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PARTICLE PRESSURES IN FLUIDIZED BEDS

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TAB X

FINAL REPORT

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SUMMARY

This project studies the "particle pressure," which may be thought of as the force exerted by the particulate phase of a multiphase mixture, independently of that exerted by other phases. The project is divided into two parts, one concerning gas and the other liquid fluidized beds. Previous work on gas fluidized beds had suggested that the particle pressures are generated by bubbling action. Thus, for these gas fluidized bed studies, the particle pressure is measured around single bubbles generated in 2-D fluidized beds, using special probes developed especially for this purpose. Liquid beds are immune from bubbling and the particle pressures proved too small to measure directly. However, the major interest in particle pressures in liquid beds lies in their stabilizing effect that arises from the effective elasticity (the derivative of the particle pressure with respect to the void fraction), they impart to the bed. So rather than directly measure the particle pressure, we inferred the values of the elasticity from measurements of instability growth in liquid beds; the inference was made by first developing a "generic" stability model (one with all the normally modeled coefficients left undetermined) and then working backwards to determine the unknown coefficients, including the elasticity.

Particle Pressures about Bubbles in Two-Dimensional Gas-Fluidized Beds: Previous work has shown that bubbles are responsible for the generation of particle pressures in gas-fluidized beds. This phase of the research was aimed at trying to understand how single bubbles generate particle pressures. Initially, it was thought that the particle pressures resulted from the particle motion induced by the passage of bubbles. The results were quite surprising as they indicated that this is not the case. Instead, it appears that the bubbles absorb much of the interstitial gas from the surrounding particle bed leaving that material defluidized; the defluidized material is no longer supported by the gas-flow and must be supported over contacts with neighboring particles - i.e. they are supported by the particle pressure. In contrast, any pressures induced by particle motion are relatively insignificant. The behavior can be understood from studies of the gas flow patterns around bubbles. There is evidence that there is refluidization far below the bubble.

Note that this result requires a rethinking of how fluidized beds behave. In particular, there will be limited contact between the particles and the fluidizing gas in the defluidized regions around bubbles; yet, it is that contact that one would want to maximize in chemical reacting systems. Also, it has interesting implications about the way that multiphase systems are modeled; the particle pressures resulting from defluidization are not in anyway reflected in the particle motions, as one would expect from a particle phase motion equation, but, arise solely from the interaction terms.

Constitutive Properties of Liquid-Fluidized Beds: Particle forces, in particular a particle phase "elasticity," (i.e. the derivative of the particle pressure with respect to the void fraction) has often been called upon as possible mechanisms to account for the stability of fluidized beds. Yet the actual magnitudes of such forces are unknown. But rather than

directly measure the elasticity, this portion of the project reverses the process. First, a light attenuation technique is employed to measure the growth and propagation of instabilities within the bed. (Questions are often raised about how accurately such measurements can be calibrated, however, a relatively simple calibration technique was developed here that eliminated any uncertainty.) The measurements are employed in a generic stability analysis allowing us to back out values for all terms that normally must be modeled, including those involving particle forces. The model is "generic" in the sense that none of the modeled terms, i.e. the particle forces and the fluid-particle drag terms, are assumed. Instead they are determined as the result of this theoretical/experimental process. Preliminary results, for 1mm glassbeads, indicate that typically assumed particle-phase elasticities fall above the values obtained from this analysis. The measurements of the viscosities are of the same order as measurements in gas-fluidized beds. Also, comparing the measured values of the drag laws indicate that the Richardson-Zaki equation provides a good fit to the fluid-particle drag while the Ergun equation and the Foscolo-Gibilardo model do not.

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Egolfopoulous, F.N. & Campbell, C.S., On the structure and dynamics of unsteady, counterflowing, strained diffusion flames: diffusion limited frequency response, *J. Fluid Mech.*, **318** (1996), 1-29

Potapov, A.V. & Campbell, C.S., Computer simulation of impact induced particle breakage, *Powder Technology*, **81**, (1994) 207-216

Potapov, A.V. & Campbell, C.S., Computer simulation of hopper flows, *Physics of Fluids A*, to appear November, 1996

Potapov, A.V. & Campbell, C.S., The two mechanisms of particle breakage and the velocity effect, *Powder Technology*, Submitted

Potapov, A.V. & Campbell, C.S., Computer simulation of shear-induced particle attrition, *Powder Technology*, submitted

Potapov, A.V. & Campbell, C.S., Computer simulation of particle breakage, *Proc. Particle Technology Forum*, Denver, CO., August 17-19, 1994

Campbell, C.S. & Potapov, A.V., Computer simulation of particle fracture, *Second Particle Technology Forum*, San Diego, CA, July 15-18, 1996

Computer Simulation of Powder Flows, To appear in the second edition of the *Powder Technology Handbook*

Computer Simulation of Particle Breakage, To appear in the second edition of the *Powder Technology Handbook*

Campbell, C.S., & Rahman, K., An improved particle pressure transducer, *Measurement Science and Technology*, 3 (1992) 709-712

Rahman, K. & Campbell, C.S. Particle pressures generated around bubbles in gas-fluidized beds, in preparation.

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Two PhD have been completed. James C. Jin (1996) and Khurram Rahman, (expected 11/96)..

Particle Pressures in Fluidized Beds

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1.0 INTRODUCTION

The particle pressure may be thought of as the force per unit area exerted by the particulate phase of a multiphase mixture and, as such, reflects the total momentum transport that can be attributed to the motion of particles and their interactions. It has a direct analog in the kinetic theory of gases in which the pressure acting on a surface is visualized as a result of the impacts of molecules. The same picture can be applied to particle-fluid situations with the particles taking the place of molecules. The only difference between the two cases is that solid particles may, in addition to short-duration collisional impacts, transmit a force via long duration contacts. (E.g. the weight of a particle, or an assembly of particles, resting on a surface).

Multiphase flows have long been modeled as interpenetrating continua. In general this technique involves writing separate conservation equations for each phase. Typically, the equations for the fluid and solid phases look individually like usual single-phase equation with the addition of a coupling term, F , that describes the interphasial forces.

$$\rho_f \epsilon \left(\frac{\partial \mathbf{V}_f}{\partial t} + \mathbf{V}_f \cdot \nabla \mathbf{V}_f \right) = \nabla \cdot \boldsymbol{\tau}_f + \rho_f \epsilon \mathbf{g} + \mathbf{F} \quad (1.1)$$

$$\rho_p (1 - \epsilon) \left(\frac{\partial \mathbf{V}_p}{\partial t} + \mathbf{V}_p \cdot \nabla \mathbf{V}_p \right) = \nabla \cdot \boldsymbol{\tau}_p + \rho_p (1 - \epsilon) \mathbf{g} - \mathbf{F} \quad (1.2)$$

Here ϵ is the void fraction (the volume fraction for the fluid phase), $\mathbf{V}_f$ and $\mathbf{V}_p$ are the fluid and particle velocities, ρ_f and ρ_p are the fluid and particle densities, and $\boldsymbol{\tau}_f$ and $\boldsymbol{\tau}_p$ are the fluid and particle phase stress tensors. This work concerns itself with $\boldsymbol{\tau}_p$, which strictly should be interpreted as the stress tensor induced by the particle phase that also acts on the particle phase. One of the difficulties in accurately applying (1.1) and (1.2) is that there is very little understanding of how to model the particle phase stresses. This study both employs an experimental method for directly measuring the component of the particle stresses that is exerted normal to the walls of a fluidized bed, and, in a parallel study, infers from liquid bed measurements, the constitutive properties appropriate for these types of models.

The difficulty in measuring the particle pressure is that the total pressure exerted on a surface - the pressure that would be measured with a standard flush mounted pressure transducer - is the sum of the particle pressure and the pressure exerted by the fluid that resides in the interstices between the particles. Furthermore, in many cases in which the motion is driven by fluid pressure - for example, fluidized beds or slurry flows - the particle pressure may be a small fraction of the total. Conceptually, such a measurement is not complicated, nor is the measurement terribly difficult. Essentially all one has to do is measure the total force acting on a surface and then let that fraction due to the fluid pressure balance itself out. Campbell & Wang (1990) described a very simple transducer for this purpose. That probe consisted of a solid diaphragm that is flush mounted into the bed wall. Small holes on either side of the diaphragm admit fluid, but no particles into a

chamber behind the diaphragm. The face of the diaphragm experiences the total pressure exerted by both the particles and the fluid, while the rear experiences only fluid forces. Thus, the net deflection of the diaphragm reflects the contribution of the particle forces only. That probe has been used to make the particle pressure measurements on the vertical side walls of gas-fluidized beds that were presented in Campbell & Wang (1991) (although it could be used in many other flow situations).

Campbell & Wang (1991) showed that the particle pressures in gas-fluidized beds were generated largely by the passage of bubbles. In particular, they showed that the average particle pressure exerted on the side walls scaled with the average size of the bubble. This immediately brings to mind two questions: (1) what is it about bubbles that leads to particle pressure generation and (2) would there be measurable particle pressures in liquid-fluidized beds which, while unstable, do not bubble? This project is largely aimed at answering these two questions. To attack the first problem, we have built a two-dimensional gas-fluidized bed into which bubbles may be injected and the distribution of particle-pressure measured. For the latter, other experiments are being performed in liquid fluidized beds.

However, it soon became apparent that the particle pressures generated in liquid beds are extremely small and that direct measurements were all but impossible, even using transducers that were especially designed for the purpose. This has pointed that phase of the research into a new direction. The new approach arose from reflection on what ultimately was the utility of the current research. To a large extent, this phase of the research was motivated by interpenetrating continua multiphase-flow models such as that described by equations (1.1) and (1.2). The classic fluidization problem to which these models have been applied is to understand the stability of a uniform state of fluidization. The only way in which a fluidized bed has been shown to be stable is through a particle-phase elasticity - i.e. it is stabilized through the particle pressure. Furthermore, once instabilities develop, the particle pressures will have significant effect on the growth and propagation of voidage disturbances. Naturally, of course, the instabilities will also be influenced by the manner in which the fluid-particle drag and other terms are modeled. This, led to the development of a "generic" stability model, in which all modeled terms are left unspecified. From analyzing this model, we have developed an experimental plan that, by measuring the characteristics of voidage disturbances and comparing with the theory, we can back out appropriate values for the modeled terms. The results will, not only, yield insight into the particle pressure, but also of the fluid drag. The latter results is used to evaluate common models for these terms.

2.0 PARTICLE PRESSURES IN GAS-FLUIDIZED BEDS

2.1 A New Particle Pressure Transducer

The first achievement under this grant was to develop a new version of the particle pressure transducer. The transducer used by Campbell & Wang (1991) is shown in Figure 2.1.1a and used the deflection of a solid diaphragm to determine the pressure measurement. The displacement of the diaphragm is measured by a MTI Accumeasure capacitance probe with a 0-0.13mm range. Two screen covered holes were drilled at diametrically opposite points around the periphery to admit gas to the rear of diaphragm and balance the gas pressure. The holes were placed at the same vertical position in the channel and thus at the same gas pressure, eliminating any gas flow through the void behind the diaphragm. There are two difficulties with this design. The first is that it acts as a fluidic R-C circuit and thus behaves like a low pass filter to the gas that must flow to the rear of the diaphragm; the resistance is applied by the small size of the holes and the covering screen while the capacitance is a result of the large gas-filled cavity behind the diaphragm. The dynamic effects of these were examined experimentally in Campbell & Wang (1990). While the low-pass filter effects would not alter the time-averaged measurements given in Campbell & Wang (1991), they might alter instantaneous measurements. The second problem arises from uncertainties about the effect of lubrication forces acting between the particles and the diaphragm; i.e. could the lubrication forces slow the particle and bias the measurements for small particles?

Both these concerns can be avoided by the new probe design, shown in Figure 1b. Here the solid diaphragm is replaced by a stretched screen which is spotwelded onto a ring. At the same time, the volume behind the screen is made as small as possible. (Here the gap is 0.13mm which was determined by the range of the capacitance probe.) This is done for two reasons. First it minimizes the capacitance and secondly, it resists the tendency of the external gas to leave the bed and travel through the gap behind the screen. Using a screen for a diaphragm also has two advantages. First, the large area minimizes the resistance for the gas to flow across the diaphragm. Secondly, if large lubrication forces do come into play, they will leak through the openings and not slow the approach of the particle. Repeating the tests in Campbell & Wang (1990), Campbell & Rahman (1992) showed that the current probe has none of the dynamic characteristics of the old probe. The construction of this new probe was conceived long ago, but a suitable screen material was not available. Usual woven screens cannot hold tension when spotwelded. (We did try to attach a screen using but the probe responded in strange ways that attributed to the glue being more flexible than the screen.) The screen we used in the current probe is not woven but etched from a continuous metal sheet. It is manufactured by Buckbee-Mears Precision Etched Products Group (model #2-2-8). The holes are approximately 70 μ m in diameter and are arranged in a hexagonal pattern on a .05mm (.002in) thick sheet. This probe was developed during the first year of the project and resulted in Campbell & Rahman (1992).

Figure 2.1.1: Schematics of the old (a) and the new (b) probe designs.

2.2 Particle Pressure Measurements around Single Bubbles in a Two-Dimensional Gas-Fluidized Bed

The results of Campbell and Wang (1991) indicate that particle pressures, measured along the side wall of a gas-fluidized bed, are primarily generated by the passage of bubbles. The primary evidence lies in the observation that the average values of the particle pressure scales with the equivalent diameter of bubbles. However, it is clear that the pressure obviously cannot be uniform across the bed. In particular, the particle pressure must go to zero in the particle-free region inside the bubble. Furthermore, the side wall of the bed may be a peculiar region as the particles pushed aside by the passage of a bubble cannot cross the wall, and therefore, the walls must affect the bubble motion in their immediate neighborhood.

The easiest way to determine the distribution of the particle pressure around a bubble is to perform the experiment in a "two-dimensional" fluidized bed. This is a term used to describe beds that are extremely thin in one dimension. In such a situation, bubbles span the small breadth of the bed so that it is possible to make a measurement of the particle pressure across a bubble without actually inserting a probe in the bed. The position of bubbles may be localized by artificially injecting bubbles into a bed held near minimum fluidization.

2.3 Experimental Apparatus

Figure 2.3.1 is a schematic of the two-dimensional bed. The test section is 152cm high, 46cm wide, but only 2.5cm deep. It is fed by an air supply system that passes air through a ten inch packed bed and several layers of filter paper to cause a large pressure

Figure 2.3.1: Schematic of the two-dimensional fluidized bed.

drop and assure a uniform airflow. The air flow is usually set so that the bed is at minimum fluidization conditions. Then bubbles are injected through a porous plate covered port, located 9 inches above the distributor. The bubble injector consists of a series of plenums which are pressurized through a precision pressure regulator. Firing a solenoid valve admits pressurized air into the bed, causing a bubble to form. Different plenum pressures will discharge different quantities of gas allowing some control over the bubble size. Finally 25 ports are cut into the face of the channel to admit the particle pressure transducers. Sixteen of the ports are configured in 4 lines of 4, spaced at 6 inch intervals above the injection port; in each line, the ports are spaced 2 inches apart, spanning the area from the center of the bed to one wall. The other 9 ports (not shown) are located at intermediate locations across the bed, to allow a finer spatial resolution. Four particle pressure probes are available to be inserted in any of the ports.

Bubbles are tracked and followed by an image processing system. The image originates in an Image Technology Methods, Datavision 262, video camera. The image is sampled by a Data Translation 3851 frame grabber with 8MB of memory that is mounted inside an IBM clone computer with a 50MHz 80486 processor. The 3851 board possesses an external trigger which allows the acquisition of the images to be synchronized with the acquisition of the particle pressure data. (The data acquisition is performed in a separate 33MHz 486 IBM clone; digital outputs from that computer are used to trip a relay which injects the bubble and to fire the external trigger of the 3851 as well as to sample the particle pressure information. This tightly synchronizes the entire experimental process.) The 8MB of memory permit 25 frames to be acquired at a rapid rate and stored on the board.

Both the acquisition and interpretation of the images are controlled by Data Translation's Global-Lab Image[®] Software package. This package provides many powerful image processing options. The most useful for this phase of the project is the ability to detect and analyze "particles" (which in this context refers to bubbles). In particular, it locates the center-of-area of the bubble which we use as a reference for the bubble location, calculates the area of the bubble, the average radius from the center-of- area, and so on.

2.4 Particle Pressure Measurements

Figure 2.4.1 shows a time history of the particle pressures measured by the four probes which are located 46 cm (18 inches) above the injection point and 0, 5, 10 and 15 cm along the horizontal direction from the bed centerline (the corresponding plots are shown in order starting at the top of the figure). The test material in this case is 0.5 mm glassbeads and the bed height is 109 cm (43 inches) which places 41 cm (16 inches) of material above the probes. The figure is also labeled with the locations of where the top and bottom of the bubble crosses the probes, as well as the location where the bubble erupts from the surface of the bed. At the time it crosses the probes, the bubble has an equivalent radius of about 9 cm (see figure 2.4.2) and thus, the bulk of the bubble crosses the positions of the two innermost probes.

Figure 2.4.1: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. From top (a) to bottom (d), the probes are mounted 0, 5, 10, 15 cm from the bed centerline. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 25.0 cm (9.8 inches).

Figure 2.4.2: The effective bubble diameter as a function of time for the particle pressure measurements shown in Figure 2.4.1.

The bed is held at minimum fluidization conditions and the bubble is injected at time = 0.0 secs. (corresponding to the time axis in Figure 2.4.1) for a duration of 0.2 secs. Before the bubble is injected the particle pressure, as measured by all four probes, is approximately constant and has a relatively small value (less than 5 mm H₂O). The particle pressure along the bed centerline (Figure 2.4.1.a) is small above the bubble (time = 0.0 secs. to about 0.75 secs.); there is a short duration initial peak which corresponds to the sudden injection of gas, followed by a period (upto about time = 0.4 secs.) where the particle pressure value is roughly the same as in a undisturbed bed held at minimum fluidization conditions, but just upstream of the bubble (time = 0.4 secs. to 0.75 secs.) there is a small, though noticeable, particle pressure generation. Naturally, the particle pressure goes to zero as the bubble crosses the centerline probe (time = 0.75 secs. to 1.2 secs., Figure 2.4.1.a) as there are no particles present. Significant particle pressures are measured below the bubble (time = 0.75 secs. to bubble eruption at time = 1.8 secs.); there is an abrupt increase in particle pressure as the centerline probe encounters the bubble wake, and as the bubble continues its ascent the particle pressure steadily rises and reaches its maximum value as the bubble erupts from the bed. Figure 2.4.1.b, which shows the particle pressure measured 5 cm from the bed centerline, is almost identical to Figure 2.4.1.b. But, Figures 2.4.1.c and Figure 2.4.1.d which represent the particle pressure measured by the probes mounted 10 cm and 15 cm from the bed centerline respectively, show a somewhat different picture since these two outer probes are not crossed by the bubble; from time = 0.0 to 0.4 secs. the plots are almost the same as Figures 2.4.1.a and 2.4.1.b, but from time = 0.4 secs. to time = 0.8 secs. the particle pressure rises and is about constant though still small in magnitude; from time = 0.8 secs. (at this point the bubble has just reached the two inner probes) to bubble eruption at time = 1.8 secs. the particle pressure rises almost steadily to its maximum value at bubble eruption. All this shows that significant particle pressures are generated not just below the bubble (1.2 < time < 1.8 secs.) but also to the

sides of the bubble ($0.8 < \text{time} < 1.2$ secs.). Also, the particle pressure is continuous within the particle phase (Figures 2.4.1.c and 2.4.1.d) and only discontinuous at the bubble boundary (abrupt jump in particle pressure as seen in Figures 2.4.1.a and 2.4.1.b when the bubble wake is encountered by the two inner probes). And, the last noticeable feature is that after bubble eruption the particle pressure returns to its small minimum fluidization value for all four probe positions.

Several mechanisms have been suggested for the generation of particle pressure in a fluidized bed. Commonly, the particle pressure is attributed to the random, thermal-like, motion of particles (e.g., Batchelor, 1988) such as those measured by Kumar et al. (1990) in a liquid-fluidized bed (note that liquid-fluidized beds are bubble free and expand homogeneously). Without much supporting evidence, Campbell and Wang (1991) speculated that a similar mechanism might be responsible for the small particle pressure they in a three-dimensional gas-fluidized bed held at minimum fluidization. But they found that relatively large particle pressures were generated in freely bubbling beds whose magnitudes scaled with the bubble size. Campbell and Wang (1991) argued that the particle pressure was generated by the large scale movement of particles that were *en masse* disturbed by the passage of a bubble much like the flow pattern of a fluid about a rising gas bubble; they suggested that as the bubble rises, it pushes particles out of the way and these particles can transmit a force out to the walls that is sensed as particle pressure. In contrast, small scale local motion of particles around bubbles can be responsible for particle pressure. It is also possible that bubbles generate particle pressure by causing a disruption in the structure of the particle bed; this is not surprising, since the presence of gas bubbles in the particle phase will alter the gas flow through the particle phase.

Particle movement in gas-fluidized beds is not random, though it may appear so when the bed is bubbling vigorously. Rowe and Partridge (1962) have shown that a bubble causes a reproducible displacement of particles. They photographed a bubble is a two-dimensional column, with a camera moving with the bubble, and by choosing an appropriate exposure time were able to capture the pattern of particle motion around it. They observed that the solid flow pattern around a bubble and its wake was 'streamline' in nature, i.e. similar to a potential flow field around a solid cylinder. Also, the wake appeared to consist of a vortex pair with particle motion upwards near the vertical axis of the bubble and downwards at the outer limits of the wake. Now, the small particle pressures observed above the bubble may be, at least partially, due to particle motion induced by the rising bubble and this is applicable to all four plots shown in Figure 2.4.1 up to time = 0.8 secs. And, as the bubble crosses the two inner probes (Figures 2.4.1.a and 2.4.1.b) the particle pressure measured to the sides of the bubble by the two outer probes (Figures 2.4.1.c and 2.4.1.d, $0.8 < t < 1.2$ secs.) may also have some contribution from the particle motion. Also, the abrupt rise in particle pressure as the two inner probes (Figures 2.4.1.a and 2.4.1.b at time = 1.2 secs.) encounter the bubble wake may be due to the agitated vortical particle motion in the bubble wake. But, it is unlikely that particle motion is the major contributor to the almost steadily increasing particle pressure and its large magnitude below the bubble (all four plots in Figure 2.4.1, $1.2 < t < 1.8$ secs.). This is clear, since far below the bubble, there is almost no gross motion of the bed. Also since the particle motions are

Figure 2.4.3: Photograph of the gas flow around a 'slow' bubble rising in a two-dimensional fluidized bed, made visible by NO_2 injection. From Rowe (1971).

roughly symmetrical above and below the bubble, one would expect that the particle pressure they generate would also be roughly symmetrical, while the data shows that the particle pressures below the bubble are of much greater magnitude than those above. It is also unlikely that these large particle pressures are a byproduct of bubble eruption since they are observed much before the bubble erupts.

Now, the gas flow pattern around a bubble in a gas-fluidized bed is strongly dependent on the bubble rise velocity and the interstitial gas velocity. There are numerous theoretical models in the fluidization literature which describe the gas flow pattern around a single (Davidson (1961), Jackson (1963), Murray (1965), Collins (1965), Stewart (1968), Azevedo and Pereira (1991), Kuipers et al. (1992)). The simplest and first, though conceptually incorrect, Davidson's (1961) model is surprisingly accurate (Littman & Holmolka, 1973) and was the pioneering force which established the idea that the ratio of the bubble rise velocity, U_b , to the interstitial gas velocity, U_0 , is a parameter that defines two very different types of gas-flow patterns around bubbles in gas-fluidized beds. Now, bubbles are regions of high permeability in an otherwise low permeability medium and this will necessarily modify the gas flow so that it converges towards the bottom of the bubble, flows through it and diverges again as it flows through the roof and back into the particulate phase. For the results presented so far for the 0.5 mm glassbeads, U_b is less than U_0 and the bubble is called a 'slow' bubble with the gas, on the average, traveling through the bubble. Figure 2.4.3 shows a photograph of the gas flow around a 'slow' bubble rising in a two-dimensional gas-fluidized bed, made visible by NO_2 injection (from Rowe, 1971). There it can be seen that the passage of a bubble, attracts almost all of the gas within the bed. This can be anticipated, as the bubble represents a void through which the gas may travel with nearly zero pressure drop; in contrast, the pressure drop the gas would experience while passing through the bed would approximately equal the hydrostatic weight of the bed. Consequently, the bubble represents a very attractive alternative path and absorbs much gas from the particulate phase. This leaves surrounding areas in the bed

which are only partially fluidized (or defluidized). Figure 2.4.3 shows that the bubble attracts gas spanning the width of the bed, extending about two bubble diameters from the edge of the bubble; this would indicate that all the material on the sides of the bubble is defluidized. This figure also shows that the gas flow is disturbed (non-vertical) extending about one bubble diameter below the bubble and this indicates that material below the bubble is also defluidized. Also, although not readily apparent from Figure 2.4.3 due to the mixing of tracer gases, the majority of the gas must be leaving the top of the bubble, keeping that region fairly well fluidized.

It now appears that the major contributor to the large particle pressures, observed below and to the sides of the bubble, is the defluidization of the material in the particulate phase; this defluidization is a consequence of (i) the absorption of gas by the bubble and (ii) the deviated (non-vertical) gas flowlines caused by this gas attraction to the bubble. Once the material below the bubble is defluidized it is no longer supported by the gas flow and must simply rest on the material below it; this defluidization manifests itself in the large particle pressures observed below the bubble (all four plots in Figure 2.4.1). The steadily increasing particle pressure is related to the increasing weight of the defluidized material above the probes (note however that the maximum particle pressure is smaller than the hydrostatic value, indicating that the material is not totally defluidized but instead is partially fluidized or just defluidized). Obviously, this increasing weight is due to the fact that as the bubble moves further away from the probes, there is progressively more defluidized material above the probes. This may not be surprising as when the bubble comes closer to eruption its size rapidly increases (see Figure 2.4.2) and it is to be anticipated that a larger bubble will absorb more gas, cause more defluidization and hence generate larger particle pressures. This is in concert with the observations by Campbell and Wang (1991) which show that the particle pressures scale with the bubble size.

Now, Figure 2.4.3 indicates that the bubble deviates the gas flow over a region extending about one bubble diameter below the bubble. The results presented so far have been for a bubble which had an effective bubble diameter of 25.0 cm (spanning 55% of the bed width) just before eruption (Figure 2.4.2). Thus, at eruption the maximum height of material above the probes was about one bubble diameter in length and this spans the defluidized zone as shown in Figure 2.4.3. It is convenient to work with such large bubbles since small bubbles invariably go through a process of splitting into two or more pieces and reforming. But smaller bubbles, even though they may split, can yield much information about the domain in which the particle pressure is generated. Figures 2.4.4 and 2.4.5 show the time history of the particle pressures for two smaller bubble sizes (in each Figure, the top plot represents the bubble centerline, and subsequent plots represent positions 5, 10 and 15 cm from the bed centerline; this presentation structure shall be used for all such future Figures); for these two cases the effective bubble diameters just before eruption are 18.8 cm (spanning 41% of the bed width) and 16.1 cm (spanning 35% of the bed width) respectively. Undoubtedly, very similar particle pressure profiles are generated by the three bubble sizes; small particle pressures above the bubble and significant particle pressures to the sides and below the bubble. For the data from the two smaller bubble sizes (Figures 2.4.4 and 2.4.5) the height of material above the probes at eruption is about 1.6 and 2.1

Figure 2.4.4: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 18.8 cm (7.4 inches).

Figure 2.4.5: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 16.0 cm (6.3 inches).

bubble diameters in length. Thus, according to the picture shown in Figure 2.4.3, the probes are mounted below the span of the defluidized zone (i.e., in the zone where the gas-flow pattern has returned to normal). But the particle pressure measurements in Figures 2.4.4 and 2.4.5 still show a steadily increasing particle pressure below the bubble. It appears, as the bed is at minimum fluidization conditions, that the gas velocity generates just enough pressure drop to support the bed locally, but not enough to support the additional defluidized material being piled on from above. In other words, there is a zone of defluidized material that follows the bubble as it rises. All this accounts for (i) why the particle pressure increases until eruption, (ii) the pressures are never as large as would be expected if all the material above the probe were defluidized and (iii) why the pressures return quickly to normal after eruption. Another noteworthy characteristic of the results shown in Figures 2.4.1, 2.4.4 and 2.4.5 is that the magnitude of the maximum particle pressure decreases as the bubble size is decreased. This is not surprising, since a larger bubble will attract more gas and hence, cause more defluidization which results in a higher particle pressure.

In fact, a similar defluidization mechanism might explain the small particle pressures observed around minimum fluidization. Remember that minimum fluidization is defined as the minimum gas velocity by which the gross weight of the bed is just supported by the gross pressure difference across it. However, it is unlikely that the actual gas flow through the apparently quiescent bed is actually uniform. It is much more likely that the gas finds weak paths or channels which it preferably follows through the bed. This would leave areas that are locally defluidized within the bed. This explanation is more satisfying than the 'thermalized' particle speculation given by Campbell and Wang (1991) as no such motions are apparent within the bed.

A noticeable feature of the results presented so far is that even the smallest bubble (Figure 2.4.5) generates particle pressures that span the width of the bed. Looking carefully at Figure 2.4.3, this should not be surprising since the bubble is attracting gas from the bed almost all the way to the sidewalls. Consequently, a much larger two-dimensional bed with dimensions 234 cm in height, 117 cm in width and 1.28 cm deep was built to measure the particle pressure far to the side of the bubble; unfortunately, it was difficult to uniformly fluidize a bed with such large dimensions and a bubble, once injected, left large areas of the bed in a defluidized state. (This was evident because the particle pressures did not return to zero after the bubble had passed.) Apparently in such a large bed, there is enough room for favorable gas channels to form so that gas can avoid large sections of the bed. As a result, the data obtained from the large bed was inconsistent and unrepeatable, again though significant particle pressures were observed all the way up to the bed sidewalls, even for bubbles of the same sizes shown above. Thus it was felt that there was little additional information to be gained from going to such a large bed.

All the data presented so far have been for a bed height of 109 cm (43 inches) with the probes mounted 46 cm (18 inches) above the injector; this places 41 cm (16 inches) of material above the probes. Now, the longer the distance a bubble has to travel in the bed the more likely it is to split. And, since the most significant particle pressures are generated

Figure 2.4.6: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 18.8 cm (7.4 inches).

below the bubble, it is convenient to mount the probes at a lower position in the bed and use a shallower bed. Figures 2.4.6 shows the time histories of the particle pressure with the probes mounted 15.2 cm (6 inches) above the injector and a bed height of 68.6 cm (27 inches) which places 30.5 cm (12 inches) of material above the probes; as before, the top plot represents the probe mounted on the bed centerline and subsequent plots are for the probes mounted 5, 10 and 15 cm from the bed centerline. This figure shows the same features as before (e.g. Figures 2.4.1, 2.4.4 and 2.4.5), that is, above the bubble the particle pressure is relatively small, to the sides of the bubble it is significant and below the bubble the particle pressure steadily increases and reaches a maximum value at eruption; the effective bubble diameter just before eruption for this case is 23.1 cm (9.1 inches). (Note that the initial peak in particle pressure observed in Figure 2.4.6, at about time = 0.1 secs., is probably a consequence of bubble injection since the probes are mounted close, 15 cm, above the injector.) Very similar particle pressure profiles are observed if the bubble size is decreased, as can be seen in Figures 2.4.7 and 2.4.8 which represent bubble sizes of 13.5 cm (5.2 inches) and 9.8 cm (3.9 inches) respectively; the most obvious difference between Figures 2.4.6, 2.4.7 and 2.4.8 is that the maximum value of the particle pressure decreases as the bubble size is increased. When the bed height is increased to 83.8 cm (33 inches), placing 45.7 cm (18 inches) of material above the probes, the particle pressure profiles are again very similar to the data already presented; this can clearly be seen in Figures 2.4.9, 2.4.10 and 2.4.11 which represent bubble sizes (just before eruption) of 30.0 (11.8 inches), 16.2 cm (6.4 inches) and 13.9 cm (5.5 inches) respectively, and again the maximum magnitude of the particle pressure decreases with the bubble size.

Figure 2.4.7: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 13.3 cm (5.2 inches).

Figure 2.4.8: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 9.8 cm (3.9 inches).

Figure 2.4.9: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 30.0 cm (11.8 inches).

Figure 2.4.10: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 16.2 cm (6.4 inches).

Figure 2.4.11: The time history of the particle pressures in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 13.9 cm (5.5 inches).

The test material for all the data presented so far has been 0.5 mm glassbeads. The obvious next step is to use a material of different density. Figure 2.4.12 shows the time history of the particle pressure for a bed composed of 0.83 mm polystyrene beads (density 1050 kg/m^3 as opposed to a density of 2500 kg/m^3 for the glassbeads); the probes are mounted 15.2 cm (6 inches) above the injector and the bed depth is 83.8 cm (33 inches). The effective bubble diameter at eruption for this case is 25.7 cm (10.1 inches). The similarity of this particle pressure to the results presented in Figure 2.4.9 is evident; the particle pressure is relatively small above the bubble, it is significant below and to the sides of the bubble and increases steadily below the bubble to its maximum value at bubble eruption. (Note that the initial peak in particle pressure observed in Figure 2.4.12, at about time = 0.1 secs., is probably a consequence of bubble injection since the probes are mounted close, 15 cm, above the injector.) The noticeable difference in these results is that the magnitude of the particle pressure is considerably smaller as compared to the case of 0.5 mm glassbeads for almost the same bubble size (e.g. compare Figure 2.4.12 with 2.4.9). This is not surprising since a given volume of defluidized glassbeads will weigh more than the same volume of defluidized polystyrene beads (of course, assuming that the beads are packed in similar ways); hence, given that the density of glass is greater than that of polystyrene, the defluidization effect of glassbeads should generate higher particle pressures than that generated by polystyrene beads. But for the polystyrene beads, as was the case with the 0.5 mm glassbeads, the trend that the magnitude of the particle pressure decreases with bubble size is still valid; this is clear from Figures 2.4.13 and 2.4.14 which represent

Figure 2.4.12: The time history of the particle pressures in a bed of 0.83 mm polystyrene beads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 25.7 cm (10.1 inches).

Figure 2.4.13: The time history of the particle pressures in a bed of 0.83 mm polystyrene beads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 16.9 cm (6.7 inches).

Figure 2.4.14: The time history of the particle pressures in a bed of 0.83 mm polystyrene beads. The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 11.0 cm (4.3 inches).

bubbles with effective bubble diameters (just before eruption) of 16.9 cm (6.7 inches) and 11.0 (4.3 inches).

The solid materials used this far have generated 'slow' bubbles for which the bubble travels slower on the average than the interstitial gas. This is a condition associated with large heavy particles that require significant superficial gas velocities to fluidize. The other important case is that of 'fast' bubbles (see the gas flow pattern from Rowe, 1971, shown in Figure 2.4.15) which rise faster than the surrounding gas and are associated with small, light particles. These different behaviors occur because the rise velocity of a bubble in a fluidized bed, like the rise velocity of a bubble in a liquid, depends only on the bubble size and is independent of the particle size. Yet, the gas velocity required for minimum fluidization is strongly dependent on the particle size. Thus, for small particles it is possible that the bubbles rise velocity, U_b , is greater than the interstitial gas velocity, U_0 . This is 'fast' bubble behavior and requires that the bubble carry a quantity of gas with it as it rises; thus, a portion of the gas that travels through the bubble center recirculates through a 'cloud' that surrounds the bubble. Such a cloud is clearly seen in Figure 2.4.15 which shows a photograph of the gas flow around a fast bubble rising in a two-dimensional gas-fluidized bed, made visible by NO_2 injection. It is unfortunate that, since the bubble rises faster than the gas, a tracer gas will never catch up with the bubble and thus one cannot map out the surrounding gas flow pattern, such as that shown for a slow bubble in Figure 2.4.3; consequently, it is difficult to predict the degree of gas-flow deviation caused by a fast bubble. Figure 2.4.16 shows the time history of the particle pressure for 0.25 mm

Figure 2.4.15: Photograph of the gas flow around a 'fast' bubble rising in a two-dimensional fluidized bed, made visible by NO₂ injection. From Rowe (1971).

glassbeads (fast bubble) for a bed height of 109 cm (43 inches) with the probes mounted 45.7 cm (18 inches) above the injector. The effective bubble diameter just before eruption is 26.1 cm (10.3 inches). The plots are also marked with the locations when the bubble crosses the probes and when the bubble erupts. Below the bubble the particle pressure rises steadily and reaches a maximum value at bubble eruption; this is similar to the slow bubble case (see Figure 2.4.1). The major difference from slow bubble behavior is above the bubble where a significant particle pressure peak is observed (at about time = 0.5 secs. in Figure 2.4.16) and a smaller eruption peak. Note that bubble injection probably accounts for the initial two particle pressure peaks (up to about time = 0.2 secs.).

Now, Davidson's model (1961) can be used to estimate the cloud size around fast bubbles from the expression:

$$\frac{D_c}{D_b} = \sqrt{\frac{U_b + U_0}{U_b - U_0}}$$

Here, D_c is the cloud diameter, D_b is the bubble diameter, U_b is the bubble rise velocity and U_0 is the interstitial gas velocity. (It should be noted that any model for predicting cloud size is only reasonably accurate, and does not take into consideration real-life complications of changes in bubble shape, particles raining from the bubble roof and bubble splitting.) The bubble rise velocity, U_b , is the center of area velocity of the bubble estimated from the sequences of frames captured during the experiment. Figure 2.4.17 shows the effective bubble diameter and the cloud diameter as calculated by the above expression; the solid line represents the bubble diameter and the dashed line the cloud diameter. Note that the two lines are diverging indicating that the cloud is growing faster than the bubble, but that for most of the latter part of the growth period, the cloud has about 1.5 times the diameter of the bubble. The cloud diameter shows a steady increase in size from time = 0.7 secs. onwards which means that the bubble and cloud are carrying increasing amounts of gas; thus, there is less gas available in the particulate phase beyond

Figure 2.4.16: The time history of the particle pressures in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 26.1 cm (10.3 inches)

Figure 2.4.17: The bubble and 'cloud' diameters as a function of time for the fast bubble particle pressures shown in Figure 2.4.16. The solid line represents the effective bubble diameter and the dashed line the calculated 'cloud' diameter.

the cloud boundary and certain regions in the bed may be defluidized. Hence, the major effect of defluidization, in terms of particle pressure generation, will once again be felt below and to the sides of the bubble. But because, on the average, no gas passes through a fast bubble, it is also possible to defluidize material above the bubble which would account for the upstream particle pressure peak. Thus, the increase in cloud and bubble size appears to be responsible for the steadily increasing particle pressure below and to the side of the bubble (Figure 2.4.16). The particle pressure peak above the bubble (all four plots in Figure 2.4.16, at about time = 0.5 secs.) may be related to the peak observed in the cloud size in Figure 2.4.17 at about the same time, but it appears unlikely that an increase in the cloud size can be responsible for generating significant particle pressures above the bubble. Now, it is expected that the gas flowlines within and around the cloud will be distorted (i.e., not vertical) and, hence, this may cause some degree of defluidization; the noticeable particle pressure peak above the bubble (at time = 0.5 secs., Figure 2.4.16) could be due to the defluidized material within and to the sides of the cloud boundary. Of course, defluidization due to the distortion of gas flowlines will also play some part in the particle pressure generated below the bubble, but it is more likely that the lack of sufficient gas to fluidize the material in the particulate phase is the more important mechanism. All this suggests that even though slow and fast bubbles have very different gas flow patterns around them, they both seem to generate particle pressure in a very similar manner, that is, by defluidizing material in the particulate phase.

Now, bubbles in a bed of 0.25 mm glassbeads are very prone to splitting. This is especially true for small bubbles since they spend a longer time in the bed because of their smaller rise velocity. Thus, it is appropriate to use a shallower bed and place the probes in a lower position in the bed. This facilitates measurement of the particle pressure below the bubble, which is the region of most interest, for smaller bubble sizes. Figure 2.4.18 shows

Figure 2.4.18: The time history of the particle pressures in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 11.2 cm (4.4 inches).

Figure 2.4.19: The bubble and 'cloud' diameters as a function of time for the fast bubble particle pressures shown in Figure 2.4.18. The solid line represents the effective bubble diameter and the dashed line the calculated 'cloud' diameter.

the time history of the particle pressure for a bed height of 68.6 cm (27 inches) with the probes mounted 15.2 cm (6 inches) above the injector; this configuration places 30.5 cm (12 inches) of material above the probes. The effective bubble diameter of the bubble just before eruption is 11.2 cm (4.4 inches), that is, the bubble spans roughly 24 % of the width of the bed. The particle pressure profile below the bubble is as expected, that is, the particle pressure slowly increases and shows a maximum value at bubble eruption; this is related to the increasing cloud size as seen in Figure 2.4.19. Also, though the bubble is relatively small, its defluidization effect spans the width of the bed, thus, even the probe mounted 15 cm (6 inches) from the centerline measures a particle pressure which is of about the same magnitude as the other three probes. The Figure 2.4.18 also shows an initial particle pressure peak which is associated with the process of bubble injection, more prominent here since the injector is only 15 cm (6 inches) below the probes. At about time = 0.25 secs. (that is, the region above the bubble) another particle pressure peak is observed which is almost unnoticeable right above the bubble but becomes more pronounced and larger in magnitude to the sides of the bubble. This peak does not appear to be related to an increase in cloud size as can be seen from Figure 2.4.19. It is more likely that this peak is due to the defluidization caused by the distorted gas flowlines, in the particulate phase, above and to the sides of the bubble; it is possible that at time = 0.25 secs. there is still injected gas leakage from the bubble roof which is compensating for the defluidization effect right near the bubble top and, hence, this peak is negligible near the bed centerline and more pronounced near the bed sidewalls. An important point illustrated by Figures 2.4.18 and 2.4.16 is that the magnitude of the particle pressure generated below a fast bubble depends on the bubble size; this is similar to the slow bubble case.

All this suggests that the two important parameters responsible for the generation of particle pressure are the bubble size and the density of the solid material. This is not surprising in lieu of the results of Campbell and Wang (1991). Their results for a freely bubbling three-dimensional bed showed that the average particle pressure (see Figure 2.4.20) scaled with the equivalent bubble diameter, i.e. $P_p/\rho_p g D_e$ is approximately a constant (here P_p is the particle pressure, ρ_p is the density of the solid material, g is the gravitational acceleration and D_e is the effective bubble diameter). Figures 2.4.21, 2.4.22 and 2.4.23 show the instantaneous particle pressure scaled according to the above rule using the instantaneous value of the effective bubble diameter for three different bubble sizes; the test material is 0.5 mm glassbeads, the bed height is 109 cm (43 inches) and the probes are mounted 46 cm (18 inches) above the injector. The usual rule is followed, that is, the top plot represents the bed centerline and subsequent plots represent positions 5 cm, 10 cm and 15 cm from the bed centerline. The effective bubble diameters just before eruption are noted on the figure caption. The time axis is non-dimensionalized by scaling with the eruption time. The most important feature of these plots is that below the bubble the value of $P_p/\rho_p g D_e$ is approximately constant and this feature is observed across the span of the bed. Thus, the increasing particle pressure below the bubble as seen in Figures 2.4.1, 2.4.4 and 2.4.5 is a direct consequence of the increasing bubble size as the bubble travels upwards through the bed. This is not surprising since a larger bubble is a larger void which presents more surface area and attracts more gas from the bed, causing more material to be defluidized which is represented by an increased particle pressure.

Figure 2.4.20: Scaling of the time-averaged particle pressure with the particle density and the effective bubble diameter. Here, H_p refers to the height of the probe above the distributor. From Campell & Wang (1991).

Figure 4.2.21: Time traces of the scaled particle pressure, $P_p / \rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. From top to bottom, the probes are mounted 0, 5, 10, 15 cm from the bed centerline. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 25.0 cm (9.8 inches). (The unscaled particle pressure data is in Figure 4.2.1.)

Figure 2.4.22: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 18.8 cm (7.4 inches). (The unscaled particle pressure data is in Figure 2.4.4.)

Figure 2.4.23: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 16.0 cm (6.3 inches). (The unscaled particle pressure data is in Figure 2.4.5.)

Now, if the above is true then the value of $P_p/\rho_p g D_e$, below the bubble, should be constant and independent of the distance between the probes and the top surface of the bed. Figures 2.4.24 to 2.4.29 show the scaled particle pressure, for various bubble sizes with the probes mounted 15.2 cm (6 inches) above the injector and bed heights of 68.6 cm (27 inches) and 83.8 cm (33 inches) (the effective bubble diameters just before eruption are marked on the figure captions). These figures do indeed show that regardless of the amount of material above the probes the value of $P_p/\rho_p g D_e$ is approximately constant. The scaled particle pressure results presented so far have placed 30.5 cm, 40.6 cm and 45.7 cm of material above the probes but the value of $P_p/\rho_p g D_e$, below the bubble, is insensitive to the amount of material above the probes. This lends further support to the notion that the steadily increasing particle pressure below the bubble is a direct result of the increasing defluidized material which is generated by the growing bubble.

This is not surprising because if the expression $P_p/\rho_p g D_e$ is to have a constant value, below the bubble, a material of lower density will generate particle pressures of lower magnitude. Figures 2.4.30 to 2.4.32 show the scaled particle pressure for several different bubble sizes, but this time the test material is 0.83 mm polystyrene beads (the effective bubble diameters just before eruption are marked on the figure captions). The bubbles in a bed of 0.83 mm polystyrene beads are slow bubbles, just like those in a bed of 0.5 mm glassbeads but the density of polystyrene (1050 kg/m^3) is significantly lower than the density of glass (2500 kg/m^3). The figures do indeed show that the value of $P_p/\rho_p g D_e$ is approximately constant below the bubble and close to a value of 0.1, just as in the case of the 0.5 mm glassbeads (see Figures 2.4.20 to 2.4.28). These results indicate that the volume of material being defluidized by the bubble is the same for both the polystyrene and glassbeads, but the weight of the defluidized material is different because of the different densities of the two materials; this is why $P_p/\rho_p g D_e$ is a constant, below the bubble, for the two different materials.

Figure 2.4.33 shows the scaled particle pressure ($P_p/\rho_p g D_e$) for a fast bubble; the test material is 0.25 mm glassbeads (fast bubble), the bed height is 109 cm (43 inches) and the probes are mounted 46 cm (18 inches) above the injector (the effective bubble diameter just before eruption is 26.1 cm (10.3 inches)). It is most interesting to note that the value of $P_p/\rho_p g D_e$, below the bubble, is approximately constant at 0.1. Figure 2.4.34 shows the scaled particle pressure ($P_p/\rho_p g D_e$) for a smaller fast bubble (effective bubble diameter just before eruption of 11.2 cm (4.4 inches); the bed height in this case is 68.6 cm (33 inches) and the probes are mounted 15.2 cm (6 inches) above the injector. In the region below the bubble the scaled particle pressure is roughly constant and has a magnitude close to 0.1. Figures 2.4.35 to 2.4.37 show the scaled particle pressure with an additional 15.2 cm (6 inches) of material added to the bed for various bubble sizes, placing potentially more defluidized material above the probes; it is obvious that the value of $P_p/\rho_p g D_e$ is roughly constant at a value of 0.1 below the bubble. Thus, from Figures 2.4.33 to 2.4.37 it is seen that the defluidization below a fast bubble can be attributed to the changing bubble size and the degree of defluidization is independent of the bed height and the material above the probes. This similarity between slow and fast bubbles is remarkable since these two types

Figure 2.4.24: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 23.0 cm (9.1 inches). (The unscaled particle pressure data is in Figure 2.4.6.)

Figure 2.4.25: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 13.3 cm (5.2 inches). (The unscaled particle pressure data is in Figure 2.4.7.)

Figure 2.4.26: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 9.8 cm (3.9 inches). (The unscaled particle pressure data is in Figure 2.4.8.)

Figure 2.4.27: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 30.0 cm (11.8 inches). (The unscaled particle pressure data is in Figure 2.4.9.)

Figure 2.4.28: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 16.2 cm (6.4 inches). (The unscaled particle pressure data is in Figure 2.4.10.)

Figure 2.4.29: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.5 mm glassbeads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 13.9 cm (5.5 inches). (The unscaled particle pressure data is in Figure 2.4.11.)

Figure 2.4.30: Time traces of the scaled particle pressure, $P_p / \rho_p g D_e$, in a bed of 0.83 mm polystyrene beads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 25.7 cm (10.1 inches). (The unscaled particle pressure data is in Figure 2.4.12.)

Figure 2.4.31: Time traces of the scaled particle pressure, $P_p / \rho_p g D_e$, in a bed of 0.83 mm polystyrene beads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 16.9 cm (6.7 inches). (The unscaled particle pressure data is in Figure 2.4.13.)

Figure 2.4.32: Time traces of the scaled particle pressure, $P_p / \rho_p g D_e$, in a bed of 0.83 mm polystyrene beads (slow bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 11.0 cm (4.3 inches). (The unscaled particle pressure data is in Figure 2.4.14.)

Figure 2.4.33: Time traces of the scaled particle pressure, $P_p / \rho_p g D_e$, in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 26.1 cm (10.3 inches). (The unscaled particle pressure data is in Figure 2.4.16.)

Figure 2.4.34: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 68.6 cm (27 inches) and the effective bubble diameter just before eruption is 11.2 cm (4.4 inches). (The unscaled particle pressure data is in Figure 2.4.18.)

Figure 2.4.35: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 26.9 cm (10.6 inches).

Figure 2.4.36: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 15.8 cm (6.2 inches).

Figure 2.4.37: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, in a bed of 0.25 mm glassbeads (fast bubble). The four plots come from 4 probes mounted 15.2 cm (6 inches) above the injection point. The bed height is 83.8 cm (33 inches) and the effective bubble diameter just before eruption is 10.7 cm (4.2 inches).

of bubbles have a markedly different gas flow pattern around them. But this makes sense since roughly the same amount of gas should be absorbed by any bubble as it grows.

The two-phase theory of fluidization (Toomey and Johnstone (1952)) postulates that the excess gas flow above that required to fluidize a bed of particles passes through the bed as visible bubble flow. Though still widely used, there is ample evidence that the two-phase theory overestimates the visible bubble flow (e.g. Grace and Clift (1974)). There is debate over where this excess gas resides. Davidson and Harrison (1966) and Lockett et al. (1967) attribute this discrepancy to the throughflow velocity, i.e., the velocity of the gas relative to the bubble passing through the bubble. Pyle and Harrison (1967) and Rowe et al. (1978) suggest that an increase in the interstitial gas flow above that required for minimum fluidization explains the 'missing flow'. More recently, Yates et al (1994) have shown that fast bubbles are surrounded by a 'shell' of high voidage; this high voidage 'shell' may be the region where the 'invisible flow' resides. For, slow bubbles, Hailu et al. (1993) report that the throughflow velocity ranges from $0.8U_{mf}$ in the middle of the bed to $1.8U_{mf}$ close to the bed surface. The particle pressure measurements presented so far show that bubbles absorb much of the gas from the surrounding material causing at least local defluidization; this lends support to the importance of the throughflow velocity in explaining the deviation from the two-phase theory; but, the interstitial gas velocity may also play a part in this deviation. Now, all the results presented in this section have been for single bubbles in a bed held at minimum fluidization conditions; any natural bubbles present in the bed have been small (effective bubble diameter less than 5 cm) compared with the size of the artificially injected bubbles, and hence the effect of these natural bubbles is negligible on the particle pressure measurements. If the experiment is performed at a slightly higher fluidizing velocity and a large enough bubble is injected, the effect of the natural bubbles will still be negligible on the particle pressure measurements, but it may provide a clue to the importance of the interstitial gas flow in the 'missing flow' debate. Figures 2.4.38 and 2.4.39 depict the time history of particle pressure for beds held at 5% and 10% above minimum fluidization. Nothing extraordinarily unusual is observed in these figures and the same holds true for the scaled particle pressure ($P_p/\rho_p g D_c$) shown in Figures 2.4.40 and 2.4.41 for the same sets of measurements. The scaled particle pressure is roughly constant below the bubble and its value is about 0.1, and this is not different from the experiments performed at minimum fluidization conditions (see Figures 2.4.21 to 2.4.23). If increasing the gas velocity above minimum fluidization were to result in extra interstitial flow, this extra interstitial flow would have partially refluidized the material below the bubble resulting in lower particle pressures and a value of $P_p/\rho_p g D_c$ noticeably less than 0.1. Since, this is not the case, it does not appear that there is any extra interstitial gas available, at least far from the bubble. This lends credence to the observations of Yates et al. (1994), that the missing gas is confined to regions near the bubble.

Rathbone et al. (1989) measured transient normal and shear stresses on the surface of a tube in a freely bubbling two-dimensional gas-fluidized bed using a piezo-electric force transducer. A schematic of their stress transient shows a peak corresponding to the arrival of bubble wake particles, followed by the decaying of the stress to a small value (Figure 2.4.42). This scenario represents a different picture from the single bubble measurements

Figure 2.4.38: The time history of the particle pressures, for a fluidization velocity of 5% above minimum fluidization, in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 18.7 cm (7.4 inches).

Figure 2.4.39: The time history of the particle pressures, for a fluidization velocity of 10% above minimum fluidization, in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 19.0 cm (7.5 inches).

Figure 2.4.40: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, for a fluidization velocity of 5% above minimum fluidization, in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 18.7 cm (7.4 inches). (The unscaled particle pressure data is in figure 2.4.38.)

Figure 2.4.41: Time traces of the scaled particle pressure, $P_p/\rho_p g D_e$, for a fluidization velocity of 10% above minimum fluidization, in a bed of 0.5 mm glassbeads. The four plots come from 4 probes mounted 46 cm (18 inches) above the injection point. The bed height is 109 cm (43 inches) and the effective bubble diameter just before eruption is 19.0 cm (7.5 inches). (The unscaled particle pressure data is in figure 2.4.39.)

Figure 2.4.42: Schematic of force transient, on a tube surface in a freely bubbling fluidized bed, showing a peak corresponding to the arrival of bubble wake particles. From Rathbone et al. (1989).

performed so far. Figure 2.4.43 shows the time trace of the particle pressure in a freely bubbling bed of 0.5 mm glassbeads fluidized at 60% above minimum fluidization. Here Figure 2.4.34.a shows the time trace generated by several bubble passages while Figure 2.4.43.b shows an expanded view of the trace from a single bubble that passes directly in front of the centerline probe. The trace has entirely lost the particle pressure peak associated with eruption and, though the particle pressure is initially large after the bubble passage, it decreases steadily until another bubble passes nearby and dominates the signal. Note the similarity in shape between this trace and the schematic of the transient stress in Figure 2.4.42. Also note that though the bubble size at eruption for this trace is approximately the same as that of the single bubble traces in Figure 2.4.1, the magnitude of the maximum particle pressure is about two times smaller. This lower magnitude does not appear to be due to the larger fluidization velocity because the experiments performed at 5% and 10% above minimum fluidization velocity (see Figures 2.4.40 and 2.4.41) do not show a lower particle pressure magnitude. The picture of the gas flow pattern around a slow bubble (Figure 2.4.3) suggests that gas being absorbed by the bubble should exit from the bubble roof, hence, at least partially refluidizing any defluidized material being piled on by a leading bubble. Thus, it is very likely that the presence of other bubbles in a freely bubbling bed are the cause of the change in behavior of the particle pressure as seen in Figure 2.4.43.

Figure 2.4.44 is the time history of the particle pressure for two bubbles, the second bubble being injected 1.0 secs. after the first. The figure is also marked with the times when the bubbles cross the probes and the points at which the bubbles erupt from the bed. The particle pressure generated below the leading bubble is significantly different from the typical particle pressure profiles generated by single bubbles. The leading bubble defluidizes material to its side until the trailing bubble is injected at time = 1.0 secs. At this point the leading bubble is covering the two center probes. But once the leading bubble crosses the probes a somewhat flat particle pressure profile is generated until the leading bubble erupts at which point there is a marked reduction in the particle pressure value. This change in behavior is undoubtedly due to the presence of the trailing bubble. It appears that

Figure 2.4.43: Particle pressures generated in a freely bubbling bed of 0.5 mm glassbeads. The probes are mounted 46 cm (18 inches) above the injector, the bed height is 109 cm (43 inches), and the fluidization velocity is 60% above minimum fluidization. (a) A time trace showing several bubble passages. (b) A detail of the trace from a bubble that nearly passes up the center of the channel.

Figure 2.4.44: Particle pressures generated by two-bubble injection (the second bubble being injected 1.0 secs after the first) in a bed of 0.5 mm glassbeads, held at minimum fluidization. The probes are mounted 46 cm (18 inches) above the injector and the bed height is 109 cm (43 inches). The effective bubble diameters just before eruption for the leading and trailing bubbles are 31.2 cm (12.3 inches) and 23.4 cm (9.2 inches) respectively.

the high gas velocity exiting the top of the second bubble is partially refluidizing the defluidized material below the leading bubble. On the other hand, after the leading bubble erupts (time = 1.4 secs.) the behavior of the trailing bubble is the same as that of single bubbles; that is, it defluidizes material to its side and below it the particle pressure increases and reaches a maximum value at bubble eruption. Thus, this two-bubble injection situation simulates a freely bubbling bed and explains that the difference in particle pressure generation between a single and a freely bubbling bed is due to the presence of other bubbles.

All this raises the issue of the marked difference between single bubble experiments and measurements performed in a freely bubbling bed. Artificially injected single bubble experiments are very popular in the fluidization community (e.g. Littman and Homolka (1970), Yates et al. (1994), etc.) and they may yield a wealth of information about single bubbles. But care must be taken in comparing a single bubble in a fluidized bed and bubbles in a freely bubbling bed. As has been shown above the interaction of bubbles can result in significantly different behavior.

3 PARTICLE PRESSURES IN LIQUID FLUIDIZED BEDS

It is most likely that, given the small magnitudes of the particle pressures in a liquid bed, it will be nearly impossible to directly measure the small pressures that occur near the neutral stability point. This is unfortunate, as those are the particle pressures that affect the stability of the bed. It is well known that fluidized beds are prone to linear voidage instabilities such as those shown in Figure 3.0.1. It has been shown many times (e.g. Anderson & Jackson (1968), Garg & Pritchett (1975), Batchelor (1988)) that an elasticity of the particle phase can stabilize the bed. Jackson (1985) argues that the large value required of the elasticity could not be generated by particle fluctuations and stability must arise from some other mechanism. Here though, we came up with another idea to indirectly measure the particle elasticity in a manner that is particularly appropriate for inclusion in stability models.

The idea arose from a class project performed by Chengzhen Jin, one of the students supported by this grant. He performed a survey of the various stability analysis and (although not required for the assignment) derived one of his own that was a hybrid of the many models he studied. This analysis, like its predecessors, models the particle and fluid phases as separate, but coupled continua, (e.g. like equations 1.1 and 1.2,) makes assumptions about the forms of the constitutive behavior, the interfacial drag, particle elasticity, bulk viscosity, etc. and predicts the onset of instability, the initial growth rate and velocities of voidage disturbances. Of course, the results depend on the assumed forms of the constitutive behavior. We realized that this process could be reversed. I.e. one could create a generic stability analysis (i.e. one that make as few assumptions as possible). Then by comparing the theory with measured properties such as the wave growth and velocity, back out appropriate values for the modeled terms. As will be shown in the following, the model may be reduced to different wave equations in the near and far field and making measurements in those locations yield additional information. (The measurements of El-Kaissy & Homisy (1976) confirm different near and far-field behaviors.)

A similar idea was used by Ham et al. (1990). They measured the location of the onset of instability in a small fluidized bed and extracted the particle-phase elasticity from an analysis based on Batchelor's (1988) theory, although they still made assumptions about the nature of the drag law. By measuring

Figure 3.0.1: Voidage wave appearance in water-fluidized bed, 2mm glass beads, $\epsilon_0 = 0.496$

additional properties we can extend this idea to determine all other modeled properties of the stability analyses. Measurements of the drag and viscosity will be valuable in themselves as they allow the evaluation of various models that are in common use.

3.1 The Generic Stability Model

3.1.1 Governing equations for fluidized beds

As the continuum approach has been the backbone of fluidization theory, and there is already experimental evidences to support it's accuracy (Homsy, *et al.* 1980), continuum two-phase equations are used in this study. The model includes two continuity equations and two momentum equations. Without any modeling of force terms, they can be written in a manner similar to that of Jackson (1985):

Conservation of Mass for the fluid and particle phases:

$$\frac{\partial \varepsilon}{\partial t} + \text{div}(\varepsilon \mathbf{u}_f) = 0 \quad (3.1.1)$$

$$\frac{\partial (1 - \varepsilon)}{\partial t} + \text{div}[(1 - \varepsilon) \mathbf{u}_p] = 0 \quad (3.1.2)$$

Conservation of momentum:

$$\rho_f \varepsilon \left(\frac{\partial \mathbf{u}_f}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f \right) = \varepsilon \nabla \cdot \mathbf{T}_f - \mathbf{F}_I + \mathbf{F}_{bf} \quad (3.1.3)$$

$$\rho_p (1 - \varepsilon) \left(\frac{\partial \mathbf{u}_p}{\partial t} + \mathbf{u}_p \cdot \nabla \mathbf{u}_p \right) = (1 - \varepsilon) \nabla \cdot \mathbf{T}_f + \mathbf{F}_I + \mathbf{F}_{bp} + \nabla \cdot \mathbf{T}_p \quad (3.1.4)$$

where $\mathbf{T}_f$ is fluid stress tensor, $\mathbf{T}_p$ is particle stress tensor, $\mathbf{F}_I$ represents the interaction force between fluid-phase and particle-phase. $\mathbf{F}_{bf}$ is the body force acting on the fluid phase, and $\mathbf{F}_{bp}$ is the body force acting on the particle phase.

Regarding the particle stress tensor, we choose a general model for particle-particle interaction forces which include both elastic component and viscous component, as is commonly used in theories (Anderson and Jackson, 1968, Batchelor, 1988, among others). By assuming that there is no other body force except gravity, and by combining the two momentum equations to eliminate the fluid stress tensor, we can get following simplified equations for one

dimensional flow.

Conservation of Mass:

$$\frac{\partial \varepsilon}{\partial t} + u_f \frac{\partial \varepsilon}{\partial z} + \varepsilon \frac{\partial u_f}{\partial z} = 0 \quad (3.1.5)$$

$$-\frac{\partial \varepsilon}{\partial t} - u_p \frac{\partial \varepsilon}{\partial z} + (1 - \varepsilon) \frac{\partial u_p}{\partial z} = 0 \quad (3.1.6)$$

Combined conservation of momentum:

$$\begin{aligned} (1 - \varepsilon) \varepsilon \rho_p \left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} \right) - (1 - \varepsilon) \varepsilon \rho_f \left(\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} \right) \\ = F_I - [\varepsilon(1 - \varepsilon)(\rho_p - \rho_f)g] + \varepsilon E_p \frac{\partial \varepsilon}{\partial z} + \varepsilon \eta_p \frac{\partial^2 u_p}{\partial z^2} \end{aligned} \quad (3.1.7)$$

where the z direction is chosen to be opposite to the direction of gravity. E_p represents the particle phase effective elasticity, η_p represents the particle phase effective viscosity, and F_I is the interacting force in the z -direction.

3.1.2 Stability analysis of homogeneous fluidization

Let ε_0 , u_{f0} , u_{p0} , F_{I0} be the void fraction, velocities and the interaction force that correspond to a state of uniform fluidization, and u_0 be the superficial fluid velocity. Under those conditions: $\varepsilon_0 = \text{const}$, $u_{f0} = u_0/\varepsilon_0$, $u_{p0} = 0$. Let the flow then be disturbed by ε' , u'_f , u'_p , F'_I , i.e:

$$\begin{aligned} \varepsilon &= \varepsilon_0 + \varepsilon' \\ u_f &= u_{f0} + u'_f \\ u_p &= u_{p0} + u'_p \\ F_I &= F_{I0} + F'_I \end{aligned}$$

Applying to equation (3.1.5) ~ (3.1.7), we get:

$$\frac{\partial \varepsilon'}{\partial t} + u_{f0} \frac{\partial \varepsilon'}{\partial z} + \varepsilon_0 \frac{\partial u'_f}{\partial z} = 0 \quad (3.1.8)$$

$$-\frac{\partial \varepsilon'}{\partial t} - u_{p0} \frac{\partial \varepsilon'}{\partial z} + (1 - \varepsilon_0) \frac{\partial u'_p}{\partial z} = 0 \quad (3.1.9)$$

$$\begin{aligned}
& (1 - \varepsilon_0)\varepsilon_0\rho_p\left(\frac{\partial u'_p}{\partial t} + u_{p0}\frac{\partial u'_p}{\partial z}\right) - (1 - \varepsilon_0)\varepsilon_0\rho_f\left(\frac{\partial u'_f}{\partial t} + u_{f0}\frac{\partial u'_f}{\partial z}\right) \\
& = F_{I0} + F'_I - [\varepsilon_0(1 - \varepsilon_0)(\rho_p - \rho_f)g] - (1 - 2\varepsilon_0)(\rho_p - \rho_f)g\varepsilon' \\
& \quad + \varepsilon_0 E_{p0}\frac{\partial \varepsilon'}{\partial z} + \varepsilon\eta_{p0}\frac{\partial^2 u'_p}{\partial z^2} \quad (3.1.10)
\end{aligned}$$

The interaction force consists of the drag and added mass forces between fluid phase and particle phase:

$$F_I = F_d(\varepsilon, u_f - u_p) + \theta(\varepsilon)\rho_f \frac{d_p}{dt}(u_f - u_p) \quad (3.1.11)$$

and

$$F'_I = \alpha \frac{F_{d0}}{u_{f0}}(u'_f - u'_p) - \beta \frac{F_{d0}}{\varepsilon_0}\varepsilon' + \theta_0\rho_f \frac{d}{dt}(u'_f - u'_p) \quad (3.1.12)$$

Here F_d is the drag force and the second term in equation (3.1.11) represents the added mass effect. In equation (3.1.12), $\alpha = \frac{u_{f0}}{F_{d0}} \frac{\partial F_d}{\partial (u_f - u_p)}$ is the drag slope vs slip velocity, $\beta = -\frac{\varepsilon_0}{F_{d0}} \frac{\partial F_d}{\partial \varepsilon}$ is the drag slope vs voidage and θ_0 is the added mass coefficient. Here $\frac{d_p}{dt}$ is material time derivative of the particle phase:

$$\frac{d_p}{dt} = \frac{\partial}{\partial t} + u_p \frac{\partial}{\partial x}$$

There are two other alternatives proposed by Homsy (1980) and Anderson & Jackson(1968). The difference arises because the velocity of the convective term can be any one of: $u_p, u_f, u_f - u_p$. As this term represents the effect of particle acceleration on the total force of the particle phase, the particle phase velocity should be most appropriate for the convective term, so the particle phase material time derivative stated above is adopted in this study.

Now under uniform state conditions:

$$\begin{aligned}
& u_{p0} = 0 \\
& F_{I0} = \varepsilon_0(1 - \varepsilon_0)(\rho_p - \rho_f)g
\end{aligned}$$

and eliminating u'_f and u'_p , we get:

$$A \frac{\partial^2 \varepsilon'}{\partial t^2} + B \frac{\partial^2 \varepsilon'}{\partial z \partial t} + C \frac{\partial^2 \varepsilon'}{\partial z^2} + F \frac{\partial \varepsilon'}{\partial t} + G \frac{\partial \varepsilon'}{\partial z} = E \frac{\partial^3 \varepsilon'}{\partial z^2 \partial t} \quad (3.1.13)$$

where:

$$\begin{aligned}
A &= \rho_p \varepsilon_0 + \rho_f (1 - \varepsilon_0) + \frac{\rho_f}{\varepsilon_0(1-\varepsilon_0)} \theta_0 \\
B &= 2\rho_f (1 - \varepsilon_0) u_{f0} + 2\rho_f \frac{u_{f0}}{\varepsilon_0} \theta_0 \\
C &= \rho_f (1 - \varepsilon_0) u_{f0}^2 - \varepsilon_0 E_{p0} \\
F &= \alpha \frac{F_{d0}}{u_{f0} \varepsilon_0 (1 - \varepsilon_0)} \\
G &= \alpha \frac{F_{d0}}{\varepsilon_0} + \beta \frac{F_{d0}}{\varepsilon_0} + (1 - 2\varepsilon_0)(\rho_p - \rho_f)g \\
F_i &= \frac{\varepsilon_0 \eta_{p0}}{1 - \varepsilon_0}
\end{aligned}$$

Equation (3.1.13) can be used to study the stability of uniformly fluidized beds. Please note here that all of the coefficients in equation (3.1.13) are determined by five parameters: drag-slope (α, β), effective elasticity (E_{p0}), effective viscosity (η_{p0}) and added mass coefficient (θ_0).

3.2 The Inverse Instability Problem

The inverse instability problem that will be described in the following is used to identify undetermined parameters involved in the stability analysis and is the main objective of this study. Based on the equation (3.1.13), we know that there are five unknown parameters: $\alpha, \beta, E_{p0}, \eta_{p0}$ and θ_0 . Up to this time, there has been no systematic effort made to measure these parameters.

Based on early experimental studies on the instability waves (Anderson & Jackson, 1968, El-Kaissy & Homsy, 1976), it is clear that the waves are growing in spatial domain rather than in the temporal domain. Although these spatial and temporal growth rates are related simply by the wavespeed of the instability waves, it will be much more convenient and accurate to directly use the spatially growing wave formula. So in this section and afterwards we will adopt spatially growing small disturbance formula: $\varepsilon' = e^{\sigma z + i(\omega t - kz)} = e^{i\omega t} e^{z(\sigma - ki)}$. Applying this to equation (3.1.13), we get:

$$\begin{aligned}
&A(-\omega^2 + B(k\omega + a\omega i) + C(a^2 - k^2 - 2aki) + F(i\omega) + G(a - ik) = \\
&E(i\omega(a^2 - k^2) + 2a\omega k)
\end{aligned}$$

which has a real part:

$$A(-\omega^2) + Bk\omega + C(a^2 - k^2) + Ga = E2a\omega k \quad (3.2.1)$$

and an imaginary part:

$$Baw - C2ak + F\omega - Gk = E\omega(a^2 - k^2) \quad (3.2.2)$$

Here again, as in equation (3.1.13), all of the coefficients (A, B, C, F, G, E) are function of five unknown parameters ($\alpha, \beta, E_{p0}, \eta_{p0}$ and θ_0):

Rewriting (3.2.1) and (3.2.2) based on $\alpha, \beta, E_{p0}, \eta_{p0}$ and θ_0 (the five unknowns), we get:

Real part:

$$\alpha_1 \alpha + \beta_1 \beta + E_1 E_{p0} + \eta_1 \eta_{p0} + \theta_1 \theta_0 = R_1 \quad (3.2.3)$$

Imaginary part:

$$\alpha_2 \alpha + \beta_2 \beta + E_2 E_{p0} + \eta_2 \eta_{p0} + \theta_2 \theta_0 = R_2 \quad (3.2.4)$$

where

$$\begin{aligned} \alpha_1 &= \frac{F_{d0}}{\varepsilon_0} \sigma; \alpha_2 = \frac{F_{d0}}{u_{f0}} \left(\frac{\omega}{\varepsilon_0(1-\varepsilon_0)} - \frac{u_{f0}}{\varepsilon_0} k \right); \beta_1 = \frac{F_{d0}}{\varepsilon_0} \sigma; \beta_2 = -\frac{F_{d0}}{\varepsilon_0} k \\ E_1 &= -\varepsilon_0(\sigma^2 - k^2); E_2 = 2\varepsilon_0 \sigma k; \eta_1 = -\frac{2\varepsilon_0 \sigma \omega k}{1-\varepsilon_0}; \eta_2 = -\frac{\varepsilon_0 \omega(\sigma^2 - k^2)}{1-\varepsilon_0} \\ \theta_1 &= 2\rho_f \frac{u_{f0}}{\varepsilon_0} k \omega - \frac{\rho_f}{\varepsilon_0(1-\varepsilon_0)} \omega^2; \theta_2 = 2\rho_f \frac{u_{f0}}{\varepsilon_0} \sigma \omega \\ R_1 &= (\rho_p \varepsilon_0 + \rho_f(1-\varepsilon_0)) \omega^2 - 2\rho_f(1-\varepsilon_0) u_{f0} k \omega - \rho_f(1-\varepsilon_0) u_{f0}^2 (\sigma^2 - k^2) \\ &\quad - (1-2\varepsilon_0)(\rho_p - \rho_f) g \sigma \\ R_2 &= -2\rho_f(1-\varepsilon_0) u_{f0} \sigma \omega + 2\rho_f(1-\varepsilon_0) u_{f0}^2 \sigma k + (1-2\varepsilon_0)(\rho_p - \rho_f) g k \end{aligned}$$

Here, it can be seen that all of the coefficients in equation (3.2.3) and (3.2.4) are functions of the instability wave properties (ω, σ, U). Furthermore, every instability wave property will provide two equations: (3.2.3) and (3.2.4). To solve for the five unknowns ($\alpha, \beta, E_{p0}, \eta_{p0}$ and θ_0), we need at least three instability wave properties to get five equations for the five unknowns. In general, if we can experimentally measure N ($N \geq 3$) sets of instability wave properties: $[\omega_i, \sigma_i, U_i], i = 1, \dots, N$, then it is possible to form $2N$ linear systems of equations to solve for these five unknowns by applying least square error technique. I.e:

$$M = \begin{bmatrix} \alpha_{1i} & \beta_{1i} & E_1 & \eta_{1i} & \theta_{1i} \\ \alpha_{2i} & \beta_{2i} & E_2 & \eta_{2i} & \theta_{2i} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{1N} & \beta_{1N} & E_{1N} & \eta_{1N} & \theta_{1N} \\ \alpha_{2N} & \beta_{2N} & E_{2N} & \eta_{2N} & \theta_{2N} \end{bmatrix} \quad R = \begin{bmatrix} R_{1i} \\ R_{2i} \\ \vdots \\ R_{1N} \\ R_{2N} \end{bmatrix} \quad X = \begin{bmatrix} \alpha \\ \beta \\ E_{p0} \\ \eta_{p0} \\ \theta_0 \end{bmatrix}$$

Then :

$$X = M^+ R \quad (3.2.5)$$

where M^+ is the pseudoinverse of matrix M , and the X obtained here is the least square error solution.

To test this procedure, five values for the unknown parameters are assumed that are physically meaningful in the sense that they result in wave properties close to those experimentally observed. More detailed discussion about variation range of the unknown parameters will be given in next section. One set of the assumed values that gives the reasonable wave properties is:

$\alpha = 1.45$; $\beta = 2.5$; $E_{p0} = 40.0 \text{ N/m}^2$; $\eta_{p0} = 1.2 \text{ Ns/m}^2$; $\theta_0 = 5.0$
with running conditions (from one of the actual measurements) as following:

particle diameter	$d_p = 1\text{mm}$
particle density	$\rho_p = 2523\text{kg/m}^3$
mean voidage	$\epsilon_0 = 0.508$
superficial velocity	$u_0 = 1.929\text{cm/s}$
minimum fluidization velocity	$u_{mf} = 1.07\text{cm/s}$
terminal velocity	$u_t = 13.24\text{cm/s}$

Table 3.2.1: Running conditions

Figure 3.2.1 and figure 3.2.2 show the corresponding growthrates, σ , and wavespeeds, U , as functions of the wave number k . The three points on the figures are randomly chosen conditions about which to test the accuracy of the calculation of the inverse instability problem. Table 3.2.2 shows the chosen values of the wave properties.

f, Hz	$k, 1/\text{cm}$	$\sigma, 1/\text{cm}$	$U, \text{cm/s}$
1.1294152205	1.5	0.0838122850	4.7308834130
1.7312796525	2.5	0.1067996517	4.3511803501
1.9940385886	3.0	0.0559942858	4.1763046539

Table 3.2.2: Wave properties for assumed parameters

From the information in Table 3.2.2, we can form six linear equations to solve for the six unknown constitutive properties. If the inversion procedure is accurate, these will yield the parameters that we initially assumed. Figure 3 shows the results of the solution of the equations by a straight forward matrix inversion. In this plot, the x axis represent number of significant digits to which the wave properties are known. The dashed lines represent the true (assumed) values, and the solid lines represent the solutions by equation (3.2.5). The two lines never converge, even if the properties are determined to 16 significant digits. (From the experiments, it will be lucky to determine the properties to two significant digits.) Based on these results, it is clear that the procedure cannot correctly identify any of the parameters. The reason can be easily

Figure 3.2.1: The amplitude growthrate distribution for assumed parameters

Figure 3.2.2: The wavespeed distribution for assumed parameters

understood from the condition number of the matrix, $C = \|M\| \cdot \|M^{-1}\|$, which is in the order of 10^5 . (In some other cases, the condition number is even higher in the range of $10^6 \sim 9$). This means that these linear systems of equations are ill-conditioned and virtually unsolvable by any normal mathematical procedure.

Figure 3.2.3: The solution from linear equations

There are some techniques available to deal with ill-conditioned problems (Rust and Burrus, 1972), but most are valid only for symmetric matrices and unfortunately, this system of equations does not generate a symmetric matrix. An alternative dynamic programming technique has been provided by Bellman *et al.* (1965). But during the investigation, it was found that the solution by dynamic programming significantly depends on the initial values (estimation) which is needed for the iteration procedure and the selective coefficients on the process, resulting in an error that is still too large to be acceptable.

3.2.1 Optimum parameter identification method (OPIM)

To overcome this problem, it is first necessary to understand underlying reason for the ill-conditioned behavior. An examination of the above matrix

indicates that the problem arises because the measured wave properties, especially wavespeeds, do not vary greatly and all lie in a narrow range. As a result, the row vectors composed of coefficients from equation (3.2.3) or (3.2.4) are nearly the same leading to an ill-conditioned system.

Because there is no reliable direct mathematical method available, we must employ physical intuition to guide us to the solution. Clearly, we can reduce the condition number by reducing the rank of the matrix and keeping only two row vectors made of coefficients from two different equations (3.2.3 and 3.2.4) in the matrix. This reduces the number of linear equations to two, which will yield at most two for the parameter. The other three parameters are determined by a best fit technique. To do this, the three other three parameters are varied through all physically acceptable values to determine the best fit. In the following, those three parameters will be referred to as fitting parameters.

To apply this best fit technique, we must select the three fitting parameters. As the technique involves searching the parameter space to determine the best fit, it is best to choose parameters whose values are physically bounded, thus limiting the search range. From the empirically supported observation of Wallis, 1969, α is not strongly dependent on ϵ , and thus can be calculated from the drag-slope for an isolated sphere. If we follow Batchelor (1988), then the value α is given by the formula (see also Ham *et al.* 1990):

$$\alpha = 1 + \frac{0.63\sqrt{Re}}{4.90 + 0.63\sqrt{Re}} \quad (3.2.6)$$

where Re is defined as superficial Reynolds number: $Re = \frac{u_0 d_p \rho_f}{\mu_f}$. From equation (3.2.6), it is clear that

$$\lim_{Re \rightarrow 0} \alpha = 1$$

and

$$\lim_{Re \rightarrow \infty} \alpha = 2$$

But we can limit the value further. In a fluidized bed, the maximum Reynolds number to be reached will be terminal Reynolds number $Ret = \frac{u_t d_p \rho_f}{\mu_f}$ (u_t is terminal velocity), and minimum Reynolds number will be the one corresponding to the minimum fluidization velocity, i.e: $Remf = \frac{u_{mf} d_p \rho_f}{\mu_f}$ (u_{mf} is the minimum fluidization velocity), so the maximum value for α will be:

$$\alpha_{max} = 1 + \frac{0.63\sqrt{Ret}}{4.90 + 0.63\sqrt{Ret}}$$

and the minimum value will be:

$$\alpha_{min} = 1 + \frac{0.63\sqrt{Remf}}{4.90 + 0.63\sqrt{Remf}}$$

These two limiting values are reliable because all the Reynolds numbers in which the measurements are taken fall well within the range: $[Remf, Ret]$. Setting the bounding values α_{min} and α_{max} will not only reduce computational time, but also eliminate non-physical solutions to the numerical procedure by OPIM.

Parameter β can be examined based on existing drag models such as the Ergun model and is expected to be in the range from 0 to 5 based both on all models available and wide range of numerical testing. There is not much information on the added mass coefficient, some studies (Stkinson & Kytomaa, 1992) believe that added mass coefficient is between 0 and 0.5, but other studies such as Homay *et al.*, (1980) showed wider range on the order of 10 or so. In an attempt to cover all possibilities, the added mass coefficient was assumed to lie anywhere in the range from 0 to 20. There is also very little information on particulate phase elasticity. The only model available is given by Foscolo-Gibilardo (1987). Also Homay *et al.* (1980) have estimated the value to be of the order of 10. The particulate phase effective viscosity is believed to be around several Ns/m^2 from the earlier measurements by Schugerl *et al.* (1961).

As stated above, we have reliable knowledge on drag slope α and β , so they are the first choices for the fitting parameters. But the only thing we know about other parameters is that they should be all nonnegative. So there is nothing to give a special preference to remaining three parameters. Arbitrarily, we decided to use the added mass coefficient θ_0 as the final fitting parameter knowing only that its expected value is between 0 and 20.

Now assume that we have N measured wave properties: $[\omega_{0i}, \sigma_{0i}, U_{0i}]$, $i = 1, \dots, N$, then an error can be defined as:

$$\Phi = \sum_{i=1}^N \left(\frac{|\sigma_i - \sigma_{0i}|}{|\sigma_{0i}|} \right) + \sum_{i=1}^N \left(\frac{|U_i - U_{0i}|}{|U_{0i}|} \right) \quad (3.2.7)$$

$$\text{for } f_i = f_{0i}$$

where σ_i and U_i are growth rates and wavespeeds, respectively, determined from the given system parameters at the same frequencies as the measured ones (the frequency is exactly known from the measurements).

we define subspace of X , M and R as:

$$X_0 = \begin{bmatrix} \alpha \\ \beta \\ \theta_0 \end{bmatrix} \quad X_s = \begin{bmatrix} E_{p0} \\ \eta_{p0} \end{bmatrix} \quad M_s = \begin{bmatrix} E_{s11} & \eta_{s11} \\ E_{s21} & \eta_{s21} \end{bmatrix} \quad R_s = \begin{bmatrix} R_{s11} \\ R_{s21} \end{bmatrix}$$

where

$$\begin{aligned} R_{s11} &= R_{11} - \alpha\alpha_{11} - \beta\beta_{11} - \Theta_0\theta_{11} \\ R_{s21} &= R_{21} - \alpha\alpha_{21} - \beta\beta_{21} - \Theta_0\theta_{21} \end{aligned}$$

The linear system of equations then reduces to:

$$M_s X_s = R_s \quad (3.2.8)$$

Here the X_0 is the vector composed of fitting parameters, and will be varied (assumed) in the given range discussed above to best fit the measured wave properties. X_s is the vector composed of remaining parameters, and will be determined by equation (3.2.8).

The optimum parameter identification procedure can be summarized as following:

$$\begin{aligned} &\forall \{\omega_{0i}, \sigma_{0i}, U_{0i}\} \text{ and } X_0, \quad \exists X_s = M_s^{-1} R_s \\ &\text{so that } X = X_s \cup X_0. \\ &\forall X = [\alpha \ \beta \ E_{p0} \ \eta_{p0} \ \theta_0]^T, \\ &A, B, C, F, G, E \text{ in Eq. (3.2.1) and (3.2.2) can be determined,} \\ &\text{so for } \omega_i = \omega_{0i}, i = 1, 2, \dots, N, \\ &[\sigma_i, U_i] \text{ can be solved from Eq. (3.2.1) and (3.2.2).} \end{aligned}$$

From which

$$\Phi(X) = \sum_{i=1}^N \left(\frac{|\sigma_i - \sigma_{0i}|}{|\sigma_{0i}|} \right) + \sum_{i=1}^N \left(\frac{|U_i - U_{0i}|}{|U_{0i}|} \right)$$

can be calculated. The optimum solution for the system is:

$$X_{opt} : \Phi(X_{opt}) = \text{minimum}[\Phi(X)] \text{ for all } X$$

Table 3.2.3 present the pseudocode for this optimum parameter identification procedure.

3.2.2 Numerical testing and examples

To test the optimum parameter identification procedure described in the previous section, we again assume the same set of parameters as the ones to illustrate the ill-conditioned nature of the problem ($\alpha = 1.45$; $\beta = 2.5$; $E_{p0} =$

```

 $\Phi_{min} = 100000;$ 
for  $\alpha_n = \alpha_{min}, \alpha_{max}, \Delta\alpha$ 
  for  $\beta_n = \beta_{min}, \beta_{max}, \Delta\beta$ 
    for  $\theta_{0n} = 0, \theta_{max}, \Delta\theta_0$ 
      solve for  $E_{p0n}$  and  $\eta_{p0n}$  from Eq 3.2.8
      based on these  $X = [\alpha_n, \beta_n, \theta_{0n}, E_{p0n}, \eta_{p0n}]^T$ 
      for every  $\omega_i, i = 1, 2, \dots, N$ , solve for  $[\sigma_i, U_i]$  from Eq 3.2.1 and 3.2.2.
      calculate:  $\Phi(X) = \sum_{i=1}^N (\frac{|\sigma_i - \sigma_{0i}|}{|\sigma_{0i}|}) + \sum_{i=1}^N (\frac{|U_i - U_{0i}|}{|U_{0i}|})$ 
      if  $\Phi(X) < \Phi_{min}$  then
         $\Phi_{min} = \Phi(X)$ 
         $\alpha_{opt} = \alpha_n$ 
         $\beta_{opt} = \beta_n$ 
         $\theta_{opt} = \theta_{0n}$ 
         $E_{opt} = E_{p0n}$ 
         $\eta_{opt} = \eta_{p0n}$ 
      endif
    end for
  end for
end for

```

OPTIMUM SOLUTION: -

```

 $\alpha = \alpha_{opt}$ 
 $\beta = \beta_{opt}$ 
 $\theta_0 = \theta_{opt}$ 
 $E_{p0} = E_{opt}$ 
 $\eta_{p0} = \eta_{opt}$ 

```

Table 3.2.3: Pesudocode for optimum parameter identification procedure

40; $\eta_{p0} = 1.2$; $\theta_0 = 5.0$). The corresponding running conditions and the wave properties were given in the Table 3.2.1 and Table 3.2.2 respectively. As the OPIM is basically, a fitting technique, the larger the number of wave observations, the more accurate the results. But on the other hand the range of variable parameters, particularly the frequency of waves that can be forces, is so limited that it is impractical to use more than a few observations for each test condition. Thus, different numbers of wave properties $[\omega, \sigma, U]$ were tested to determine the minimum number of measurements needed to clearly and uniquely identify the system parameters $[\alpha, \beta, E_{p0}, \eta_{p0}$ and $\theta_0]$. We followed this procedure, first assuming that the wave properties were known exactly and later incorporating a degree of error in the measurements to see how the error affects the final results.

3.2.3 Wave properties with no error

To use equation (3.2.8) requires at least one complete wave observation $[\omega_1, \sigma_1, U_1]$ plus additional information that may take the form of addition growthrate, frequency or wavespeed information. Three different cases, i.e. three different choices for the information, were tested to determine the minimum requirements for system identification when there are no significant errors incurred from the measurements.

Table 3.2.4 shows the wave observations used in these three different cases. Note that only one complete wave observation is used in case I, while two complete wave observations are used in case II. In case III, only one wavespeed is used in the procedure, but two additional growthrate measurements are added to compensate for the shortage in wavespeed observations. (Case III will be a most likely scenario to follow; as the wavespeed changes little, which in turn led to the ill-conditioned nature of the inversion matrix, it makes little sense to use more than one wavespeed measurement.) For all of the cases studied, the ranges for $[\alpha, \beta, \theta_0]$ are varied through $\alpha_{min} \leq \alpha \leq \alpha_{max}$; $0.0 \leq \beta \leq 5$; $0.0 \leq \theta_0 \leq 20$ using increments: $\Delta\alpha = 0.001$; $\Delta\beta = 0.01$; $\Delta\theta_0 = 0.01$. These allow for both reasonable computational time and acceptable accuracy. The results that correspond to the minimum value of Φ are given in Table 3.2.4.

Figures 3.2.4 - 3.2.6 show the error Φ distribution for the three different cases. Note that Φ indicates the error in three independent variables (α, β, θ) , but the figures are practically limited to a 3-D frame which permits Φ to be plotted against only two of the three variables. Thus, several projections will be used. In Figures 3.2.5, ??, the error Φ is projected in (α, θ) domain by selecting the minimum error for all β :

$$\Phi^p(\alpha, \theta) = \text{minimum}[\Phi(\alpha, \beta, \theta), \text{ for all } \beta]$$

case number	observations			results by OPIM				
	f, Hz	$\sigma, 1/cm$	$U, cm/s$	α	β	E_{po}	η_{po}	θ_{po}
I	1.129	0.0838	4.73	many	many	many	many	many
II	1.129	0.0838	4.73	1.327	2.29	36.58	1.10	4.53
	1.731	0.1068	4.35					
III	1.129	0.0838	4.73	1.462	2.52	40.03	1.215	5.02
	1.731	0.1068	—					
	1.994	0.0560	—					
True values				1.45	2.5	40.0	1.20	5.0

Table 3.2.4: Examples for optimum parameter identification method (OPIM) with no observation errors

As well as projections into the various planes which better demonstrate the optimum values for these parameters.

To clearly show the distribution, the figure is plotted with a negative sign ($-\Phi^p$) which means that the optimum solutions appears as a peak and the vertical range is adjusted for each of the different cases. Figure 3.2.4 shows Φ^p for case I, in which only a single wave observation is used. Here the distribution is flat indicating that there are many solutions that fit that single observation in the physically acceptable parameter ranges. This means that only one complete wave observation is not sufficient to identify the system parameters. Figure 3.2.5 shows the function Φ^p distribution for case II (two complete wave observations). From the figure, we can see one peak (minimum error); this means that one clear solution can be determined from the given wave observations. Figure 3.2.6 shows the function Φ^p distribution for case III (one complete observation plus two additional growthrates). Based on this result, it is clear that even with only one wavespeed, if combined with additional growthrate information, we can still uniquely determine the parameters by OPIM.

All of the results obtained from case II and case III are summarized in Table 3.2.4, and those results are compared with the corresponding true values. Based on these examples, Case II gives higher error than Case III on the solutions. Thus Case III (one wavespeed and two growthrates) appears to be the minimum requirements if there is no significant error involved in the measurements. Of course, these are minimum requirements and any additional information (growthrates and/or wavespeeds corresponding to different frequencies) will improve both the accuracy and reliability of the solutions.

Figure 3.2.4: The error distribution for case I. As there are no noticeable peaks, there are no optimum parameter values that can be determined from the single wave observation used here

Figure 3.2.5: The error distribution for case II. Here enough information is present to make parameter determinations. The optimum values for α and θ are apparent at the maxima in the two projections shown at the bottom of the page

Figure 3.2.6: The error distribution for case III. Again enough information is present to make parameter determinations. The optimum values for α and θ are apparent at the maxima in the two projections shown at the bottom of the page

3.2.4 Wave measurements with error

Practically, all of the measured wave properties will contain some errors in their measurements. To test the response of the procedure to the error in the wave measurements, we added different amounts of error to the wave properties, all of which are close to what might be expected in actual measurements. (The error ranges were determined by the scatter in the actual data.)

case #	observations			results by OPIM				
	f, Hz	$\sigma, 1/cm$	$U, cm/s$	α	β	E_{p0}	η_{p0}	θ_{p0}
II	1.129	$0.0838 \pm 10\%$	$4.73 \pm 2\%$	1.306~	2.21~	21.47~	0.61~	2.97~
	1.731	$0.1068 \pm 10\%$	$4.35 \pm 2\%$	1.4600	2.63000	84.80	1.73	9.68
III	1.129	$0.0838 \pm 10\%$	$4.73 \pm 2\%$	1.436~	2.42~	25.95~	1.067~	3.43~
	1.731	$0.1068 \pm 10\%$	—	1.500	2.670	53.94	1.53	6.75
	1.994	$0.0560 \pm 10\%$	—					
True values				1.45	2.5	40.0	1.20	5.0

Table 3.2.5: Examples from the optimum parameter identification method (OPIM) with observation errors

Table 3.2.5 shows the wave observations and the corresponding error in the observations. No error has been applied to the frequency because the measured frequency is exactly equal to the forced frequency when the waves are forced in fluidized bed. The optimum parameter identification procedure can be performed after the errors have been added to the original data. If we have N_U measurements on the wavespeed and N_σ measurements on the growthrate, then after adding the errors, we will have $3^{N_U} 3^{N_\sigma}$ different combinations to get the solutions by OPIM. The computational work by OPIM will also be increased by $3^{N_U} 3^{N_\sigma}$ times. To reduce the computational time, the step sizes are increased to $\Delta\alpha = 0.01$; $\Delta\beta = 0.01$; $\Delta\theta_0 = 0.01$ for this procedure. The results in Table 3.2.5 show the range of the solutions obtained after all of the different combinations are processed.

As is apparent, these errors in the measurements can lead to very large errors (up to 400

3.3 Measurement of Forced Waves

There are only few efforts have been made to measure the instability wave properties (wave frequency, wavespeed, wave amplitude). Anderson and Jackson (1969), El-Kaissy and Homsy (1976) used light attenuation techniques and measured the dominant frequency distribution, total amplitude and wavespeed distribution in naturally fluidized beds. The limitation on these measurements

is that the dominant frequency measured varies in very short distances and the frequencies other than dominant frequency still present significant energy in the signal. As a result, the corresponding amplitude and wavespeed measurements can not be correlated to one frequency.

To accomplish the objective of measuring the single frequency wave information, we decided to develop a technique to force the waves in liquid fluidized beds and take the measurements on the forced waves.

3.3.1 Concentration measurement and integral calibration method

Concentration measurements are made in the cylindrical fluidized bed shown in Figure 3.3.1. The distributor is constructed of a porous plate composed of $20\mu\text{m}$ particles. This ensures a huge pressure drop and thus, a uniform fluidization velocity. The instantaneous concentration is determined using a light attenuation technique similar to ones used by Anderson & Jackson (1969) and El-Kaissy & Homsy (1976). A nearly constant intensity light is provided by two small ALS 4mW diode lasers. Each signal is sensed by a Melles-Griot silicon photodiode that is placed at the end of a 6 mm diameter and 20 mm long cylindrical channel that has been painted black to eliminate noise from scattered light. The 35.56 mm spacing between the two measurement points was chosen by trial and error to eliminate any cross talk between the channels. The attenuation of each beam provides an instantaneous measurement of the particle concentration and cross-correlating the signals provides the time delay used to determine the wavespeed.

The difficulty with all light attenuation systems is the calibration for concentration. In particular, the problem is to create a system with a known particle concentration upon which such a calibration might be based. We have developed a technique that nearly eliminates this uncertainty by basing the calibration, not on the local concentration which is somewhat uncertain, but on the total volume of particles in the bed which is easily determined from the weight of particles divided by their density.

Like all attenuation systems, the light intensity decreases exponentially as it crosses the bed. That is, the intensity I_x , at a distance x within the bed, is related to the initial intensity I_0 , by:

$$I_x = I_0 e^{-(\lambda_a + \lambda_s)\nu x} \quad (3.3.1)$$

where λ_a and λ_s are the appropriate, but unknown constants for the absorption and scattering of the light by particles respectively, and ν is the solid concentration. (Note, as we use clear water for fluidizing medium, the attenuation by the water is insignificant. Also, as the photodiodes are placed at the end

Figure 3.3.1: A schematic of experimental apparatus

of a blackened tube, little scattered light does not reach the diode detectors and is lost just as if it were absorbed by the particles.) This then gives a key to the solid concentration, ν , i.e:

$$\nu = \frac{\ln I_0}{(\lambda_a + \lambda_s)D} - \frac{\ln I_D}{(\lambda_a + \lambda_s)D} \quad (3.3.2)$$

where I_D is the light intensity at the measurement point and D is the inside diameter of the tube. Thus, as the signal s obtained from the photodiode should be proportional to the intensity I_D , it should correlate to the solid concentration as:

$$\nu = A \ln s + B \quad (3.3.3)$$

A preliminary calibration (which assumed that the average concentration in the bed equaled the concentration at the center of the expanded bed) confirmed this form for the calibration. The next step in the calibration is to make measurements all along the depth H of the bed and integrate (3.3.3) along the depth. Thus if V_p is the total volume of particles in the bed, then the average solid concentration is:

$$\bar{\nu} = \frac{V_p}{\frac{\pi D^2 H}{4}} \quad (3.3.4)$$

from integration of (3.3.3):

$$\bar{\nu} = \frac{1}{H} \int_0^H \nu dy = A \frac{1}{H} \int_0^H \langle \ln s \rangle dy + B = A \bar{\ln s} + B \quad (3.3.5)$$

where

$$\bar{\ln s} = \frac{1}{H} \int_0^H \langle \ln s \rangle dy$$

is the space-average of $\langle \ln s \rangle$ along the height of the bed, and $\langle \ln s \rangle$ is the time-average of the logarithm of the signal derived from the photodiode at each position y . Since V_p is known, if we have more than two measurements for different bed height H (different mean solid concentration), then we are able to determine the coefficients A and B by least square techniques.

Forced waves of given frequencies are generated by fluctuating the superficial velocity either by varying the pump speed or by a piston located in the inlet of the fluidized beds (see Figure 3.3.1).

Table 3.3.1 and Table 3.3.2 shows the glass beads and configuration of fluidized beds used in the experiments. To compensate for the greater attenuation for the smaller beads, and allow the use of the same light source and photodiode detectors, a smaller diameter bed was used for 0.5mm glass beads.

Set No.	Glass beads	$\rho_p, Kg/m^3$	d_p, mm	dominant d_p, mm
A	Potters B	2500	0.5 ~ 0.6	0.5
B	Potters A100	2523	1.0 ~ 1.18	1
C	Potters A200	2485.7	1.4 ~ 2	2

Table 3.3.1: Glass properties used in the experiments

Set No.	dominant d_p, mm	bed diameter D, mm	ratio: D/d_p	$u_{mf}, cm/s$	ϵ_{mf}
A	0.5	12.7	25.4	0.52	0.389
B	1	25.4	25.4	1.07	0.402
C	2	25.4	12.7	2.65	0.402

Table 3.3.2: Fluidized bed configuration used in the experiments

3.3.2 Forced waves in fluidized bed

The concern in forcing waves is that the natural behavior of fluidized beds might be artificially destroyed and the resulting wave properties would be irrelevant to the instabilities of naturally fluidized beds. (Note, theoretically this should not be a problem as long as the waves are in the linear growth regime.) To assure that the forced waves are representative of natural instabilities, we compared their growthrates to the growthrates of various waves with the same frequencies in a naturally fluidized bed. The growthrates in a naturally fluidized bed are found by tracking the amplitude corresponding to the specified frequency along the fluidized bed (i.e. the signal from the naturally fluidized beds was sent through a band pass filter of width 0.2 Hz about specified frequency). Figure 3.3.2 shows the results of the comparison. Note that the slopes of the natural and forced waves are identical within the initial linear growth regime.

Based on these results, it is clear that the linear growthrates of the forced waves are the same as those of natural waves in fluidized beds. This implies that the behavior of the waves are not altered by forcing. In addition, it clearly shows that the linear growth near the distributor is not dependent on the strength of the initial perturbations. On the other hand, we can see that the non-linear region appears to be effected by the forcing.

Typical signals obtained from forced waves are shown in Figure 3.3.3. The corresponding power spectra are shown in Figure 3.3.4, and the cross-

Figure 3.3.2: Comparison of amplitude growth between natural waves and forced waves

correlation signals are shown in Figure 3.3.5 which are needed to get the time delay between two transducers and determine the wave velocity. From the power spectra measurements, it can be seen that, in addition to the peak frequency signal near the distributor, there appears to be the secondary frequency signal corresponding to the frequency far away from the distributor.

We tried to force waves of as many frequencies as possible, but found that it was not possible to force waves of arbitrary frequency. Only waves with frequencies close to the dominant frequency that occurs in naturally fluidized bed could be forced. Waves of significantly different frequencies are difficult or impossible to generate by small perturbations of the superficial velocity. For particles of 1mm and 2mm glass beads, we choose to force one wave close to the highest dominant frequency (the dominant frequency near the distributor) in naturally fluidized beds, and another one close to the lowest dominant frequency (the dominant frequency far away from the distributor). For 0.5 mm glass beads, we found that it is impossible to force waves for frequencies higher than dominant lowest frequency found in naturally fluidized beds, so only waves of a single frequency were forced. To satisfy the condition that the perturbations be small, the fluctuations of superficial velocity is limited within 2 – 5% of the mean velocity.

The amplitude, wavespeed and wave frequency are plotted as a function of height above the distributor in Figure 3.3.6 - 3.3.13. Clearly, the measured wave frequencies for all intents and purposes are identical to the forced frequencies. The amplitudes of the waves are well reproducible if same initial perturbations are forced, and the error in the amplitude is between 1 – 3%. When the strengths of the initial perturbations are forced, the amplitude is different, but the growthrate could be reproduced to within 10%. Notice also that a nearly constant wavespeed is measured near the distributor, and the variations of the wavespeed in these regions are between 1 – 2%.

For all of the experiments, forcing the waves at a constant frequency results in linear growth of the amplitude and nearly constant wavespeed in the region near the distributor. These wave properties can be applied to the generic linearized instability analysis.

3.3.3 Wave properties for set A

Figure 3.3.6 - Figure 3.3.8 shows the amplitude, velocity and frequency distributions of forced waves along the height of the fluidized beds for set A (0.5mm glass beads). Five voidages ranging from 0.530 to 0.507 are covered. These are flow ranges where clear and coherent linear waves can be forced. Beyond these regions, it was found that the fluidized bed does not respond to the forced

Figure 3.3.3: Representative voidage signals of forced waves from a bed of 1mm glassbeads at a void fraction, $\epsilon = 0.526$. (a) 7.2 cm above the distributor, (b) 65cm above the distributor.

Figure 3.3.4: Corresponding power spectra. (a) 7.2 cm above the distributor, (b) 65cm above the distributor.

Figure 3.3.5: Corresponding cross-correlation output, $\epsilon = 0.526$. (a) 7.2 cm above the distributor, (b) 65 cm above the distributor.

waves very clearly.

Linear amplitude growth can be observed up to 20 ~ 30 cm above the distributor and are used to find the linear growth rate corresponding to the forcing frequencies. The frequencies are stable up to near 40 cm, and then small variations are observed for all of the flow rates. The wavespeed is nearly constant up to a height of 15 cm for the smallest voidage and up to 40 cm for the largest voidage. After those regions, the wavespeeds decrease slightly with the height. Please note the consistency between frequency variation and wavespeed variation, i. e.: wavespeed decreases with increasing frequency. For all of the voidages, it was found to be easy to force 1 Hz waves. Also note that the wavespeed increases monotonically with increasing voidage.

3.3.4 Wave properties for set B

The amplitude, velocity and frequency distributions of forced waves for set B (1 mm glass beads) are shown in Figure 3.3.9 - 3.3.11 for five voidages ranging from 0.485 to 0.526. Again, this is the voidage range where the clear and coherent forced waves can be detected.

For the smallest voidage ($\epsilon_0 = 0.485$), the amplitudes grow linearly up to 50 cm for 0.74 Hz, and to 20 cm for 1 Hz frequency. Wavespeeds are stable up to nearly 20 cm, and then increase slightly with height. The frequencies

Figure 3.3.6: Wave properties measured from set A for $\epsilon_0 = 0.530$ and $\epsilon_0 = 0.536$

Figure 3.3.7: Wave properties measured from set A for $\epsilon_0 = 0.541$ and $\epsilon_0 = 0.546$

Figure 3.3.8: Wave properties measured from set A for $\epsilon_0 = 0.507$

Figure 3.3.9: Wave properties measured from set B for $\epsilon_0 = 0.485$ and $\epsilon_0 = 0.495$

Figure 3.3.10: Wave properties measured from set B for $\epsilon_0 = 0.508$ and $\epsilon_0 = 0.516$

Figure 3.3.11: Wave properties measured from set B for $\epsilon_0 = 0.526$

are relatively stable throughout the fluidized bed. For $\varepsilon_0 = 0.495$ similar behavior can be seen for the wavespeeds. But for the amplitudes, it was found that linear growth regions are reduced to the first 40cm for 0.8Hz, and to below 15cm for the 1.4Hz frequency; in addition, the 1.4 Hz frequency is only stable up to 10 cm, and then the frequency generally decreases with height. For $\varepsilon_0 = 0.508, \varepsilon_0 = 0.516, \varepsilon_0 = 0.526$, the extent of linear growth regions are further reduced with increasing voidage. In addition, it is found that the size of the linear growth regions decreases with increasing frequency at any given voidage. For the forced frequencies, the lower frequency remained the dominant frequency nearly throughout the fluidized bed, but the higher frequency remained dominant only to about 10cm above the distributor. The wavespeeds for these three flowrates are constant up to 10 to 15cm, and then slightly increase with height except for $\varepsilon_0 = 0.516$ where the wavespeed decreases slightly.

These results show that the higher the frequency that is forced, the less the impact on the regions far away from the distributor in the sense that the conditions far from the distributor are more or less the same (as in an unforced bed). In other words, the higher frequency waves only have significant impact on the flow near the distributor.

3.3.5 Wave properties for set C

These 2mm beads are the largest beads tested in these experiments. For these beads, the signals from the detectors are different from the signals derived from other two sets in that these signals contained large random peaks. These arose because the large glass beads acted like convex lenses and focused the light as bright spots onto the photodiodes. This high frequency noise is eliminated by utilizing a moving 0.08s average filter which is long enough to eliminate the noise without altering the measuring signal (around 1Hz).

The amplitude, velocity and frequency distributions of forced waves along the height of the fluidized beds are shown in Figure 3.3.12 - 3.3.13 for four voidages ranging from 0.471 to 0.507. Again, these are the voidage regions where the clear and coherent forced waves can be detected.

The results show similar behavior to those of set B, but the extent of the linear growth regions are further reduced to within 20 cm above the distributor for all of the voidages. It is also found that for higher voidages $\varepsilon_0 = 0.496, 0.507$, the linear growth regions can only be observed for about 10cm above the distributor. In addition, it is found that the higher frequencies are stable only up to 10cm above the distributor while lower frequencies are much more stable and remain dominant throughout the bed. The wavespeeds increase slightly

Figure 3.3.12: Wave properties measured from set C

Figure 3.3.13: Wave properties measured from set C

after the stable regions near the distributor for all of the voidages.

3.3.6 Summary and Discussion

All the experiments presented above show linear growth regions near the distributor. This implies that the waves are growing independently in the initial stages of the growth, and the interactions between the waves are negligible. After these very limited linear growth regions, the higher frequency waves are decaying for short ranges while lower frequency waves reach approximately stable conditions. In addition, the lower frequency waves growing from the distributor take over from the higher frequency waves and eventually, the bed reaches a stable condition corresponding to a low frequency. This indicates that the dominant waves far away from the distributor will always be the relatively low frequency waves, and are not affected by any high frequency disturbances near the distributor. These results are qualitatively consistent with the results by El-Kaissy and Homsy (1976).

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ϵ_0	$u_0, cm/s$
1.0	0.049359	2.57	0.530	1.08
1.0	0.043851	2.62	0.536	1.12
1.0	0.039104	2.66	0.541	1.15
1.0	0.038406	2.69	0.546	1.17
1.0	0.037000	2.72	0.551	1.20

Table 3.3.3: Forced wave properties and flowing conditions for set A

The reason behind the transformation from high frequency to low frequency, linear growth regions to nonlinear growth regions can be understood from these experimental results. The change in the wave structure, and the break up of the wave train are a partial cause of the nonlinear waves (El-Kaissy and Homsy, 1976), but these are not the most important feature. As different frequency disturbances are generated in the distributor, and as the wavespeed changes with frequency, so the disturbances with different frequencies will travel with different wavespeeds, so eventually, faster lower frequency waves will capture the slower higher frequency waves. At that point, two waves will merge. These result in a "zig-zag" motion which is clearly observed in the experiments. The ultimate source for the wave clusters and wave break-ups (El-Kaissy and Homsy, 1976) is believed to be wave capturing and the subsequent "zig-zag" motions of the interacting waves.

The wave properties are obtained in the linear growth regions where the wavespeeds and frequencies are also stable. The spatial growth rates can be

easily measured by taking the slopes of linear amplitude growth in the amplitude *vs* height plots. The forced wave properties for set A are summarized in Table 3.3.3, only one forced wave is measured for each of the five runs which is the one frequency can successfully be forced in 0.5 mm glass beads. Table 3.3.4 shows the forced wave properties of five runs for set B; two wavespeeds and growthrates are measured for every voidage for set B. Table 3.3.5 shows the forced wave properties for set C; two wavespeeds and growthrates are measured for every voidage in set C.

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ϵ_0	$u_0, cm/s$
0.740	0.052364	4.11	0.485	1.71
1.0	0.065300	3.96		
0.800	0.049600	4.28	0.495	1.76
1.434	0.119980	3.99		
1.205	0.093600	4.70	0.508	1.93
1.795	0.160700	4.34		
0.801	0.037000	5.01	0.516	2.05
2.00	0.157620	4.41		
1.434	0.086380	5.01	0.526	2.15
2.00	0.149400	4.64		

Table 3.3.4: Forced wave properties and flowing conditions for set B

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ϵ_0	$u_0, cm/s$
0.756	0.06000	4.91	0.471	3.46
1.240	0.01102	4.56		
0.800	0.06508	5.22	0.481	3.61
1.522	0.03265	4.80		
0.792	0.07842	5.59	0.496	3.87
1.794	0.15245	5.03		
0.765	0.05606	5.88	0.507	4.06
2.00	0.17918	5.31		

Table 3.3.5: Forced wave properties and flowing conditions for set C

Figure 3.4.1: Supplemental wave properties measured from set B

3.4 Results and Comparison

In addition to the forced wave measurements, we also obtained some supplemental growthrate information by tracking, where possible, nearly constant frequency waves that appeared naturally (i.e. at other than the forced frequency). When such an amplitude shows clear linear growth near the distributor, the growthrate is used as supplemental information in the Inverse Instability Problem, thus improving the accuracy of the Optimum Parameter identification method. These waves are tracked by filtering the signal through a tight notch filter with a width of 1% of the frequency of interest. Not all frequencies show clear linear growth even near the distributor, so these supplemental growthrates were found only by considering many possible frequencies. Figure 3.4.1 shows an example from 1mm glass beads (set B) for the supplemental growthrate measurements.

The wave properties are obtained in the linear growth regions where the wavespeeds and frequencies are also stable. All the wave properties for set A are summarized in Table 3.4.1, only one wavespeed is measured, and three growthrates are obtained for every voidage in this set. Table 3.4.2 shows the wave properties of five runs for set B; here two wavespeeds and three growthrates are measured for every voidage for set B. Table 3.4.3 shows the wave properties for set C; two wavespeeds and three growthrates are measured for every voidage in set C.

Based on the experimental wave properties and supplemental growthrate information presented above, the system parameters (α , β , E_p , η_p and θ) for the three different particle sizes at various voidages can be determined by the Optimum Parameter Identification Method. The error estimation is done in the same fashion as section 3.2.1 by applying 10% error in growthrate for all of the runs, and 2% error in wavespeed. No error has been applied to forced wave frequencies as the frequency measured is exactly same as the forced frequency, but 1% error in frequencies for supplemental growthrate measurements has been applied based on the fluctuation on the frequency which is tracked along the height of the bed. The error bar on the mean voidage is based on the bed height fluctuation during the measurements at the mean voidage conditions, and it is 0.2% for set A, 0.3% for set B and set C.

3.4.1 Drag slopes in fluidized beds

The derivatives of the drag forces versus slip velocity are presented in figure 3.4.2 ~ figure 3.4.4. The derivatives of the drag force versus void fraction are presented in figure 3.4.5 ~ figure 3.4.7. Comparison has been made with

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ε_0	$u_0, cm/s$
1.0	0.049359	2.57	0.530	1.08
1.434	0.067874	—		
1.988	0.040178	—		
1.0	0.043851	2.62	0.536	1.12
1.522	0.069034	—		
2.287	0.029238	—		
1.0	0.039104	2.66	0.541	1.15
1.777	0.074220	—		
2.393	0.039870	—		
1.0	0.038406	2.69	0.546	1.17
1.496	0.070388	—		
2.530	0.055700	—		
1.0	0.037000	2.72	0.551	1.20
1.706	0.068454	—		
2.701	0.025500	—		

Table 3.4.1: Wave properties and flowing conditions for set A

the three most popular drag models: the Ergun model (1952), the Richardson-Zaki model (1954) and the Foscolo-Gibilaro model (1984, 1987). The Ergun and Richardson-Zaki models are summarized in Shook and Roco (1991) as

Ergun:

$$F_{erg} = \left[1.75 + \frac{150\mu_f(1-\varepsilon)}{d_p\rho_f(u_f - u_p)\varepsilon} \right] \frac{\rho_f(u_f - u_p)^2(1-\varepsilon)}{d_p} \quad (3.4.1)$$

Thus

$$\alpha_{erg} = \frac{u_{f0}}{F_{erg0}} \left[\frac{\partial F_{erg}}{\partial(u_f - u_p)} \right]_{\varepsilon=\varepsilon_0} =$$

$$\frac{u_{f0}}{F_{erg0}} \frac{\rho_f}{d_p} \left[3.5u_{f0} + \frac{150\mu_f(1-\varepsilon_0)}{\rho_f d_p \varepsilon_0} \right] (1-\varepsilon_0) \quad (3.4.2)$$

and

$$\beta_{erg} = -\frac{\varepsilon_0}{F_{erg0}} \left[\frac{\partial F_{erg}}{\partial \varepsilon} \right]_{\varepsilon=\varepsilon_0} = \frac{\varepsilon_0}{F_{erg0}} \left[1.75 \frac{\rho_f u_{f0}^2}{d_p} + \frac{150\mu_f u_{f0}(1-\varepsilon_0^2)}{d_p^2 \varepsilon_0^2} \right] \quad (3.4.3)$$

where

$$F_{erg0} = \left[1.75 + \frac{150\mu_f(1-\varepsilon_0)}{d_p\rho_f u_{f0}\varepsilon_0} \right] \frac{\rho_f u_{f0}^2(1-\varepsilon_0)}{d_p}$$

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ε_0	$u_0, cm/s$
0.740	0.052364	4.11	0.485	1.71
1.0	0.065300	3.96		
1.314	0.033997	—		
0.800	0.049600	4.28	0.495	1.76
1.434	0.119980	3.99		
2.002	0.037518	—		
1.205	0.093600	4.70	0.508	1.93
1.795	0.160700	4.34		
2.361	0.080050	—		
0.801	0.037000	5.01	0.516	2.05
2.00	0.157620	4.41		
2.525	0.050895	—		
1.434	0.086380	5.01	0.526	2.15
2.00	0.149400	4.64		
3.10	0.055729	—		

Table 3.4.2: Wave properties and flowing conditions for set B

f, Hz	$\sigma, 1/cm$	$U, cm/s$	ε_0	$u_0, cm/s$
0.756	0.06000	4.91	0.471	3.46
1.240	0.01102	4.56		
1.070	0.04330	—		
0.800	0.06508	5.22	0.481	3.61
1.522	0.03265	4.80		
1.170	0.07780	—		
0.792	0.07842	5.59	0.496	3.87
1.794	0.15245	5.03		
2.420	0.08894	—		
0.765	0.05606	5.88	0.507	4.06
2.00	0.17918	5.31		
3.421	0.08876	—		

Table 3.4.3: Wave properties and flowing conditions for set C

Richardson-Zaki:

$$F_{rz} = \frac{3C_{Ds}(1-\varepsilon)\rho_f(u_f - u_p)^2}{4d_p\varepsilon^{1.7}} \quad (3.4.4)$$

so:

$$\alpha_{rz} = \frac{u_{f0}}{F_{erg0}} \left[\frac{\partial F_{erg}}{\partial(u_f - u_p)} \right]_{\varepsilon=\varepsilon_0} = 2 + \frac{u_{f0}}{C_{Ds0}} \left[\frac{\partial C_{Ds}}{\partial(u_f - u_p)} \right]_{\varepsilon=\varepsilon_0} \quad (3.4.5)$$

and

$$\beta_{rz} = -\frac{\varepsilon_0}{F_{rz0}} \left[\frac{\partial F_{rz}}{\partial \varepsilon} \right]_{\varepsilon=\varepsilon_0} = \frac{1.7 - 0.7\varepsilon_0}{1 - \varepsilon_0} - \frac{\varepsilon_0}{C_{Ds0}} \left[\frac{\partial C_{Ds}}{\partial \varepsilon} \right]_{\varepsilon=\varepsilon_0} \quad (3.4.6)$$

where

$$F_{rz0} = \frac{3C_{Ds0}(1-\varepsilon_0)\rho_f u_{f0}^2}{4d_p\varepsilon_0^{1.7}}$$

$$C_{Ds} = \frac{24}{Re_s}$$

$$\text{For } Re_s = \frac{\varepsilon d_p(u_f - u_p)\rho_f}{\mu_f} < 0.2,$$

$$C_{Ds} = \frac{24}{Re_s} (1 + 0.15Re_s^{0.687}) \quad \text{For } 0.2 < Re_s < 1000,$$

$$C_{Ds} = 0.44$$

$$\text{For } Re_s > 1000$$

Foscolo-Gibilaro:

$$F_{fg}(\varepsilon, u_f - u_p) = (1 - \varepsilon)\varepsilon^{\frac{4.8-3.8n}{n}}(\rho_p - \rho_f)g \left(\frac{u_f - u_p}{u_t} \right)^{4.8} n \quad (3.4.7)$$

so:

$$\alpha_{fg} = \frac{u_{f0}}{F_{fg0}} \left[\frac{\partial F_{fg}}{\partial(u_f - u_p)} \right]_{\varepsilon=\varepsilon_0} = \frac{4.8}{n} \quad (3.4.8)$$

and

$$\beta_{fg} = -\frac{\varepsilon_0}{F_{fg0}} \left[\frac{\partial F_{fg}}{\partial \varepsilon} \right]_{\varepsilon=\varepsilon_0} = \frac{3.8 - 2.8\varepsilon_0}{1 - \varepsilon_0} - \frac{4.8}{n} \quad (3.4.9)$$

The terminal velocity and the exponent n are found from experiment.

Figure 3.4.2 presents the velocity derivative α for set A (0.5mm glass beads) which is the smallest size tested in this study. The measurements covered a voidage range from 0.530 to 0.55. The Richardson-Zaki model predicts the

value with reasonable accuracy on this parameter, while Ergun's model predicted slightly lower values than the experiments. But on the opposite side, the Foscolo-Gibilaro model predicted much higher values than seen experimentally. The variations of α with void fraction is small. Figure 3.4.3 presents the derivative α for set B (1.0mm glass beads) which is the intermediate size tested. The mean voidage ranges from 0.48 to 0.53. A slightly larger variation with voidage is observed and α is seen to increase almost linearly with voidage. Again, most of the data points lie between Foscolo-Gibilaro model and Richardson-Zaki model, and Ergun's model again predicted lower values than the measurements. Figure 3.4.4 presents the derivative α for set C (2.0mm) which is the largest size tested with the voidage ranging from 0.47 to 0.51. Here the Ergun model give very reasonable accuracy, while the Richardson-Zaki model is slightly lower than experiments. The Foscolo-Gibilaro model again predicted much larger values than were measured. The data points show a clear increasing trend as the voidage is increased.

Figure 3.4.5 shows the voidage derivative β for set A. The results show that all of the models predict higher values than are observed experimentally, with the closest coming from the Ergun model which still overpredicts the data by about 15%. The variation with voidage is again small. Figure 3.4.6 shows the derivative β for set B. The Ergun model still gives reasonable agreement, and the other two models are again higher than the measurements. Figure 3.4.7 shows the derivative β for set C. All of the models predicted much higher values than the experiments for set C. The value β generally increases with increasing voidage.

A tremendous discrepancy between the measured and predicted values of β is observed especially for the largest glass beads. This suggests that the discrepancy might be partially due to finite size effects (i.e. that the particle size is a significant fraction of the wavelength, i.e. here $d_p = 2.0mm$ and wavelength $\lambda \sim 20mm$, so $\frac{d_p}{\lambda} \sim 0.1$). One correction for this effect has been proposed by Singh and Joseph (1995). This was incorporated in our instability analysis, but unfortunately, the resulting correction is too small to account for the observed discrepancy (although it does act in right direction). Nevertheless there might still be some value in considering finite particle sizes in the particle motion equation.

3.4.2 Particulate effective elasticity

The particle phase effective elasticities are shown in figure 3.4.8 - figure 3.4.10. The only theoretical prediction of this quantity comes from the Foscolo-Gibilaro

Figure 3.4.2: Drag derivative vs slip velocity for set A

Figure 3.4.3: Drag derivative vs slip velocity for set B

Figure 3.4.4: Drag derivative vs slip velocity for set C

Figure 3.4.5: Drag derivative vs voidage for set A

Figure 3.4.6: Drag derivative vs voidage for set B

Figure 3.4.7: Drag derivative vs voidage for set C

model (1984, 1987) which predicts:

$$Ep_{fg} = 3.2gd_p(1 - \varepsilon)(\rho_p - \rho_f) \quad (3.4.10)$$

Figure 3.4.8 shows measurements of particulate effective elasticity for set A (0.5mm). The data show a clear decreasing (almost linearly decreasing) trend as the voidage is increased. Here, the Foscolo-Gibilario model predicts lower values than the measurements. In addition, the elasticities measured decrease much more rapidly than predicted by the Foscolo-Gibilario model. Figure 3.4.9 shows particulate effective elasticities for set B (1.0mm). For most of the data points, the Foscolo-Gibilario model gives reasonable agreement except for the one point at lowest voidage. The measured data again decreases more rapidly than the Foscolo-Gibilario model. Figure 3.4.10 shows the results for set C (2.0mm). Here the Foscolo-Gibilario model gives higher values than the experiments, but both the models and measured data decrease with approximately same slope as the voidage increases except at the lowest voidage.

From the Foscolo-Gibilario model, the particulate effective elasticity is proportional to the particle diameter, and decreases linearly with increasing voidage. But based on the comparison between measurements and Foscolo-Gibilario model, Foscolo-Gibilario model predicts higher elasticities for larger beads, and lower elasticities for smaller beads, indicating that the measured values are not increased correspondingly (proportionally) when the particle diameter is increased; instead, all of the data falls roughly into the same range. This strongly indicates that the elasticities are roughly independent of particle diameter. In addition, the measured data decrease much faster than Foscolo-Gibilario model especially for smaller beads.

The current results are the first effort to measure the particle elasticity at various voidages. Qualitatively, all data proved that the particulate effective elasticity generally decreases with increasing voidage. In that they agree with all of the theoretical studies (Clift, 1989, among others). The relationship between the elasticity and other parameters can only be revealed after a complete dimensional analysis which will be discussed in detail in section 3.5.

3.4.3 Particulate effective viscosity

The particulate phase effective viscosities are shown in figure 3.4.11 ~ figure 3.4.13. No comparison with theory is available for this parameter other than for the viscosity of suspensions (which are much smaller than any of these measurements).

Figure 3.4.11 presents the particulate effective viscosities for set A. The results show that the effective viscosities are in the range of 1 to 3 poise, and

Figure 3.4.8: Particulate effective elasticity for set A

Figure 3.4.9: Particulate effective elasticity for set B

Figure 3.4.10: Particulate effective elasticity for set C

decrease modestly with increasing voidage. Figure 3.4.12 shows the results for set B. The effective viscosities for set B ranges from 5 to 25 poise and also decrease with increasing voidage. Figure 3.4.13 shows the results for set C. The data ranges from 5 to 25 poise and decrease rapidly with increasing voidage.

Based on the results of set A and set B, the particulate effective viscosity seems to increase with increasing particle diameter, but the results of set B and set C lie roughly in the same range. This indicates that the particulate effective viscosity is sensitive to particle diameter for small beads, but is not sensitive to the particle diameter for large beads ($d_p > 1mm$ in this case). Here we should particularly note that the voidage ranges for three different beads sizes are different. The larger the bead size, the lower the voidage range. All of the data proves that the particulate effective viscosity decreases with increasing voidage, and effective viscosity for the larger beads pretend to decrease more rapidly with increasing voidage.

The results presented here are approximately in the same order as those measured by Schugerl *et al.* (1961) in a concentric cylinder viscometer which contained a gas-fluidized bed, and as inferred from the motion of bubbles by Grace (1970) also in a gas-fluidized bed. To our knowledge, the current measurements are the first such determinations made in liquid fluidized beds. This

Figure 3.4.11: Particulate effective viscosity for set A

Figure 3.4.12: Particulate effective viscosity for set B

Figure 3.4.13: Particulate effective viscosity for set C

may provide some insight into the mechanism of viscosity generation. If one were to assume that the values should scale with the viscosity of the interstitial fluid, (as one would expect for a suspension) then the relative viscosities, η_p/μ_f measured here are off by orders of magnitude from their corresponding values for gas fluidized beds. However, if one assumes that these viscosities are primarily a result of momentum transport induced by particle motion and interactions, the values should be relatively independent of the fluid; in that case, the results in gas and liquid fluidized beds should be similar just as observed in these results. This outcome was also predicted by Anderson and Jackson (1968).

3.4.4 Added mass effects

Figure 3.4.14 - Figure 3.4.16 show the results for the added mass coefficients. The values are from several to 10s, and decrease with increasing voidage for all of the measurements. From the results, the larger beads intend to have stronger dependency on the voidage than smaller beads.

These are probably the most controversial results as they conflict with most of the theoretical studies which state that the added mass coefficient should be in the range of $0 \sim 0.5$ (Atkinson and Kytomaa 1992, Dodemand, *et al.* 1995),

Figure 3.4.14: Added mass coefficient for set A

Figure 3.4.15: Added mass coefficient for set B

Figure 3.4.16: Added mass coefficient for set C

except for the values estimated by Homsy, *et al.* (1980) which is roughly in the same order as ours. There are no quantitative values available for comparison although there are some theoretical analyses of the added mass effect (Geurst, 1991, Drew, *et al.* 1979).

The trends of the added mass term also conflicts with most of the theoretical studies which state that the added mass term will increase with increasing voidage, while our measurements show that the added mass coefficients decrease with increasing voidage. Remember that the added mass reflects the mass of fluid that must be accelerated and carried along as a particle accelerates. The theoretical results that indicate that the added mass should increase with voidage are implicitly assuming that only fluid, and not surrounding particles are accelerated along with a given particle. Under those conditions, the surrounding particles, which do not accelerate, simply fill up space so that might otherwise be occupied by fluid, which would accelerate. Consequently, the larger the voidage, the more fluid that is available, and the added mass coefficient should increase with voidage until it reaches a value of 0.5, which correspond to a single sphere in an infinite fluid under conditions of potential flow. The large magnitudes of the added mass terms seen here indicate that the particles play an important role in the added mass process.

An understanding of the added mass term will fundamentally depend on

how an individual particle experiences its surroundings. One very simple approach will be to treat the surrounding mixture as a fluid with a corrected density that reflects the presence of the particles: $\rho_m = \epsilon \rho_f + (1 - \epsilon) \rho_p$. As the particles have roughly two-and-a-half times the density of the interstitial fluid, this would, by itself, indicate an increase in the local density and thus in the density of the material accelerated along with the particle. Furthermore, the apparent density will decrease as the voidage increases in concert with the current observations. But for the most closely packed spheres studied here, the voidage is about 0.48, which indicates a specific gravity of the mixture to be about 1.78, which would increase the added mass coefficient from 0.5 to 0.9, much less than the values on the order of 10 seen here. Therefore, some other effect must also arise from the presence of the particulate phase.

One answer is to note that the particulate phase has a large effective viscosity and thus cannot be considered to be a potential flow as is assumed in the standard added mass analyses. But it is probably more physically meaningful (although undoubtedly related) to realize that the presence of the particle phase introduces the particle diameter as an additional length scale into the problem. This scale may manifest itself in the added mass coefficient by increasing the extent of the material that would be accelerated along with an accelerating particle. This may be physically pictured by realizing that an accelerating particle will collide with its neighbors which both causes them to accelerate and so extends the influence of the original particle diameter an additional particle diameter deeper into the material. Those neighboring particles will themselves collide with neighbors and extend the influence of the original particle still deeper into the surrounding material. This also accounts for the severe drop in the added mass coefficient with voidage; increasing the voidage increases the interparticle spacing, thus decreasing the number of collisions with neighboring particles and, with it, decreasing the "reach" of a particle into the surrounding material.

At first glance this may seem inconsistent with the results as the value of the added mass coefficient does not appear to depend on particle size. But remember that physically, the added mass coefficient represents the fraction of a particle volume of the surrounding material that is accelerated along with the original particle. Thus, the fact that the added mass coefficient does not change significantly with particle diameter indicates that the volume of accelerated material still scales with the particle volume.

3.5 Summary and Discussion

To summarize the results obtained in the preceding sections, we tried to formulate the results in nondimensional form, and relate them to the nondimensional variables of the problem. By choosing $\{u_0, d_p, \rho_f\}$ as scaling parameters for velocity, length and density, respectively, appropriate dimensionless quantities are:

$$V_f = u_f/u_0, V_p = u_p/u_0, Z = z/d_p, T = tu_0/d_p, F = \frac{F_I}{\mu_f u_0/d_p^2}$$

The resulting governing equations in nondimensional form then become:

Conservation of Mass:

fluid phase:

$$\frac{\partial \varepsilon}{\partial T} + V_f \frac{\partial \varepsilon}{\partial Z} + \varepsilon \frac{\partial V_f}{\partial Z} = 0 \quad (3.5.1)$$

particle phase:

$$-\frac{\partial \varepsilon}{\partial T} - V_p \frac{\partial \varepsilon}{\partial Z} + (1 - \varepsilon) \frac{\partial V_p}{\partial Z} = 0 \quad (3.5.2)$$

Combined conservation of momentum:

$$\begin{aligned} & (1 - \varepsilon) \varepsilon \rho \left(\frac{\partial V_p}{\partial T} + V_p \frac{\partial V_p}{\partial Z} \right) - (1 - \varepsilon) \varepsilon \left(\frac{\partial V_f}{\partial T} + V_f \frac{\partial V_f}{\partial Z} \right) \\ & = \frac{1}{Re} F - \frac{1}{Fr} [\varepsilon(1 - \varepsilon)(\rho - 1)] + \varepsilon \tilde{E}_p \frac{\partial \varepsilon}{\partial Z} + \varepsilon \tilde{\eta}_p \frac{\partial^2 V_p}{\partial Z^2} \end{aligned} \quad (3.5.3)$$

$\tilde{E}_p = \frac{E_p \rho}{u_0^2 \rho_f}$	is the dimensionless particulate effective elasticity,
$\tilde{\eta}_p = \frac{\eta_p}{\rho_f u_0 d_p}$	is the dimensionless particulate effective viscosity,
$Re = \frac{\rho_f u_0 d_p}{\mu_f}$	is the particle Reynolds number,
$Fr = \frac{u_0^2}{g d_p}$	is the particle Froude number,
$\rho = \frac{\rho_p}{\rho_f}$	is the density ratio.

and F is a function of α, β and θ , where α, β and θ already in dimensionless form.

Assuming that the parameters $\alpha, \beta, \theta, \tilde{E}_p, \tilde{\eta}_p$ depend on the flow conditions, and not upon one another, it is found by dimensional analysis that the state of fluidization will be determined by at most four dimensionless variables. From

ϵ_0	void fraction
$\rho = \frac{\rho_p}{\rho_f}$	density ratio
$Re = \frac{\rho_f u_0 d_p}{\mu_f}$	particle Reynolds number
$Fr = \frac{u_0^2}{g d_p}$	particle Froude number

equation 3.5.3, we can choose following independent variables which determine the conditions of fluidized beds:

Based on the information above and the fact that the density ratios are nearly constant for all of these experiments, all the material constants (dimensionless system parameters, $\alpha, \beta, \theta, \tilde{E}_p, \tilde{\eta}_p$) are expected to be functions of three dimensionless variables (ϵ_0, Re, Fr). We then combined the material constants and these three dimensionless variables to best fit the data. In the following plots, the axes are chosen so as to generate straight lines.

Figure 3.5.1 shows the correlated function and data points measured for drag-slope against slip velocity. Here, the correlation function for the drag derivative α is given by:

$$\alpha = 9.712 \frac{0.63\sqrt{Re}}{4.9 + 0.63\sqrt{Re}} \frac{\epsilon^2}{Fr^{0.3} Re^{0.1}} - \frac{3.53 \times 10^{-2}}{Fr^{0.7} Re^{0.1}} \quad (3.5.4)$$

It is clearly shown that the drag-slope α increases with increasing voidage and Reynolds number, but decreases with increasing Froude number.

Figure 3.5.2 shows the correlated function and data points for drag-slope β . The correlation function for the drag derivative β is given by:

$$\beta = 0.0425 \frac{Re^{0.22}}{Fr^{0.87}} \left(\frac{\epsilon}{1 - \epsilon} \right)^{4.8} \quad (3.5.5)$$

As might be expected, the drag-slope β has a similar form to that for α . It increases with increasing voidage and Reynolds number, and decreases with increasing Froude number.

The advantage of these correlation functions is that they are based on an instability problem and measurements of unsteady and local information. In contrast, most existing drag models are based on the mean force balance (average information). So the correlation function presented by this study will hopefully provide more reliability for unsteady and local drag derivatives.

Figure 3.5.1: Correlation function for α

Figure 3.5.2: Correlation function for β

Based on the instability theory, α and β will be the main factor to determine the continuity wavespeed; more specifically, the continuity wavespeed is proportional to the ratio of β to α (equation ??). Based on the correlation function (3.2.6) and (3.5.5), both α and β have similar functional forms of dimensionless variables.

If $\beta > \alpha$, i.e. if the void fraction gradient is larger than the slip velocity gradient, small voidage changes will cause large changes in the drag force, and therefore the system will be more unstable (i.e. have a higher continuity wavespeed). On the other hand, if $\beta < \alpha$, small relative velocity changes will cause large changes in drag force, and therefore the system pretend to be more stable (i.e. have a smaller continuity wavespeed).

Figure 3.5.3 shows the correlated function and data points measured for the particulate dimensionless effective elasticity. The final correlation function is given by:

$$\tilde{E}_p = 17.2 \left(\frac{1-\varepsilon}{\varepsilon} \right) + 3.56 \times 10^{-6} \frac{Re^{0.5}}{Fr^{4.63}} \left(\frac{1-\varepsilon}{\varepsilon} \right) \quad (3.5.6)$$

Figure 3.5.3: Correlation function for the dimensionless effective elasticity

The dimensionless elasticity increases with increasing Reynolds number, but decreases with increasing Froude number and voidage. Based on the instability theory, the elasticity will be the main factor in determining the

Figure 3.5.4: Correlation function for the dimensionless effective viscosity

dynamic wavespeed; specifically, the larger the effective elasticity, the larger the dynamic wavespeed. From the correlation function (3.5.6), the effective elasticity decreases with increasing Froude number. As a result, the larger the Froude number, the smaller the dynamic wavespeed which makes the system more unstable.

Figure 3.5.4 shows the correlation function and corresponding data points measured for dimensionless particulate effective viscosity. Here the correlation function is:

$$\tilde{\eta}_p = 1.6 \times 10^4 e^{-136.4 Fr} Re^{1.5} Fr^{1.5} \sqrt{\left(\frac{1-\epsilon}{\epsilon} \right)} \quad (3.5.7)$$

It is shown that the dimensionless effective viscosity increases with increasing Reynolds number, but decreases with increasing voidage. In addition, the effective viscosity increases with increasing Froude number as long as the Froude number is small ($Fr < 1.1 \times 10^{-2}$), but decreases with increasing Froude number for larger Froude number ($Fr > 1.1 \times 10^{-2}$). From equation (3.5.7), the effective viscosity decreases with increasing voidage and quickly reaches zero at $\epsilon = 1$. This implies that the viscous force between the particles strongly depends on void fraction (or the mean distance among the

particles), and it will only be important when the particles are close to each other (low void fraction).

Figure 3.5.5 shows the correlation function and data points measured for the added mass coefficient. Here, the correlation function is given by:

$$\frac{\theta}{1-\epsilon} = 0.5 + 3.4 \times 10^{-3} \frac{Re^{0.62}}{Fr^{2.02}} \left(\frac{1-\epsilon}{\epsilon} \right)^{2.88} \quad (3.5.8)$$

Note that $\frac{\theta}{1-\epsilon}$ equals the added mass coefficient for an isolated sphere when $\epsilon = 1$. It is shown that the added mass coefficient increases with increasing Reynolds number, but decreases with increasing Froude number and voidage; in addition, it is also seen that the coefficient varies strongly with voidage. Based on these results, the added mass coefficients will start larger than 0.5, decrease with increasing voidage and reach limiting value of 0.5 at $\epsilon = 1$.

Figure 3.5.5: Correlation function for the added mass coefficient

The results shown above conflicts with most of other studies (Atkinson and Kytomaa 1992, Dodemand *et al.* 1995, Drew *et al.* 1979) where the added mass coefficients are predicted to increase with increasing voidage. Only Lamb's (1954) "cell model" showed a decreasing in the added mass coefficient with increasing voidage.

These results indicate that accelerating a particle in fluidized beds accelerates a larger surrounding mass than would be seen in an infinite fluid. This

may only be accounted for by the presence of surrounding particles which are accelerated along with fluid and extend the reach of the accelerating particle further in the flow.

It should be noted that these correlations are based on a very limited range of void fractions, and may not be applicable outside the range studied. But an additional caveat must be applied to these results. The generic stability model upon which all this information is based assumes that gravity, drag, elasticity, viscosity and added mass can be used to represent all the forces that particles experience. Admittedly, these account for all of the forces included in all fluidized bed stability models that have appeared to this time. But that doesn't mean that additional effects might also be present in a real fluidized bed that are not accounted for and that any missing effects must have somehow been absorbed into the framework of the assumed model. For example, we neglected added mass effects in the first generic model that we analyzed, but realized that something was probably quite wrong because the drag gradients were all very different from the generally accepted models. It would have been interesting to have analysed a more complicated wave equation which contained additional terms (whose physical significance would not be immediately apparent), to see if it yielded closer fits to the various correlations. Unfortunately, that would increase the number of undetermined coefficients and thus increase the number of wave observations necessary to fit those coefficients. As it was, the number of possible observations was barely adequate to determine the number of coefficients in the current model. Thus, it must be understood, that the quantities measured here might not represent the drag gradients elasticity etc. and have no greater physical significance than just being the best choices to use in a stability model of the form employed here.

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