DAMAGE BEHAVIOR OF ALIGNED AND RANDOM FIBER REINFORCED COMPOSITES FOR AUTOMOTIVE APPLICATIONS

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ABSTRACT

Damage constitutive models based on micromechanical formulation and combination of micromechanical and macromechanical effects are presented to predict progressive damage in aligned and random fiber reinforced composites. To estimate the overall elastoplastic damage responses, an effective yield criterion is derived based on the ensemble-volume averaging process and the first-order effects of eigenstrains due to the existence of discontinuous fibers. Progressive interfacial fiber debonding models are subsequently considered in accordance with a statistical function to describe the varying probability of fiber debonding. First, an effective elastoplastic constitutive damage model for aligned fiber reinforced composites is proposed. A micromechanical damage constitutive model for two- and three-dimensional random fiber reinforced composites is then developed. Finally, the complete progressive damage constitutive model is implemented into finite element code DYNA3D to simulate the dynamic inelastic behavior and the progressive damage of the composite structures. This allows prediction of the mechanical response of large composite structures and eliminate the need for expensive large-scale experiments. The computational capability also plays a significant role in the optimizing the design of complex mechanical systems composed of composite materials.

KEYWORDS: Micromechanical damage constitutive models, Elastoplastic damage responses, Random fiber reinforced composites, Finite-element implementation

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1. INTRODUCTION

The goal to provide lighter-weight, more fuel-efficient automobiles capable of greater crashworthiness provides an incentive for continued development of advanced materials. Carbon fibers used in composites with a lightweight matrix, generally epoxy resin composites have attracted worldwide attention and hold great promise, but are in general significantly more brittle compared with other polymer composites. It is well known that organic matrix fiber-reinforced composites are very susceptible to impact damage, especially at low velocities. Low velocity impact can cause significant damage inside the composites in terms of delaminations and matrix cracks. Such damage is very difficult to detect and may cause significant reduction in the strength and stiffness of the materials. Light-weight random carbon fiber polymer matrix composite has the potential to satisfy the crashworthiness requirements, and because of its relatively low cost compared with other composite materials, it is a strong candidate for automobile applications.

In general, traditional continuum mechanics is based on the continuity, isotropy and homogeneity of materials. It cannot directly solve the problem for heterogeneous composites, since, microscopically, fibers or particles are present within the composites and have a significant effect on the mechanical properties of materials. Hence, micromechanics based models have been developed to solve the problem on a finer scale and to relate mechanics of materials to their microstructure. The derivation of the constitutive equations in form of a phenomenological parameter model from entirely micromechanical considerations creates a foundation for a rigorous analysis of composite structures.

Recently, a micromechanical analysis based on the modified Mori-Tanaka method was performed by Meraghi and Benzeggagh [1] to address the effect of matrix degradation and interfacial debonding on stiffness reduction in a random discontinuous-fiber composite. Their modeling was developed through a methodology of experimental identification of basic damage mechanisms, which involves amplitude analysis of acoustic emission and microscopic observations. Tohgo and Weng [2] and Zhao and Weng [3,4] proposed progressive interfacial damage models for ductile matrix composites. They used Weibull [5] probability distribution function to describe the probability of particle debonding. Most recently, Ju and Lee [6] developed a micromechanical damage model to predict the overall elastoplastic behavior and damage evolution in ductile matrix composites. In their derivation, to estimate the overall elastoplastic-damage behavior, an effective yield criterion was derived based on the ensemble-volume averaging procedure and the first-order effects of eigenstrains stemming from the existence of inclusions.

Following the work of Zhao and Weng [3,4] and Ju and Lee [6], we first propose an effective elastoplastic constitutive damage model for aligned fiber reinforced composites. A micromechanical damage constitutive model for three- and two- dimensional random fiber reinforced composites is then developed. The governing field equations and over all yield function for aligned-fiber orientations are averaged overall orientations to obtain the constitutive relations and yield function of two- and three- dimensional random fiber reinforced composites. In our derivation, fibers are assumed to be elastic (prolate) spheroids.
which are embedded in a ductile polymer matrix. Furthermore, the ductile matrix behaves elastoplastic under arbitrary three-dimensional loading/unloading histories. All fibers are assumed to be non-interacting for dilute composite medium and initially embedded firmly in the matrix with perfect interfaces. After the interfacial debonding between fibers and the matrix, these partially debonded fibers are regarded as equivalent, transversely isotropic inclusions. It is worth mentioning that since the scope of this work is to predict the overall damage behavior of composites globally, the local microcrack propagation and void nucleation at the interfaces are ignored in our derivation. However, it is possible to extend the proposed damage model to accommodate local damage evolution once new damage growth model and failure criterion are developed based on rigorous experiments. Finally, the complete progressive damage constitutive model is implemented into finite element code DYNA3D to simulate the dynamic inelastic behavior and the progressive damage of the composite structures.

2. OVERALL ELASTOPLASTIC BEHAVIOR OF COMPOSITES

2.1 Effective elastic moduli and elastoplastic behavior of composites with aligned discontinuous fibers

Let us start by considering an initially perfectly bonded two-phase composite consisting of a matrix (phase 0) with bulk modulus \( k_0 \) and shear modulus \( \mu_0 \), and aligned discontinuous, randomly dispersed (prolate) spheroidal fibers (phase 1) with bulk modulus \( k_f \) and shear modulus \( \mu_f \). When spheroidal inclusions (discontinuous fibers) are aligned in the \( l \)-direction, the composite as a whole is transversely isotropic. Subsequently, as loadings or deformations are applied, some fibers are partially debonded (phase 2), and these partially debonded fibers are regarded as equivalent, transversely isotropic inclusions. Following Zhao and Weng [3,4], a partially debonded fiber can be replaced by an equivalent, perfectly bonded fiber which possesses yet unknown transversely isotropic moduli. The transverse isotropy of the equivalent fiber can be determined in such a way that (a) its tensile and shear stresses will always vanish in the debonded direction, and (b) its stresses in the bonded directions exist, since the fiber is still able to transmit stresses to the matrix on the bonded surfaces.

By the help of the exterior-point Eshelby's tensor of an ellipsoidal inclusion, the effective elastic stiffness tensor of aligned (in the \( x_l \)-direction) can be explicitly derived as

\[
(C_{e})_{ijkl} = F_{ijkl}(t_1, t_2, t_3, t_4, t_5, t_6)
\]

where the parameters \( t_1, ..., t_6 \) are given in Lee and Simunovic [7].

We now consider the overall elastoplastic responses of progressively debonded fiber-reinforced composites which initially feature perfect interfacial bonding between fibers and the matrix in two-phase composites. It is known that partial interfacial debonding may occur in some fibers under applied loading. Therefore, an original two-phase composite may gradually become a three-phase composite consisting of the matrix, perfectly bonded fibers and partially debonded fibers. In what follows, we will regard partially debonded fibers as equivalent, perfectly bonded transversely isotropic fibers. For simplicity, the von Mises yield
criterion with isotropic hardening law is assumed here. Extension of the present framework to
general yield criterion and general hardening law, nevertheless, is straightforward.

An effective yield criterion is derived based on the ensemble-volume averaging process and
first-order effects of eigenstrains due to the existence of spheroidal (prolate) fibers. The
effective yield criterion, together with the assumed overall associative plastic flow rule and
hardening law, constitutes the analytical foundation for the estimation of effective
elastoplastic behavior of ductile matrix composites. By collecting and summing up all the
current stress norm perturbations produced by any typical perfectly bonded fiber and any
typical partially debonded fiber, and averaging over all possible locations, the ensemble-
averaged current stress norm at any matrix point can be derived as

\[ <H> = \sigma : \mathbf{T} : \sigma \]

where \( \sigma \) is the macroscopic stress and the positive fourth-rank tensor \( \mathbf{T} \) is defined as

\[ \mathbf{T} = P^T \cdot T \cdot P = F_{ijkl}(T_1, T_2, T_3, T_4, T_5, T_6) \]

and the fourth-rank tensor \( P \) and the parameters \( T_1, \ldots, T_6 \) of fourth-rank tensor \( \mathbf{T} \) are given in
Lee and Simunovic [7]. More detail of elastoplastic stress-strain relationship for partially
debonded three-phase aligned fiber reinforced composites can be seen by Lee and Simunovic
[7].

2.2 Effective elastic moduli and elastoplastic behavior of randomly oriented composites
Consider the composite models, in which spheroidal fibers with an aspect ratio of \( \alpha \) (the ratio
of length to diameter) are uniformly dispersed and randomly oriented in two- or three-
dimensional space. The average process over all orientations upon governing constitutive
field equations is performed to obtain the constitutive relations and the overall yield function
for randomly oriented composites.

The local axes of an inclusion are denoted by the primed coordinate system and the fixed or
material axes by the unprimed one. With no loss in generality, we let axis 1' be the symmetric
axis of the spheroid and 3' lie in the 2-3 plane. Denoting \( Q_{ij} \) as the directional cosine between
the i-th primed and j-th unprimed axes, we have

\[ x'_i = [Q_{ij}]x_j \]

where the transformation matrix for two- and three-dimensional orientation have the form of

\[
\begin{bmatrix}
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta \\
1 & 0 & 0
\end{bmatrix} \quad \text{for two-dimensional orientation}
\]
\[
\begin{bmatrix}
\sin\theta \sin\phi & \cos\theta & \sin\theta \cos\phi \\
\cos\theta \sin\phi & -\sin\theta & \cos\theta \cos\phi \\
\cos\phi & 0 & -\sin\phi
\end{bmatrix}; \quad \text{for three-dimensional orientation}
\]

in which \(\theta\) is the angle between \(x_j\) and \(x_j'\) and \(\phi\) is the angle between \(x_3\) and \(x_3'\). Any second-rank tensor, e.g., stress tensor, can be transformed as

\[
\sigma'_{ij} = Q_{ik} Q_{jk} \sigma_{kl}
\]

The symbols \((\cdot)^{2D}\) and \((\cdot)^{3D}\) are used to define the two- and three-dimensional orientational averaging process, respectively, as

\[
(\cdot)^{2D} = \int_0^{\pi} (\cdot) P(\theta) \, d\theta
\]
\[
(\cdot)^{3D} = \int_0^{\pi} \int_0^{2\pi} (\cdot) P(\theta, \phi) \sin \theta \, d\theta \, d\phi
\]

where \(P(\theta)\) and \(P(\theta, \phi)\) are the probability density functions. In the special case of uniformly random orientation, we have \(P(\theta) = 1/\pi\) and \(P(\theta, \phi) = 1/(2\pi)\).

Accordingly, the constitutive relations and overall yield function of two- and three-dimensional random fiber reinforced composites can be derived by performing orientational averaging process over all orientations upon the governing field equations and overall yield function for aligned-fiber orientations given in Section 2.1.

### 3. EVOLUTIONARY INTERFACIAL DEBONDING

The progressive interfacial debonding may occur under increasing deformations and influence the overall stress-strain behavior of randomly oriented, discontinuous fiber reinforced composites. After the interfacial debonding between fibers and the matrix, the debonded fibers lose the load-carrying capacity in the debonded direction and are regarded as partially debonded fibers. Within the context of the first-order (noninteracting) approximation, the stresses inside fibers should be uniform. For convenience, following Zhao and Weng [3,4], we employ the average internal stresses of fibers as the controlling factor. The probability of partial debonding is modeled as a two-parameter Weibull process; see Ju and Lee [6]. Assuming that the Weibull statistics governs, we can express the cumulative probability distribution function of fiber debonding (damage), \(P_d\), at the level of hydrostatic tensile stress as:
where \((\sigma_{m})_l\) is the hydrostatic tensile stress of the fibers, the subscript denotes the fiber phase, and \(S_o\) and \(M\) are the Weibull parameters.

### 4. FINITE ELEMENT IMPLEMENTATION

Impact simulation requires very small time step and thus the developed damage models are implemented into explicit finite element code DYNA3D to simulate the dynamic inelastic behavior and the progressive damage of the composite materials. Furthermore, the model will be used for simulation of dynamic loading problems. The experimental results will be provided for comparison with the simulation models.

The methodology used in this work is based on the well-known strain-driven algorithm in which the stress history is to be uniquely determined by the given strain history mainly because of its computational efficiency in the framework of explicit time integration computer program DYNA3D. The two-step operator splitting methodology is also adopted here to split the elastoplastic loading process into the elastic predictor and the plastic corrector. More detail of strain driven algorithm, micromechanical iterative algorithm for the progressive damage model and 3-D return mapping algorithm can be seen by Ju and Lee [6].

The authors are currently working on the implementation of the proposed damage models into DYNA3D and will show the dynamic inelastic behavior and progressive crushing in composite structures under impact loading in our presentation.

### 5. EXPERIMENTAL COMPARISONS AND NUMERICAL SIMULATIONS

In order to assess the validity of the proposed micromechanical framework, we now compare our analytical predictions with bounds based on Halpin-Tsai micromechanics equations (Halpin and Kardos, [8]). One of the advantages of the Halpin-Tsai equations is that they cover both the particulate reinforced case (fiber aspect ratio=unity, lower bound) and the continuous fiber case (fiber aspect ratio=infinity, upper bound). Indeed, one can mathematically express the equation limits as the rule of mixtures for continuous fibers and modified Kerner equation for spherical reinforcement. We will consider the following constituent elastic properties for carbon fiber polymer matrix composites: \(E_p=3\) GPa, \(\nu_p=0.35\), \(E_f=380\) GPa and \(\nu_f=0.25\). Figure 1 shows the predicted effective (normalized) Young's moduli in the fiber direction \(E_f/E_o\) of aligned fiber reinforced composites at various fiber volume fractions \(\phi_f\). We plot the theoretical predictions in Figure 1 based on Halpin-Tsai's bounds and
the proposed method with various fiber aspect ratios. Clearly, our analytical predictions are well within the Halpin-Tsai's bounds.

To illustrate the elastoplastic behavior of the present damage constitutive framework, our present damage model considering interfacial debonding is exercised for the case of aligned carbon fiber polymer matrix composites. The material properties used in these simulations are: $E_0=3.0\,\text{GPa}$, $\nu_f=0.35$, $E_f=380\,\text{GPa}$, $\nu_f=0.25$, $\sigma_0=125\,\text{MPa}$, and hardening parameters $h=400\,\text{MPa}$ and $q=0.5$. In addition, to implement the proposed probabilistic micromechanics based on Weibull function into the present constitutive models, we need to estimate the values of Weibull parameters $S_0$ and $M$. For simplicity, we assume the Weibull parameters to be: $S_0=81.75\,\sigma_y$ and $M=40$. Figure 2 shows the effect of the shape of fibers on the mechanical behavior of aligned fiber reinforced composites with the same fiber volume fraction and it clearly shows that the elastoplastic behavior of the composites is strongly dependent upon the shape of fibers.

We further compare our prediction with the experimental data provided by Mergahni and Benzeggagh [1] for three-dimensional random fiber composites in Figure 3. Figure 4 shows the predicted evolution of debonded fiber volume fraction versus strain corresponding to Figure 3. Since our formulation does not consider inter-fiber interaction, the stress-strain curve for the present prediction is lower than that based on experiment in early stage. As the strain increases, the effect of damage becomes dominant. The stress-strain curves corresponding to the present prediction and the experiment will intersect each other, because the proposed damage constitutive model includes the interfacial debonding only. Therefore, it is concluded that the interaction effect among constituents must be considered in modeling damage behavior of composites for moderately and extremely high fiber volume fraction. Furthermore, damage mechanisms, such as matrix cracking, void nucleation, etc., must be included in our damage constitutive models to offer more realistic damage predictions. Finally, the present model does not account for other damage mechanisms nor impact since these effects are beyond the scope of the present stage of this research. In spite of these limitations, the agreement between the present predictions and experiments is encouraging for possible use of the proposed damage constitutive models for predicting the progressive damage in composite structures.

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6. REFERENCES

Figure 1. The comparison between the proposed predictions with various fiber aspect ratios and Halpin-Tsai's bounds for effective Young's modulus in the fiber direction vs. fiber volume fraction.

Figure 2. Effect of the shape of fibers on the overall uniaxial elastoplastic behavior of aligned fiber reinforced composites.
Figure 4. The comparison between the present prediction and experimental data (Meraghni and Benzeggagh, 1995) for overall uniaxial tensile responses of randomly oriented discontinuous fiber composites at initial fiber volume fraction of 0.5.

Figure 5. The predicted evolution of debonded fiber volume fraction versus strain corresponding to Figure 3.
Figure 3. The comparison between the present prediction and experimental data (Meraghni and Benzeggagh, 1995) for overall uniaxial tensile responses of randomly oriented discontinuous fiber composites at initial fiber volume fraction of 0.5.

Figure 4. The predicted evolution of debonded fiber volume fraction versus strain corresponding to Figure 3.