Title: PRECISION ESTIMATES FOR TOMOGRAPHIC NONDESTRUCTIVE ASSAY

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(SUMMARY)
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Requirements for effective safeguards during the transition to environmental management at nuclear material production facilities within the DOE complex are deriving improvements in the accuracy of nondestructive assay (NDA) techniques. An important aspect of the transition is the need for facilities to terminate safeguards on waste materials, thus reducing the cost for safeguards at the facility. Requirements for the termination of safeguards on candidate waste material have been established by DOE to minimize the potential for diversion or theft of nuclear material. Because heterogeneous waste and residue materials are stored in large containers such as 208-L drums, conventional assay techniques such as segmented gamma scanning (SGS) that were developed to assay small samples cannot always provide accurate measurements. Consequently, facilities using the conventional NDA instrumentation may be limited in their ability to discard waste materials in compliance with DOE requirements.

One technique being applied to improve the accuracy of assays of waste in large containers is computerized tomography (CT). Research on the application of CT to improve both neutron and gamma-ray assays of waste is being carried out at Los Alamos National Laboratory.\textsuperscript{1,2} For example, tomographic gamma scanning (TGS) is a single-photon emission CT technique that corrects for attenuation of gamma rays emitted from the sample using attenuation images from transmission CT.\textsuperscript{1,3} By accounting for the distribution of emitting material and correcting for attenuation of the emitted gamma rays, TGS is able to achieve highly accurate assays of radionuclides in medium-density waste. The spatial resolution used in TGS (~2 in.) was selected to minimize bias while satisfying common facility throughput requirements (> 8 drums per 8-hour shift).\textsuperscript{1}
While CT, in theory, resolves the problem of assay accuracy, the mathematical complexity of the CT reconstruction gives rise to a challenging task: developing methods to estimate the precision of tomographic assays. Because statistical variation can contribute significantly to the total uncertainty of assays, the development of reliable precision estimates is important for both the safeguards termination and waste management problems. To further explore this problem, the specific example of precision estimators for TGS will be examined.

The process of determining the mass of special nuclear material (SNM) by TGS can be divided into four distinct steps: (1) interpolation of measured transmissions, (2) reconstruction of attenuation images, (3) reconstruction of attenuation-corrected emission images, and (4) calculation of mass. In a well-designed experiment, the third step introduces the greatest statistical variation in the assay result. However, because the first two steps involve the solution of a nonlinear system, transmission measurement errors can be amplified considerably for objects that are nearly opaque, causing large variations in the assay. To develop a reliable estimator of assay precision, both transmission and emission uncertainties must be considered.

The starting point for the development of a precision estimator for TGS assays is the emission reconstruction algorithm, which can be cast as a maximum-likelihood (ML) problem. The TGS emission reconstruction problem can be stated as

\[
\text{minimize} \quad -\log L(x|d,s) \\
\text{subject to} : \quad x \geq 0
\]

where \( L \) is the likelihood function, \( x \) is a vector of parameters that describes the emission image and \( d \) is the emission data set. The effect of attenuation image statistics is encoded by the vector \( s \) which describes the effect of transmission uncertainties on the distribution of mean emission counts. The mass of SNM in the sample or in a selected sample volume
is \( M = q^r x^* \), where \( q \) is a vector of weights that depend on the image geometry and \( x^* \) is the solution of the ML problem.

The precision estimation problem involves determining the effect of perturbations of \( d \) and \( s \) on \( x^* \). To determine the uncertainty in \( M \), correlations between elements of \( x^* \) must be determined. In TGS, image parameter correlations are significant. As a result, the complete covariance (error) matrix for \( x^* \) must be generated to estimate assay precision. In the case of TGS, there can be as many as 1500 correlated image parameters, resulting in a covariance matrix with over 2 million entries. For unconstrained optimization, the covariance matrix is the inverse of the Hessian of the likelihood-function at \( x^* \). The emission problem can be transformed to an equivalent unconstrained problem with a simple transformation.

Because of the scale of the emission reconstruction problem and limitations on computational resources, we have explored approximate methods for estimating the covariance of the solution vector. A promising technique that we have evaluated approximates Problem A with an equivalent least-squares problem. The covariance matrix for the approximate problem can be constructed from the singular value decomposition (SVD) of the emission weight matrix.

To meet analysis throughput and storage requirements, we have implemented sparse SVD techniques that target only the top singular triplets of the emission weight matrix. Sparse SVD techniques minimize memory requirements for the emission weight matrix and, with a small number of singular triplets, can represent primary features of the weight matrix. The primary benefit for variance propagation is that the sparse SVD is fast and covariance terms can be computed on the fly when estimating the uncertainty in \( M \). The SVD-based estimator is being validated using replicate simulated and experimental TGS assays.
References


