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A PROPOSAL FOR THE DEVELOPMENT OF TECHNIQUES
FOR THE IMPLEMENTATION OF CONTROL TO
EXPERIMENTAL CHAOTIC SYSTEMS

Final Technical Report

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FINAL REPORT

1 CONTROLLING COMPLEXITY

The following papers resulted from our proposed work in controlling complex dynamical systems. These systems include those with many attractors and complicated basin structures, to the point where relatively small perturbations can be used to move from one basin to another. Related to this area is our discovery of and subsequent work on Wada basin boundaries, where three or more attractors have basin boundaries that are intertwined so that every point on the boundary between two of the basins is also on the boundary of all other basins; such systems have unusual flexibility for the type of control considered here.

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2 COMMUNICATING WITH CHAOS

The following papers concern our proposed work on using controlled chaotic signals for communication. For the transmitting site, we demonstrated how to use small perturbations of a dynamical system to achieve a desired coding sequence in its symbolic dynamics. For the receiving site, we discussed conditions under which a dynamical system with input from the transmitted signal will synchronize with the transmitter in order to output the desired symbol sequence. One important discovery we made was that by controlling the dynamics of the transmitter on an appropriate chaotic saddle, we can reduce the incidence of transmission errors without significantly degrading the throughput of the system, as measured by its topological entropy.

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12. E. Rosa and E. Ott, Mixed basin boundary structures of chaotic systems, *Phys. Rev. E* 59 (in press).

3 TARGETING

In the following papers we have continued to explore the theory and applications of the method for controlling chaos we proposed in 1990, with particular emphasis on the problem of targeting a system from one state to another. Our original method of chaos control is to convert chaotic motion to one of the infinite number of unstable time-periodic motions contained within the chaotic attractor by making only small time-dependent perturbations to an available system parameter. In targeting, on the other hand, one rapidly takes the system state from an initial point to some specified targeting region in phase space by applying small controls. The purpose of targeting is not only to control the final state of a system, but also to drastically reduce the amount of time spent in a transient chaotic state before the controlled state is reached. Thus we have also considered the problems of optimizing cost (in time, energy, *etc.*) to reach a given final state, and of optimizing the controlled state. In the latter case, our main result is that the optimal controlled state is typically a low-period unstable periodic orbit, which validates the approach commonly taken for pragmatic reasons in applications of chaos control.

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16. M. S. Baptista, Targeting applying e-bounded orbit correction perturbations, *Int. J. Bifurc. Chaos* (to appear).

4 DETERMINISTIC MODELING

The following papers resulted from our proposed study of deterministic modeling and its limitations. We have considered the extent to which the property of shadowing, known to hold for hyperbolic systems, holds also for nonhyperbolic systems, which are common in applications. Shadowing concerns how well a numerically computed trajectory of a dynamical system compares with an exact trajectory of the system, and also how differences between a physical system and a deterministic model can make

the evolution of the system differ from that of the model. We have identified specific mechanisms that can cause shadowing to fail, and offered methods for quantifying the time scale over which shadowing can still be expected to hold in the presence of nonhyperbolicity. We have also considered other phenomena that are impediments to predictability in deterministic modeling. In particular, we have continued to investigate the incidence and properties of riddled and intermingled basins, in which a dynamical system with a symmetry can have multiple attractors whose basins of attraction are intertwined at all scales; if there is any uncertainty in the initial condition of a trajectory, no matter how small, then each of the attractors has a positive probability of attracting the trajectory. In addition, we have also addressed basic questions of model building using experimental data, and, in particular, the theory of embedding for chaotic dynamical systems.

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2. E. Kostelich, J. A. Yorke, Z. You, Plotting stable manifolds: error estimates and noninvertible maps, *Physica D* 93, 210–220 (1996).
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5 OTHER RESEARCH

Other research supported by DOE over the last three years, reflected by the following papers, includes among other topics work on: the interface between chaotic and stable periodic behavior as system parameters vary; bifurcations of non-smooth systems that describe impact oscillators; phenomena that occur in quasiperiodically forced systems; and the fractal and topological properties of chaotic invariant sets, in particular those arising in fluid flow.

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5.0.1 BOOKS

The following books resulted in part from research supported by DOE over the last 16 years.

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2. C. Grebogi and J. A. Yorke, eds., *The Impact of Chaos on Science and Society*, United Nations University Press, Tokyo, 1997.
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4. E. Ott, *Chaos in Dynamical Systems*, Cambridge University Press, 1993.
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