We discuss the simultaneous resummation of threshold and recoil enhancements to partonic cross sections due to soft radiation. Our method is based on a refactorization of the parton cross section near its partonic threshold. It avoids double counting, conserves the flow of partonic energy, and reproduces either threshold or recoil resummation when the other enhancements are neglected.

1 Introduction

Power corrections are phenomenologically significant in many QCD hard-scattering cross sections for which the operator product expansion is not directly available. Examples that have received considerable attention include event shapes in electron-positron annihilation and transverse momentum distributions in Drell-Yan cross sections. In each of these cases, a perturbative description of the cross section leads to integrals of the form

$$I_p \equiv Q^{-p} \int_0^Q d\mu \mu^{p-1} \alpha_s(\mu)$$

with $Q$ the hard scale and $p \geq 1$. In perturbation theory with a fixed coupling, $I_p$ is just a number, but when the coupling runs, the integral becomes ill-defined at its lower limit. This observation requires us to introduce a minimal set of power corrections of the form $\lambda_p/Q^p$, one for each ambiguous $I_p$ that we encounter. The perturbative expression is cut off, or otherwise regularized to make it finite without changing the set of exponents $p$. The values of the coefficients $\lambda_p$ are then to be found by comparison with experiment; they depend on the nature of the perturbative regularization that is employed. In any case, it is only the sum of regularized perturbation theory and power corrections that has physical meaning.

The first step in this process is to show that in some self-consistent approximation the cross section at hand may be written in terms of integrals like the $I_p$ above. In many cases, this step involves the resummation of logarithms associated with soft gluon emission, for which the eikonal approximation is
useful. In this talk⁴, we discuss an expression for the eikonal approximation in hadronic collisions, where the analysis of power corrections through the running coupling is particularly transparent.

2 The Eikonal Cross Section

To be specific, we discuss the eikonal approximation as it appears when partons $a$ and $b$ combine through an electroweak current, such as the Drell-Yan annihilation of quark with antiquark to a lepton pair or gluon fusion to a Higgs boson,

$$
\sigma_{ab}^{(eik)}(q) = \int d^4x \ e^{ix\cdot q} \langle 0 | W_{ab}(x) W_{ab}(-0) | 0 \rangle.
$$

(1)

The operators $W_{ab}$ are defined by

$$
W_{ab}(0) = \Phi^\dagger_{\beta'}(0) \Phi^\dagger_{\beta}(0),
$$

(2)

in terms of nonabelian phase operators for $a$ and $b$. $\Phi_{\beta}(0) = P \ e^{-i\zeta \int_0^\infty d\lambda \ A(\lambda \beta)}$, with lightlike velocities $\beta$ and $\beta'$, $\beta \cdot \beta' = 1$.

The eikonal cross sections reproduce the logarithms, as singular as $(Q/q_0) \alpha_s^2 \ln^{n-1}(q_0/Q)$ and $(q_0/q_T) \alpha_s^2 \ln^{n-1}(q_T/Q)$, that characterize the edges of the partonic phase space at which the energy of radiation, $q_0$, or its total transverse momentum, $q_T$, vanish. The resummation of these logarithms is most convenient in terms of transforms,

$$
\tilde{\sigma}_{ab}^{(eik)}(N, b) = \int d^4q \ e^{-Nq_0 - ib \cdot q} \sigma_{ab}^{(eik)}(q).
$$

(3)

In the transformed functions we find logarithms at each order up to $\alpha_s^2 \ln^{n-1} N$ and $\alpha_s^2 \ln^{n-2}(bQ)$, which exponentiate. The exponentiation of energy logarithms is known as threshold resummation ⁵⁶⁷, of transverse momentum logarithms as $k_T$ resummation ⁷⁸.

3 Exponentiation

Transforms of the eikonal cross section may be written in exponential form on the basis of algebraic considerations that have been known for a long time,

$$
\tilde{\sigma}_{ab}^{(eik)}(N, b) = \exp \left[ E_{ab}^{(eik)}(N, bQ, \epsilon) \right],
$$

(4)

where the exponent is an integral over functions $w_{ab}$, sometimes called "webs" ⁹, which are defined by a modified set of diagrammatic rules,

$$
E_{ab}^{(eik)} = 2 \int_0^Q d^4k \ \frac{d^{4-2\epsilon}k}{\Omega_{1-2\epsilon}} \times w_{ab} \left( k^2, k \cdot \beta \cdot \beta', \mu^2, \alpha_s(\mu^2), \epsilon \right) \times \left( e^{-N(k_0/Q)+ik \cdot b} - 1 \right).
$$

(5)

The variable $k$ in this expression may be thought of as the momentum contributed by the web to the final state. The webs factor from each other under the transforms, and indeed in any symmetric integral over phase space. ¹¹

Webs have a number of restrictive properties. At fixed $k$, they are invariant under rescalings of the velocities in the eikonal phases, which corresponds to boost invariance under the axis defined by the two. In addition, at any fixed order, the web function has only one overall collinear and IR divergence, from $k_T \to 0$ and $k_0 \to 0$, respectively: Finally, the web functions have no overall renormalization:

$$
\frac{d}{d\mu} w_{ab} \left( k^2, k \cdot \beta \cdot \beta', \mu^2, \alpha_s(\mu^2), \epsilon \right) = 0
$$

(6)

Using boost invariance in the large-$N$ limit, we find that the exponent takes the form

$$
E_{ab}^{(eik)} = 2 \int_0^Q d^4k_T \ \frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}} \times \int_0^{Q^2-k_T^2} dk^2 \ w_{ab} \left( k^2, k_T^2 + k^2 \right)
$$

(7)
2 Electroweak annihilation

These processes are characterized at lowest order by \( ab \to V \), with \( a,b \) partons and \( V \) an electroweak final state of mass \( Q \) and transverse momentum \( Q_T \). Because of their relative simplicity and phenomenological interest, these processes have been studied intensely for resummation purposes. Partonic threshold is at \( z \equiv Q^2/\hat{s} = 1 \) and \( Q_T = 0 \), and the singular functions to be resummed are plus-distributions in \( 1 - z \) and \( Q_T/Q \). Our method generalizes that of Ref. 5, and organizes these distributions in the cross section. This is done by refactorizing partonic cross sections into short-distance functions, \( \sigma_{ab}^{(H)} \), sensitive only to the hardest scale, \( Q \), and matrix elements of appropriate operators that absorb all dependence on \( (1 - z)Q \) and \( Q_T \), to leading power in \( 1 - z \).

\[
\frac{d\sigma_{ab}^{(H)}}{dQ^2(d^2Q_T)} = \frac{1}{S} \sigma_{ab}^{(H)}(Q^2, \alpha_s(Q^2))
\]

\[
\times \int d\omega d^2k_b R_{ab}(\omega, k_b, Q)
\]

\[
\times \int d\omega d^2k_a R_{a}(\omega_a, k_a, Q)
\]

\[
\times \int \delta(1 - Q^2/\hat{s} - (1 - \omega_a) - (1 - \omega_b) - w_s)
\]

\[
\times \delta^2(Q_T - k_a - k_b - k_s) + Y_f.
\]  

(1)

where

\[ R_{ff}(x, k, 2p_0) = \]

\[
\frac{1}{4\sqrt{2N_c}} \int \frac{d\lambda}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} e^{-i\lambda p_0 + i\lambda b \cdot k}
\]

\[
\times \langle f(\lambda^+, b, 0^-) \gamma^+ q_f(0) | f(p) \rangle
\]  

(2)

is a partonic quark density at fixed energy and transverse momentum, and \( U_{ab} \) a purely eikonal function, depending on parton velocities \( \beta \), and on the soft radiation's energy \( \omega, Q \). The \( x \)'s and \( k \)'s are defined by (2). The delta functions in the triple convolution of (1) relate the singular behavior of the various functions, and \( Y_f \) represents the matching to finite order. The all-order behavior of the above functions and the cross section can be analyzed through their eikonalized equivalents. The eikonal cross section can be expressed as an exponent, \( E_{ab} \), an integral over diagrammatically-defined functions referred to as “webs”, \( \omega_{ab} \).

\[
E_{ab} = 2 \int d^4x \frac{d^2Q_T}{Q_{1-2\epsilon}}
\]

\[
\times \omega_{ab} \left( k^2, \frac{k \cdot \beta}{\beta}, \frac{\beta'}{\beta}, \xi, \alpha_s(\mu^2), \epsilon \right)
\]

\[
\times e^{-N(k_a/Q) - ikb - 1}.
\]  

(3)

The integral is over the energy and transverse momentum contributed by each web to the final state. The \( k \)-dependence of \( \omega_{ab} \) follows from the invariance of the eikonal cross section under rescalings of \( \beta \) and \( \beta' \).

3 Single-particle inclusive

For definiteness we consider prompt photon production, but the method sketched below is more general 3. A similar refactorization as in (1) holds for single-particle inclusive cross sections at high \( p_T \). In particular, it contains the same \( R_{fff} \) functions. The arguments supporting this refactorization are somewhat more involved than for (1), but reveal that only initial state radiation contributes to \( Q_T \) (the transverse momentum of the 2 \( \to 2 \) parton cms frame) 7. In the inclusive cross section, final-state interactions require threshold resummation only. In contrast to electroweak annihilation, \( Q_T \) is an unobserved variable, akin to \( 1 - z \). Thus, the jointly resummed prompt photon \( p_T \) spectrum may be written as an integral over \( Q_T \) of a “profile function” 3

\[
P_{ij}(N, Q_T, Q) = \int d^3b e^{-ibQ_T e^{E_f, j, \gamma}}
\]  

(4)

where the exponential exhibits the joint resummation, as

\[
\frac{d^2\sigma_{AB \to \gamma}}{dp_T} \overset{(\text{resum})}{=} \]


References


