Title: UTILIZATION OF BAETEN'S DEAD-TIME CORRECTION FORMALISM FOR MULTIPLICITY COUNTING

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Abstract

We evaluate a dead-time correction formalism for double and triple coincidences in multiplicity counting. Denoting $S$, $D$, $T$ and $Q$ the "true" (zero dead-time) values for singles, doubles, triples and quads, the measured values are $S(\delta)$, $D(\delta)$, $T(\delta)$ ($\delta$ is the dead-time parameter). The three correction factors relating $S(\delta)$, $D(\delta)$, $T(\delta)$ to $S$, $D$, $T$ depend on $S$, $D$, $T$ and $Q$. Thus, determining $S$, $D$ and $T$ from the measured values involves solving a non-linear equation system and assuming a point-model value for $Q$. We will describe the accuracy by which $S$, $D$ and $T$ (and consequently, the mass, multiplication, and alpha) of fissile material samples can be determined using this formalism.

INTRODUCTION

A novel dead-time correction has been developed by Baeten\(^1\). This method relates the dead-time correction applied to the singles, doubles and triples rates in neutron multiplicity counting\(^2\), to the "true" values (assuming no dead-time) of the singles ($S$), doubles ($D$), triples ($T$) and quads ($Q$) rates. For convenience, we reproduce these formulae:

\[
\frac{S(\delta)}{S} = \left(1 - \frac{D\delta}{S\tau}\right)e^{-\delta S} \tag{1}
\]

\[
\frac{D(\delta)}{D} = \left(1 - 2S\delta - 2\frac{T}{D\tau}(1 + \frac{f_2}{f_1})\delta\right)e^{-2\delta S} \tag{2}
\]

The integrals $f_i$ are given by

\[
f_i = \frac{1}{i}e^{-\frac{i}{\tau}}\left(1 - e^{-\frac{i}{\tau}}\right)
\]

\[
\frac{T(\delta)}{T} = \left[1 - 2S\frac{D}{T}f_{1,1}\delta - \frac{D^2}{T}\delta\left(1 + \frac{f_{1,2}}{f_{2,1}} + \frac{f_{1,1}}{f_{2,1}}\right)\right] \left[-3S\delta - 3\frac{Q}{T\tau}\delta\left(1 + \frac{f_{3,1}}{f_{2,1}} + \frac{f_{3,2}}{f_{2,1}}\right)\right] e^{-3\delta S} \tag{3}
\]
The integrals $f_{i,j}$ are given by

\[ i \neq j \neq 0 \rightarrow f_{i,j} = \frac{2}{j!} \left[ e^{-\frac{P+P}{\tau}} \frac{1}{j-i} \left( e^{-\frac{(j-i)P+\delta}{\tau}} - e^{-\frac{(i-j)P}{\tau}} \right) + e^{-\frac{P}{\tau}} \left( e^{-\frac{(j-i)P}{\tau}} - e^{-\frac{P}{\tau}} \right) \right] \]

\[ i = j \neq 0 \rightarrow f_{i,i} = \frac{2}{i!} \left[ e^{-\frac{P+P}{\tau}} \left( W - \delta \right) + e^{-\frac{P}{\tau}} \left( e^{-\frac{P+P+\delta}{\tau}} - e^{-\frac{P}{\tau}} \right) \right] \]

\[ j = 0 \rightarrow f_{i,0} = \frac{2}{i^2} \left( e^{-\frac{P+P+\delta}{\tau}} - e^{-\frac{P}{\tau}} \right) + \frac{2}{i} \left( W - \delta \right) e^{-\frac{P}{\tau}} \]

where $\delta$ is the dead-time parameter, $\tau$ - Decay Time, $P$ - Predelay time, $W$ - Gate width. We note that $S$, $D$, $T$ and $Q$ are the full rates not limited by the gate fraction. It is assumed that the dead-time is "updating", i.e., if a second pulse arrives during the time the system is dead ($\tau$ after the pulse), the dead-time is extended by $\tau$.

Typically, there are two contributions to the dead-time of multiplicity systems. The first is the dead-time of the $^3$He proportional counters and is typically of the order of microseconds. The second, is the dead-time of the electronic circuitry and is of the order of 50 ns. The effect of electronics dead-time is usually mitigated by the use of derandomizing circuits ("derandomizers"). These circuits can be used for banks of detectors and their output combined using a second layer of derandomizers. The existing formalism does not contend with either of these two effects. However, given its relative simplicity, we consider it important to validate the formalism and determine its limitations. If successful, we could later examine its applicability to these more complex issues.

**MONTE CARLO SIMULATION**

We have written a Monte Carlo simulation code that allows us to examine the effect of different sampling schemes, perform error analysis, and determine dead-time effects. The general logic of the code is

1. Input data that include the fission and ($\alpha$,n) neutron rates are used to generate a time-sequence of neutrons. A fission event is produced at random in time. The number of emitted neutrons is sampled from the frequency distribution. A capture time (based on an exponential capture time) is sampled and its detection (sampled based on the input detector efficiency) is determined for each neutron. The ($\alpha$,n) neutrons are also generated with a random rate, and the detection likewise sampled.

2. A dead-time correction is applied to the neutron sequence. Neutrons arriving within a time of $\tau$ (determined by input) of a previous neutron are removed.

3. A multiplicity fold distribution is formed and the reduced moments ($S$, $D$, $T$ and $Q$) are calculated.
Results of the code have been compared to a figure-of-merit analytical code, and to other results to validate it.

METHOD OF ANALYSIS

The Monte Carlo code uses input values of the $^{240}$Pu mass ($m_{240}$), multiplication $M$, and $(\alpha,n)$-to-fission-neutron-ratio $\alpha$. Running the code provides "measured" values of the singles, doubles and triples denoted $S(\delta)$, $D(\delta)$ and $T(\delta)$ considering the effect of dead-time. We need to invert eqs. (1), (2), and (3) to determine $S$, $D$, and $T$. This inversion also requires the quads $Q$. We use the point-source model\textsuperscript{2} iteratively as follows. Starting with the initial values $S(\delta)$, $D(\delta)$, and $T(\delta)$, we calculate $m_{240}$, $M$, and $\alpha$ from the point model equations. From these, we calculate $S$, $D$, $T$ and $Q$, and the dead-time correction factors (eqs. 1-3). We now recalculate $S$, $D$, and $T$ from $S(\delta)$, $D(\delta)$, and $T(\delta)$ using the new correction factors, recalculate $m_{240}$, $M$ and $\alpha$ from the point model equations, and continue to iterate until the process converges (the iterated values of $S(\delta)$, $D(\delta)$, $T(\delta)$ coincide with the Monte Carlo output values). Typically for the cases we have examined, convergence occurs within 4-5 iterations.

RESULTS

Figure 1 presents a direct comparison of the live time as determined by the Monte Carlo code to values obtained from eqs. 1-3, for doubles and triples. Monte Carlo results for quads are

![Figure 1: Fractional live time of doubles (diamonds), triples (triangles) and quads (circles) as a function of the singles dead-time. The lines are Baeten's calculations - upper line for doubles and lower line for triples.](image-url)
presented for completeness. The statistical errors in the Monte Carlo calculations are smaller than the point sizes, except where depicted. The dead-time parameter used in these calculations was 0.1 μsec. The highest rate of detected neutrons was $1.55 \times 10^6$ neutrons/sec. Evidently, the corrections provide a good (but not perfect) description of the dead-time corrections for live times above a value of 60%. The equations overestimate the correction at higher values (higher rates).

The important quantity we infer from multiplicity counting is the mass of $^{240}$Pu ($m_{240}$). Figure 2 shows how accurate the iterative procedure is when used to determine $m_{240}$ — for singles dead-times of 2% and above (equivalent to 9% dead-time in doubles and 20% dead-time in triples) significant biases are evident. It remains to be seen in future studies whether a small (artificial) adjustment of the dead-time parameter would enable us to obtain more accurate corrections and consequently, reliable assay values for $m_{240}$.

![Figure 2: Systematic error in the determination of the $^{240}$Pu mass (relative to the standard deviation in the Monte Carlo calculation) using the iterative process we described. At smaller values of the singles live time, the $m_{240}$ value is essentially indeterminate.](image)

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REFERENCES