Are Net Discount Rates Stationary?:
Some Further Evidence

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Michael Nieswiadomy
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ABSTRACT

Gamber and Sorensen provide evidence suggesting that the net discount ratio experienced a level shift in the mean between 1977 and 1981. If such a shift occurred, the nonlinearity in the data shows up as a failure to reject the null hypothesis that a unit root is present; that is, the series is I(1). In this reply, evidence is presented—the Phillips-Perron test and a univariate version of the Stock-Watson q-test—suggesting that the net discount ratio is stationary. Hence, the mean is constant. In addition, if one extends the analysis to include the 1989 through 1993 period, the net discount ratio appears to be reverting.

Introduction

In a previous article in this Journal (Haslag, Nieswiadomy, and Slottje, 1991), we provided evidence consistent with the notion that the net discount ratio is a stationary time series. Gamber and Sorensen’s comment reexamines this issue and claims that the net discount ratio is nonstationary. Using techniques developed in Balke and Fomby (1991), the nonstationarity reflects a nonlinearity in the time series; more precisely, a mean shift is identified as having occurred sometime during the period 1977 through 1981. The upshot of the Gamber-Sorensen analysis is that we substantially underestimated the post-mean shift by using the full sample mean.

Gamber and Sorensen’s primary concern is that we chose to use only four lagged values of the change in the net discount ratio in augmented Dickey-Fuller specifications. Gamber and Sorensen proceed, selecting the “optimal” lag length by applying both the Akaike Information Criterion and a procedure seeking the longest lag specification in which the coefficient is significant. Although our lag-length selection procedure was ad hoc, the procedures em-

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ployed in Gamber and Sorensen do not really address the critical issue. Including extra lags is one way to reduce serial correlation in the residuals. Since the Dickey-Fuller tests assume that errors are not serially-correlated errors, the appropriate lag length is one in which the observed residuals are not serially correlated.¹ The procedures employed by Gamber and Sorensen focus on the tradeoff between efficiency and explanatory power. They report, but do not discuss, their tests for serial correlation. In effect, Gamber and Sorensen’s real concern is presumably that the parameter of interest is imprecisely estimated.

In this reply, the econometric evidence is reconsidered in light of the serial correlation issue. Our evidence focuses on two issues: serially-correlated errors and robustness, using alternative unit root tests. In one sense, the results in Haslag, Nieswiadomy, and Slottje (1991) are flawed; the ad hoc lag-length specification suffers from serially-correlated errors. Using lag length specifications in which the errors are not serially correlated, the augmented Dickey-Fuller test fails to reject the null hypothesis that a unit root is present. Alternative unit root tests, however, strongly reject the unit root hypothesis: more specifically, the Stock-Watson (1988) q-statistic and the Phillips-Perron (1988) test for unit roots strongly reject the null hypothesis that a unit root is present.² Based on such mixed evidence, we cannot reject the hypothesis of nonstationarity.

In addition, when one includes data over the 1989 through 1993 period, the net discount ratio has been rising and is now nearly identical to the full sample mean. In light of the evidence from the last four years, Gamber and Sorensen’s conclusion that a mean shift in the net discount ratio occurred seems less compelling. Thus, the evidence still suggests that the net discount ratio is stationary and has not experienced a (permanent) mean shift. Thus, the interpretations made in HNS are still appropriate.

The Evidence

Table 1 reports the results obtained from augmented Dickey-Fuller specifications. The general form of the test is to estimate a regression of the form

\[ \Delta x_t = a + bx_{t-1} + \sum_{i=1}^{n} c_i \Delta x_{t-i} + u_t, \]  

where \( \Delta \) = the difference operator,  
\( x = (1+g)/(1+r) \) = the net discount ratio with \( g \) as the growth rate of real wages and \( r \) the (ex post) real interest rate,  
\( u \) = the error term, and  
\( a, b, \) and \( c \) = parameters.


² Schwert (1989) uses Monte Carlo analysis to examine the usefulness of various unit root tests when a moving average component is present. The Phillips-Perron test tends to frequently identify unit roots in stationary series when the moving average term is large. For the net discount ratio, the moving average component appears to be small.
The equations reported in Table 1 differ only by the number of lagged values, \( n \), in equation (1). The by-now familiar test-statistic is calculated under the null hypothesis that \( b = 0 \); that is, a unit root is present.

<table>
<thead>
<tr>
<th>Number of Lags</th>
<th>Unit Root Test Statistic</th>
<th>Serial Correlation Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-3.49*</td>
<td>3.01*</td>
</tr>
<tr>
<td>10</td>
<td>-2.35</td>
<td>0.72</td>
</tr>
<tr>
<td>12</td>
<td>-2.23</td>
<td>0.13</td>
</tr>
<tr>
<td>17</td>
<td>-2.53</td>
<td>0.01</td>
</tr>
</tbody>
</table>

* Significant at the 5 percent level.

As in Haslag, Nieswiadomy, and Slottje, the sample period is 1964 through 1989. The results are reported only for the three-month Treasury bill. The results for the six-month, one-year, and three-year notes are qualitatively similar and are available from the authors upon request.

The results presented in Table 1 indicate that the null hypothesis that \( b = 0 \) can be rejected when only four lagged values of \( \Delta x \) are included in the specification. However, the third column in Table 1 reports the Breusch-Godfrey statistic calculated under the null hypothesis that the \( u \)’s are not serially correlated. According to the Breusch-Godfrey statistic, the residuals are serially correlated. Thus, the evidence suggests that the residuals are serially correlated in the regression with only four lagged values of \( \Delta x \), casting doubt on the validity of the inference. Indeed, when one includes more lags (10, 12, or 17), the evidence suggesting serial correlation fades, but a unit root is suggested. Thus far, the evidence concurs with Gamber and Sorensen’s findings.

Our next question is to consider additional tests for unit roots. The low power of the augmented Dickey-Fuller test is demonstrated in DeJong et al. (1992) and is chiefly responsible for the proliferation of unit root tests. Specifically, the tests proposed by Stock and Watson and Phillips and Perron are employed. The test results for the 1964 through 1989 period are reported in the top half of Table 2. At each lag length, both the Stock-Watson \( q \)- and Phillips-Perron statistics strongly reject the notion that a unit root is present in the net discount ratio, contradicting the augmented Dickey-Fuller results.

In addition, Figure 1 plots the net discount ratio over the 1964:4 through 1993:5 period. Gamber and Sorensen argue that the mean net discount ratio shifted and, consequently, that using the full sample is inappropriate. Since

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3 It should be noted that these results differ somewhat from those presented in Haslag, Nieswiadomy, and Slottje. One reason is that the consumer price index was rebenchmarked since we initially conducted our analysis.
Table 2
Results from Alternative Unit Root Tests

<table>
<thead>
<tr>
<th>Number of Lags</th>
<th>Stock-Watson</th>
<th>Phillips-Perron</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964-1989 Sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-47.32*</td>
<td>-41.93*</td>
</tr>
<tr>
<td>12</td>
<td>-47.02*</td>
<td>-45.06*</td>
</tr>
<tr>
<td>17</td>
<td>-44.98*</td>
<td>-51.43*</td>
</tr>
<tr>
<td>1964-1993 Sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-48.08*</td>
<td>-43.86*</td>
</tr>
<tr>
<td>12</td>
<td>-48.23*</td>
<td>-47.35*</td>
</tr>
<tr>
<td>17</td>
<td>-46.89*</td>
<td>-54.63*</td>
</tr>
</tbody>
</table>

Note: The 5 percent critical value for the q-statistic is -27.9, and the Phillips-Perron (T(p-1) test is -21.8.
* Significant at the 5 percent level.

Figure 1
Net Discount Ratio 1964-1993, Monthly

1989, the net discount ratio has been rising and now appears close to the average level that existed in the pre-1980 sample. Figure 1 suggests that a single, sample-wide mean is appropriate and is corroborated by the unit root test results.

Gamber and Sorensen make an important point in asking what is the “optimal” forecast of the net discount ratio? The evidence does not suggest that a permanent shift in the mean occurred. However, the first two coefficients for the autocorrelation function are 0.90 and 0.82, suggesting that there is substantial persistence in the net discount ratio. Although the evidence supports the notion that the net discount ratio is stationary, there is an important implication for forecasts of the net discount ratio. If one is primarily interested in near-term forecasts of the net discount ratio, then an “optimal” forecast would take into account any persistence. If one is trying to predict the net discount ratio
at some distant point in the future, the time series seems to revert to its (appar-ently) constant mean.

**Summary and Conclusion**

Gamber and Sorensen raise an important issue in the unit root literature and apply their question to the issue of whether the net discount ratio is stationary. As they demonstrate, the specification of lag lengths in the augmented Dickey-Fuller regression can be important. The appropriate guide should be that the residuals from the regression are not serially correlated. In HNS, four lags were used. The evidence, however, suggests that this ad hoc lag length selection was too short to satisfy not-serially-correlated errors criterion. Based on only the augmented Dickey-Fuller tests, longer lag lengths would not lead to rejection of the hypothesis that a unit root is present in the net discount ratio. In this reply, however, two additional pieces of evidence are considered. First, other unit root tests strongly reject the presence of a unit root. Second, plotting the data reveals that the net discount ratio has been rising over the last several years and is approximately equal to the full sample mean initially reported in Haslag, Nieswiadomy, and Slottje. This additional evidence leads us to conclude that significant nonlinearities are not present in the net discount ratio and that the variable can be represented as a stationary time series.

The Gamber-Sorensen comment raises an important, heretofore unresolved, methodological issue. Plotting the time series is useful, but temporary trends can show up as nonlinearities in the time series representations. In short, there is an inherent danger in identifying a recent experience as a permanent shift in the time series.4 Such an identification means that the series is subject to permanent shocks in the future. Clearly, the series might revert, making such an identification wrong. In the case of the net discount ratio, the series has some persistence and can (temporarily) deviate from its mean for periods long enough to appear to have experienced a permanent shift. Indeed, an “optimal” forecast of the net discount ratio should take this persistence into account.

Overall, we maintain that the additional evidence supports our original conclusion that the net discount ratio is a stationary series.

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4 Christiano and Eichenbaum (1990) argue that the most important feature of a nonstationary series is that the forecast error variance is infinitely large as the forecast horizon becomes infinitely long.
References


Christiano, Lawrence J. and Martin Eichenbaum, 1990, Unit Roots in Real GNP: Do We Know, Do We Care? Carnegie-Rochester Conference Series on Public Policy, 32: 7-62.


