flow rates of the present experiment $\dot{g}_g$ is the 1-atm. equivalent value converted from the rota-meter reading at the inlet of the test section.

<table>
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<th>$j_f$ [m/s]</th>
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3.2 Drift Flux Model Analysis: Estimation of Distribution Parameter

In this section, the distribution parameter $C_o$, in the drift flux model is evaluated with all the data obtained in the given test section. Two approaches based on both theoretical assumption and experimental measurements are taken in the estimation of $C_o$ and $<\langle V_g \rangle>$. The accurate estimations of $C_o$ and $<\langle V_g \rangle>$ are important in view of evaluating the theoretical $a_t$ transport equation against the experimental data, because the drift flux model can be employed in model evaluation for the axial development of the void fraction. A more detailed explanations on how the drift flux model was employed in the model evaluation is discussed in the following section.

In principle, the one-dimensional drift flux model is given by

$$\frac{\langle j_g \rangle}{\langle \alpha \rangle} = C_o \langle j \rangle + \langle \langle V_g \rangle \rangle \quad (46)$$

Here, $C_o$ is the distribution parameter defined by

$$C_o = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle} \quad (47)$$

which describes the distribution of the dispersed phase in a given two-phase flow. Hence, when the flow reaches a steady state condition, drift flux model can be employed.
in the analysis of the given system by plotting $\langle j \rangle$ versus $\langle j_g / \alpha \rangle$ with known $C_o$ value.

In the previous study, based on both experimental results and on the assumption that $C_o$ depends on the density ratio and the liquid Reynolds number, Ishii[10] suggested that

$$C_o \equiv \gamma \left( 1.2 - 0.2 \sqrt{\frac{\rho_g}{\rho_f}} \right); \text{round tube, and}$$

$$C_o \equiv \gamma \left( 1.35 - 0.35 \sqrt{\frac{\rho_g}{\rho_f}} \right); \text{rectangular channel}$$

where $\gamma$ is a factor accounting for the vapor generation, $\Gamma_g$, such that

$$\gamma = \left( 1 - e^{-m(a)} \right) \text{ when } \Gamma_g > 0; \text{ and } \gamma = 1, \text{ otherwise}$$

However, the $C_o$ value may differ from the values suggested above depending on the given flow path geometry. As can be seen in equation (46), accurate estimation of $C_o$ value is of great importance in evaluating the given system, and the geometry of the flow path may have significant effect in this value. In view of this, two different approaches are taken in determining the $C_o$ in the present study.

The first approach in determining the $C_o$ is based on the assumption that the liquid velocity profile is similar to that of turbulent flow such that

$$v_f \equiv V_o \left( 1 - \frac{x}{W} \right) \left( 1 - \frac{y}{G} \right)$$

where $V_o$ is the center or maximum liquid velocity, and $W$ and $G$ are the width and the gap measured from the center, respectively. Considering that the minimum Reynolds number for the present experimental conditions is approximately 60,000, the turbulent velocity profile assumption is reasonable. In fact, when measured $<j_f>$ is compared with the area-averaged value of the local $(1-\alpha)v_f$ calculated by equation (51) with $\alpha$ acquired by the probe measurement, the percent differences between $<j_f>$ and $<(1-\alpha)v_f>$ in all of the flow conditions are within the measurement error of $\pm 10\%$. As the local $v_f$ is estimated, other local parameters are estimated by

$$j_x = \alpha v_f; \quad j_f = (1-\alpha)v_f; \quad j = j_x + j_f$$
where the local \( \alpha \) and \( v_g \) are obtained from the probe measurements. Thus, the \( C_o \) can be readily calculated by the definition given by equation (47).

The second approach in estimating \( C_o \) is based on the theoretical model. From equation (46), \( C_o \) can be obtained by

\[
C_o = \frac{\langle j_x \rangle / \langle \alpha \rangle - \langle \langle V_{\gamma} \rangle \rangle}{\langle j \rangle}
\]  

(53)

provided that \( \langle \langle V_{\gamma} \rangle \rangle \) is known. Here, the \( \langle j_g \rangle \), \( \langle \alpha \rangle \), and \( \langle j \rangle \) can be obtained from the measurement. Therefore, employing the drift velocity for the distorted particles given by Ishii[10] as

\[
\langle \langle V_{\gamma} \rangle \rangle = \sqrt{2 \left[ \frac{\sigma g \Delta \rho}{\rho_f^2} \right]} (1 - \langle \alpha \rangle)^{1/3}
\]  

(54)

and substituting it into equation (53), \( C_o \) can be obtained.

In employing the two different approaches discussed above, it should be noted that the accuracy of the first approach depends on the accuracy in the estimation of local liquid velocity via the local void fraction measurement, whereas the second approach relies on the accuracy of the flow rate measurement. Considering that the current conductivity probe employs the sensor of diameter less than 0.1-mm, the errors associated with the void fraction measurement are minimal. In fact, throughout the previous benchmark experiments, the difference in the void fraction measurements between the conductivity probe and several other different measurement techniques was less than \( \pm 5\% \). Hence, it is desirable to use the first approach in estimating the \( C_o \). This is true especially for flow conditions where \( \langle j \rangle \) is low, where small errors in the \( C_o \) estimation would propagate and result in significant errors for high \( \langle j \rangle \) conditions.

Therefore, in the present study, following approaches are taken in evaluating the experimental data with the drift flux model;

Step 1. Estimate local \( v_L \) by equation (51)

Step 2; Calculate local \( j_r, j_g \), and \( j \) using experimentally obtained \( \alpha \) and \( v_g \).

Step 3; Estimate the \( C_o \) by equation (47)

Step 4; Determine \( \langle \langle V_{\gamma} \rangle \rangle \) based on the \( C_o \) estimated in the previous step
Step 5; Calculate $\langle V_{gi} \rangle$ by model given by (54)

Step 6; Estimate the $C_0$ by equation (53)

Step 7; Compare the two $C_0$ values from Steps 3 and 6, and determine the final $C_0$ based on the best fit with the experimental data

Step 8; Adjust $\langle V_{gi} \rangle$ based on the $C_0$ modified in Step 7.

The final result obtained after performing the calculation steps described above is shown in Figure 3, where the data are plotted in x and y coordinates of $<j>$ and $<j_g>/<\alpha>$. In this figure, the data obtained at $L/D_H=8.02$ are not plotted accounting for the errors associated with the entrance effect. Lines by three different $C_0$ values, two obtained from two different approaches and the average of the two, are shown in the figure. The $\langle V_{gi} \rangle$ in the figure is the average value of those calculated for each flow condition examined. The average values for $C_0$ and $\langle V_{gi} \rangle$ are 1.076 and 0.195-m/s, respectively.

Figure 3. Drift flux model versus experimental data

4. MODEL EVALUATION

In this section, the results obtained in the comparison between the data and the model predictions are presented. To simplify the model for comparison against data, the interfacial area transport equation given earlier by equation (45) was spatially averaged over the flow cross-sectional area. This process assumes all parameters exhibit nearly uniform profiles along the cross-sectional flow area, so that the covariance terms are
negligible. This assumption was verified in most of the flow conditions examined that the profiles of $\alpha$, $a$, and $v_g$ remained nearly uniform across the flow cross-section. However, in some cases, the characteristic wall-peak phenomena were also observed. In such cases, the one-dimensional approach may introduce errors in the $a_i$ estimation, and a fully three-dimensional approach should be taken. Nevertheless, in the scope of the present study, only the one-dimensional approach was employed for simplicity.

Also, $R_{ph}$ term was omitted from the model to reflect the fact that the experiments were performed under adiabatic air/water two-phase conditions. Furthermore, recalling that the first term in the right-hand side of equation (45) originates from the volume source, it can be expressed in terms of pressure drop. Therefore, assuming that the ideal gas law is valid and the flow condition is isothermal, we have

$$\frac{2\langle a_i \rangle}{3} \left( \frac{\partial}{\partial z} \langle \alpha \rangle \langle v_g \rangle \right) = -\frac{2\langle a_i \rangle \langle v_g \rangle}{3} \left( -\frac{\partial p}{\partial z} \right)$$

Thus, combining equation (56) and the interfacial source/sink terms described earlier, the steady state one-dimensional interfacial area transport equation is given by

$$\frac{\partial}{\partial z} \left( \langle a_i \rangle \langle v_g \rangle \right) = -\frac{2\langle a_i \rangle \langle v_g \rangle}{3} \left( -\frac{\partial p}{\partial z} \right)$$

$$+ C_{n} \frac{2}{18} \left( \langle a_i \rangle \langle a_i \rangle \right) \sqrt{1 - \frac{W_{cr}}{W_e}} \exp \left( -\frac{W_{cr}}{W_e} \right)$$

$$- C_{ncr} \frac{1}{3\pi} \langle a_i \rangle \langle u_i \rangle \left[ 1 - \exp \left( -\frac{\langle \alpha \rangle_{max}^{1/3} \langle \alpha \rangle_{min}^{1/3} - \langle \alpha \rangle^{1/3}}{\langle \alpha \rangle_{max}^{1/3} - \langle \alpha \rangle_{min}^{1/3}} \right) \right]$$

$$- C_{we} C_{D} \frac{1}{3\pi} \langle a_i \rangle \langle u_i \rangle$$

(57)

where the bubble number density, $n$, and the bubble Sauter mean diameter are replaced by

$$n = \psi \frac{a_i}{\alpha^2} \quad \text{and} \quad D_{nm} = \frac{6\alpha}{a_i}$$

(58)
Here, the $a_i$ weighted bubble interfacial velocity was approximated by the void weighted gas-phase velocity by noting from the experimental data that the bubble size across the flow area at a given axial level was nearly uniform.

In evaluating the model with the experimental data, the adjustable coefficients in the model should be specified. In order to determine the coefficients accurately, the dominant bubble interaction mechanism for the given experiment is identified based on the flow conditions. For example, in flow conditions where the liquid superficial velocity is high (Runs 7, 8, and 9) the bubble interaction is assumed to be strongly affected by the turbulence. In Run 8, for instance, where both the liquid and gas superficial velocities are high, the dominant mechanisms can be random collision coalescence and turbulent impact break-up. Furthermore, the turbulent impact break-up becomes more important in Runs 7 and 9, where the flow conditions reach the finely dispersed bubbly flow condition. On the other hand, near the flow regime transition boundary with high void fraction condition, such as Run 6, the turbulent effect is assumed to be weaker compared to Runs 7, 8, and 9, whereas the wake entrainment mechanism becomes more significant.

Based on such analogy, the coefficients are determined by finding the values that yield best agreement for all of the available data sets. In the random collision coalescence model, the constant $C$ that accounts for the effective range of influence of eddies in driving bubbles to collisions is assumed to be 3. Furthermore, in estimating the critical Weber number, it was varied from 2.3 to 12 based on the studies by Prince and Blanch[11] and Ishii and Chawla[12]. In the present study, the critical Weber number of 8 is found to yield the best results. The coefficients adequate for the present loop geometry are then determined as

- Wake Entrainment; $C_{WE}$=0.042
- Random Collision; $C_{RC}$=0.003; $C$=3.0; $\alpha_{max}$=0.75
- Turbulent Impact; $C_{TI}$=0.026; $We_{cr}$=8.0

The coefficients given above significantly differ from the ones given by Wu et al.[6]. This stems from several facts as follows; First, the geometry of the test section is different for the two studies. Secondly, in their study, the bubble expansion due to pressure drop ($EXP$) was not considered, which plays a major role in the $a_i$ change in the
low pressure system as will be discussed later. Furthermore, the model presented in this study was further improved from the preliminary model given in their study. These include: criterion for critical Weber number in the turbulent effect and inclusion of the drag effect in the estimation of bubble rise velocity in the $WE$ model.

Furthermore, as can be seen in equation (57), four variables should be known in solving the $a_i$ transport equation. To furnish these variables and to evaluate over the entire length of the test section, the analysis package consisting of the transport equation with its constitutive models and coefficients and the drift flux model was employed together with the test data. The experimental data at the first measurement location were used to set the initial values. The vapor fraction was calculated from the drift flux model using the values of $C_o$ and $\langle V_g \rangle$ as determined in the previous section. The other variables ($\langle v_g \rangle$, $\langle v_f \rangle$, $\langle a_i \rangle$) were taken from interpolations of the experimental data. Since the drift flux model was based on all the available data from the given test geometry, this was thought to be the most reliable approach available.

The evaluation results are shown in Figure 4. In the figures, the results are categorized into two figures based on the flow conditions, such that the results from the bubbly flow conditions (Runs 1 through 5) are plotted in Figure 4(a), and those from the transition conditions are plotted in Figure 4(b). Here, the $\pm 10\%$ error bars are drawn from the experimental data points.

As shown in the figures, the model prediction and the experimental data agree well within the measurement error of $\pm 10\%$ in most flow conditions. In flow conditions where liquid superficial velocity is high (Runs 7 and 9) the agreement is remarkable. In Run 6, the model predicts very well up to $L/D_h=88.22$, but fails at $L/D_h=141.70$. Considering the fact that the Run 6 was performed in the flow regime transition condition from bubbly to cap-bubbly regimes, and recalling that the occasional cap bubble appearance was observed in this flow condition, the deviation may stem from the measurement error in the application of the double-sensor probe. Since the double-sensor probe is designed only for small bubbles, when cap bubbles are encountered it may over-predict the interfacial area concentration. This is due to the misinterpretation of the interfacial velocity measured by the two sensors, and the application of the four-sensor
probe becomes necessary in such flow conditions. Another fact that attributes to such deviation may be due to the limitation of one-group interfacial area transport equation, which accounts only for the particle interactions among small dispersed bubbles. The deviation between the prediction and the data can be also observed in Runs 2 and 3. In such flow conditions, the flows are in bubbly flow regime, and the deviations between the experimental data and predictions are hard to explain.

Figure 4. Model evaluation against the data. (a) Bubbly flow conditions, (b) Flow conditions near transition boundaries between bubbly and cap-bubbly. * denotes the data obtained in a longer test section.

The Run 8 is somewhat unique considering not only the appearance of cap bubbles at $L/D_h=141.70$, but also the entrance effect at the mixture inlet up to $L/D_h=88.22$. It is interesting to note that the model prediction passes through the mid-point of the experimental data. However, accounting for the errors associated with the measurement in Run 8, it is hard to evaluate the results. Nevertheless, accounting for the possible measurement errors in boundary flow conditions, overall agreement between the model and the data is quite acceptable.

In order to examine the contributions from each mechanism and their sensitivity to the flow conditions, the estimation made by each mechanism is also investigated. In Figure 5, the characteristic results from the bubbly condition (Run1) and bubbly to cap-
bubbly transition condition (Run6) are examined. In these flow conditions, it can be found from the results that the dominant mechanism are the \( WE \) and \( EXP \) mechanisms, whereas \( TI \) mechanism plays no role. This is due to the fact that the turbulent velocity is not high enough to reach the critical Weber number in both flow conditions. On the other hand, contribution from the \( RC \) mechanism becomes more significant as the void fraction and the turbulence increases (from Run 1 to Run 6). In the two given flow conditions, the \( EXP \) mechanism serves as the only source for \( a_i \).

![Figure 5. Sensitivity analysis for individual mechanism in Run 1 and Run 6.](image)

In view of the contribution from \( TI \) mechanism, the results from the Run 7 and Run 9 are plotted in Figure 6. It is clear from the figures that the contribution from \( TI \) significantly increases as the liquid superficial velocity increases from Run 7 to Run 9. In Run 9, in particular, the \( TI \) contribution is clearly dominant, and it is much higher than that due to \( EXP \). Also interesting to note in Run 7 is that the \( TI \) becomes important from \( z \geq 2 \) m, which demonstrates the development of the two-phase flow parameters along the flow direction. In both flow conditions the contributions from \( RC \) and \( WE \) are small compared to those from \( TI \) and \( EXP \). However, it is noteworthy that the \( RC \) becomes more important than \( WE \) as the flow condition changes from Run 7 to Run 9. As noted earlier, this is due to the increase in turbulent effect, which promotes the \( RC \) mechanism.
Therefore, in flow conditions where turbulent effect is high, it is $TI$ that is dominant in source terms, while $RC$ becomes dominant to $WE$ among sink terms.

Figure 6. Sensitivity analysis for individual mechanisms in highly turbulent conditions (Run 7 and Run 9).

5. SUMMARY AND CONCLUSION

In effort of establishing the closure relation for the $a_i$ in the two-fluid model, the interfacial area transport equation is presented with detailed formulation procedures. When this approach is applied to the system analysis code employing the two-fluid model, it not only eliminates the numerical instability which can be encountered as a result of employing the static experimental correlations, but also improve the accuracy of the code prediction.

In view of establishing the database, the local data are acquired in an adiabatic air-water co-current vertical loop by the state-of-the-art miniaturized double-sensor conductivity probe with measurement error of approximately ±10%. In order to highlight the bubble transport phenomena, two test sections of the same geometry with different heights are employed with its maximum developing length extending up to $L/D_s=141.70$.

In the course of evaluating the acquired data, the data are compared with the drift flux model. In the estimation of $C_0$, two approaches based on both theory and experiments are
presented. First, the $C_o$ is calculated by estimating the local liquid velocity based on the turbulent velocity profile assumption. Secondly, the estimation of $C_o$ is done based on the theoretical calculation of $\langle \langle V_{gi} \rangle \rangle$. The average value of $C_o$ from the two approaches is used for the evaluation of the data. With the average $C_o$ value of 1.076, the agreement between the data and the drift flux model was within percent difference of $\pm 10\%$ in general.

The one-group interfacial area transport equation accounting for the bubble transport in bubbly flow condition is evaluated against the experimental data obtained in a wide range of bubbly flow conditions including the flow regime transition conditions. In the evaluation, the model is simplified into the steady state one-dimensional form by applying the area-average to the local formulation, and the predictions made by the model are compared with the area-averaged data. A good agreement is observed in overall results, where most of the results fall within the measurement error of $\pm 10\%$.

The model is further analyzed by evaluating the individual contribution from each source or sink term. In the analysis, active fluid particle interactions are clearly demonstrated in all of the flow conditions examined. It was also shown that the dominant mechanism varies depending on the flow conditions. In general, the random collision coalescence and the turbulent impact break-up mechanisms are found to be dominant in the highly turbulent flow conditions, whereas the wake induced coalescence and the bubble expansion mechanisms are dominant in the bubbly or bubbly to cap-bubbly transition flow conditions.

Some deviations are observed at the highest measurement locations for the flow conditions near the flow regime transition boundaries. Considering that occasional cap bubbles appear in such flow conditions, the deviations are believed to be due to both the application of double-sensor probe and the limitation of one-group model which accounts only for the small bubble transport. Accounting for the agreement in the wide range of flow conditions examined and the entrance effect associated with the unique two-phase mixture inlet geometry, it can be concluded that the present model serves well in estimating the $a_i$ transport in the given experimental conditions.
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REFERENCES
6. S. Kim, Interfacial area transport equation and measurement of local interfacial characteristics, *Ph.D. Thesis*, School of Nuclear Engineering, Purdue University, Dec. 1999
