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Computation with Inverse States in a Finite Field $\mathbb{F}_p$: The Muon Neutrino Mass, the Unified Strong-Electroweak Coupling Constant, and the Higgs Mass

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**Computation with Inverse States in a Finite Field $\mathbb{F}_p$:
The Muon Neutrino Mass, the Unified Strong-Electroweak Coupling
Constant, and the Higgs Mass**

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ABSTRACT

The construction of inverse states in a finite field $\mathbb{F}_p$ enables the organization of the mass scale with fundamental octets in an eight-dimensional index space that identifies particle states with residue class designations. Conformance with both CPT invariance and the concept of supersymmetry follows as a direct consequence of this formulation. Based on two parameters (P_α and g_α) that are anchored on a concordance of physical data, this treatment leads to (1) a prospective mass for the muon neutrino of ~ 27.68 meV, (2) a value of the unified strong-electroweak coupling constant $\alpha^* = (34.26)^{-1}$ that is physically defined by the ratio of the electron neutrino and muon neutrino masses, and (3) a see-saw congruence connecting the Higgs, the electron neutrino, and the muon neutrino masses. Specific evaluation of the masses of the corresponding supersymmetric Higgs pair reveals that both particles are superheavy ($>10^{18}$ GeV). No renormalization of the Higgs masses is introduced, since the calculational procedure yielding their magnitudes is intrinsically divergence-free. Further, the Higgs fulfills its conjectured role through the see-saw relation as the particle defining the origin of all particle masses, since the electron and muon neutrino systems, together with their supersymmetric partners, are the generators of the mass scale and establish the corresponding index space. Finally, since the computation of the Higgs masses is entirely determined by the modulus of the field P_α , which is fully defined by the large-scale parameters of the universe through the value of the universal gravitational constant G and the requirement for perfect flatness ($\Omega = 1.0$), the see-saw congruence fuses the concepts of mass and space and creates a new unified archetype.

I. Introduction

The concept of mass [1] has been an everlasting challenge to physical theory [2]. In general, an organization of the particle mass scale is sought that exhibits certain salient features. Four prominent desired characteristics are (A) the equivalence of particle P and antiparticle $\bar{P}$ masses ($m_P = m_{\bar{P}}$), a demand of CPT invariance [3,4], (B) an expression of supersymmetry [5], a principle invoking fermion (f) /boson (b) pairing that presently exists as a very attractive theoretical hypothesis, (C) the existence of the Higgs particle [6], the entity which introduces mass into the Standard Model [7], and (D) a basis for the value of the unified strong-electroweak coupling constant α^* , the physical parameter regulating non-gravitational interactions [8].

Taken together, the two features (A) and (B) of the mass spectrum noted above incorporate three forms of two-particle associations. The requirement of CPT separately pairs both fermi (e.g. e^+/e^-) and bose (e.g. π^+/π^- and π^0/π^0) species. Supersymmetry connects the fermi and bose particle genres through the formation of corresponding supersymmetric pairs (P/P_{ss}). Therefore, these two classes of relationships produce three pairings (f/f, b/b, and b/f) yielding a system in which each particle P has both an antiparticle $\bar{P}$ and a supersymmetric partner P_{ss} . The overall result is the construction of the particle mass scale [9] with fundamental quartets ($P, \bar{P}, P_{ss}, \bar{P}_{ss}$). Two previous studies [9,10] explicitly illustrate the manner in which this mass scale can be organized. A leading prediction of these analyses is a prospective value of 0.808 meV for the mass m_{ν_e} of the electron neutrino ν_e , a magnitude that is consistent with currently available data [11].

The present work demonstrates that the three classes of pairings associated with CPT and supersymmetry given above follow as immediate consequences of the definition of a new class of particle states designated as inverse states. This description additionally associates each particle state P with a corresponding inverse state P_{in} through a specific relationship of the particle masses. The outcome is an affiliation of four particles ($P, P_{ss}, P_{in}, (P_{in})_{ss}$), normally, but not necessarily, with nondegenerate mass values, which are further related to a corresponding quartet of antiparticles. The final result is (1) the formation of an octet of states that generally represents four distinct mass values and (2) the construction of a corresponding eight-dimensional index space that provides a system of particle classification.

This theoretical picture can be implemented by incorporating the previous analyses [9,10] into the mathematical structure of a finite field [12]. Since an algebraic field possesses the operation of division, this extension permits the definition of inverses and the corresponding construction of a set of units $\{U\}$. An important consequence is the fact that these units comprise a group structure [12]. As shown in the development below, we will be primarily concerned with the subset of fields $\mathbb{F}_p$ in which $p \equiv 1 \pmod{4}$, the physically motivated case [9,10,13].

The ability to define inverse states leads to several new statements concerning the organization of the mass scale and satisfaction of the four desired characteristics (A–D) listed above. Chief among the specific findings are (1) the muon neutrino mass with a predicted value of

$$m_{\nu_\mu} = 27.68 \text{ meV}, \quad (1)$$

(2) the conclusion that the unified strong-electroweak coupling constant α^* is physically defined through the electron/muon neutrino mass ratio by

$$\alpha^* = \frac{m_{\nu_e}}{m_{\nu_\mu}} = (34.26)^{-1}, \quad (2)$$

(3) the identification of uniquely specified candidate values for the masses of a supersymmetric doublet that corresponds to the Higgs particle, and (4) a see-saw congruence that connects the electron neutrino mass, the muon neutrino mass, and the masses of the supersymmetric Higgs pair [6–8,14]. Explicit evaluation of the pair of Higgs masses indicates that both particles are in the superheavy regime ($>10^{18}$ GeV). Since the computation of the Higgs masses is intrinsically divergence-free, no renormalization [6] is necessary. The stated values of this set of fundamental parameters are consistent with the ranges of their potentially expected magnitudes [6–8,11,14–17] and all currently available experimental data. The overarching conclusion that follows from the form of the see-saw relation derived in this analysis is that the concepts of space [18,19] and mass [1,2] are fundamentally inseparable. The results state directly that these two physical archetypes constitute an identity.

II. Theoretical Development

A. Elementary Properties of Finite Fields $\mathbb{F}_p$

A commutative division ring is designated as a field [12]. For any prime integer P , the field $\mathbb{F}_p$ consists of the set of residue classes

$$\{[0]_p, [1]_p, [2]_p, \dots, [P-1]_p\}. \quad (3)$$

Computations performed with elements $[x]_p$ of $\mathbb{F}_p$ are conducted with the customary rules of arithmetic and reduction modulo the prime P . Since all primes possess primitive roots [20], power residue systems of enumeration in $\mathbb{F}_p$ can be established and the properties of congruences [21] are central. Physically anchored procedures of counting based on these concepts were used in earlier studies to describe the organization of the particle mass scale [9,10] and the phenomenon of γ -ray bursts [22].

The existence of the operation of division enables the definition of an inverse $[x]_p^{-1}$ for every element $[x]_p$ of $\mathbb{F}_p$ except $[0]_p$. Hence, a set of units $\{U\}$, defined by the statement

$$[x]_p [x]_p^{-1} = [1]_p, \quad (4)$$

exists and it is known that these units $\{U\}$ form a multiplicative group [12]. Furthermore, a simple general procedure exists for the calculation of the inverses $[x]_p^{-1}$ based on Fermat's Little Theorem [23]. Specifically, Fermat's result states that

$$\forall x, \gcd(x, P) = 1, [x]_p^{P-1} = [1]_p, \quad (5)$$

from which it follows directly that

$$[x]_p^{-1} = [x]_p^{P-2}. \quad (6)$$

Eq.(6) explicitly shows that an inverse $[x]_p^{-1}$ generally depends on both the value of x and the modulus P .

B. Specific Mathematical Properties of Inverses

All inverses can be computed directly through Eq.(6). However, a restricted subset of inverses has a special elementary formulation. Anticipating the need for this result below, we state the lemma specifying this set of residues and give the corresponding proof.

Lemma (1): Let P_α be prime, B_x be a divisor of $P_\alpha - 1$, and $(B_x)_{ss} = P_\alpha - B_x$.

$$\text{Then, } [(B_x)_{ss}]_{P_\alpha}^{-1} = [(P_\alpha - 1)/B_x]_{P_\alpha}.$$

Proof: By definition, $[(B_x)_{ss}]_{P_\alpha}^{-1} = [(B_x)_{ss}]_{P_\alpha}^{P_\alpha - 2}$.

$$\begin{aligned} \text{Since } (B_x)_{ss}^{P_\alpha - 2} &= (P_\alpha - B_x)^{P_\alpha - 2} = \sum_{i=0}^{P_\alpha - 2} (-1)^i \binom{P_\alpha - 2}{i} P_\alpha^{P_\alpha - 2 - i} B_x^i \\ &= -B_x^{P_\alpha - 2} + k \cdot P_\alpha \text{ for some integer } k, \end{aligned}$$

$$\text{we have } (B_x)_{ss}^{P_\alpha - 2} \equiv -B_x^{P_\alpha - 2} \pmod{P_\alpha}. \quad (7)$$

$$\text{Suppose } B_x^{P_\alpha - 2} \equiv A \pmod{P_\alpha}, \quad (8)$$

$$\text{then } -B_x^{P_\alpha - 2} \equiv P_\alpha - A \pmod{P_\alpha}. \quad (9)$$

In order to evaluate the constant A from Eq.(8), we write

$$1 \equiv B_x^{P_\alpha - 1} \equiv B_x^{P_\alpha - 2} \cdot B_x \equiv AB_x \pmod{P_\alpha}, \text{ if } \gcd(B_x, P_\alpha) = 1.$$

Therefore, $AB_x = 1 + k_x \cdot P_\alpha = (k_x + 1) + k_x(P_\alpha - 1)$ for some integer k_x .

Then $A = \frac{k_x + 1}{B_x} + k_x \frac{P_\alpha - 1}{B_x}$. Since $\frac{P_\alpha - 1}{B_x}$ is an integer by assumption, $\frac{k_x + 1}{B_x}$ must also be an integer.

Let $\frac{k_x + 1}{B_x} = N$, where N is an integer.

$$\begin{aligned} \text{Then we have } A &= N + (NB_x - 1) \frac{P_\alpha - 1}{B_x} \\ &= N + N(P_\alpha - 1) - \frac{P_\alpha - 1}{B_x} \\ &= NP_\alpha - \frac{P_\alpha - 1}{B_x}. \end{aligned}$$

From Eq.(9),

$$-B_x^{P_\alpha-2} \equiv P_\alpha - NP_\alpha + \frac{P_\alpha - 1}{B_x} \equiv \frac{P_\alpha - 1}{B_x} \pmod{P_\alpha},$$

and from Eq.(7)

$$[(B_x)_{ss}]_{P_\alpha}^{-1} = [-B_x]_{P_\alpha}^{P_\alpha-2} = [(P_\alpha - 1)/B_x]_{P_\alpha}. \quad (10)$$

Q.E.D.

Hence, B_x must be a divisor of $P_\alpha - 1$.

Lemma (1) specifies a subset of residue classes $\{B_x\}$ whose corresponding supersymmetric [9,10] partners $(B_x)_{ss}$ possess inverse states of the special form given by Eq.(10). The number of elements of the set $\{B_x\}$ is given by the arithmetic function $d(P_\alpha - 1)$, the function [24] which represents the number of divisors of the integer $(P_\alpha - 1)$. On the basis of an earlier study [13], $d(P_\alpha - 1) \cong 2.32 \times 10^{11}$. By inspection of Eq.(10), all members of $\{B_x\}$ obey the congruence

$$[B_x]_{P_\alpha} [(B_x)_{ss}]_{P_\alpha}^{-1} \equiv -1 \pmod{P_\alpha}. \quad (11)$$

The subset $\{g\}$ of elements of $\mathbb{F}_{P_\alpha}$ whose members are primitive roots of the modulus P_α also enjoy a select status [9,10,21,22], since these integers serve a generating function. Of particular significance will be the restricted subset $\{g_x\}$ of primitive roots such that

$$\{g_x\} = \{g\} \cap \{B_x\}. \quad (12)$$

Specifically, it will be shown below that the subset $\{g_x\}$ relates directly to both the magnitude of the unified strong-electroweak coupling constant α^* and the values for the masses of the electron and muon neutrinos. We note further that since $\{g_x\}$ can be empty, as the prime 41 demonstrates [21], the demand for the existence of a nonempty set $\{g_x\}$ constitutes an additional mathematical requirement for acceptable physical values [13] of the prime P_α .

In order to explore the needed characteristics of primitive root inverse mappings, we will initially limit our treatment to moduli of the form $P \equiv 1 \pmod{4}$ and pairs of primitive roots (g_α, g_β) that sum to the modulus P_α , the condition that expresses supersymmetry [9,10]. Further, since the Euler function $\phi(P - 1)$ is always even [24–26] for any prime $P > 3$, and a result of Gauss [27] certifies that all primitive roots of primes of the genre $P_\alpha \equiv 1 \pmod{4}$ can be arranged in supersymmetric pairs [9,10] such that

$$g_\alpha + g_\beta = P_\alpha, \quad (13)$$

it follows from Lemma (2) below that the inverse state operation given by Eq.(6) always maps a primitive root pair (g_α, g_β) according to the pattern

$$(g_\alpha, g_\beta) \rightarrow (g'_\alpha, g'_\beta) \quad (14)$$

with

$$g_\alpha \neq g'_\alpha \quad (15)$$

and

$$g_\alpha + g_\beta = g'_\alpha + g'_\beta = P_\alpha. \quad (16)$$

Therefore, primitive root pairs perform transform to different primitive root pairs with both pairs in conformance with Eq.(13), the postulated mass relationship expressing supersymmetry [9,10,22]. This is an extension of the Q-pairing pattern described in an earlier study [9] of the mass scale. We now prove this important statement.

Lemma (2): Assume $g_\alpha + g_\beta = P_\alpha$, g_α, g_β are primitive roots of P_α , and P_α prime.

Let the inverses of g_α and g_β respectively be B_α and B_β .

Then, (1) $B_\alpha + B_\beta = P_\alpha$,

(2) B_α and B_β are primitive roots of P_α .

Proof: (1) can be proved by using Lemma (1) of an earlier study [9] whose proof was therein given.

Let $x = P_\alpha - 2$, an odd number.

By the previously demonstrated lemma [9], we have

$$g_\alpha^{P_\alpha-2} + g_\beta^{P_\alpha-2} \equiv 0 \pmod{P_\alpha}$$

and

$$B_\alpha + B_\beta = P_\alpha.$$

To show that both B_α and B_β are primitive roots of P_α , we use the relation

$$g_\alpha \cdot B_\alpha \equiv 1 \pmod{P_\alpha},$$

the defining statement of the inverse mapping.

For any $y \leq P_\alpha - 1$,

$$g_\alpha^y \cdot B_\alpha^y \equiv 1 \pmod{P_\alpha}$$

$$g_\alpha^{P_\alpha-y-1} \cdot g_\alpha^y \cdot B_\alpha^y \equiv g_\alpha^{P_\alpha-y-1} \pmod{P_\alpha}$$

$$\Leftrightarrow g_\alpha^{P_\alpha-1} \cdot B_\alpha^y \equiv g_\alpha^{P_\alpha-y-1} \pmod{P_\alpha}$$

$$\Leftrightarrow B_\alpha^y \equiv g_\alpha^{P_\alpha-y-1} \pmod{P_\alpha}, \text{ since } g_\alpha^{P_\alpha-1} \equiv 1 \pmod{P_\alpha}.$$

The residue of B_α is in one-to-one relation with the residue of $g_\alpha^{P_\alpha-y-1}$. Hence, B_α is a primitive root. The corresponding result for B_β follows *mutatis mutandis*.

Q.E.D.

It follows from Lemmas (1) and (2), if either g_α or g_β is a member of $\{g_x\}$, the $(g_\alpha, g_\beta) \rightarrow (g'_\alpha, g'_\beta)$ inverse transformation is immediately calculable on the basis of the divisor structure of $P_\alpha - 1$ through Eq.(10).

C. Mass Parameter Inverse Mapping Characteristics

Scale invariance of the representation of the mass parameters is a fundamental characteristic of the power residue formulation [9,10]. Therefore, the basic features of the inverse state mapping described above can be fully illustrated with an example utilizing a prime modulus of small magnitude. Table I displays the inverse state primitive root pairings for the prime modulus $P_\alpha = 37$. We observe that one primitive root ($g_\alpha = 2$) is a divisor of $P_\alpha - 1 = 36$, so that $[g_\beta]_{37}^{-1} = 18$, the result given directly by Eq.(10). In order to show the gross characteristics of the scaling of the pairing pattern on the magnitude of the modulus, a second example of the primitive root inverse mapping is given in Table II for $P_\alpha = 73$ and it is seen that the qualitative features of the transformation are unaltered.

Two new conserved quantities are revealed by the primitive root inverse mappings illustrated in Tables I and II. The statements of the conserved magnitudes are presented in Table III. It is conjectured that these relationships correspond to the conservation of internal attributes of the particle under the inverse state operation. Further work [28] is underway to elucidate the potential physical interpretations of these conserved quantities in terms of the subgroup structure of the units of $\mathbb{F}_{P_\alpha}$.

An explicit picture of the quartet of residues representing the octet of physical particle masses $\{P, \bar{P}, P_{ss}, \bar{P}_{ss}, P_{in}, \bar{P}_{in}, (P_{in})_{ss}, (\bar{P}_{in})_{ss}\}$ can be made by further examination of the example for $P_\alpha = 37$. For this illustration, we will take $(g_\alpha, g_\beta) = (2, 35)$, one of the primitive root pairs identified in Table I. Fig. (1) shows the spectrum of these primitive roots in conjunction with the corresponding inverse state pairs $(g'_\alpha, g'_\beta) = (19, 18)$. These four generators can now be used jointly to organize both (a) the particle mass parameters and (b) the corresponding index space for state classification.

Inspection of Fig. (1) shows that the operators corresponding to the supersymmetry O_{ss} and inverse state O_{in} transformations commute, so that

$$O_{in}O_{ss} = O_{ss}O_{in}. \quad (17)$$

We also have

$$O_{in}^2 = O_{ss}^2 = 1, \quad (18)$$

the unity operator, and the inverses

$$[O_{in}]^{-1} = O_{in}, \quad [O_{ss}]^{-1} = O_{ss}, \quad (19)$$

so that $\{O_{in}, O_{ss}\}$ constitutes an abelian group.

Table IV comprehensively illustrates the mappings of the residues that arise from the four primitive roots shown in Fig. (1). The particle states, supersymmetric partners, and corresponding inverse states that occur through organization by the quartet of primitive roots are shown. In general, each residue possesses four power indices $\{t_i; i=1,2,3,4\}$ and four corresponding modulus indices $\{k_i; i=1,2,3,4\}$,

parameters which define the coordinates of the 8-dimensional space in terms of residue classes $[x]_{P_\alpha}$ designations.

Examination of Table IV demonstrates that the Q- and D-pairing motifs developed in an earlier study [9] remain. The extension of the pattern with inverse states simply adds new relationships that the four-dimensional structure could not reveal. Specifically, for power index t odd, a supersymmetric pair is coupled to a corresponding inverse supersymmetric pair (e.g. $t = 7$, $(20, 17) \leftrightarrow (13, 24)$) that preserves the Q-pairing structure. All supersymmetric primitive root pairings fall in this class, as described in Section II.B. Likewise, for power index t even, the degenerate pattern exhibited by D-pairing [9] is transferred to the corresponding inverse state pair which also exhibits D-pairing. The appropriate conjugate panels [9] are always related by a shift in the power index t by the transformation $t \rightarrow t + (P_\alpha - 1)/2$, the relation established in the earlier work [9].

INVERSE STATE PROPERTIES

Power Residue Systematics ($P_\alpha = 37$)

t	1	2	3	4	5	6	7	8	9	10
	2 35	4 4	8 29	16 16	32 5	27 27	17 20	34 34	31 6	25 25
1	13 24	26 26	15 22	30 30	23 14	9 9	18 19	36 36	35 2	33 33
2	29 8	21 21	5 32	10 10	20 17	3 3	6 31	12 12	24 13	11 11
3	22 15	7 7	14 23	28 28	19 18	1 1				

Table I: The systematics of the power residue counting system represented by $(g_\alpha, g_\beta, P_\alpha) = (2, 35, 37)$. The parameter t represents the respective integral power indices t_α and t_β . In green are the residues (B_α) for $g_\alpha = 2$ and correspondingly (B_β) in red for $g_\beta = 35$. Two patterns are apparent; the one for t odd always yields $B_\alpha + B_\beta = P_\alpha = 37$, the value coincident with $(g_\alpha + g_\beta = P_\alpha = 37)$, the condition governing supersymmetry. The remaining entries for t even always represent the degenerate condition $B_\alpha = B_\beta$. Conjugate panels are shifted $t \rightarrow t + (P_\alpha - 1)/2$ uniformly in both cases such that the respective sums give P_α . Hence, both patterns provide pairing which represents the prime modulus $P_\alpha = 37$. Every residue has an associated inverse in a field. Shown below is the subclass corresponding to the primitive root pairings for prime $P_\alpha = 37$ established by constructing units in the finite field $\mathbb{F}_{37}$. The primitive roots are $\{2, 5, 13, 15, 17, 18, 19, 20, 22, 24, 32, 35\}$.

STATES

(g_α, g_β)

(2, 35)

(5, 32)

(13, 24)

INVERSE STATES

(g'_α, g'_β)

(19, 18)

(15, 22)

(20, 17)

The inverse operation in $\mathbb{F}_{37}$ relates conjugate panels for two distinct sets of primitive roots, thereby conferring a unique status on these four residues. For example, the inverse mapping (1) satisfies supersymmetry ($g_\alpha + g_\beta = g'_\alpha + g'_\beta = P_\alpha = 37$) and (2) the pair $(g_\alpha, g_\beta) = (2, 35)$ is transformed to $(g'_\alpha + g'_\beta) = (19, 18)$, another primitive root pair. Therefore, two distinct quartet manifolds are related by the inverse mapping for primitive root states. Hence, four (2,18,19,35) distinct masses are related together with a corresponding quartet of antiparticle states. Therefore, the existence of inverse states give the mass scale an extended foundation through the construction of octets and a corresponding eight-dimensional index space which classifies the particle states with residue class designations. These octets involve (A) particle/antiparticle (CPT), (B) particle/supersymmetric particle (Supersymmetry), and (C) particle/inverse particle (Unitary symmetry) relations. Generally, $[g_\alpha][g_\beta]^{-1} \equiv -1 \pmod{P_\alpha}$ and only t odd states are involved in the primitive root mappings.

Power Residue Systematics ($P_\alpha=73$)

$$g_\alpha = 34, g_\beta = 39, g_\alpha + g_\beta = P_\alpha$$

t	1	2	3	4	5	6	7	8	9	10
0	34	61	30	71	5	24	13	4	63	25
1	39	61	43	71	68	24	60	4	10	25
2	47	65	20	23	52	16	33	27	42	41
3	26	65	53	23	21	16	40	27	31	41
4	7	19	62	64	59	35	22	18	28	3
5	66	19	11	64	14	35	51	18	45	3
6	29	37	17	67	15	72	39	12	43	2
7	44	37	56	67	58	72	34	12	30	2
8	68	49	60	69	10	48	26	8	53	50
9	5	49	13	69	63	48	47	8	20	50
10	21	57	40	46	31	32	66	54	11	9
11	52	57	33	46	42	32	7	54	62	9
12	14	38	51	55	45	70	44	36	56	6
13	59	38	22	55	28	70	29	36	17	6
14	58	1								
15	15	1								

Primitive root pair $\rightarrow$ its inverse

$\{5, 68\} \rightarrow \{44, 29\}$
 $\{11, 62\} \rightarrow \{20, 53\}$
 $\{13, 60\} \rightarrow \{45, 28\}$
 $\{14, 59\} \rightarrow \{47, 26\}$
 $\{15, 58\} \rightarrow \{39, 34\}$
 $\{31, 42\} \rightarrow \{33, 40\}$

power index $\rightarrow$ power index mapping sum

$5 \rightarrow 67$ 72
 $59 \rightarrow 13$ 72
 $7 \rightarrow 65$ 72
 $61 \rightarrow 11$ 72
 $35 \rightarrow 37$ 72
 $55 \rightarrow 17$ 72

Table II: Systematics of the inverse states for the prime $P_\alpha = 73$ represented for the primitive roots

$g_\alpha = 34$ and $g_\beta = 39$.

NEW STATES and NEW CONSERVATION LAWS

- Inverse States Define a New Symmetry Based on Supersymmetry Preserving Primitive Root Mapping
 - $(g_\alpha, g_\beta) \longrightarrow (g'_\alpha, g'_\beta)$
 $g_\alpha + g_\beta = P_\alpha, \quad g'_\alpha + g'_\beta = P_\alpha, \quad g_\alpha \neq g'_\alpha$
- Inverse State Transformations Define New Conservation Laws

- Power Index t

$$\begin{array}{ccc} (g_\alpha, g_\beta) & \longrightarrow & (g'_\alpha, g'_\beta) \\ t & \longrightarrow & t' \end{array}$$

$t + t' = P_\alpha - 1$

- Modulus Index k

$k_\alpha g'_\alpha + k'_\alpha g_\alpha \equiv 0 \pmod{P_\alpha}$ $k_\beta g'_\beta + k'_\beta g_\beta \equiv 0 \pmod{P_\alpha}$
--

Presumably indicate conservation of internal attributes

- One New Fundamental State Classification (Symmetry)
and

Two New Conserved Quantities

Table III: Inverse states define a new symmetry based on a supersymmetry preserving mapping that can be achieved with $P_\alpha \equiv 1 \pmod{4}$.

Fig. (1): Example of the spectrum of primitive roots governing the octet state manifolds for $P_\alpha = 37$. The primitive roots are $(g_\alpha, g_\beta) = (2, 35)$; the corresponding supersymmetric inverses of (g_α, g_β) are $(g'_\alpha, g'_\beta) = (19, 18)$.

Residue Mappings for Supersymmetric Primitive Root Pairs ($P_\alpha = 37$)

$$[g_\alpha]_{37} = 2, [g_\beta]_{37} = 35, [g_\alpha]_{37}^{-1} = 19, [g_\beta]_{37}^{-1} = 18$$

$\begin{array}{c} g \\ \hline r \end{array}$	35	2	18	19
1	35	2	18	19
2	4	4	28	28
3	29	8	23	14
4	16	16	7	7
5	5	32	15	22
6	27	27	11	11
7	20	17	13	24
8	34	34	12	12
9	6	31	31	6
10	25	25	3	3
11	24	13	17	20
12	26	26	10	10
13	22	15	32	5
14	30	30	21	21
15	14	23	8	29
16	9	9	33	33
17	19	18	2	35
18	36	36	36	36
19	2	35	19	18
20	33	33	9	9
21	8	29	14	23
22	21	21	30	30
23	32	5	22	15
24	10	10	26	26
25	17	20	24	13
26	3	3	25	25
27	31	6	6	31
28	12	12	34	34
29	13	24	20	17
30	11	11	27	27
31	15	22	5	32
32	7	7	16	16
33	23	14	29	8
34	28	28	4	4
35	18	19	35	2
36	1	1	1	1

3 classes generated by the quartet of inverse state-related primitive roots
 $\rangle = (19,18)$ involving the residues B, their supersymmetric partners B_{ss} ,
 se state pairs B_{in} and $(B_{in})_{ss}$. See Fig. (1) for reference.

III. Computational Findings

A. Muon Neutrino Mass

The results derived in Section II above can be applied to several specific computations. The use of Eq.(10) permits the determination of a mass parameter which we prospectively identify as the muon neutrino ν_μ mass m_{ν_μ} . An earlier study [9] gave an electron neutrino mass m_{ν_e} of ~ 0.808 meV, a value corresponding to the mass parameter [9,10]

$$g_\alpha = 2q = 2 \cdot 3^2 \cdot 7 \cdot 13 \cdot 31 \cdot 37 \cdot 47 \cdot 53 \cdot 59 \cdot 71 \cdot 73 \cdot 83 \cdot 107 \cdot 109 \cdot 113 \cdot 137 \cdot 139 \cdot 149. \quad (20)$$

Direct evaluation [9,10,13] showed that g_α is a primitive root of the modulus P_α , the vital property that lets it serve as a generator. Furthermore, since g_α is manifestly [13] a member of the divisor set $\{B_x\}$ of the integer

$$P_\alpha - 1 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot 43 \cdot 47 \cdot 53 \cdot 59 \cdot 61 \cdot 67 \cdot 71 \cdot 73 \cdot 79 \cdot 83 \cdot 89 \cdot 97 \cdot 101 \cdot 103 \cdot 107 \cdot 109 \cdot 113 \cdot 127 \cdot 131 \cdot 137 \cdot 139 \cdot 149 \cdot 151, \quad (21)$$

Eq.(10) of Lemma (1) can be used to compute directly the inverse of

$$g_\beta = P_\alpha - g_\alpha. \quad (22)$$

This gives

$$[g_\beta]_{P_\alpha}^{-1} = \frac{r}{2} = 2 \cdot 5^2 \cdot 11 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 41 \cdot 43 \cdot 61 \cdot 67 \cdot 79 \cdot 89 \cdot 97 \cdot 101 \cdot 103 \cdot 127 \cdot 131 \cdot 151 \quad (23)$$

in which the integer r is defined in an earlier study [13]. Accordingly, on the basis of the previous work [9,10,13], we immediately recognize the specific magnitude of $[g_\beta]_{P_\alpha}^{-1}$ as a factor in the known result [13]

$$qr = (2q)(r/2) = P_\alpha - 1. \quad (24)$$

These earlier analyses [9,10,13] also demonstrated the important finding that the fine-structure constant α could be uniquely represented as

$$\alpha = \frac{q}{r} = (137.03599926808)^{-1} \quad (25)$$

and that this value is in full agreement with the measured magnitude [29,30].

In summary, Eqs. (22) and (24) can be respectively restated as

$$[g_\alpha]_{P_\alpha} + [g_\beta]_{P_\alpha} \equiv 0 \pmod{P_\alpha}, \quad (26)$$

the additive relation representing supersymmetry [9,10] in the field $\mathbb{F}_\alpha$, and

$$[g_\alpha]_{P_\alpha} [g_\beta]_{P_\alpha}^{-1} \equiv -1 \pmod{P_\alpha}, \quad (27)$$

a corresponding multiplicative condition. However, the unusual factor structure exhibited by $[g_\beta]_{P_\alpha}^{-1}$ in Eq.(23), specifically, that $[g_\beta]_{P_\alpha}^{-1} \in \{B_x\}$, requires the special condition that $[g_\alpha]_{P_\alpha} \in \{B_x\}$ hold, an outcome legislated by Eq.(10). Only a very rare fraction of the primitive roots of P_α will satisfy this exceptional requirement and, without conformance to this condition, the crucial statements given in Eqs.(24) and (25) would not follow. For the physically relevant values [9,10,13] of P_α , this fraction is less than one in $\sim 10^{47}$.

The magnitude, smoothness, and factor structure of $[g_\beta]_{P_\alpha}^{-1}$ correspond to a light propagating particle [13] whose properties would resemble those of the electron neutrino which has been prospectively identified [9,10] with a mass parameter of g_α and a physical mass of ~ 0.808 meV. From the symmetry expressed by Eq.(27), we will provisionally assign $[g_\beta]_{P_\alpha}^{-1}$ as the mass number of the muon neutrino ν_μ . This gives the physical mass corresponding to the muon neutrino ν_μ as

$$m_{\nu_\mu} \cong 27.68 \text{ meV}, \quad (28)$$

a value that is consistent with both the available data and the range expected for this particle [11].

B. Unified Strong-Electroweak Coupling Constant α^*

The results presented in Section III.A above lead directly to an expression for the magnitude of the unified strong-electroweak coupling constant α^* . A previous study [9] derived the relationship

$$(\alpha^*)^{-1} = \frac{P_\alpha - 1}{g_\alpha^2} \cong 34.26 \quad (29)$$

on the basis of characteristics of the mass scale. However, the introduction of the concept of inverse states, the results given in Eqs.(20), (22), (23), and (26), and the identification of m_{ν_μ} with the quantity $[g_\beta]_{P_\alpha}^{-1}$, enable Eq.(29) to be immediately recast in the form of the fundamental mass ratio

$$\alpha^* = \frac{m_{\nu_e}}{m_{\nu_\mu}} = \frac{[g_\alpha]_{P_\alpha}}{[g_\beta]_{P_\alpha}^{-1}} \cong (34.26)^{-1}. \quad (30)$$

The expression given in Eq.(30) has several attractive properties. First, it gives a magnitude that stands in good correspondence with the anticipated range [7,8,16,17] of α^* . Further, it simultaneously relates the utterly basic quantity α^* mathematically to the ratio of two primitive roots that are members

of a restricted set $\{g_x\}$ of special elements in the field $\mathbb{F}_{p_\alpha}$ and physically to the mass ratio of two fundamental particles. Moreover, the integers $[g_\alpha]_{p_\alpha}$ and $[g_\beta]_{p_\alpha}^{-1}$, along with their corresponding respective supersymmetric partners $[g_\beta]_{p_\alpha}$ and $[g_\alpha]_{p_\alpha}^{-1}$, are all primitive roots by Lemma 2. This enables them to serve collectively as the generators of the mass scale and the organizers of the eight-dimensional index space that classifies all particle states. Fig. (2), in parallel with the model shown in Fig. (1), illustrates the relationships connecting these four primitive roots. Basic physical entities and correspondingly fundamental mathematical quantities unite to (a) construct the particle mass scale and (b) specify the magnitude of the cardinal physical parameter governing their interactions.

MASS SCALE OCTET GENERATORS

P_α, m_{mo}

$P_\alpha \equiv 1(\text{mod } 4)$

Fig. (2): Illustration of the primitive root states that organize the mass scale, construct the eight-dimensional index space, and specify the unified strong-electroweak coupling constant α^* . The magnitude of P_α represents the monopole mass m_{mo} , E_0 denotes the energy unit, and v_1 a boson neutrino with a mass equal to E_0 . See references [9,10,13,22] for further discussion.

C. Definition and Computation of the Higgs Mass

The analysis presented above demonstrates how the definition of inverse states extends the previously developed four-dimensional index space [9] to eight dimensions, thereby enriching its classificational capacity. Inspection of the pattern represented by the inverse states immediately identifies a fundamental degeneracy. For a particle P , it is the condition of the equality of the mass parameters for the supersymmetric P_{ss} and inverse P_{in} states; hence,

$$B_{ss} = B_{in}, \quad (31)$$

a statement signaling a point of symmetry. Furthermore, this degeneracy only occurs for two critical values of the power index

$$t_{\pm} = \frac{P_{\alpha} - 1}{2} \pm \frac{P_{\alpha} - 1}{4}. \quad (32)$$

From Eq.(32) it is apparent that the existence of the degeneracy given by Eq.(31) requires $P_{\alpha} \equiv 1 \pmod{4}$, a condition made necessary earlier by several other considerations [9,10,13]. Accordingly, this degeneracy appears in Table IV for $t = 9$ and $t = 27$. In these two cases, the residues represent the equivalent supersymmetric pairs (6, 31) and (31, 6). It can be further shown that the magnitudes of these residues are unique, since they are invariant over the full set of primitive roots of the prime P_{α} . It follows that these two residues (e.g. 6 and 31) depend solely on the value of the prime modulus P_{α} , the large scale parameter in the counting system, and obey the quadratic congruence

$$x^2 \equiv -1 \pmod{P_{\alpha}}. \quad (33)$$

Indeed, it can be shown [23] that Eq.(33) has a solution if and only if $P_{\alpha} \equiv 1 \pmod{4}$, the condition stipulated by Eq.(32). Finally, these two residues always have order 4, hence, the quartic congruence

$$x^4 \equiv 1 \pmod{P_{\alpha}} \quad (34)$$

simultaneously holds.

The unique status of these residues that is established by Eqs.(31), (32), and (33), in alliance with the fact that they depend only on the modulus P_{α} , whose value is fully determined by the large scale parameters of the universe [13], nominates these supersymmetric states as candidates for the Higgs particle. With this assignment, we can write from Eq.(33) the mass parameter of the Higgs system as

$$B_{\text{Higgs}}^2 \equiv -1 \pmod{P_{\alpha}}. \quad (35)$$

We now explore the consequences of this prospective identification.

Consider the definition of the Higgs mass presented in Fig. (3). The defining statement given by Eq.(31) is expressed by the simultaneous fulfillment of two conditions, one additive and the other

multiplicative. These conditions respectively relate (1) the supersymmetric (X,Y) pair mass distribution and (2) the inverse state definition. They are

$$[X]_{P_\alpha} + [Y]_{P_\alpha} \equiv 0(\text{mod } P_\alpha) \quad (36)$$

and

$$[X]_{P_\alpha} [Y]_{P_\alpha}^{-1} \equiv 1(\text{mod } P_\alpha) . \quad (37)$$

Satisfaction of Eqs.(36) and (37) leads to the statement given in Eq.(31), the degeneracy of the P_{ss} and P_{in} states and the specification of the supersymmetric residue pair uniquely dependent upon the modulus P_α .

D. See-Saw Mass Congruence

The results expressed in Eqs.(27) and (35) immediately lead to the relation

$$[g_\alpha]_{P_\alpha} [g_\beta]_{P_\alpha}^{-1} - [B_{\text{Higgs}}]_{P_\alpha}^2 \equiv 0(\text{mod } P_\alpha) \quad (38)$$

and the see-saw statement

$$[g_\alpha]_{P_\alpha} [g_\beta]_{P_\alpha}^{-1} \equiv [B_{\text{Higgs}}]_{P_\alpha}^2 , \quad (39)$$

HIGGS MASS DEFINITION

Fig. (3): Defining condition for the supersymmetric Higgs particle.

a fundamental expression that connects the electron neutrino mass, the muon neutrino mass, and the masses of the supersymmetric Higgs pair. Importantly, since Eq.(39) is written as a congruence, it holds simultaneously for the masses of both particles comprising the supersymmetric Higgs system.

An explicit evaluation of the Higgs masses can be made, since g_α and P_α are known [9,10,13] from Eqs.(20) and (21), and the critical power indices $t_\pm$ are given by Eq.(32). In order to simplify the calculation, we can use the smallest primitive root of P_α , which has the value $g_{\min} = 14$, since the residues corresponding to the supersymmetric Higgs pair are independent of the primitive root and depend only on the modulus P_α . Hence, we evaluate the congruence

$$\frac{P_\alpha - 1}{g_{\min}^4} \equiv B_{\text{Higgs}} \pmod{P_\alpha} \quad (40)$$

and obtain the mass parameter of the Higgs boson as

$$B_{\text{Higgs}}^b = 5295067887317048385691240239639344309098972470130691907921115, \quad (41)$$

an integer composed of the factors

$$5, 13, 31, 109, 677, 25717, 165235181, 3158418359, \\ 5645180551, 470014220572174240309. \quad (42)$$

The mass parameter presented in Eq.(41) gives a physical mass of $\sim 9.56 \times 10^{18}$ GeV. The corresponding supersymmetric Higgs fermion is then found directly by the condition for supersymmetry through

$$B_{\text{Higgs}}^f = P_\alpha - B_{\text{Higgs}}^b \\ = 1464518162437886934175466847230415731421582430740004414836186, \quad (43)$$

a mass parameter having the factor structure

$$2, 3, 29, 37, 344353, 204901219, 14323448690905379, \\ 225085635424845454780404199 \quad (44)$$

and representing a physical mass of $\sim 2.64 \times 10^{18}$ GeV. Consequently, both members of the supersymmetric Higgs doublet possess masses $> 10^{18}$ GeV. These results given by Eqs. (41) and (43) were confirmed by a calculation using the primitive root g_α . Therefore, since the two neutrino species have a lower bound on their masses (< 30 meV) and the Higgs particles are both superheavy ($> 10^{18}$ GeV), we find that Eq.(39) contains no representative of an intermediate scale (~ 100 GeV) mass. This outcome contrasts sharply with customary see-saw relationships [31] developed originally from Lagrangian field theory [11,32,33]. The modular see-saw given by Eq.(39) expresses a fundamentally different physical conclusion. Further, since the mathematical procedures used to compute the Higgs

mass with Eq.(40) are entirely divergence-free, no renormalization is necessary, a second result that highlights the contrast with findings derived by customary quantum field theoretical methods [6-8,34,35].

The see-saw congruence given by Eq.(39) expresses a deep connection between the concepts of space and mass. In particular, Eq.(39) states that the Higgs system specifies the set of generators $\{g_\alpha, g_\beta, g_\alpha^{-1}, g_\beta^{-1}\}$ of the mass scale illustrated in Fig. (2). The mass parameters corresponding to the supersymmetric Higgs doublet cannot be generators themselves, since they satisfy the quadratic congruence given by Eq.(35) and are thereby barred from status as primitive roots of P_α . Further, since the magnitudes of the Higgs residues are determined in Eq.(40) solely by the value of the modulus P_α , the see-saw congruence founds the generators of the particle masses directly on the large scale characteristics of space. Hence, the cosmic see-saw creates a new archetype by expressing the fusion of the concepts of space and mass.

E. Higgs Mass and the Many-Body Problem

Quantum field theoretical procedures have been applied to the computation of the Higgs mass [6-8,34,35]. Electroweak theory requires quantum loop corrections that are intrinsically divergent and lead to an estimate of the Higgs mass that is in the superheavy regime ($> 10^{16}$ GeV). Basically, the underlying theory has a many-body character and these corrections are a manifestation of that feature. Hence, the calculation of the Higgs mass in quantum field theory is a basic many-body problem, an outcome that reflects the fundamental nature of the Higgs mechanism and our concept of the vacuum [19].

The result expressed by Eq.(40) for the mass parameters of the supersymmetric Higgs doublet is an exact value obtained by mathematical procedures that do not admit a perturbative approach [13]. With the assumption that the problem is properly formulated, the only result obtainable is the unique correct solution. In the approach using finite fields that leads to Eq.(40), this occurs because of the large number of hidden constraints that are present. Basically, the solution is gridlocked by the combined influence of the magnitude and the factor structure of the integer. This exceptional intolerance to computational approximation follows as a consequence of the profound influence of unity, at any number scale, on the factor structure of integers. Therefore, since there exists no neighborhood about the exact result in which an approximation is meaningful, the procedures of perturbation theory are invalid.

We conclude with a statement of the obvious conjecture, namely, that the counting procedure intrinsic to the theoretical framework provided by a finite field automatically and perforce yields the exact solution to a properly posed many-body problem. Further, if this conjecture holds for the Higgs particle, we may expect that is valid for any system, potentially even proteins. In the case of proteins, the verification of this conjecture would allow a fundamental reformulation of the protein folding problem, a central question of immense importance in biology and medicine.

III. Conclusions

The construction of inverse states in a finite field $\mathbb{F}_{P_\alpha}$ enables the organization of the mass scale with fundamental octets in an eight-dimensional index space that classifies particle states with residue class designations. Conformance with both CPT invariance and the concept of supersymmetry follows as a direct consequence of this formulation. Based on two parameters (P_α and g_α) that are anchored on a concordance of physical data embracing both large scale cosmic parameters and small scale couplings [9,10,13], this analysis leads to (1) a prospective mass for the muon neutrino of ~ 27.68 meV, (2) a value

of the unified strong-electroweak coupling constant $\alpha^* = (34.26)^{-1}$ that is physically defined by the electron and muon neutrino mass ratio, and (3) a see-saw congruence connecting the Higgs, the electron neutrino, and the muon neutrino masses. Specific evaluation of the masses of the corresponding supersymmetric Higgs pair reveals that both particles are superheavy ($>10^{18}$ GeV). No renormalization of the Higgs masses is necessary, since this theoretical approach intrinsically excludes the possibility of divergences. Further, the Higgs fulfills its conjectured role through the see-saw relation as the particle defining the origin of all particle masses, since the mass parameters of the electron and muon neutrino systems, together with their supersymmetric partners, are the generators of the mass scale and the corresponding index space. Finally, since the computation of the Higgs masses is entirely determined by the modulus P_α , which is fully defined by the large-scale parameters of the universe through the value [36] of the universal constant G and the requirement [9,10,13,37] for perfect flatness ($\Omega = 1.0$), the cosmic see-saw congruence provides an explicit unifying statement for the concepts of mass [1,2] and space [18,19]. As inseparable entities in this picture, these two physical archetypes constitute an identity. This outcome mirrors the fundamental connection between space and geometry in general relativity.

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