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Abstract.

A model for void coalescence for high strain rate ductile damage in metals is presented. The basic mechanism is void linking through an instability in the intervoid ligament. The formation probability of void clusters is calculated, as a function of cluster size, imposed stress, and strain. A wave speed limiting is applied to the cluster size enhancement of cluster growth. Due to lack of space, model formulas are merely described and not derived..

INTRODUCTION

High strain rate ductile fracture is caused on the microscopic scale by the nucleation, growth, and link up of voids. We present a model for small scale void cluster growth via the coalescence of initially existing voids, based on earlier work. [1-3]. Void nucleation for cluster growth is not included. Inertia is not considered. Hence, the model is not suited for the growth of macroscopically large cracks. Due to lack of space, formula derivations are omitted.

The general phenomenology envisioned for high and lower strain rates is as follows. At high strain rates, a random initial void configuration gives rise to a spatially disordered damage morphology, where widely separated voids have little time to communicate with each other via stress waves. In other words, when voids coalesce into a cluster, there is not time for the enhanced stress and strain fields to form at cluster boundaries. This retardation inhibits the potential for large damage clusters to grow faster than the smaller ones. As a result, the sample breaks when widespread and uncorrelated damage finally accumulates to the point where a damage surface forms, breaking the sample. At lower strain rates, the ductile damage tends to consist of flat, disk-like clusters or cracks, whose growth does benefit from the size enhancement of growth mentioned above. The sample breaks with little general damage, when the biggest crack rapidly outstrips its neighbors. It can do so because there is time for the size enhancement of stress and strain fields to occur at its periphery. Consequently, the strain to fracture in the low strain rate case is significantly less than in the high strain rate case. [1,3].

Two analytical models have been formulated to explain the point of fracture. At high strain rates, the point of fracture is modeled with random percolation theory [4]. The organized clusters at low strain rates are modeled by refining a probabilistic theory for cluster growth [5].

GENERAL PHYSICAL MODELING AND FORMULAS

The growing voids must coalesce or link up in order to form a continuous internal surface that separates the sample. This will occur in dynamic situations when the intervoid ligament undergoes a localized mechanical instability that rapidly thins it out and causes an elastic unloading in the surroundings [6-8]. Local straining is necessary to thin out the intervoid ligament once the applied stress is great enough to establish the local instability.

The average amount of external strain to add a new void to a cluster, *i.e.*, thin out the separating intervoid ligament, depends on the cluster size. It is appreciable for small clusters but can become very small for large clusters because the external strain is greatly amplified at the periphery of a large cluster.

The stress conditions triggering the local instability depend on cluster geometry and the applied stresses and were given by Thomason [6-8] for arrays of voids for the quasistatic situation. We have generalized this work to treat single voids linking to an existing void cluster with an enhanced range that comes from the enhanced stress and strain fields at the cluster periphery. We have included both shear and tensile stresses and linking strain effects.

We seek the most probable damage configuration. We assume disks of linked voids that have their major surfaces perpendicular to the direction of greatest principal stress, the "I" direction. This orientation, which is appropriate for plate impact spallation, intercepts the greatest applied stress and causes the greatest cluster size enhancement of the applied stress at the cluster periphery. The disks are assumed to be one void thick. The disks will be grown from the basic voids and will not interact. This should be accurate for the early and middle regions of damage growth.

A disk is "grown" in a stepwise fashion by adding rings of linked voids to a previously existing disk. The ring voids and the disk periphery voids are given the same model stress and strain linking ranges, which are enhanced by the disk size.

The *stress* linking range of two potential ring voids, r_σ, i. e. the threshold center to center separation below which the local intervoid instability occurs, is qualitatively based on two dimensional results of Thomason [6] for void arrays and plane strain slip line fields [9-10]:

$$r_\sigma = D\left[1 + g(\sigma_I/\sigma_y)(2r/D)\right], \qquad (1)$$

where D is the void diameter, r is the disk radius, σ_I is the effective macroscopic stress at the disk surface, σ_y is the uniaxial plastic yield stress, and g is a parameter roughly equal to 1. The factor $2r/D$ approximates the effect of the disk size to enhance the local stress/strain at the periphery of the disk. Eq. (1) embodies the approximations that r_σ is linear in the applied stress and the cluster size, r.

The effective stress, σ_I, is based on the root mean average of the forces which act perpendicular and parallel on the disk surface [11].

Once a void ligament instability is established, external strain is still required to thin it down. The thinning is assumed to depend on ε_I, the accumulated effective external strain that includes both shear and tensile components, but which is incremented only when the instability is active. For a given amount of strain ε_I, pairs of voids closer than a certain model distance, r_ε, the strain intervoid linking distance, will have coalesced. A 2D derivation of the formula for r_ε depends on the following assumptions. The effective relative displacement of material above and below the intervoid axis necessary to sever the intervoid ligament is taken as $d_t=(d/2)(1+d/(3D))$, where d is the edge to edge void separation. (For small d, the value $d_t= d/2$ was given by Thomason [6].) A strain field enhancement factor at the disk periphery of $2r/D$ is assumed. A geometric factor relating the macroscopic strain ε_I to the local ligament strain is also used. Then,

$$r_\varepsilon = D\left[1 + \eta\varepsilon_I r/D\right] \qquad (2)$$

where a reasonable guess for η is 12.

The probability of formation, $p(c)$, for a ring of circumference c around a disk of voids of radius r is given approximately by:

$$p(c) \approx \alpha \exp(-c\gamma/fD), \qquad (3)$$

where f is given by: $f = 1 + \eta\varepsilon_I r/D$.

See Domb [5]. γ is given by the nontrivial solution of the equation: $\beta\exp(-\beta) = \gamma\exp(-\gamma)$ where: $\beta = \alpha' f^3 \rho D^3$.

α' is a constant of magnitude about π, and ρ is the number density of void centers. The quantity γ is transcendental and must be approximated. For $\beta > 4$, for example, γ is close to $\beta\exp(-\beta)$. For $\beta < 0.25$, γ is close to $\beta^{1/2}-\ln\beta$.
The prefactor α is an algebraic combination of β and γ: $\alpha = (\beta-\gamma)/(\beta(1-\gamma))$

Using Eq. (3) for p(c), we can finally find the probability $P(r)$ for a disk to grow to radius r, given an initial void at r=D/2. We add rings having the sequence of radii r_i, where: $r_{i+1} = r_i + r_{\varepsilon,j}/2$.

$r_{\varepsilon,i}$ are obtained from Eq.(2) for r_ε with r set equal to r_i and r_1 set to D/2. The width of each new ring is taken to be half the linking range, as the most probable value. The probability of formation for each ring i is given by $p(c)$ with $2\pi r_i$ inserted for c, and for α, β, and f, the r value r_{i-1} is used. This is because the new disk of size r_i depends on the size of the previous disk, r_{i-1}. Thus, the probability, $P(r_i)$, for a disk to grow to a radius at least r_i is given by the product of the probabilities of formation of each incremental radius r_i:

$$P(r_i) = \prod_{k=1,j-1} \alpha_k \exp(-\gamma_k 2r_{k+1}/(Df_k)) . \qquad (4)$$

$P(r_i)$ includes all future growth processes. Each factor is a conditional probability which depends on its corresponding disk size, through f_k. The probability for a disk of radius r to exist is obtained by multiplying $P(r)$ by the probability that no further growth occurs, *i.e.* the probability that no voids exist within the link range of the disk.

Equation (4) can be exponentiated and approximated by an integral using the correspondence $\Delta i \rightarrow (2/3)d\beta/(\beta\eta\varepsilon)$ between the sum index and variable of integration. Let log(P) = $I_\alpha + I_\gamma$, where I_α, the contribution from the α_k prefactors is:

$$[2/(3\eta\varepsilon)]\int_{\beta_0}^{\beta_r}(\ln\alpha)d\beta/\beta , \qquad (5)$$

and I_γ, the contribution from the "exp(γ)" part is:

$$(\frac{4}{3}\pi/(\varepsilon\eta)^2)[-(1+\eta\varepsilon/2)\int_{\beta_0}^{\beta_r}\gamma d\beta/\beta]$$
$$+(\frac{4}{3}\pi/(\varepsilon\eta)^2)[(6\Phi_0)^{1/3}\int_{\beta_0}^{\beta_r}\gamma d\beta/\beta^{4/3}] . \qquad (6)$$

Log(P) above is a function of ε_I and β, which can be considered to be a scaled version of the disk radius, r. The scaling involves the strain ε_I and the porosity of the individual voids, $\Phi_0 = \pi\rho D^3/6$. ρ is the volume density of void centers. The integral version of log(P) above is especially useful for numerical implementation because the integrals are a function of only the single variable β and can be approximated with simple analytical forms. The upper integration limit, β_r, is β evaluated at the disk size *just previous* to the disk size of interest in the disk size series.

The integrals were approximated numerically by finding analytical approximations for γ that could be inserted in Eq. (6) and integrated to obtain elementary functions or gamma functions. The approximations used for γ for two ranges were noted earlier. The approximation for the first term in Eq.(6) worked well for Eq.(5). This procedure avoids a pitfall appearing if each integral is approximated separately after numerical integration. The two integrals often become very large and nearly cancel each other. In this case, even good approximations of the separate integrals will

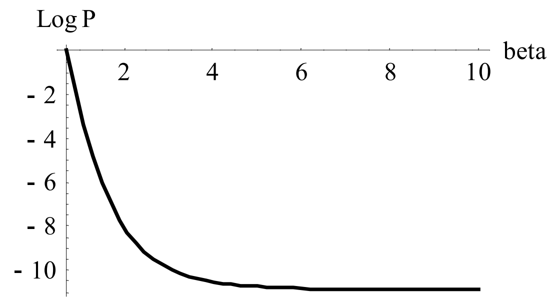

FIGURE 1. Log of void disk probability versus β for β_0=0.7, $6\Phi_0$=0.68, and ηε=0.36.

produce nonphysical results. To avoid this, it was also found useful to derive an analytical approximation to I_γ in the case that the limits of integration are nearly equal.

A graph showing log(P) is shown in Fig. 1. Note that the solution for large β, or large *r/D,* if D is less than r and is held fixed, asymptotes to a constant. Therefore, the disk probability does not die off with size but is finite. This is typical of a correlated growth process where bigger clusters link together at greater separations than smaller clusters. A cluster can grow arbitrarily large with probability one after reaching a threshold size. This leads to the result that large systems will surely break if enough linking strain has accumulated.

Time delay effects are added to the series (4) for $P(r)$ by limiting r/D in f and γ, but not in c, to Ct_k/D, where C is a release wave velocity and t_k is the total linking strain time. This will produce in $P(r)$ an exponential decay in some power of r, which will greatly curtail disk growth.

In the formulas above, D is a parameter. A law for the growth of D is needed for a complete model.

There are two fracture criteria for a computational cell. The first criterion assumes that failure occurs when the existence probability of a void disk large enough to span the cell is one: $P_I(L/2)N = 1$ where $N = \rho V$ is the number of void centers in the computational cell of volume V and edge *L*. This first fracture criterion will predominate at low strain rates where the cluster size enhancement of void linking has time to develop.

Stress linking volumes are imaginary volumes extending in every direction from the physical disk by half a stress linking range. [12,4]. The second, percolation fracture criterion is that the stress linking volumes of the disks sufficiently fill in the computational cell, so that no one dimensional path of solid and unlinked material, i.e. no strong beam of material, still exists that completely spans the cell. In this case, a sheet of stress linked voids spans the cell, the plastic flow localizes, and the cell breaks with little additional external strain. This criterion is equivalent to a random volume percolation of the disk stress linking range volumes. This percolation will occur when the sum of linking volumes per unit volume (overlaps included) equal 2.53. This criterion can be plausibly made to yield the 30% spallation porosity limit.[13] We give, without derivation, a final formula for the (second) percolation fracture criteria:

$$2.53 = 1.5\Phi_o(1 + C_1)^2\left[\overline{y}^2 + C_2'\overline{y}^3 / 2\right] \quad (7)$$

where C_I=g(σ_I/σ_y) and C'_2 = g($(\sigma_{II}+\sigma_{III})/\sigma_y$), $\overline{y}^2$ and $\overline{y}^3$ are averages of $(2r/D)^2$ and $(2r/D)^3$ over the disk cluster population. G is an adjustable parameter, and σ_I,σ_{II}, and σ_{III} are stresses in the indicated directions. With the further restriction of no coalescence or deviatoric stress, Eq. (7) yields a formula for a pressure dependent fracture porosity $\Phi_{frac} = 2.53/(1+g\Sigma_m/\sigma_y)^3$ where Σ_m is the volumetric tension.

We briefly describe without derivation some additional model parts. Probability functions for two and three void clusters were derived separately and the starting disk in the void disk series was pasted onto the 3 void cluster. This is necessary because the single and two void "clusters" are not well approximated by a disk. In doing the averages in Eq. (7) over disk and cluster sizes, it was found that, under most circumstances, including only the one, two, and three void clusters, the second disk, and the biggest, i. e. system sized, disk were sufficient. This works because, when the second disk becomes important, the disk probability has likely saturated to a constant and only the largest disk is relevant. A more complete treatment will appear in a larger paper.

CONCLUSION

In summary, we have developed a void cluster model of ductile damage based on void coalescence of existing voids. Inertia is neglected. At low loading strain rates, the biggest cluster has time to grow much more rapidly than smaller clusters to break the sample. At high loading strain rates, large clusters cannot grow any faster than smaller clusters so the sample breaks when enough clusters grow independently to form a fracture surface by random accumulation.

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