EQUILIBRIUM & NON-EQUILIBRIUM
ASPECTS OF HOT, DENSE QCD

July 17–30, 2000

Organizing Committee:
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Volume 1 - Open Standards for Cascade Models for RHIC - BNL-64722
Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD and RHIC physics through the nurturing of a new generation of young physicists.

During the first year, the Center had only a Theory Group. In the second year, an Experimental Group was also established at the Center. At present, there are seven Fellows and nine post docs in these two groups. During the third year, we started a new Tenure Track Strong Interaction Theory RHIC Physics Fellow Program, with six positions in the academic year 1999-2000; this program will increase to include eleven theorists in the next academic year, and, in the year after, also be extended to experimental physics. In addition, the Center has an active workshop program on strong interaction physics, about ten workshops a year, with each workshop focussed on a specific physics problem. Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form proceedings, which can therefore be available within a short time.

The construction of a 0.6 teraflop parallel processor, which was begun at the Center on February 19, 1998, was completed on August 28, 1998.

T. D. Lee
September 29, 2000

*Work performed under the auspices of U.S.D.O.E. Contract No. DE-AC02-98CH10886.
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Motivation & Purpose of the Workshop

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven, beginning operation this year, and the Large Hadron Collider (LHC) at CERN, beginning operation ~2005, will provide an unprecedented range of energies and luminosities that will allow us to probe the Gluon-Quark plasma.

At RHIC and LHC, at central rapidity typical estimates of energy densities and temperatures are $e \approx 10$ GeV/fm$^3$ and $T_0 \approx 300 - 900$ MeV. Such energies are well above current estimates for the GQ plasma. Initially, this hot, dense plasma is far from local thermal equilibrium, making the theoretical study of transport phenomena, kinetic and chemical equilibration in dense and hot plasmas, and related issues a matter of fundamental importance.

During the last few years a consistent framework to study collective effects in the Gluon-Quark plasma, and a microscopic description of transport in terms of the hard thermal (and dense) loops resummation program has emerged. This approach has the potential of providing a microscopic formulation of transport, in the regime of temperatures and densities to be achieved at RHIC and LHC. A parallel development over the last few years has provided a consistent formulation of non-equilibrium quantum field theory that provides a real-time description of phenomena out of equilibrium. Novel techniques including non-perturbative approaches and the dynamical renormalization group techniques lead to new insights into transport and relaxation. A deeper understanding of collective excitations and transport phenomena in the GQ plasma could lead to recognize novel potential experimental signatures. New insights into small-c physics reveals a striking similarity between small-c and hard thermal loops, and novel real-time numerical simulations have recently studied the parton distributions and their thermalizations in the initial stages of a heavy ion collision.

Recently new exciting theoretical developments in understanding cold dense plasmas revealed the possibility of novel phases of dense quark matter in which color superconductivity can occur and novel features of the possible phase diagram of QCD in the temperature-chemical potential plane had been studied. AGS at Brookhaven can study a region of large chemical potential and low temperatures which can potentially probe these new phases of QCD. Furthermore, event-by-event analysis of fluctuations had been proposed as definite experimental signatures of these new phases. These phases are not only interesting from the perspective of the GQ plasma, but also could exist at the cores of neutron stars and could have observational implications in pulsar glitches.

Clearly, a deep understanding of equilibrium and non-equilibrium phenomena described from first principles from quantum field theory is a necessary step for recognizing experimental signatures in the new generation of ultrarelativistic heavy-ion colliders.

Moreover, the theme of transport phenomena on extremely short time and spatial scales is truly interdisciplinary: in cosmology a deeper understanding of inflationary dynamics and reheating has been benefitting from the application of techniques brought to bear on the problem of GQ plasma, and the QCD phase transitions in the early universe can have an imprint in the CMB. In condensed matter physics, recent advances in femtosecond spectroscopy is beginning to probe kinetics and relaxation phenomena on unprecedented small scales and the field can benefit from developments in equilibrium and non-equilibrium techniques from the GQ plasma program.
Transport Coefficients in Hot Gauge Theories

P. Arnold
G. Moore
L. Y.

What is:
- electric conductivity
- 'flavor' diffusion constants
- shear viscosity
- bulk viscosity
- color conductivity

\[ \text{not this talk} \]

hot: \( T \gg m_8, \mu_8, \lambda_{\text{acc}} \)
\[ g^3(T) \ll 1 \]

Warning:
Qualitative:

transport coefficients \( \propto \) relevant mean free scattering time

Ex: conductivity
\[
\Delta \rho = e E \Delta t \\
\Delta \nu = \frac{\Delta \rho}{\rho_0} \sim eE \Delta t/T \\
\Delta \gamma \sim ne \Delta \nu \\
\sim T^3 \frac{e^2 \Delta t \cdot E}{T} \\
\text{accel. time } \Delta t \sim \tau_{\text{mft}}
\]

\[ \therefore \sigma \sim e^2 T^2 \tau_{\text{mft}} \quad \text{[Drude]} \]

large angle scattering rate
\[
\tau_{\text{mft}}^{-1} \sim e^4 T^2 \left[ \int \frac{d^6 q}{q^2} \right] \\
\ln T/m_0 \sim \ln \nu e
\]

\[ \therefore \sigma = O \left( \frac{T}{e^2 \ln e^{-1}} \right) \]

\[
= \frac{T}{e^2} \left[ \frac{\#}{\ln e^{-1}} + \frac{\#}{\ln^2 e^{-1}} + \ldots \right] \times (1+o(e))
\]

LLL0 \quad NLLL0

leading order
Quantitative:

Effective theory adequate for leading order

= kinetic theory for hard excitations

free dispersion relation \( p^0 = |p| \)

collision term valid for \( O(gT) \) momentum xfer.

distribution function \( f_s(p, x, t) \)

Boltzmann equation

\[
\left[ \partial_t + \nabla \cdot q - F_{\text{ext}} \cdot \frac{\delta}{\delta p} \right] f_s = -C[f]
\]

collision term

\[
(C[f])(p) = \int_{k, p, k'} |M(p, k, p, k')|^2 (2\pi)^4 \delta^4(p + k - p' - k') \times \left[ f_p f_{k'} (1 - f_p) (1 - f_{k'}) - f_p f_k (1 - f_p) (1 - f_k) \right]
\]

energy momentum tensor

\[
T^{\mu\nu}(x) = \sum_p \frac{p^\mu p^\nu}{p^0} \sum_s f_s(p, x)
\]

conserved currents

\[
J^M_a(x) = \sum_p \frac{p^M}{p^0} \sum_s (\delta_a)_s f_s(p, x)
\]
Electric conductivity \[ \mathbf{j}^{\text{em}} = \sigma \mathbf{E} \]

Linear response

\[ f(p,x) = f_0(p) + f_0 [1 - f_0] \frac{x}{h} \mathbf{E} + O(x^2) \]

\[ \xrightarrow{\text{equilibrium}} \]

\[ \xrightarrow{\text{departure from equilibrium}} \]

Boltzmann

\[ \text{LHS} = -\mathbf{F}_{\text{ext}} \cdot \frac{\partial}{\partial \mathbf{p}} f^* = \beta g f_0 (1 - f_0) \hat{\mathbf{p}} \cdot \mathbf{E} \]

\[ \text{RHS} = -\sum_{k,\mathbf{p},\mathbf{k}} \int \mathbf{l} M \mathbf{l}^2 (2\pi)^3 \delta^3 (\mathbf{p}_m - \mathbf{p}_m) \]

\[ \times f_0(p) f_0(\mathbf{k}) [1 - f_0(p)] [1 - f_0(\mathbf{k})] \]

\[ \times \left[ \chi(p) + \chi(k) - \chi(p') - \chi(k') \right] \mathbf{E} \]

\[ = -\left[ C \chi \right] \mathbf{E} \]

\[ \xrightarrow{\text{linearized collision op.}} \]

\[ \mathbf{j}^{\text{em}} = S_p \sum_{\mathbf{p}} g \hat{\mathbf{p}} f(p) = S_p \mathbf{I}(p) \chi(p) \mathbf{E} \]

\[ \therefore \sigma = \frac{1}{2} S_p \mathbf{I}(p) \chi(p) \]
Scattering processes

\[ e^+ \{ e^+ \} \rightarrow e^+ \{ e^+ \} \]

\[ e^+ \gamma \rightarrow e^+ \gamma \]

\[ e^+ e^- \rightarrow e^+ e^- \]

leading log: \[ \left| \frac{2}{2} \right| + \left| \frac{2}{2} \right| \]

neglected in nearly all previous work

beyond leading log: everything contributes
Leading log results

Conductivity $\sigma = \# \frac{T}{e^2 \ln e^{-1}}$

<table>
<thead>
<tr>
<th># leptons</th>
<th># quarks</th>
<th>$\sigma \cdot (e^2/T) \ln e^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>15.696</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>20.656</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>12.287</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>11.972</td>
</tr>
</tbody>
</table>

1 term ansatz accurate to 0.4%:

$$\sigma = \left( \frac{12^4 \beta_0^2 \pi^{-3} N_{\text{leptons}}}{3\pi^2 + 32 N_{\text{species}}} \right) \frac{T}{e^2 \ln e^{-1}}$$

Shear viscosity $\eta = \# \frac{T^3}{g^4} \ln g^{-1}$

<table>
<thead>
<tr>
<th># flavors</th>
<th>$\eta \cdot (g^4/T^3) \ln g^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.126</td>
</tr>
<tr>
<td>1</td>
<td>60.808</td>
</tr>
<tr>
<td>2</td>
<td>86.473</td>
</tr>
<tr>
<td>3</td>
<td>106.664</td>
</tr>
<tr>
<td>4</td>
<td>122.958</td>
</tr>
</tbody>
</table>

$SU(3)$:

Baryon diffusion $D = \# \frac{T}{g^4} \ln g^{-1}$

<table>
<thead>
<tr>
<th># flavors</th>
<th>$D \cdot (g^4/T) \ln g^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;0&quot;</td>
<td>16.060</td>
</tr>
<tr>
<td>1</td>
<td>14.368</td>
</tr>
<tr>
<td>2</td>
<td>12.999</td>
</tr>
<tr>
<td>3</td>
<td>11.769</td>
</tr>
<tr>
<td>4</td>
<td>10.920</td>
</tr>
</tbody>
</table>

$SU(3)$:
Full leading order (all logs) results

Keep full HTL self-energies on exchange line

\[ \Rightarrow \text{all logs included} \]

some (but not all) \( \alpha \text{(g)} \) corrections included

\[ \sigma = \frac{1}{e^2} \beta \left( m_p, \frac{m_y}{T} \right) \left[ 1 + O(\alpha) \right] \]

For \( e^2 = \frac{4\pi}{137}, \ m_{\text{f}} = \frac{e^2}{3} T^2, \ m_{\text{D}} = \frac{T^2}{3} \sum_{\text{species}} \epsilon^2 \)

<table>
<thead>
<tr>
<th># leptons</th>
<th># quarks</th>
<th>( \sigma \cdot (e^2/T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>6.420</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>8.857</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5.826</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6.301</td>
</tr>
</tbody>
</table>
QCD Thermodynamics:
from Lattice
to Quasi particles

Eric Branton
Ohio State University

collaborators: Jens Andersen (Utrecht)
Emmanuel Petitgirard (Ohio State)
Mike Strickland (Seattle)
Goal: Quantitative understanding of QCD thermodynamics at $T > T_c$ in terms of quark and gluon quasiparticles.

Motivation

1. Intuitive understanding of thermal QCD
2. Applications to dense QCD
3. Phenomenology of quark-gluon plasma at RHIC
Outline

1. Rigorous methods
   a. Lattice QCD
   b. Lattice DRQCD
   c. Weak-coupling expansion

2. Nondiagrammatic approaches
   a. Pade approximants
   b. Quasiparticle models

3. Diagrammatic approaches
   a. HTL perturbation theory
   b. Approximately self-consistent HTL resummation
Rigorous Approaches
to QCD Thermodynamics

well-defined procedure
for calculating $T(T)$
to arbitrarily high accuracy
in at least some domain of $T$

1. **Lattice QCD**
   for references, see
   in principle, all $T$
in practice, $T < 5T_c$ ?

2. **Lattice DRQCD**
   Kajantie, Rummukainen, Schaposhnikov '95
   K, Laine, R+S '96
   K, L, Peisa, Rajantie, R, S '97

3. **Weak-coupling expansion**
   Braaten, Nieto '86

   $T > 10^6 T_c$ ?
Conclusions

Two useful rigorous methods for calculating QCD Thermodynamics
1. Lattice QCD
2. Lattice DRQCD Kajantie et al.
overlapping domains in T !?

can be used to test (tune?)
less rigorous diagrammatic methods
based on quark & gluon quasi particles
diagrammatic approaches to
QCD thermodynamics

1. approximately self-consistent HTL resummation
   Blaizot, Iancu, * Robhan

   "HTL" approx.: error $\sim g^3$

   "NLO" " : error $\sim g^4$
   (but completely unstable)

2. HTL perturbation theory
   Anderson, Branton, Strickland

   1-loop approx.: error $\sim g^2$

   2-loop " : error $\sim g^4$
   allows gap equation for $m^2$
   in progress
In the $A$-ptcle case:

$$
\frac{dN_{\nu}}{d\nu} = \int dl_1 dx_1 dt_A dx_A dt'_A dx'_A \chi^{(-)}_{\nu}(x_1 t_1, x_A t_A) \\
\times (\mp i)^A \Sigma_A^<(x_1 t_1, x_A t_A; x'_1 t'_1, x'_A t'_A) \chi^{(-)}_{\nu}(x'_1 t'_1, x'_A t'_A)
$$

where $A$-ptcle source-function is

$$(\mp i)^A \Sigma_A^<(x_1 t_1, x_A t_A; x'_1 t'_1, x'_A t'_A) = \langle j^+(x'_A t'_A). j^+(x'_1 t'_1) j(x_1 t_1). j(x_A t_A) \rangle_{A \text{ irred}}$$

$$
\chi^{(-)}_{\nu}(x_1 t_1, x_A t_A) = (-i)^A \lim_{t' \to \infty} \int dx'_1. dx'_A \times G_A^-(x_1 t_1, x_A t_A; x'_1 t'_1, x'_A t'_A) \exp[-iE_\nu t'] \Phi^{(-)}_{\nu}(x'_1, x'_A)
$$

and the advanced $A$-particle Green's function

$$(\mp i)^A G_A^-(x_1 t_1, x_A t_A; x'_1 t'_1, x'_A t'_A)$$

$$
= \sum_{\sigma \in \Pi(A)} \frac{\text{sgn} \sigma}{1} \times \theta(t' - t_\sigma(1)) \cdot \theta(t_{\sigma(A-1)} - t_{\sigma(A)}) \times \left\langle \left[ \psi^+(x'_1 t'_1), \psi^+(x'_A t'_A), \psi(x_1 t_1) \right]_{\pm} \cdot \psi(x_A t_A) \right\rangle_{\pm}
$$

\begin{itemize}
  \item Inclusive $A$-ptcle yields from real-time theory
    \begin{itemize}
      \item Nonrelativistic theory
      \item $A$-ptcle sources $\langle j^+(x'_A t'_A). j^+(x'_1 t'_1) j(x_1 t_1). j(x_A t_A) \rangle_{A \text{ irred}}$
    \end{itemize}
  \item Application
    \begin{itemize}
      \item pp measurements
      \item Simultaneous pp, nn and np measurements
      \item Three-body problem for emission
      \item Numerical results
      \item Discussion: sudden vs adiabatic limits
    \end{itemize}
  \item Conclusions
\end{itemize}
Finally, when effects of symmetrization of particles with the remainder of the system are insignificant, then

$$\frac{dN_\nu}{d\nu} = \int dt \, dx_1 dx_A \, dt' \, dx'_A \, \chi^{(-)}(x_1, x_A, t) \chi^{(-)}(x'_1, x'_A, t') \times \langle j^\dagger(x'_A, t') \psi(x'_A, t'), \psi(x_1, t) \rangle_{A-\text{irred}}$$

where

$$i \frac{\partial}{\partial t} \chi^{(-)}(x_1, x_A, t) = \sum_{j=1}^{A} \left(-\frac{\nabla_j^2}{2m}\right) \chi^{(-)}(x_1, x_A, t) + \sum_{j=1}^{A} \int dx'_j \, \sum_{i<j} (x_j, x'_i, t) \chi^{(-)}(x_1, x'_j, x_A, t) + \sum_{j<k} \int dx'_j \, dx'_k \, V(x_j, x_k; x'_j, x'_k) \chi^{(-)}(x_1, x'_j, x'_k, x_A, t)$$

Simultaneous pp, nn & np Measurements

R. Ghetti, PhD '97
$^{40}$Ar + $^{197}$Au at E/A = 30 MeV/nucleon
symbols – data

Structures in nn and pp, but not in np!
**Simple Explanation**

As particles move out, proton gains momentum

\[ \Delta p_C \approx \frac{V_C}{v} \]

relative to neutron. For Au, change in the relative momentum \( \Delta q = \Delta p / 2 \approx 35 \text{ MeV/c} \) \((v \sim 0.2c)\), is large compared to the range in \( q \) where the correlation is enhanced:

![Graph](image)

np wavefunction in the vicinity of the emitting source:

\[ i \frac{\partial}{\partial t} \Phi = - \frac{\nabla^2_1}{\lambda_1} + \frac{\nabla^2_2}{\lambda_2} \Phi + V_{pn}(r_1 - r_2) \Phi + \Sigma_p^{-}(r_1, t) \Phi + \Sigma_n^{-}(r_2, t) \Phi \]

where

\[ \Sigma_{p}^{-} = V_C + V_N - i W_N \]

and \( \Phi(-) \rightarrow e^{i(P(r_1 + r_2)/2 + E(t - \ell_1))} \phi_{q}^{(-)}(r_1 - r_2) \)

as \( r_{1,2} \rightarrow \infty, t \rightarrow \infty \)

**Three-Body Problem**

Stationary eq:

\[ E \Phi = - \left( \frac{\nabla^2_1}{2m} + \frac{\nabla^2_2}{2m} \right) \Phi + V_{pn}(r_1 - r_2) \Phi + \Sigma_p^{-}(r_1) \Phi + \Sigma_n^{-}(r_2) \Phi \]

7-dimensional eq. assuming spherical symmetry:

Angular momentum expansion?:

Intrinsic and pair CM momenta couple.

\( \rightarrow \) For every total \( LM \) coupled eqs in \( r_1, r_2, \ell_1, \ell_2 \) and \( m_1 \).

No transparency. Boundary conditions??

Anything else?

Possibility:

\[ \Phi(r, R) = \chi_p(r_1) \chi_n(r_2) \phi(r, R) e^{-i qr} \]

where the 1-particle wavefunctions \( \chi \) satisfy the 1-particle eqs

\[ E \chi = -\frac{\nabla^2}{2m} \chi + \Sigma \chi \]

Then eq for intrinsic wavefunction dependent on CM position:

\[ \frac{1}{M} \nabla_R (\log \chi_p \chi_n) \nabla_R \phi_q \]

\[ = -\frac{q^2}{2\mu} \phi_q - \frac{\nabla^2}{2\mu} \phi_q + V(r) \phi_q \]

\[ = \frac{\nabla^2}{2M} \phi_q + \frac{i}{\mu} \nabla_r (\log (\chi_p \chi_n)) (q + i \nabla_r) \phi_q \]

Slow variation of intrinsic wavefunction with CM position + largest variation of 1-particle wavefunction along the tof CM momentum.

p starts slower than n towards Au.
Should be slowed down more, i.e. \( q \) should increase on approach to the nucleus.

Relative wavefunction at the distance \( R = 25 \text{ fm} \) from Au:
SUDDEN VS ADIABATIC LIMITS

- Naive expectation (BDS)
- Full adjustment of the intrinsic wavefunction to external changes = the adiabatic limit.

Intrinsic wavefunction actually changes VERY LITTLE

Conditions for sudden and adiabatic limits formulated normally for discrete states.

Wave spends at short distances the time:

\[
\tau = 2 \frac{d\delta}{dE}
\]

where \(\delta\) - phase shift. Wavefunction can adjust itself if:

\[\tau \ll \Delta t\]

where \(\Delta t\) - time for significant changes in the exterior. If

\[\tau \gg \Delta t\]

no adjustment can be expected.
In the case at hand $\delta \approx -a_{pn}^s q$, where $a_{pn}^s = -23.7 \text{ fm}$. Then

$$\tau \approx -\frac{2a_{pn}^s m_N}{q} \sim 2(a_{pn}^s)^2 m_N$$

for relevant $q \sim 1/|a_{pn}^s|$. Time for significant changes given by

$$\Delta \eta_p \sim 2q \approx F \Delta t$$

i.e.

$$\Delta t \sim \frac{2R}{V(R)a_{pn}^s}$$

The wavefunction can adjust if

$$R \gg R_c = a\sqrt{Z\alpha m_N} = 190 \text{ fm}$$

\[ Z = 73 \]

**Conclusions**

1. Many-body theory allows to formulate cleanly the problem of $A$-particle emission in a complicated heavy-ion reaction.

2. $A$-particle sources can be defined in terms of expectation values.

3. Relative np wavefunction undergoes only modest changes in the field of heavy nuclei.

4. The lack of a structure in the correlation function measured by CHIC is not explained.

5. The fate of a 2-ptcle scattering wavefunction generally depends on $d\delta/dE$. 
Calculating the viscosity
(the hard way)

- What is the viscosity?
  - relation to Boltzmann eqn
  - relation to field theory

- Need for summation of graphs beyond one loop
  - via 3 point function
  - via 4 point function

- Are ladder graphs enough?
  - why they are important
  - how non-ladders may contribute
  - how to test
What is the viscosity?

- Consider a system with no conserved particle number perturbed slightly from equilibrium

\[
<T_{ij}> = \delta_{ij} <P> \\
- \frac{\eta}{\langle\varepsilon+P\rangle} \left[ \nabla_i <T_{ij}^0> + \nabla_j <T_{ij}^0> - \frac{2}{3} \delta_{ij} \nabla^0 <T^0> \right] \\
- \frac{\zeta}{\langle\varepsilon+P\rangle} \delta_{ij} \nabla^0 <T_{ij}^0>
\]

\[\Gamma <T^{xy}> = 0\]

\(\varepsilon = T^{00}\) = energy density

\(P = \text{local equilibrium pressure}\)

\(\eta: \text{shear viscosity}. \text{Characterizes relaxation of transverse momentum density fluctuations (proportional to 2 body elastic scattering mean free path)}\)

\(\zeta: \text{bulk viscosity}. \text{Characterizes departure from equilibrium during a uniform expansion (proportional to mean free path of particle number changing processes)}\)

\[\frac{\gamma}{\langle\varepsilon+P\rangle}: \text{diffusion constant}. \text{Characterizes shear fluctuations}\]
Field theory approach

Kubo formula

$$\gamma = \frac{B}{20} \lim_{q_0 \to 0} \lim_{\vec{q} \to 0} \sigma(Q)$$

$$\sigma(Q) = \int dx e^{iQ \cdot (x-y)} <\Pi_{em}(x) \Pi_{em}(y)>$$

$$\Pi_{em} = \nabla_x \Phi \nabla_y \Phi - \frac{1}{3} \delta_{em} (\nabla \Phi)^2$$

In $\phi^4$ theory, use the Keldysh formalism

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$$

$$D_{11}(x-y) = -i <T [\phi(x) \phi(y)]>$$

$$D_{12}(x-y) = -i <\phi(y) \phi(x)>$$

$$D_{21}(x-y) = -i <\phi(x) \phi(y)>$$

$$D_{22}(x-y) = -i <T [\phi(x) \phi(y)]>$$

$T$ = time ordering

$\hat{T}$ = anti time ordering

$$D_R = D_{11} - D_{12}$$

$$D_A = D_{11} - D_{21}$$

$$D_{11} - D_{12} - D_{21} - D_{22} = 0$$
How to calculate $\sigma(\omega)$?

1. via 3 point functions

$$\Gamma^{\text{em}}_{c\bar{a}a} = T_c \, T_b \, T_a \, \delta_{ab} \, \delta_{cb} \, \Gamma^{\text{em}}(k, k) \quad \text{for } a = 1, 2$$

$$\Gamma^c_{c\bar{a}a}(z, x, y) = \langle T_c \left[ \Phi_c(z) \, T_b \, \Gamma^{\text{em}}(x) \, \Phi_a(y) \right] \rangle$$

Consider

$$\begin{array}{cc}
\begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array} & - & \begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array} \\
\begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array} & - & \begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array} \\
\begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array} & + & \begin{array}{c}
\begin{array}{c}
1 \\
2
\end{array}
\end{array}
\end{array}$$

$$\bullet = \Gamma^{\text{em}}$$

$$\blacksquare = \Gamma^c$$
Ladder graphs

Can generate an infinite summation of ladder graphs either via the 3 pt fn

\[ \begin{array}{c}
\text{\textbullet} - k - \\
\hline
\end{array} = \begin{array}{c}
\text{\textbullet} + \\
\hline
\end{array} + \begin{array}{c}
\text{\textbullet} - k - \\
\hline
\end{array} \]

or via the 4 pt fn

\[ \begin{array}{c}
\hline \\
\end{array} = \begin{array}{c}
\hline \\
\end{array} + \begin{array}{c}
\hline \\
\end{array} \]

However, in the $Q \rightarrow 0$ limit there is an IR divergence. As with Feynman, replace bare propagators (on the horizontal rails) by effective propagators

\[ \frac{1}{p^2} \rightarrow \frac{1}{p^2 - \Sigma(p)} \]
Consider the 3 pt fn

\[ + + \]

The equation can be massaged into

\[ I_{em}(k,k) \sim \int dPdP'dR \, \delta(P+P'-R-k) \times \]
\[ \times \int \rho^{(0)} \rho^{(0)} \rho^{(0)} \left[ B_{em}(\rho) + B_{em}(\rho') - B_{em}(\kappa) - B_{em}(\rho'') \right] \times \]
\[ \times (1+n\rho)(1+n\rho') \eta \rho (1+n\kappa)^{-1} \]

\[ B_{em}(\rho) = \frac{1}{\rho^2} I_{em}(\rho,\rho) B(\rho) \]
\[ B(\rho) = \frac{\Pi_R(\rho)}{\text{Im} \Sigma_R(\rho)} \]
\[ \rho^{(0)} = 2\pi \varepsilon(\rho \omega) \delta[p^2 - m_{th}^2] \quad m_{th}^2 = m^2 + \text{Re} \Sigma_R \]

Use this equation to find \( B \), and plug into

\[ \gamma = \frac{B}{15} \int dk \, k^2 n\kappa (1+n\kappa) B_k \rho_k^{(0)} \]

to find \( \gamma \). The result agrees with Ien (using the imaginary time approach), which in turn agrees with the form coming from the Boltzmann equation.
Non-interacting Non-relativistic Bose Gas

\[ L = \int d^3x \left( i \frac{\partial}{\partial t} \psi^* + \frac{1}{2m} \nabla^2 \psi - \nabla (\mu - \nabla \psi) \right) \psi^* \]

+ (chemical potential term)

Why? Note that eq. of motion

\[ (i \frac{\partial}{\partial t} + \frac{1}{2m} \nabla^2 - \nabla (\mu - \nabla \psi)) \psi = 0 \]

is the Schrödinger eq.!

Path integral \( \rightarrow \) 2nd quantization of Schrödinger eq.

\[ \hat{\psi}(x,t) = \sum_n \hat{a}_n \psi_n(x) e^{-i \omega_n t} \]

\( \hat{a}_n \) annihilation operator for particles in mode \( n \)

\( \psi_n(x) \) eigen-states

\( \omega_n \) eigen-energies

\( V(x) = 0 \) \( \hat{\psi}(x,t) = \int_k \hat{a}_k e^{ik \cdot x - i\omega_k t} \)

Number operator:

\[ \hat{N} = \int_x \hat{\psi}^* \hat{\psi} = \sum_n \hat{a}_n^\dagger \hat{a}_n \]

Chemical potential term:

\[ \mu \hat{N} = \mu \int_x \psi^* \psi \]

\[ L = \int d^3x \left( i \frac{\partial}{\partial t} \psi^* + \frac{1}{2m} \nabla^2 \psi^* - \nabla (\mu - \nabla \psi) \right) \psi^* \]
Include 2-body potential $U(x-y)$ [adequate in dilute limit]

$$L_{\text{int}} = -\frac{i}{2} \int \psi^* \nabla \psi U(x-y) \psi^* \psi(y)$$

In low-energy limit (wavelength $\gg a$)

$$U(x-y) \rightarrow \frac{4\pi a}{m} \delta(x-y)$$

$a = $ scattering length (can be measured!)

Diluteness expansion: $a \ll a^{\frac{1}{3}}$

Focus on $V(x) = 0$ case

$$L \rightarrow \int d^3 x \left[ \psi^*(i\partial_t + \frac{1}{2m} \nabla^2 + \mu - V(x)) \psi - \frac{2\pi a}{m} (\psi^* \psi)^2 \right]$$

Looks just like relativistic $\phi^n$ theory with $\phi^* (\partial_t^2 - \nabla^2) \phi \rightarrow \psi^* i\partial_t \psi$.

\[
\begin{array}{ccc}
\psi & \rightarrow & -\mu(t) < 0 \\
\psi & \rightarrow & -\mu(t) > 0 \\
\text{condensate: } <\psi> \neq 0 \\
\text{COLDER} & \longrightarrow & \text{WARMER}
\end{array}
\]

Why does $<\psi>$ measure the "condensate"?

Contribution of $k=0$ to $n = <\psi^* \psi>$

$$n_0 = <\psi>^* <\psi>$$

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Go to Euclidean time formalism.

Note that all but $\omega = 0$ modes decouple at large distance:

$$\int_0^\beta d\tau L_E \rightarrow \beta \int d^3x \left[ \psi^* \left(-\frac{1}{2m} \nabla^2 - \mu_{\text{eff}}\right) \psi + \frac{2\pi a}{m} (\psi^* \psi)^2 \right]$$

Note that all corrections turn out to be higher-order in diluteness expansion.

Rescale $\psi_0 = \sqrt{\frac{2m}{\beta}} \phi$

$$\int_0^\beta d\tau L_E \rightarrow \int d^3x \left[ |\nabla \phi|^2 + r |\phi|^2 + u |\phi|^4 \right]$$

An $O(2)$ field theory

$$u = 4\pi a m T$$

$$r = -2m \mu_{\text{eff}}$$

Diluteness expansion $a \ll n^{1/3} \Rightarrow u \text{ "small"}$
A goal: Calculate $T_c$ as a function of $n$
(to leading non-trivial order in interactions)

Recall $n = \langle \psi^* \psi \rangle \sim \langle \phi^* \phi \rangle$

$$n \propto \langle \phi^* \phi \rangle = \begin{array}{c}
\circ \quad + \quad \circ \quad + \quad \circ \quad + \quad \ldots
\end{array}$$

$$\Delta n \propto \Delta \langle \phi^* \phi \rangle = \begin{array}{c}
\circ \quad + \quad \circ \quad + \quad \ldots
\end{array}$$

But perturbation theory breaks down. What to do?

Will discuss two methods to solve 3-dim. theory

1. Large $N$

2. Lattice simulations
**PRELIMINARY RESULT:**

\[ \frac{\Delta \langle \phi^2 \rangle}{u} = -0.00112 \pm 3\% \text{ (stat. + finite } l\text{)} \]
\[ \pm \% \text{ (finite } \alpha_{\text{lat}}\text{)} \]

with \( \% \sim 6\% \).

**COMPARISON**

Lattice (preliminary): \( \frac{\Delta T_c}{T_c} = 1.2 \text{ an } 1''^3 \)
Leading-order \( 1/N \): \( \frac{\Delta T_c}{T_c} = 2.3 \text{ an } 1''^3 \) \( (+100\%) \)
Next-order \( 1/N \): \( \frac{\Delta T_c}{T_c} = 1.7 \text{ an } 1''^3 \) \( (+40\%) \)
Space-time is preferable to momentum space for certain aspects of quantum field theory such as lattice gauge theory and the short distance operator product expansion. Here the asymptotic behavior of the photon propagator $D^{\mu \nu}(t, \vec{r})$ in the region $r \to \infty$ and $t \to \infty$ with fixed ratio $r/t < 1$ is used to understand the behavior of the electron self-energy near the mass-shell in various temperature regimes. (1) At zero temperature the free photon propagator falls like $1/t^2$ and this causes the electron self-energy to have a branch point at the mass-shell. (2) At low temperature, $0 < T \ll m_e$, the free photon propagator falls exponentially at large time and this causes the electron self-energy to have a simple pole at the mass-shell. (3) At high temperature, $m_e \ll T$, the resummed hard-thermal-loop propagator for the photon falls like $T/r$ even in the deep time-like region and this causes the electron to have a logarithmically divergent damping rate.
2. Electron self-energy to one loop

\[ \Sigma(P) = i e^2 \int \frac{d^4 K}{(2\pi)^4} D^{\mu\nu}(K) \gamma^\mu S(P + K) \gamma^\nu \]

Electron propagator

\[ S(P \pm K) = \frac{\gamma \cdot (P + K) + m}{(P + K)^2 - m^2 + i\epsilon} \]

For soft $K$ approximate as

\[ S(P + K) \approx \frac{\gamma \cdot P + m}{P^2 - m^2 + 2P \cdot K + i\epsilon} \]

Use exponential form

\[ \frac{1}{P^2 - m^2 + 2P \cdot K + i\epsilon} = -i \int_0^\infty \frac{dt}{2p_0} e^{it(P^2 - m^2 + 2P \cdot K + i\epsilon)/2p_0} \]

\[ \Sigma(P) = e^2 \gamma^\mu (\gamma \cdot P + m) \gamma^\nu \int_0^\infty \frac{dt}{2p_0} e^{it(P^2 - m^2 + i\epsilon)/2p_0} \mathcal{D}^{\mu\nu}(t, \nu t) \]

Behavior near mass shell controlled by photon propagator

\[ \mathcal{D}^{\mu\nu}(t, \nu t) \] in space-time as $t \to \infty$. 

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Define $\Pi(P) = \frac{1}{4E} \text{Tr} \left( (\gamma \cdot P + m) \Sigma(P) \right)$.

Near mass shell $p_0 \approx E$ the fermion propagator is

$$S'(P) \approx \frac{\text{Numerator}}{p_0 - E - \Pi(P)}$$

where the self-energy is

$$\Pi(P) = \frac{e^2}{E^2} \int_0^\infty dt \, e^{it(p_0 - E + i\epsilon)} \, P_\mu P_\nu D^{\mu\nu}(t, \vec{v}t)$$

1. Self-energy at zero temperature

Use zero-temperature photon propagator. In Feynman gauge

$$D^{\mu\nu}(t, \vec{r}) = \frac{-ig^{\mu\nu}}{4\pi^2(t^2 - r^2)}$$

The electron self-energy near mass-shell depends on $D^{\mu\nu}(t, \vec{v}t)$:

$$\Pi(P) = \frac{-ie^2}{4\pi^2} \int_{t_0}^\infty \frac{dt}{t^2} e^{it(p_0 - E + i\epsilon)} \to \frac{-e^2}{4\pi^2} (p_0 - E) \ln(p_0 - E)$$

where $t_0$ is an UV regulator.

The denominator of the electron propagator becomes

$$p_0 - E - \Pi(P) \to p_0 - E + \frac{\alpha}{\pi} (p_0 - E') \ln(p_0 - E')$$

$\alpha/\pi$ is infrared anomalous dimension.

This result is well-known.
2. Self-energy at low temperature \( (0 < T \ll m_e) \)

**Puzzle:** Exact momentum-space calculation at one-loop shows that for nonzero temperature, \( 0 < \pi T \ll m_e \), the electron self-energy does not contain a term \((p_0 - E) \ln(p_0 - E)\).

The singularity is a simple pole. (HAW, Phys. Rev. D 59)

**Space-time explanation:** In Feynman gauge the free, thermal propagator is

\[
D^{\mu \nu}(t, \vec{r}) = g^{\mu \nu} \frac{iT}{4\pi r} \left[ \frac{1}{e^{2\pi T(t+r)} - 1} - \frac{1}{e^{2\pi T(t-r)} - 1} \right] - g^{\mu \nu} \frac{1}{8\pi r} \left[ \delta(t + r) - \delta(t - r) \right]
\]

(See HAW, hep-ph/0007072 for this and results in other gauges.)

The fermion self-energy is

\[
\Pi(P) = \frac{ie^2m^2T}{8\pi vE^2} \int_{t_0}^{\infty} \frac{dt}{t} \left[ \frac{e^{it(p_0 - E + i\epsilon)}}{e^{2\pi Tt(1+v)} - 1} - \frac{e^{it(p_0 - E + i\epsilon)}}{e^{2\pi Tt(1-v)} - 1} \right]
\]

This integral converges at \( p_0 = E \) and so do all its derivatives with respect to \( p_0 \). The self-energy analytic at \( p_0 = E \).

The electron propagator has a simple pole for any small, but nonzero, temperature.
3. Self-energy at high temperature ($\pi T \gg m$)

For hard fermions can neglect resummed vertex functions.

Resummed fermion propagator introduces thermal $m = eT/\sqrt{8}$.

Need HTL photon propagator $*D^{\mu\nu}(t, \vec{r})$ in space-time.

<table>
<thead>
<tr>
<th>Type</th>
<th>Deep space-like</th>
<th>Deep time-like</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free massless</td>
<td>$T/r$</td>
<td>$(T/r) \exp(-2\pi T [t - r])$</td>
</tr>
<tr>
<td>HTL $*D^{00}$ pole</td>
<td>$m_g/r$</td>
<td>$m_g^{1/2} / t^{3/2}$</td>
</tr>
<tr>
<td>HTL $*D^{00}$ cut</td>
<td>exp small</td>
<td>$T/(m_g^2 t^3)$</td>
</tr>
<tr>
<td>HTL $*D^{ij}$ pole</td>
<td>$1/(m_g r^3)$</td>
<td>$m_g^{1/2} / t^{3/2}$</td>
</tr>
<tr>
<td>HTL $*D^{ij}$ cut</td>
<td>$T/r^{(a)}$</td>
<td>$T/r^{(a)}$</td>
</tr>
</tbody>
</table>

(*) A magnetic mass changes both these to $T/(m_{mag}^2 r^3)$

The cut contribution to $*D^{ij}$ in the deep space-like limit is

$$r > t : {*D}^{ij \text{cut}}(t, \vec{r}) \to -\frac{iT}{8\pi r} (\delta^{ij} + \hat{x}^i \hat{x}^j) + \frac{iT}{4\pi m_g^2 r^3} (-\delta^{ij} + 3\hat{r}^i \hat{r}^j).$$

In the deep time-like limit the leading behavior is the same:

$$t > r : {*D}^{ij \text{cut}}(t, \vec{r}) \to -\frac{iT}{8\pi r} (\delta^{ij} + \hat{r}^i \hat{r}^j) - i \frac{b_t^{\text{cut}}(\bar{x})}{t^3} (-\delta^{ij} + 3\hat{r}^i \hat{r}^j)$$

$$- i \frac{b_t^{\text{cut}}(\bar{x})}{t^3} (\delta^{ij} - \hat{r}^i \hat{r}^j).$$

where $\bar{x} = r/t$ is fixed.
The fact that the space-like behavior is $T/r$ is trivial to verify in the Matsubara formalism. The fact that the time-like behavior is also $T/r$ is very non-trivial. (The free thermal propagator falls exponentially in the deep time-like region.)

Fermion self-energy is

$$\Pi(p_0, \vec{p}) = \frac{-ie^2Tv}{4\pi} \int_{t_0}^{\infty} \frac{dt}{t} e^{it(p_0 - E + i\epsilon)}$$

This diverges at $p_0 = E$ like $\ln(p_0 - E)$.


Fourier transform

$$\Pi(t, \vec{p}) = \frac{-ie^2Tv}{4\pi} \frac{e^{-iEt}}{t}$$


For nonabelian, magnetic mass changes asymptotic behavior of $^\star D^{\mu\nu}(t, \vec{v}t)$ from $T/r$ to $1/t^3$. Then $\Pi(P)$ is analytic and damping rate is finite.
Resonant decay of parity odd bubbles in hot hadronic matter

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PLB 484 (2000) 58 (hep-ph/00 05 051)
The Model

\[ \mathcal{L}_{\text{eff}} = \frac{f_{\pi}^2}{4} \left[ \text{tr}(\partial_\mu U \partial^\mu U^+) + \text{tr}(MU + MU^+) \right] - \frac{\alpha}{N_c} \left( \theta - \frac{i}{2} \text{tr}(\ln U - \ln U^+) \right)^2 \]

in the following: \( N_c = 3 \), "real world" \( \theta = 0 \)

meson fields \( U = \exp(i\phi/f_{\pi}) \)

with \( \phi = \sqrt{\frac{2}{3}} \eta_1 1 + \left( \begin{array}{ccc} \pi^0 + \frac{\eta_8}{\sqrt{3}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta_8}{\sqrt{3}} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}} \eta_8 \end{array} \right) \)

mass matrix \( M_{ij} = \mu_i^2 \delta_{ij} \)

with \( \mu_1^2 - \mu_2^2 \equiv m_\pi^2, \)
\( \mu_3^2 \equiv 2m_K^2 - m_\pi^2 \)
\( \mu^2 \equiv (\mu_1^2 + \mu_2^2 + \mu_3^2)/3 \)
\( \simeq 0.171 \, \text{GeV}^2 \)

topological susceptibility \( a = 2N_f \lambda_{YM}/f_{\pi}^2 \)
\( \simeq 0.726 \, \text{GeV}^2 \)

pion decay constant \( f_{\pi} \simeq 93 \, \text{MeV} \)
Temperature dependence of singlet effective potential

\[ V_{\text{eff}} \left( \frac{\eta}{f} \right) = \mu^2 f^2 \left( -\cos \frac{\eta}{f} + \frac{a}{2\mu^2} \left( \frac{\eta}{f} \right)^2 \right) \]

with \( \eta \equiv \eta_1 \) and \( f \equiv \sqrt{3/2} f_\pi \)

- \( T = 0 \)
  - \( a/\mu^2 > (a/\mu^2)_{sp} \)

- \( T = T_{sp} \)
  - \( a/\mu^2 = (a/\mu^2)_{sp} = 0.217 \)
  - \( (\eta/f)_{sp} = 4.493 \)

- \( T_{sp} < T < T_d \)
  - \( 0 < a/\mu^2 < (a/\mu^2)_{sp} \)

Kharzeev, Pisarski, Tytgat 98

parity odd metastable states

- \( T = T_d \)
  - \( a/\mu^2 = 0 \)
Application to heavy ion collision:

\[ T = 0 \]

little BANG

\[ T = T_d \]

\[ T_{sp} < T < T_d \]
field trapped in false vacuum

\textit{CP-odd bubble in hadronic phase}

\[ T = T_{sp} \approx 0.86 T_d \]

\textit{decay of bubble:}
field starts \textit{rolling down}
energy transfer to fluctuations

\[ \Rightarrow \text{particle production} \]

see also: K. Buckley, T. Fugleberg and A. Zhitnitsky

\textit{inflation, DCC, axion production}
Evolution equations

decompose field into expectation value and fluctuations:
(e.g. D. BOYANOVSKY, H. DE VEGA et al., A. LINDE et al.)

\[ \eta(x, t) = \varphi(t) + \chi(x, t) \]

with \( \langle \chi(x, t) \rangle = 0 \)

Hartree-type approximation \( \chi^3 \rightarrow 3\langle \chi^2 \rangle \chi \) and \( \chi^2 \rightarrow \langle \chi^2 \rangle \)

\[
\frac{d^2}{d\tau^2} \varphi(\tau) + \left( 1 - \frac{\langle \chi^2 \rangle}{2f^2} \right) \sin \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2} \frac{\varphi(\tau)}{f} = 0
\]

dimensionless time \( \tau \equiv \mu t \)

initial conditions: \( \frac{\varphi(0)}{f} \leq \frac{\varphi_{sp}}{f} = 4.493 \) and \( \frac{\dot{\varphi}(0)}{f} = 0 \)

expectation value of fluctuations

\[
\langle \chi^2 \rangle_{\tau} \equiv \int \frac{d^3 k}{(2\pi)^3} |\chi_k(\tau)|^2
\]

\[
\frac{d^2}{d\tau^2} \chi_\kappa(\tau) + \left( \kappa^2 + \left( 1 - \frac{\langle \chi^2 \rangle}{2f^2} \right) \cos \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2} \right) \chi_\kappa(\tau) = 0
\]

dimensionless momentum \( \kappa \equiv k/\mu \)

initial conditions: \( \chi_\kappa(0) = \frac{1}{\sqrt{2\omega_\kappa}}, \dot{\chi}_\kappa(0) = -i \sqrt{\frac{\omega_\kappa}{2}} \)
Characteristics of the evolution equations

\[ \frac{d^2 \varphi(\tau)}{d\tau^2} + \left( 1 - \frac{\langle \chi^2 \rangle}{2f^2} \right) \sin \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2} \frac{\varphi(\tau)}{f} = 0 \]

\[ \frac{d^2 \chi_\kappa(\tau)}{d\tau^2} \left( \kappa^2 + \left( 1 - \frac{\langle \chi^2 \rangle}{2f^2} \right) \cos \frac{\varphi(\tau)}{f} + \frac{a}{\mu^2} \right) \chi_\kappa(\tau) = 0 \]

\[ \omega^2_\kappa(\tau) \]

coupled non-linear evolution equations:

- zero mode transfers energy to mode functions via time dependent frequency \( \omega_\kappa(\tau) \)
  \( \Rightarrow \) particle production

- mode functions react back on zero mode via fluctuations ("back reaction")
  \( \Rightarrow \) damping
Solving the evolution equations II

Numerical solution of system including back reaction

\[ \mu(0) \simeq 412 \text{ MeV}; \quad \mu(T_{sp}) \simeq 676 \text{ MeV} \]

\[ f(T_{sp}) \simeq f(0) \]

\[ \frac{\varphi(0)}{f} = 4.2 \]
Particle number density

\[ n(t) \quad \text{[fm}^{-3}] \]

\[ \text{t[fm]} \]

\[ \mu(T_{\tau}) \simeq 676 \text{ MeV} \]
\[ \mu(0) \simeq 412 \text{ MeV} \]

inclusion of back reaction destroys parametric resonance
Momentum spectrum

of produced particles at $t_{end} \approx 10 \, fm$

for $\mu(T = 0) \simeq 412 \, MeV$

$k_{max} \simeq 190 \, MeV$

does not fit Bose-Einstein distribution

$m = 0, \, T = 1.16 \, GeV$

$m = 0, \, T = 130 \, MeV$
Summary

We investigated the decay of metastable states with broken CP-symmetry which are expected to form in hot hadronic matter.

- amplification of the low momentum modes by parametric resonance

- back reaction must be taken into account ⇒ amplification of $\eta'$ by parametric resonance suppressed

- for $t \approx 10 \, fm$
  
  - particle density $n(t) \approx 0.7 \, fm^{-3}$
  
  - correlation length $\xi \approx 2.4 \, fm$
  
  - soft, non-thermal momentum spectrum with $k_{max} \approx 190 \, MeV$
The Quantum Field Limit
Of Classical Field Dynamics

Berndt Müller
Duke University

BNL – RIKEN, July 20, 2000
BNL Talk

Introduction

GR + EM are at odds: how to resolve?
Can EM emerge as LE limit of some deterministic theory?

\[ U(t) = \exp(\frac{it}{\hbar} \mathbf{E} \cdot \mathbf{r}) \]

Chiral fermions in 3 dim.
Not generalizable. But maybe requires information loss.

There space

Long-time dynamics contracts to "discrete" limit cycles,
or equivalence classes of microstates.
EM is dynamics of these equivalence classes.


classical

4-D (lattice) gauge theory

\[ E = 2(N_c^2 - 1) \frac{T}{a^3} + O(g^2) \]
\[ S = \frac{1}{a^3} + O(g^2) \]

Approach to (microcanonical) equilibrium: KS entropy

\[ h_{KS} = g^2 T \left( \frac{L}{a} \right)^3 \quad \rightarrow \quad T_{eq} = \frac{S_{eq}}{h_{KS}} \sim \frac{1}{g^2 T} \]

Note \( \dim[g^2] = \frac{1}{E} \)
\[ \dim[A] = \frac{1}{E} \]
\[ \mathcal{A} = \int d^3 x (\partial A - gA^2)^2 \]

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There space volume
\[ \det \left( \frac{\partial \vec{X}(t)}{\partial \vec{X}(0)} \right) = \text{const.} \]

But focus on locally expanding part:
\[ \chi(\vec{x}(t)) = \frac{d}{dt} \ln \det \left( \frac{\partial \vec{x}(t)}{\partial \vec{x}(0)} \right) \text{ expanding} \]

For a globally hyperbolic system (strong chaos) \( h_{KS} = \langle \chi(\vec{x}) \rangle \),
\[ h_{KS} = \lim_{t \to \infty} \frac{1}{t} \int_0^t dt \chi(\vec{x}(t)) = \sum_{j=1}^{d-1} \lambda_j \quad (\lambda_j > 0) \]

For Anosov systems (phase space compact), fluctuations are Gaussian:
\[ h(t) \text{ for finite } t \text{ is Gaussian distributed with} \]
\[ \langle h(t) \rangle = h_{KS} \quad \text{and} \quad \Delta h(t)^2 = \frac{P''(t)}{t} \]

Ensemble either phase space or ergodic. Define
\[ a(t) = \langle \chi(\vec{x}(t)) \chi(\vec{x}(0)) \rangle - (h_{KS})^2 \]
then
\[ P''(t) = \int_{-\infty}^t dt \ a(t) \]
Numerically:
\[ P''(t) = C (L/a)^3 \quad \text{with} \quad C = 0.69 \pm 0.004 \]
\[ h_{KS} = 0.34 L^3 \quad (\text{for } E/a^2 = 5.4) \]

Note also that:
\[ \Delta h(t)^2 / h_{KS} \sim \frac{1}{L^3 t} \]

Weak coupling:
\[ h_{KS} \sim (g^2 T) L^3 \]

\( \rightarrow \) On long distances /times \( SU(2) \) classical dynamics generates a Gaussian process.
**Dimensional reduction**

**Scales:** Electric: \( \omega_p^2 \sim g^2 \sum \frac{T}{k} \rho_k^2 \sim \frac{g^2 T}{a} \) UV divergent

Autocorr. scale: \( \frac{1}{T_{eq}} = \tau_{eq} \sim g^2 T \sim \tau_{mag} \)

At scales longer than \( T_{eq} \): Dynamics described by

\[
\sigma \frac{dA}{dt} = -D \times B + \eta \quad \text{with} \quad \langle \eta_x, \eta_y \rangle = 2 \alpha T \delta(x-x')
\]

**Color conductivity:** \( \sigma \sim \frac{\omega_p^2}{g} \)

**Quantum theory:** \( \sigma \sim \frac{T/k}{\ln (d_{mag}/d_{el})} \sim \frac{T/k}{\ln \frac{1}{g^2 a}} \)

Classically: \( \sigma \sim \frac{1}{a \ln \frac{1}{g^2 a}} \)

Observers sensitive to long distances/time scales:

\[
\langle \Theta(A) \rangle = \int dA \frac{O(A)}{P(A)} e^{-\frac{W[A]}{T}} \text{ with } W[A] = \int d^3x \frac{1}{2} B^2(x)
\]

From Fokker-Planck Eq,

\[
\sigma \frac{\partial}{\partial t} P[A] = \int d^3x \frac{\delta}{\delta A} \left( T \frac{\delta P}{\delta A} + \frac{\delta W}{\delta A} P[A] \right)
\]

as stationary solution \( A(x,t) \rightarrow A(x) \).

\[
B_i = \frac{1}{2} \varepsilon_{ijk} F_{(4)}^{jk} = \frac{1}{2} \varepsilon_{ijk} f_{(3)}^{jk} \quad \text{where} \quad f_{(3)}^{jk} \text{ is full 3-D tensor.}
\]

However, \( F_{(4)}^{jk} \) and \( f_{(3)}^{jk} \) have different dimensions,

\[
\left[ \int d^3x \left( F_{(4)}^{jk} \right)^2 \right] = \text{Energy} \quad \left[ \int d^3x \left( f_{(3)}^{jk} \right)^2 \right] = \text{Energy}
\]

Define \( f_{(3)}^{jk} = \frac{1}{a} F_{(4)}^{jk} \), then

\[
\frac{1}{T} W(A) = \frac{1}{T} \int d^3x \int d^3x \left( f_{(3)}^{jk} \right)^2 \equiv \int d^3x \left( \frac{1}{T_{(3)}} S_{(3)}^{(3)} [A] \right) = \int d^3x \left( S_{(3)}^{(3)} [A] \right).
\]
Thus in 3 (euclid.) dimensions

\[ \langle \Theta \rangle = \int \mathcal{DA} \ e^{- \frac{1}{\hbar^{(3)}} \mathcal{S}^{(3)}} \Theta(A) \]

with \( \frac{\hbar^{(3)}}{\alpha T} = \alpha T \). Rescaling also generates a new 3-D coupling constant

\[ g^{2}_{(3)} = \frac{g^{2}}{\alpha} = \frac{g^{2}T}{\hbar^{(3)}} \]

3-D observer measures \( \hbar^{(3)} \) and \( g^{2}_{(3)} \), from which she can deduce a length scale

\[ \Lambda_{\text{mag}} = g^{2}_{(3)} \hbar^{(3)} = g^{2}T \]

but cannot determine \( \alpha \) or \( T \) alone.

\hline

**Speculations:**

5D \( \rightarrow \) 4D ?

5-D lattice gauge theory (classical) has not been studied, but there are good reasons to believe it is chaotic.

How small can \( \alpha \) be? To ignore gravity microscopically,

\[ \alpha \gtrsim GT \rightarrow \hbar \gtrsim \alpha T^{2} \quad \text{or} \quad T \approx \frac{\hbar}{\pi g^{2}} \approx 17p_{\text{e}} \]

and similarly \( \alpha \gtrsim h \). Planck scale.

\hline

Information loss is essential ingredient. \( A(x,t) \rightarrow \overset{\text{smoothed over distances } (g^{2}T)^{-1}}{A(x)} \).
The Renormalization-group Method Applied to Transport Equations

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The renormalization group (RG) method is applied to deduce a slow dynamics from transport equations, i.e., Boltzmann equation and Langevin equations. We deduce a hydrodynamic equation from the Boltzmann equation and also Fokker-Planck equations from Kramers equations which correspond to Langevin equations with multiplicative noise.

Following [1], we start with derivation of an exact RG equation for asymptotic analysis of evolution equations, which is analogous to the Wilsonian RG equations in statistical physics and quantum field theory. It is clarified that the perturbative RG method constructs invariant manifolds successively as the initial value of evolution equations, thereby the meaning to set \( t_0 = t \) is naturally understood where \( t_0 \) is the arbitrary initial time. We show that the integral constants in the unperturbed solution constitutes natural coordinates of the invariant manifold.

We show that the RG equation determines the slow motion of the would-be integral constants in the unperturbed solution on the invariant manifold. We emphasize that the underlying structure of the reduction by the RG method as formulated in the present work turns out to completely fit to the universal one elucidated by Kuramoto[2] a decade ago. We indicate that the reduction procedure of evolution equations has a good correspondence with the renormalization procedure in quantum field theory.

We work out two simplest examples from ordinary differential equations; the damped harmonic oscillator and the Rayleigh oscillator which has a limit cycle. Then we examine a generic non-linear equation in which the linear operator in the unperturbed equation has degenerate zero eigenvalues, to show how a reduction of dynamics is described in the RG method: We explicitly show how the invariant or attractive manifold and the reduced dynamics on it are naturally deduced perturbatively in the RG method.

As good examples of this generic system, we examine the hydrodynamic limit of the Boltzmann equation and the reduction of the Kramers equation to Fokker-Planck equation. In the latter case, we discuss on the way how the Ito-Storatovitch dilemma is resolved when the Langevin equation with multiplicative noise is extended to Newton equation with an inertia.

[1]The original title was “The Renormalization-group Method Applied to Kinetic Equations”. However, I feel now that the changed title reflects more the content of the talk.
References

   See also, T. Kunihiro, Prog. Theor. Phys. 94(1995)503; (E) ibid., 95(1996)835;

The Renormalization Group Method
Applied to Kinetic Equations

T. Kunihro (YITP)
Equilibrium & Non-equilibrium
Aspects of Hot, Dense QCD
July 19th - July 20th, 2000
Riken - BNL workshop

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§ RG method for reduction
inv. manifold & dynamics on the manifold
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§ Summary

Ref. Prog. Theor. Phys. 94, 503 (75)
99, 199 (99)
Suppl 2, 459 (98)

§ Boltzmann Equation → Fluid dynamics
§§ Boltzmann eq.
\[ \frac{\partial f}{\partial t} + \nabla \cdot \mathbf{v} f = \mathcal{L}[f] \]
\[ \mathcal{L}[f] = \int dv \int dv' \int dv'' f(v,v') \mathcal{W}(v,v',v'') \left\{ f(v',v) + f(v,v') - f(v,v') f(v',v) \right\} \]

\[ \mathcal{W}(v,v',v'') = \mathcal{W}(v',v,v'') \]

A Conserved Quantities:
1. The particle number \[ \int dv \, \mathcal{C}[f] = 0 \]
2. Total Momentum \[ \int dv \, \mathcal{V}[f] = 0 \]
3. Energy \[ \int dv \, \mathcal{E}[f] = 0 \]

§§ Collision invariants: \[ q(v) \]
\[ \int dv \, \mathcal{C}[f] q(v) = 0 \]

\[ H[\mathbf{f}] = \int dv \, f \mathcal{H} \]
\[ J_{\mathbf{v}} = \int dv \, \mathbf{v} f \mathcal{J} \]
\[ \partial_{t} H + \nabla \cdot \mathbf{v} J_{\mathbf{v}} = \int dv \, \mathcal{C}[f] \mathcal{J} \mathcal{J} \mathcal{H} \]

If \( \partial_{t} J_{\mathbf{v}} \) is conserved, \( H \) is conserved.

\[ \phi + \mathbf{v} + c \mathbf{v}^2 \leftrightarrow f \text{ is Maxwellian} \]
Maxwell distribution \( f_n \):

\[
f(n,v,t) = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m(v-u)^2}{2kT} \right]
\]

e^a \sim n, \text{ etc.}

\[\underline{\text{§§ A derivation of fluid dynamical eq. from Boltzmann eq. by the RG method}}\]

\[\text{Discretize } v \rightarrow v_i\]

\[\text{Write } f_i(v_i,u_i,t) = f_i : \left( f_i(v_i,t) \right)\]

\[
\left[ \begin{array}{l}
\text{Boltzmann eq. } \frac{\partial f_i}{\partial t} = (\mathcal{E} f_i), -v_i \cdot \frac{\partial f_i}{\partial v_i}
\end{array} \right]
\]

\[
\text{where } (\mathcal{E} f_i) = \sum_{j,k} f_i(u_j,v_j,v_i) (f_{i,j} - f_{j,i}) \delta_{ik}
\]

To extract slow and low wave length motion, of

\[
\text{We have } \frac{\partial f_i}{\partial t} = (\mathcal{E} f_i) - \varepsilon \psi_i \cdot \frac{\partial f_i}{\partial x_i}
\]

suitable form to apply perturbation theory.

(i) Expand the solution around \( t = t_0 \).

\[
f_i(u,v,t; t_0) = f_i^{(0)}(u,v,t; t_0) + \varepsilon f_i^{(1)}(u,v,t; t_0) + \cdots
\]

\[
z.e. f_i(u,v,t; t_0) = f_i^{(0)}(u,v,t; t_0)
\]

\[
f_i^{(1)}(u,v,t; t_0) = f_i^{(1)}(u,v,t; t_0)
\]

(ii) \( O(\varepsilon) \)

\[
\frac{\partial f_i^{(0)}}{\partial t} = (\mathcal{E} f_i^{(0)})
\]

with \( z.e. f_i^{(0)}(u,v,t; t_0) = \tilde{f}_i(u,v,t_0) \)

We are interested in a slow motion as \( t \rightarrow \infty \), so we choose the stationary sol. \( \frac{\partial f_i^{(0)}}{\partial t} = 0 \) as the 0-th order sol.

\[\text{i.e. } \mathcal{E} f_i^{(0)} = 0 \text{ coll. inv. } f_i^{(0)} = \text{ local Maxwellian!} \]
The choice of I.C.

\[ f^0(t; t_0) = A^x \Omega \dot{\gamma} \frac{\partial f^0}{\partial x} \]

then

\[ \tilde{f}^0(t; t; t_0) = -(t-t_0) \tilde{P} \nabla \cdot \frac{\partial f^0}{\partial x} + A^x \nabla \cdot \frac{\partial f^0}{\partial x} \]

up to order,

\[ \tilde{f}(r_t; t; t_0) = \tilde{f}(r, t; t_0) + \varepsilon \left[ -(t-t_0) \tilde{P} \nabla \cdot \frac{\partial f^0}{\partial x} + A^x \nabla \cdot \frac{\partial f^0}{\partial x} \right] \]

R.G. eq. : \[ \frac{\partial \tilde{f}}{\partial t} \bigg|_{t=t_0} = 0 \]

\[ \frac{\partial \tilde{f}^0}{\partial t} \bigg|_{t=t_0} + \varepsilon \tilde{P} \nabla \cdot \frac{\partial f^0}{\partial x} = 0 \]

- master eq. for fluid dynamic of \( f \)

Today the inner products with \( m, \dot{m} m, \dot{m}^2 \).

\[ \varepsilon \partial_e P + \nabla \cdot \tilde{P} = 0 \]

\[ \varepsilon (\dot{P} \dot{u}_i) + \frac{\partial}{\partial t} (P \dot{u}_i \dot{u}_j) + \frac{\partial^2}{\partial x^2} P = 0 \]

\[ \varepsilon (P \dot{u}^2 + e) + \frac{\partial}{\partial x} \left( (P \dot{u}^2 + e + \tilde{P}) \dot{u}_i \right) = 0 \]

\[ f = m \int d\dot{v} f^{(v)}(v, \dot{v}), P \dot{u}_i = m \int d\dot{v} \dot{u}_i f^{(v)}, \varepsilon = \int d\dot{v} \dot{u}^2 \dot{v} - m \int d\dot{v} f^{(v)} \]

\[ PV = n k T \quad \text{Energy of no dissipation up to this order.} \]
\[ \text{PQCD (} T \geq 2T_c \text{; } N_c=3) \]

KK-Laine-Rummukainen-Schröder

hep-ph/0007109

Braaten-Nieto ....

\[ \int d\phi e^{-\int d\tau dx L(\phi)} \]

How far can one go analytically with controlled good accuracy?

Minimise numerics

All the contents of the talk are in the above hep-ph + a forthcoming longer paper; here only the main result is given:
Starting point

\[ e_0 = e_1 \log y_0 + e_2 \]

from \[ \langle A_0^2 \rangle \]

\[ 4\text{-loop effects} \]

[prominent!]

\[ \frac{p}{p_0} \]

\[ \frac{T}{\Lambda_{\text{MS}}} \]

4d lattice
Non-equilibrium quantum plasmas, quantum kinetics and the dynamical renormalization group.

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We study the real time nonequilibrium dynamics in hot quantum plasmas both in QED and scalar theories. We derive the relaxation of scalar, gauge and fermion fields implementing the dynamical renormalization group and using the hard thermal loop (HTL) approximation. We derive quantum kinetic equations from a quantum field theory diagrammatic perturbative expansion improved by a resummation via the dynamical renormalization group. This method resums secular terms in real time and leads directly to the quantum kinetic equation. We compute photon production taken into account transient process at one loop (order $e^2$). Such contribution grows only logarithmically with time instead of linearly as the two loop contribution does. However, this new one-loop photon production contribution gives comparable effects at the time scales to be observed at RHIC.
Transverse gauge field $A_T(x^i, t, e)$

$\Box A_T(x^i, t) + \int d\delta^4 x \, T_{\mu\nu}(x^i, e-t) A_{\mu}(\delta x) = 0$

Fourier modes $A_T(\vec{x}, t) = \int d^4 k \, e^{-i \vec{k} \cdot \vec{x}} A_T(\vec{k}, e)$

$\frac{\partial^2}{\partial t^2} + k^2 A_T(\vec{k}, t) + \int d\delta^4 k \, T_T(\vec{k}, e-t) A_T(\vec{k}, e) = 0$

we choose $A_T(\vec{x}, 0) = 0 \quad t \leq 0$, linear response

$A_T(\vec{x}, 0)$ initial data

Laplace Transform $\tilde{A}_T(\vec{k}, s) = \int_0^\infty e^{-s t} A_T(\vec{x}, t) dt$

$\frac{s^2 + k^2 + \tilde{T}_T(\vec{k}, s)}{s^2 + k^2 + \tilde{T}_T(\vec{k}, s)} \tilde{A}_T(\vec{k}, s) = \left[ 1 + \frac{k^2}{s^2 + k^2 + \tilde{T}_T(\vec{k}, s)} \right] A_T(\vec{x}, 0)$

$A_T(\vec{k}, s) = \frac{1}{s} \left[ 1 - \frac{k^2}{s^2 + k^2 + \tilde{T}_T(\vec{k}, s)} \right] A_T(\vec{x}, 0)$

Asymptotic behavior:

$\alpha_T(\vec{x}, t) = -\frac{\alpha_T(\vec{x}, 0)}{t}$, end-points dominate

$\alpha_T(\vec{x}, t) = 1 - \frac{\alpha_T(\vec{x}, 0)}{t}$, non-exponential decay

$\alpha_T(\vec{x}, t) = \alpha_T(\vec{x}, 0)$, oscillating effect
\[ \text{sec}(\theta) = \sum_{\omega(wk^2)} \rho(k, \omega) \left( e^{i\omega t} - e^{-i\omega t} \right) \quad \omega = \sqrt{\omega^2 + k^2} \]

**Secular terms**

<table>
<thead>
<tr>
<th>Threshold behaviour ( \rho(k, \omega) )</th>
<th>Secular terms from ( \text{sec}(\theta) )</th>
<th>DRG Resummed field amplitude</th>
</tr>
</thead>
</table>
| \( 4 \omega(w-k^2) \) | \( -\text{d}w \rho(w \omega^2) \left( \frac{t_0}{t} \right)^1 \) | \( e^{\omega w} \left( \frac{t_0}{t} \right)^1 \)
| \( T = 0 \) | | |
| \( 8 \omega T \theta(\omega-w) \) | \( \text{d}w \rho(w \omega^2) \left( \frac{t_0}{t} \right)^1 \) | \( e^{\omega w} \left( \frac{t_0}{t} \right)^1 \)
| \( \text{Soft poles} \sim -e^{\frac{2}{T} \omega w} \) | \( e^{\omega w} \left( \frac{t_0}{t} \right)^1 \) | \( e^{\omega w} \left( \frac{t_0}{t} \right)^1 \)
| \( T \gg 0 \) | \( \frac{\omega}{T} \) | \( \frac{\omega}{T} \)
| \( \text{Hard limit} \) | \( \gamma^2 \chi \frac{A(\omega-w)}{2T^2} \) | \( \gamma^2 \chi \frac{A(\omega-w)}{2T^2} \)
| \( \frac{1}{\pi} \) | \( \int \frac{d\omega}{\pi} \) | \( \int \frac{d\omega}{\pi} \)

\( \frac{1}{\pi} \) is the charge density of fermions.

\( \rho \) is the charge density of fermions.

\( \omega = \sqrt{\omega^2 + k^2} \)

The harder the singularity at threshold, the stronger grow the secular terms.

**Photon production**

\[ \int \frac{d\omega}{\omega^2} \frac{dN_{\omega}}{d\omega^2} \left( \frac{t_0}{t} \right)^1 = \frac{2g}{C_{\text{QED}}} \alpha(T) \]

\[ k(t-w) > \frac{5}{18} \pi \frac{T^2}{\omega^2} \left[ \log \frac{28(1-c)}{1} + Y - 1 \right] \theta \left( \frac{t}{t_0} \right) \]

\( \theta \sim \frac{1}{k+T} \quad \text{Semihard photons} \)

\( \text{Photons in Quantum Chromodynamics (QCD)} \)

**Off-shell**

\( \text{One-loop effect} = 0 (32) \)

\( \text{On-shell results = Two loops} \)

\( \text{Hard photons} \)

\[ k \left( \frac{dN_{\omega}}{d\omega^2} \right) = \frac{1}{4A_{\text{QCD}}} \frac{a}{\pi^2} \left( \frac{t_0}{t} \right)^1 \left( \frac{t_0}{t} \right)^1 \left( \frac{t_0}{t} \right)^1 \left( \frac{t_0}{t} \right)^1 \]

\( \alpha = \frac{3}{4} \)

\( g = \text{QCD coupling} \)

**Bauer et al.**

For \( t \rightarrow 0 \) on shell dominates, but...

**The lifetime of the QGP at RHIC**

is estimated to be \( 40 - 50 \) for...
\[ \frac{\langle k \bar{k} | h_{\pi}^\nu | k \bar{k} \rangle}{\langle k \bar{k} | h_{\pi}^\nu | k \bar{k} \rangle_{\text{on-shell}}} \]

\[ t \text{ (fm/c)} \]

\[ \begin{align*}
\text{OFF-SHELL} & = \bar{q} \to \bar{q}+\gamma, \quad \bar{s} \to \bar{t}+\gamma \\
\text{ON-SHELL} & = q+g \to \bar{t}+\gamma, \quad 1+\bar{s} \to 0+\gamma
\end{align*} \]

\[ a_s = 0.4, \quad \hbar = T = 200 \text{ MeV} \]

\[ q^2 = 5 \]

SY Wang, P. Bogomolny, H. Ide, Vegha, and D.S. Lee

hep-ph/0005223
Domain Walls and Cosmology at the QCD scale.

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Abstract

QCD was shown to have a nontrivial vacuum structure due to the topology of the $\theta = \theta + 2\pi n$ parameter. Specifically, the transitions $\theta \to \theta + 2\pi n$ are homotopically distinct—the homotopy classes are described by the winding number $n$. These transitions can be described exclusively in terms of QCD degrees of freedom (which are identified with the $\eta' \sim (\phi_u + \phi_d)$ meson) without involving the dynamical axion field. As a result of this nontrivial transition, quasi-stable QCD domain walls will appear. I construct and discuss the physics of these QCD domain walls. I argue that QCD domain walls, though classically stable, are unstable on the quantum level due to the tunneling. Therefore, this object is irrelevant for study at the present epoch. The lifetime is short enough that no cosmic QCD domain walls remain today, however, short-lived QCD domain walls might play an important role in evolution of early universe and may be detectable in energetic collisions such as those at RHIC. We also argue that the interactions with baryons make domain walls ferromagnetic. This feature may play an important role in physics shortly after the QCD phase transition in early Universe. In this case the domain walls formed at the QCD phase transition can serve as the magnetic seeds of the primordial galactic magnetic fields.
\[ L = \frac{1}{8} \int \pi^2 \left( \partial_i \varphi_u \right)^2 + \frac{1}{8} \int \pi^2 \left( \partial_i \varphi_d \right)^2 + E \cos \left( \frac{\Sigma \varphi_i}{N_c} + 2 \pi \right) + \sum_i m_i \varphi_i \cos \varphi_i \]

2. In the chiral limit \((m_u = m_d = 0)\), an analytical solution can be found:

\[ \varphi_{\text{singlet}} = \varphi_u + \varphi_d \]

\[ \varphi_{\text{pair}} = \varphi_u - \varphi_d \]

\[ \varphi_s (\infty) = 2 \pi \]

\[ \varphi_s (-\infty) = 0 \]

\[ \left\{ \begin{array}{l}
\varphi_s = 4N_c \tan^{-1} \sqrt{\frac{\pi}{4N_c} e^{\mu(z-z_0)}} \quad z-z_0 < 0 \\
\varphi_s = 2\pi - 4N_c \tan^{-1} \sqrt{\frac{\pi}{4N_c} e^{-\mu(z-z_0)}} \quad z-z_0 > 0
\end{array} \right. \]

\[ \mu^2 = \frac{2E}{N_c^2} \quad \text{in the chiral limit} \]

\[ \mu^2 = m_{\pi}^2 \]

\[ \int \pi^2 m_i^2 = \frac{4N_c}{N_f} E + \frac{4}{N_f} \sum m_i |<\bar{u}_i u_i>| + O(m_0^2) \]

This equation relation follows from the effective action.
3. The wall surface tension

\[ \sigma = \left[ \frac{1}{8} \int \pi \left( \frac{\partial \Phi}{\partial z} \right)^2 + \frac{1}{8} \int \pi \left( \frac{\partial \Phi}{\partial \bar{z}} \right)^2 + \right. \\
+ E \left( 1 - \cos \frac{\Phi \tau_{\Phi} - 2 \tau_{\Phi} \eta}{N_c} \right) + m_{\Phi} \frac{\Phi \tau_{\Phi} - 2 \tau_{\Phi} \eta}{N_c} \left( 1 - \cos \Phi \tau_{\Phi} \right) + \right. \\
+ m_{\Phi} \frac{\Phi \tau_{\Phi} - 2 \tau_{\Phi} \eta}{N_c} \left( 1 - \cos \Phi \tau_{\Phi} \right) \left. \right] \, d\bar{z} \]

In the limit \( m_\Phi = m_\bar{\Phi} = 0 \) where the analytical solution is known, \( \sigma \) is given by

\[ \sigma = \frac{4}{\sqrt{2}} \pi \frac{\sqrt{\langle G_{\mu \nu}^2 \rangle}}{G_{\mu \nu}} \left( 1 - \cos \frac{\Phi \tau_{\Phi}}{2N_c} \right) + O(m_{\Phi}) \]

\[ \sim (200 \, \text{MeV})^3 \]

4. The QCD domain wall is actually \( \Sigma \) domain wall:

a) width \( \sim m_\Phi \)

b) main contribution to \( \sigma \) comes from \( \Phi \) field

5. Initially small domain walls start to grow, forming clusters
1. We create a hole as a fluctuation.

2. Path: $\Delta \varphi = 2\pi$

It means that there is a "string-like" configuration (the edge of the hole).

3. \[
\frac{P}{ST} \sim \sqrt{\frac{S_0}{2\pi}} \int e^{-S_0} d\varphi. \text{ Set}
\]

3 zero modes instead of 4 (Coleman).

\[
\frac{x^2 + y^2 + z^2}{r^2} = R^2
\]

4. \[
S_1 = \frac{4\pi}{3} R^2 \delta - \frac{4\pi}{3} R^3 \delta \quad \delta: \text{wall tension}
\]

5. \[
R_c = \frac{2\alpha}{\delta}, \quad S_0 = \frac{16\pi}{36^2} x^3
\]
6. \[ g = \frac{4N_c}{12} \frac{e^{2}}{\pi} \left< \frac{6d_{s}}{32\pi} G_{m}^{2} \right> \text{ (1 - \cos \frac{\pi}{2N_c})} \approx (200 \text{ MeV})^3 \]

\[ \omega = \sqrt{\left< \frac{6d_{s}}{32\pi} G_{m}^{2} \right>} \approx (280 \text{ MeV})^2 \]

\[ R_c \sim \frac{2\omega}{\omega} \sim \frac{1}{200 \text{ MeV}} \sim 4 \text{ fm} \]

\[ S_0 \sim 120 \quad e^{-S_0} \ll 1. \quad \frac{p}{T} \sim \left( \frac{L^2}{4\pi \rho} \right) e^{-S_0} \left( \frac{S_0}{2\pi} \right)^{3/2} \]

7. **Life-time of the QCD domain wall**

\[ t \sim \frac{\text{Hubble size}}{c} \sim \frac{30 \text{ km}}{3 \times 10^{10} \text{ cm/s}} \sim 10^{-5} \text{ s}. \]

Moral: QCD scale "penetrates" to the cosmology scale due to \( e^{-S_0} \ll 1 \), such the QCD domain wall lives for a long time.
QCD at Finite Density of Isospin: from pion to $\bar{q}q$ condensation

- Motivation and Formulation
- Positivity and Inequalities
- Small $\mu$ - pion condensation
- Large $\mu$ - $\bar{q}q$ Cooper pairing
- Quark-hadron continuity
  - Confinement
- Phase Diagrams
\[ \mathcal{L} = \frac{1}{4} \int \text{Tr} \left[ \partial_{\mu} \Sigma \partial_{\nu} \Sigma^+ - 2m_{\pi}^2 \text{Re} \Sigma \right] \]

\[ \text{two phenom. parameters} \]

\[ \mu_{\Sigma} \neq 0: \quad \delta_0 \Sigma \rightarrow \delta_0 \Sigma = \delta_0 \Sigma - \frac{1}{2} \mu_{\Sigma} (\tau_3 \Sigma - \Sigma \tau_3) \]

"vector pot-l" coupled to \( I_3 \)

\[ \Sigma \rightarrow U \Sigma U^+ \]

\( U \) generated by \( I_3 = \frac{1}{2} \tau_3 \)

\[ \mathcal{L}(\Sigma) \text{ as a function of } \mu_{\Sigma} \text{ determines:} \]

- vacuum alignment (minimum of \( \mathcal{L}(\Sigma) \))
- masses/mixings of mesons (curvatures)

2 phases as function of \( \mu_{\Sigma} \):

(i) \( |\mu_{\Sigma}| < m_{\pi} \) - normal phase (vacuum)

(ii) \( |\mu_{\Sigma}| > m_{\pi} \) - Pions Bose condense (\( \pi^- \) for \( \mu_{\Sigma} < 0 \)) (\( \pi^+ \) for \( \mu_{\Sigma} > 0 \))

In the language of \( \Sigma \):

\[ \Sigma = \cos \alpha + i (\tau_1 \cos \phi + \tau_2 \sin \phi) \sin \alpha \]

with \( \cos \alpha = \frac{m_{\pi}^2}{\mu_{\Sigma}^2} \) (\( \alpha = 0 \) in normal phase).

Degeneracy w.r.t. \( \phi \). Why?

\( \mathcal{L}(\Sigma) \) has one remaining \( U(1)_V \) sym. (\( I_3 \) rots)

\( \alpha \neq 0 \Rightarrow \Sigma \) breaks this \( U(1)_V \Rightarrow \text{Goldstone} \)
- SMALL $\mu_I$ (contd.)

Spectrum: ($\mu_I < 0$)

\[ |\mu_I| = m_\pi \quad |\mu_I| \ll m_\rho \]

\[ m_\pi \]

\[ \pi^+ \quad \pi^0 \quad \pi^- \]

Goldstone

normal \quad pion superfluid

Density of isospin (eq. of state):

\[ n_I = f_\pi^2 \mu_I \left( 1 - \frac{m_\pi^4}{m_\rho^4} \right) \]

\[ |\mu_I| - m_\pi \ll m_\pi \Rightarrow n_I \sim f_\pi^2 (|\mu_I| - m_\pi) \]

linear in $|\mu_I| - m_\pi$ as in nonrelativistic Bose gas (with repulsion).
Equation of State

\[ n_i = \int \mu_i \left( 1 - \frac{m_i^4}{\mu_i^4} \right) \] 

\[ P = \int n_i d\mu_i \quad \rho = \mu_i n_i - P \quad \] 

\[ \frac{P}{\rho} = \frac{\mu_i^2 - m_i^2}{\mu_i^2 + 3m_i^2} \]
Large \( \mu_x \)

\( \mu_x \gg \Lambda_{QCD} \)

Quarks/gluons become relevant (weakly interacting) degrees of freedom.

Neglecting int.: 2 Fermi spheres: \( \bar{u} \) and \( d \)

of radii \( 1/\mu x^{1/2} \).

Interaction is attractive in color singlet \( \bar{u}d \)

\( \Rightarrow \) Cooper pairing and condensation:

\[ \langle \bar{u}s\bar{d} \rangle \neq 0. \]

Gap:

\[ \Delta = \frac{\mu x}{A} g^{-5} e^{-c/g} \]

\[ c = \frac{3\alpha^2}{2} = \frac{\text{Csc} \sqrt{2}}{\sqrt{2}} \]

\( 1g \) attraction is stronger in color singlet \( \bar{u}d \) than in \( \bar{s} ud \)

* Instantons choose parity odd channel

\[ \langle \bar{u}s\bar{d} \rangle \text{ over } \langle \bar{u}d \rangle \] (in agreement with inequalities !)

Quark-Hadron continuity

Small \( \mu_x \) — BEC of \( \pi^- \)

Large \( \mu_x \) — BCS of \( \bar{u}s\bar{d} \)

Same symmetries, same q. numbers \( \Rightarrow \) same phase?

Conjecture (a la Schäfer-Wilczek):

no phase transition between \( \pi^- \) BEC and \( \bar{u}s\bar{d} \) BCS.

Hadron d.o.f. \( \Rightarrow \) smoothly with \( \mu_x \) \( \Rightarrow \) quark d.o.f.
PHASE DIAGRAMS

Want: \( T \) vs \( \mu_I \) vs \( \mu_B \)

Can: \( T \) vs \( \mu_I \)

Sketch: \( \mu_I \) vs \( \mu_B \)

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Properties of Gluons in Color Superconductors

Dirk H. Rischke
RIKEN-BNL Research Center
Brookhaven National Laboratory
Debye and Meissner masses

\[ m_D^2 = -\Pi_{aa}^{\infty}(0), \quad m_M^2 = \Pi_{ab}^{\infty}(0), \quad m_\eta^2 = \frac{N_f}{6\pi^2} g_\eta^2 \]

\[ N_f = 2: \quad \phi^{ij} = \epsilon^{ijk} \phi^k, \quad \phi^k \sim \delta^k^3 \]

<table>
<thead>
<tr>
<th>( a )</th>
<th>( m_D^2 ) [ 3 \ m_\eta^2 ]</th>
<th>( m_M^2 ) [ m_\eta^2 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1, 2, 3 )</td>
<td>0!</td>
<td>0</td>
</tr>
<tr>
<td>( 4 - 7 )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( 8 )</td>
<td>1</td>
<td>( \frac{1}{3} )</td>
</tr>
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</table>

\[ N_f = 3: \quad \phi^{ij}_{gh} = \epsilon^{ijk} \epsilon_{gh} \phi^k_h, \quad \phi^k_h \sim \delta^k_h \]

<table>
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<tr>
<th>( a )</th>
<th>( \frac{21 - 8 \ln 2}{54} )</th>
<th>1</th>
<th>( \frac{21 - 8 \ln 2}{54} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 - 8 )</td>
<td>0.2862</td>
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</table>

Glueon self-energy at nonzero $p_0, \mathbf{p}$:

$$\Pi_{aa}^{\circ\circ}(p_0, \mathbf{p}) = \text{Re} \Pi_{aa}^{\circ\circ}(p_0, \mathbf{p}) + i \text{Im} \Pi_{aa}^{\circ\circ}(p_0, \mathbf{p})$$

$$\Pi_{aa}^{t_1}(p_0, \mathbf{p}) = (\delta^{ij} - \hat{\mathbf{p}}^i \hat{\mathbf{p}}^j) \Pi_{aa}^{t_1}(p_0, \mathbf{p}) + \hat{\mathbf{p}}^i \hat{\mathbf{p}}^j \Pi_{aa}^{t_1}(p_0, \mathbf{p})$$

$$\Pi_{aa}^{t_1, t_2}(p_0, \mathbf{p}) = \text{Re} \Pi_{aa}^{t_1, t_2}(p_0, \mathbf{p}) + i \text{Im} \Pi_{aa}^{t_1, t_2}(p_0, \mathbf{p})$$

(Do not consider $\Pi_{aa}^{\circ\circ}$ at this point, as it is irrelevant for the solution of the gap equation.)

For now, focus on $N_f = 2, T = 0$:

$\Rightarrow$ Only consider $a = 1, 2, 3, 8$ ($a = 4, \ldots, 7$ do not enter gap equation).

$$\Pi_{11}^{\mu\nu} = \Pi_{22}^{\mu\nu} = \Pi_{33}^{\mu\nu} \Rightarrow \text{Only consider } a = 1, 8.$$  

$$\Pi_{88}^{\mu\nu} = \frac{2}{3} \Pi_{0}^{\mu\nu} + \frac{1}{3} \Pi_{1}^{\mu\nu}$$

\text{HDL self-energy}

$$\Pi_{11}(p_0, \mathbf{p}) = \frac{p_0^2}{p^2} \Pi_{11}^{\circ\circ}(p_0, \mathbf{p}) , \quad \Pi_{1}^{\circ\circ}(p_0, \mathbf{p}) \neq \frac{p_0^2}{p^2} \Pi_{11}^{\circ\circ}(p_0, \mathbf{p})$$

$\Rightarrow$ Imaginary parts: analytically computable (in terms of elliptic integrals)

Real parts: have to be computed numerically
$\text{Im } \Pi_{ab}^{\mu\nu}(p_0, p)$ in 2-flavor CSC

Glueon momentum $p = 4\phi$
Conclusions

Color-superconducting condensate affects gluon propagation for gluon energies and momenta $p_\mu, p \gg x\phi, x \gg 1$.

- It does not affect it for $p_\mu, p \lesssim q\mu \gg \phi$.

Outlook

How does this change the solution of the gap equation? $\phi = 2\mu b \exp \left[ -\frac{c}{g} \right]$

Do $b, c$ change from their HDL values? $b_{\text{HDL}} = 256\pi^4 \left( \frac{2}{N_f} \right)^{5/2} q^{-5}$, $c_{\text{HDL}} = \frac{3\pi^2}{\sqrt{2}}$
Quark Stars from
Perturbative QCD

Eduardo S. Fraga

Nuclear Theory - BNL

Work done in collaboration with Jürgen Schaffner-Bielich (RIKEN-BNL)
Summary

1. Introduction and motivation

Brief history:

- 1932:
  Chadwick discovers the neutron
  Landau predicts the existence of neutron stars

- 1934:
  Baade and Zwicky suggest connection with supernovae explosions

- 1939:
  Tolman, Oppenheimer and Volkoff perform the 1st neutron star theoretical calculations (TOV equations)

- 1967:
  Bell and Hewish discover neutron star as a radio pulsar

- 1963:
  Gell-Mann and Zweig propose the quark model for hadrons

- 1965:
  Ivanenko and Kurdgelaidze present the hypothesis of quark stars in super-dense stars

- 1975:
  Collins and Perry suggest that matter at the high densities found in neutron star cores is a quark soup due to hadrons overlap (asymptotic freedom - 1973); expect: superfluidity and superconductivity

2. Usual approach to quark stars and strange stars

3. Quark stars from perturbative QCD

4. Discussion of results

5. Conclusion and future work
- 1975/78:
  Many papers on quantum-hadron phase transition, high-density regime and neutron stars (Baym & Chin, Kislinger & Morley, Baluni, Freedman & McLerran, ...)

- 1978:
  Freedman and McLerran develop the first systematic quark star phenomenology by using high-density perturbative QCD.

- 1984:
  Witten proposes the idea of stable strange matter, i.e., quark matter rather than nuclear matter might be the ground state of QCD at finite baryon number (Followed by studies by Farhi and Jaffe) → Quark matter in neutron star cores → Strange stars!

- 1986:
  Haensel, Zdunik and Schaeffer / Alcock, Farhi and Olinto discuss in detail the phenomenology of self-bound strange stars

...:
Hybrid stars, different families of neutron stars, strange stars, etc. (see, e.g., Glendenning, Compact Stars)

Physical description (basics):

Neutron stars, strange stars, hybrid stars, ... too many possibilities. How to decide?

Astronomical observables:

- $M$: total mass
- $R$: total radius
- $I$: moment of inertia

Example (Bombaci, astro-ph/0002524):

![Graphical representation of neutron star properties](image)
How to calculate them - TOV equations:

Einstein's equations + static + spherically symmetric stars:

\[
\frac{dp}{dr} = -\frac{GM(r)c(r)}{r^2} \left[ 1 + \frac{\rho(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^2 p(r)}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}
\]

\[
\frac{dM}{dr} = 4\pi r^2 \epsilon(r) ; \quad M(R) = M
\]

Given \( M(0) = 0 \), \( c(0) = c_0 \), and the equation of state (EoS) \( p = p(\epsilon) \), one can integrate the TOV equations from the origin until the pressure \( p(r) \) becomes zero at \( r = R \).

Given \( p = p(\epsilon) \), \( M(\epsilon_0) \) defines a family of stars. Extrema in \( M(\epsilon_0) \) signal gravitational instability (theorem, see Weinberg) \( \rightarrow \) maximum mass.

Then: different types of stars (neutron, strange, ...) give different EoS \( \rightarrow \) different astronomical output.

Remark: The usual approaches to quark stars rely on the MIT bag model and provide results that depend strongly on the bag parameter \( B \). It would be nicer to obtain results which depend on quantities like \( \alpha_s \) or \( \beta(\alpha_s) \) instead.

2. Usual approach to quark stars and strange stars

Physical picture:

- Strange matter described by a Fermi gas of \( u \), \( d \), and \( s \) quarks, and electrons, where the region the quarks live in is characterized by a constant energy density \( B \).

- Star temperature \( \ll \) typical chemical potentials \( \rightarrow T = 0 \)

- Chemical equilibrium:

\[
d \rightarrow u + e^- + \overline{\nu_e}
\]

\[
u + e^- \rightarrow d + \nu_e
\]

\[
s \rightarrow u + e^- + \overline{\nu_e}
\]

\[
u + e^- \rightarrow s + \nu_e
\]

\[
s + u \rightarrow d + u
\]

Then:

\[
\mu_d = \mu_s \equiv \mu
\]

\[
\mu_u + \mu_q = \mu
\]
- Overall charge neutrality:

\[ \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0 \]

Then: There is only one independent chemical potential.

\[ n_i = -\frac{\partial \Omega_i}{\partial \mu_i} \]

\( \Omega_i \): thermodynamic potential (free gas + eventual \( O(\alpha_e) \) corrections, \( \alpha_e = \text{const} \))

For the simple case \( m_u = m_d = m_s = 0, \alpha_s = 0, \) and \( \mu_e = 0 \) (\( n_e/n_u \approx 0 \)), the EoS is:

\[ p(\epsilon) = \frac{1}{3}(\epsilon - 4B) \]

Remark: For intermediate values of \( m_u \), the correction is less than 4%. The EoS is dominated essentially by \( B \)! 

Global properties of strange stars (from TOV equations):

\[ p = -B - \sum_i \Omega_i ; \quad \epsilon = B + \sum_i (\Omega_i + \mu_i n_i) ; \quad B^{1/4} = 145 \text{ MeV} \]

\[ M_{\text{max}} = \frac{0.0258}{G^{3/2}} R^{1/2} \approx 2M_{\text{sol}} \]

\[ R_{\text{max}} = \frac{0.095}{G^{1/2}} R^{1/2} \approx 11 \text{ km} \]

\[ \epsilon_{\text{c,max}} = 19.2B \approx 2 \times 10^{18} \text{ g/cm}^3 \approx 8\epsilon_0 \]

\[ \epsilon_{\text{surf}} = 4B \approx 4 \times 10^{14} \text{ g/cm}^3 \approx 2\epsilon_0 \]

\[ \epsilon_0 = \epsilon_{\text{nucleus}} \approx 2.5 \times 10^{14} \text{ g/cm}^3 \]

\[ c_{\text{surf}} = c_{\text{solid Fe}} \approx 7.8 \text{ g/cm}^3 \]

Remarks:

- Results depend crucially on \( B \).
- Very different patterns for NS and SS. However, for \( M \approx 1.4M_{\text{sol}} \) most of the properties are almost the same.
- Surface details are complicated and will not be discussed.
Plots from Alcock, Faedi and Olinto (86)

Fig. 1.—Density ($\rho$) vs. radius ($r$) for strange stars of mass (a) 0.53 $M_\odot$, (b) 1.35 $M_\odot$, (c) 1.95 $M_\odot$, and (d) 1.99 $M_\odot$.

Fig. 2.—Total mass ($M$) vs. central density ($\rho_c$) for stable strange stars.
3. Quark stars from perturbative QCD

Physical picture:

- Star is made only of a gas of u, d and s quarks.

- Strong interaction in this dense quark-gluon plasma is taken into account perturbatively up to $O(\alpha_s^2)$; $\alpha_s = g^2/4\pi$.

- $\alpha_s$ is allowed to run according to the renormalization group equation.

- No bag constant is introduced.

- Star temperature $< \ll$ typical chemical potentials $\rightarrow T = 0$

- Charge neutrality + chemical equilibrium:

$$\mu_u = \mu_d = \mu_s = \mu$$

Then: Use high-density zero-temperature perturbative QCD. The typical densities one can find inside quark stars allow for a sensible use of perturbation theory [Pisarski & Rischke (2000); cold, dense quark matter].

Warning: It is not our aim to provide a realistic and accurate description of the phenomenology related to quark stars by using such a crude model. We intend to highlight the essential difference between the bag equation of state approach (strong dependence on $B$) and the perturbative QCD approach (dependence on $\alpha_s$, $\beta(\alpha_s)$, etc.).
Remark: Nevertheless, it is natural to expect that the results obtained from bag models should correspond to some limit in this more fundamental approach.

The thermodynamic potential:

The thermodynamic potential of a plasma of massless quarks and gluons with $N_c$ colors and $N_f$ flavors upto $O(a^2)$, in the MOM subtraction scheme and using the Landau gauge, is given by [Freedman & McLerran (77)]

\[
\Omega_{\text{MOM}} = \sum_i \Omega_{i}^{(0)} \left\{ 1 - \left( \frac{\alpha_s^{\text{MOM}}}{4\pi} \right) \left( \frac{3N_c}{N_c} \right) + \left( \frac{\alpha_s^{\text{MOM}}}{4\pi} \right)^2 \left( \frac{3N_f}{N_c} \right) \times \right.
\]

\[
\times \left[ C - \frac{\mu^2}{\mu_0^2} \left( 2 \ln \left( \frac{\alpha_s}{4\pi} \right) - D \right) - F_1(\mu) \right] \}
\]

\[
\Omega_{i}^{(0)} = -\mu_i^4 N_c / (12\pi^2) \quad \text{(ideal gas, 1 flavor, massless, } T = 0) \]

\[
F_1(\mu) = -2 \frac{\mu_i^2}{\mu_0^2} \ln \frac{\mu_i^2}{\mu_0^2} + \frac{1}{2} \sum_{i \neq j} \left[ \frac{2}{3} \left( \frac{\mu_i - \mu_j}{\mu_i \mu_j} \right) \ln \frac{\mu_i^2 - \mu_j^2}{\mu_i \mu_j} \right] + \frac{8}{3} \frac{\mu_i \mu_j (\mu_i^2 + \mu_j^2) \ln \left( \mu_i^2 + \mu_j^2 \right) - 2}{3} \left( \frac{\mu_i^2 - \mu_j^2}{\mu_i \mu_j} \right) \ln \frac{\mu_i}{\mu_j} \]

\[
C = -2.250N_c + 0.409N_f - 3.697 - \left( \frac{4.24 \pm 0.12}{N_c} \right) \quad ; \quad D = 0.476
\]

\[
\alpha_s^{\text{MOM}} = \alpha_s^{\text{MOM}}(\mu_0) \quad ; \quad \mu_0 : \text{Euclidean subtraction point at which the charge is defined.}
\]

Translation from MOM to $\overline{\text{MS}}$ [Celmaster & Sivers (81); Raczka & Raczka (89)]

\[
\alpha_s^{\text{MOM}} = \alpha_s^{\overline{\text{MS}}} \left[ 1 - A \frac{\alpha_s^{\overline{\text{MS}}}}{\pi} \right]
\]

\[
A = -6.623 + 0.854N_f
\]

Then: Up to this order in the expression of $\Omega$, the translation between schemes corresponds to a shift in the constant of the 2nd order term of $\Omega^{\text{MOM}}$.

Final form for $\Omega$ in the $\overline{\text{MS}}$ scheme:

\[
\Omega(\mu) = \frac{N_f N_c^4}{4\pi^2} \left\{ 1 - 2 \left( \frac{\alpha_s}{\pi} \right) - \left( \frac{\alpha_s}{\pi} \right)^2 \left[ G + N_f \ln \frac{\alpha_s}{\pi} \right] \}
\]

\[
G = N_f \ln N_f + \frac{4}{3}(N_f - 1) \ln 4 - \frac{1}{2} (\tilde{C} + N_f D)
\]

\[
\tilde{C} = C + 4A \quad ; \quad D = 3.249
\]

The coupling $\alpha_s(\mu_0)$ runs according to

\[
\alpha(\mu_0) = \frac{4\pi}{\beta_0 \ln \left( \mu^2 / \Lambda_{\overline{\text{MS}}}^2 \right)} \left[ 1 - 2 \beta_1 \ln \left( \mu^2 / \Lambda_{\overline{\text{MS}}}^2 \right) + \frac{4\beta_1^2}{\beta_0^2 \ln^2 \left( \mu^2 / \Lambda_{\overline{\text{MS}}}^2 \right)} \left( - \frac{1}{2} \right) + \frac{\beta_2}{6 \beta_0^2} - \frac{5}{4} \right]
\]

\[
\beta_0 = 11 - 2N_f / 3 \quad ; \quad \beta_1 = 51 - 19N_f / 3
\]

\[
\beta_2 = 2857 - 5033N_f / 9 + 325N_f^2 / 27 \quad ; \quad \Lambda_{\overline{\text{MS}}} = 364.834 \text{MeV}
\]
Choice of $\mu_0$:

Since the only physical scale of the system is given by the Fermi momentum of the interacting massless quarks, we choose $\mu_0 = \mu$.

Another “physically motivated” choice: $\mu_0 = \mu_B = 3\mu \rightarrow$ leads to results that reproduce the features obtained by using the bag model approach.

From the knowledge of $\Omega(\mu)$:

\[ p = p(\mu) = -\Omega(\mu) \]
\[ \varepsilon = \varepsilon(\mu) = -p(\mu) + \mu n \]

Then: TOV equations $\rightarrow$ astrophysical features that follow can be written in terms of $\alpha_s, \beta(\alpha_s), \ldots$

In fact, one could even define a “bag function”:

\[ B(\mu) = \frac{[\varepsilon(\mu) - 3p(\mu)]}{4} = \frac{1}{2} \left( \frac{N_f \mu^4}{4\pi^2} \right) \beta(\alpha_s) \left\{ 1 + \alpha_s \left[ G + \frac{N_f}{2} N_f \ln \left( \frac{\alpha_s}{\pi} \right) \right] \right\} \]

4. Discussion of results

Equation of state:

\[ \varepsilon_0 \approx 2.5 \times 10^{14} \text{ g/cm}^3 \]
Contributions to the pressure:

Total mass x central energy density:

In the usual bag model approach:

\[ M_{\text{max}} \approx 2M_{\odot} \]

\[ \varepsilon_c^{\text{max}} \approx 2 \times 10^{15} \text{ g/cm}^3 \approx 3\varepsilon_0 \]

\[ \rho_c \approx 2.5 \times 10^{16} \text{ g/cm}^3 \]
Quark stars from perturbative QCD have the following features:

- They are smaller ($R \approx 3 \text{ km}$), less massive ($M \approx 0.5 M_{\odot}$), and denser ($\varepsilon \approx 80 \varepsilon_0$) than bag model quark stars and neutron stars. \textit{Very different phenomenology!}

- Phenomenological results depend on $\alpha_s$ and $\beta(\alpha_s)$, not on $B$.

- They represent a new class of stars (new branch in $p = p(c)$).

Related problems:

- More complicated and realistic models.

- Relation to MACHOS (Massive Compact Halo Objects)? [Best fit mass $\approx 0.5 M_{\odot}$]

- Relation to color superconductivity, CFL phase, etc?

In the usual bag model approach:

$$M_{\text{max}} \approx 2 M_{\odot}$$

$$R_{\text{max}} \approx 11 \text{ km}$$
The Electrical Conductivity in a QED Plasma
--Luis Bettencourt and Emil Mottola, Los Alamos

A calculation of the electrical conductivity of a high temperature plasma of $e^\pm$ and photons in QED in the real time formalism is described. The methods employed in this calculation are applicable to nonequilibrium processes in other field theories, and offer an interesting prototype of extracting the hydrodynamic limit from a microscopic quantum theory.

Linear response about thermal equilibrium is the simplest such real time process and the abelian QED case is the simplest gauge theory, where weak coupling methods should be applicable. Nevertheless a non-trivial resummation of perturbation theory is required to extract the weak coupling result for the DC conductivity, due the extreme sensitivity of the relevant long wavelength, large time hydrodynamic limit to infrared effects in the microscopic theory.

We propose a specific truncation of the infinite Schwinger-Dyson hierarchy of correlation functions in QED to just the electron/positron and photon two-point functions, together with the 3-point vertex function, which satisfies the relevant Ward identities. This set of S-D equations is the next non-trivial order beyond the leading order in large N which includes Landau damping and Hard Thermal Loops. The S-D equations are renormalizable and include both hard and soft scattering processes as well as the fermion damping rate in a self-consistent way. The Boltzmann equation with two-to-two scattering processes in the collision kernel can be derived from our S-D equations in a certain limit, but our approach is to keep careful track of the errors made in passing to the Boltzmann limit from the microscopic point of view. The method of solution of the S-D equations in real time in the high temperature QED plasma is sketched and the final results will appear shortly.
Electric Conductivity in High T

QED

L. Bottencourt
E.M.

Motivations:
1) Non-equilibrium (i.e. real time) Dynamics of Gauge Theories
2) Origin of Irreversibility from Microscopic Field Theory

Linear Response = simplest example of real time processes in relaxation/transport

QED = simplest (abelian) gauge theory

- Weak coupling $\alpha \ll 1$
- Should be calculable
- But long-time hydrodynamic behavior is beyond leading order $1/N$ Hartree Hard Thermal Loops

$\Rightarrow$ Non-trivial but solvable prototype problem of Nonequilibrium Field Theory

(Also relevant to early universe, mag. fields)
Boltzmann Eq.
\[ \frac{\partial}{\partial t} + \vec{\nabla} \cdot \vec{V} + e\vec{E} \cdot \vec{V} \] \( n(x, \vec{p}; t) = C \text{ollision Integral} \)

Baym-Heiselberg: \[ \left| \chi \right|^2 \] \text{only} \n
(w. HTL's resummed photon propagator)

\( n(x, \vec{p}; t) = n_{\text{eq}} (\vec{p}) + \delta n (\vec{p}, t) \)

\( c\vec{E} \cdot \vec{V}_{\text{eq}} = \mathcal{L}[n_{\text{eq}}] \cdot \delta n \)

\( \mathcal{L}[\text{Linearized Collision Int.}] \)

\( \delta n \propto \vec{E}^2 \)

\( \delta \langle \psi \rangle = e \int \frac{d^3 \vec{p}}{(2\pi)^3} \nabla \delta n(\vec{p}) \propto \vec{E}^2 \)

Coefficient is \( \sigma \)

Should give right answer \( (?) \) \text{ iff}

- All relevant scattering processes in \( C \)
- No IR divergences encountered
- Linearized Boltzmann Eq. solved exactly
- Mean Free Path \( cT \gg m^{-1}, T^{-1} \)
  (In high temp. , leading log in \( \alpha \rightarrow 0 \) only)
Conductivity in QED (Begin again...)

- \[ \langle j^\mu(x) \rangle = -ie^2 \text{Tr} \left\{ \gamma^\mu \gamma(x,x) \right\}_A \]
  \[ = 0 \quad \text{in equilibrium} \]

- \[ \Delta(x,y) \quad \text{exact } e^z \text{ Green's fn.} \]

- \( \delta \langle j^\mu(x) \rangle = -\int d^4x' \Pi^{\mu\nu}(x,x') \delta A_\nu(x') \quad (\text{Kudela}) \)
  \[ \Pi^{\mu\nu}(x,x') \quad \text{exact vertex fn.} \]

- \[ \Gamma^\mu = \frac{\delta A^{-1}}{\delta A_\mu} \quad \text{exact vertex fn.} \]

Current Cons. \( \partial_\mu \Pi^{\mu\nu}(x,x') = 0 \)

Ward Identity requires \( \Gamma^\mu \neq \delta^\mu \)

if \( A^{-1} \neq G^{-1} \) (free)

- \[ \Pi^{\mu\nu} = \Pi^T \Pi^{\mu\nu} + \Pi^L \Pi^{\mu\nu} \]

- \[ \Pi^{10} = \frac{\delta^i \delta^0 \Pi^L}{\omega^2-k^2} \quad \text{(translational, rotational, inv.)} \]

- \[ \sigma(\omega) = -\frac{1}{\omega} \lim_{k \to 0} \text{Im} \Pi^L(\omega,k) = \lim_{k \to 0} \frac{\text{Im} k \cdot \Pi^{10}(\omega,k)}{k^2} \]

- \[ \sigma_{\text{dc}} = \lim_{\omega \to 0} \sigma(\omega) = -\lim_{\omega \to 0} \left\{ \frac{1}{\omega} \lim_{k \to 0} \text{Im} \Pi^L(\omega,k) \right\} \]
Perturbation Theory Inadequate

- $\text{Im } \mathcal{O}_{k=0|\omega<\omega_0} = 0$ (lowest order or HTL resummed)

- $\text{Im } \mathcal{O}_{k=0|\omega<\omega_0} = \infty \quad \sigma_{BC} = \infty$ (!)

Technical Reason: Pinching poles = product of on-shell $S$-fns. $[S(\omega-E_p)]^2$ at finite temp.

Physical Reason: No fermion damping included in HTL approximation $\gamma_e = 0$, $\tau_e = \frac{1}{\gamma_e} = \infty$.
Electrons/Positrons live forever (Fermi Golden Rule: $[S(\omega-E_p)] \rightarrow S(\omega-E_p)\tau_e$)

$\Rightarrow$ $e^\pm$ Damping Essential:

- $\mathcal{G}^{-1} = G^{-1} + \Sigma$
  - Self-energy needed

(exact)

- $\Sigma = -i \mathcal{D}_{\mu\nu} \gamma^\mu \gamma^\nu$

Need consistent (gauge invariant) hierarchy truncation of Schwinger-Dyson Eqs.

In $\Sigma$, Try

- $\mathcal{D}_{\mu\nu}^{-1} \rightarrow \mathcal{D}_{\mu\nu}^{-1} = \mathcal{D}_{\mu\nu}^{-1} - i \text{tr} \{ \gamma_\mu G \gamma_\nu G \}$

  - Included HTL/leading order $1/N$
  - To control co-linear div.s
Gauge Invariance - Ward Identities

$$D_{\mu}^{-1} = d_{\mu}^{-1} - i \text{ tr } \{ \gamma_\mu G_A \gamma_5 G_A \}$$

is transverse:

$$D_{\mu}^{-1} \gamma^\nu = \delta^\nu_\mu D_{\nu}^{-1} = 0 \quad \text{(projector)}$$

Vertices here are bare vertices since

$$\frac{\delta G_A^{-1}}{\delta A^\mu} = \gamma_\mu$$

$D_{\mu}^{-1}$ can be inverted on transverse vectors

$$D_{\mu}^{-1} = (D_{\mu}^{-1})^{-1} \quad \text{to which an arbitrary longitudinal part can be added:}$$

$$D_{\mu\nu} = D_{\mu\nu}^L + D_{\mu\nu}^G$$

$$D_{\mu\nu}^G = k_\mu F_\nu + k_\nu F_\mu + k_\mu k_\nu \Lambda_\nu \quad , \quad k^\mu F_\mu = 0$$

Gauge Dependent Part, $F_\mu, \Lambda_\nu$ are gauge fns. drop out of $\Sigma$

but

$$A^{-1} = G_A^{-1} + i \gamma^\mu \gamma^5 \gamma_\nu D_{\mu\nu}$$

generates the non-trivial vertex

$$\Gamma_\mu = \frac{\delta A^{-1}}{\delta A^\mu} = \gamma_\mu - i \gamma^5 \gamma_\nu \Gamma_\mu \gamma_5 \gamma_\nu D_{\nu\rho}$$

$$\omega_\mu = \omega^\gamma + \omega^D$$
Effective Action \( (2\pi) \)

\[
S = S_{cl} - i \text{Tr} \log (G_0 \cdot \mathcal{A}^{-1})
+ \frac{i}{2} \text{Tr} \log (d \cdot \mathcal{D}^{-1})
+ \frac{i}{2} \text{Tr} (\mathcal{D}^{-1} \mathcal{D} - 1) + \Gamma_2 (\mathcal{D}, \mathcal{D} = D)
\]

\[
G^{-1}_A = i (\mathcal{A} - i \mathcal{A} + m), \quad G^{-1}_0 = G^{-1} \bigg|_{\mathcal{A} = 0}
\]

\[
d^{-1}_{\mu \nu} = -\frac{1}{e^2} (g_{\mu \nu} \Box - \partial_{\mu} \partial_{\nu})
\]

Variational Principle:

\[
\frac{\delta S}{\delta \mathcal{A}} = 0 = i \left\{ \mathcal{A}^{-1} - G^{-1}_A - i \frac{\delta \Gamma_2}{\delta \mathcal{D}} \right\}
\]

\[
\frac{\delta S}{\delta \mathcal{D}} = 0 = -\frac{i}{2} \left\{ \mathcal{D}^{-1} - D^{-1} + 2i \frac{\delta \Gamma_2}{\delta \mathcal{D}} \right\}
\]

\[
\mathcal{D}^{-1} = D^{-1} = d^{-1}_{\mu \nu} - i \text{tr} \left\{ g_{\mu \nu} G_A \right\}
\]

\[
\Gamma_2 = \frac{1}{2} \text{tr} (g^{\mu \nu} \gamma^\nu \gamma^\mu) D_{\mu \nu}
\]

\[
\frac{\delta \Gamma_2}{\delta \mathcal{A}} = i \gamma^\mu \gamma^\nu D_{\mu \nu} = \Sigma
\]

\[
\Sigma^{-1} = G^{-1} + \Sigma
\]
Extend to fully Non-equilibrium Processes

\[ \partial_{\mu} F^{\mu\nu} = -ie^2 \text{Tr} \{ \gamma^\nu [A^\mu, Y_A] \} \]

\[ d_A^{-1} = \gamma^\mu - iA^\mu + m - iD_{\mu
u} \gamma^\nu \]

\[ D_{\mu
u} = d_{\mu
u} - i \text{tr} \{ d_{\mu A} \gamma^\nu, G_A \} \]

- Similar to gap eqns. solved in high \( \mu \) QCD
- Follows from an action principle,
- Has bounded energy, free energy, (Improved Eq. of St)
- Satisfies appropriate Ward identities,
- Gives correct long-time hydrodynamic limit,
- Incorporates fermion damping rate,
- Goes beyond \( \hbar^2 \), Hartree, HTL's, S-D hierarchy
- QED relativistic generalization of resummation techniques in plasma physics, condensed matter, many-body problem
- Incorporates smooth soft mean fields \( A^\mu \) and hard scattering consistently
- Works for other theories like \( \lambda Ph \)

\[ \text{Extend to non-abelian gauge theory?} \]

Stay tuned....
Abstract

Conventional thermal perturbation theory yields a series expansion in powers and logarithms of the coupling $g$ with poor convergence properties, caused in particular by contributions associated with collective effects such as screening. Usually only the effects of static Debye screening are utilized in resumming perturbation theory. A more complete description should involve the full hard thermal/dense loop (HTL/HDL) propagators. It is shown that in a self-consistent $\Phi$-derivable two-loop approximation of the thermodynamic potential one can derive simple effectively one-loop expressions for the entropy and the quark densities which are formally UV finite. Gauge independent and UV finite results are obtained by restricting the propagators to those of HTL/HDL perturbation theory up to order $g^3$. The results for entropy and density need not be truncated into a polynomial in $g$ (with poor convergence properties), but can retain numerically important HTL/HDL effects that are formally of higher order in $g$. The resummation in entropy and density exhibits an interesting reorganisation of LO and NLO contributions: The LO interaction terms are caused entirely by the so-called asymptotic thermal masses at hard momentum scales, as given by the light-cone values of HTL/HDL self-energies. The plasmon effect $\sim g^3$, which usually comes from soft momentum scales only, is found to arise mostly (to 75%) from NLO corrections to the hard asymptotic masses, calculable from HTL/HDL perturbation theory. Numerically, including the NLO corrections to the asymptotic masses through an approximately self-consistent gap equation achieves remarkable agreement with lattice data at $T > 2T_c$ while being perturbatively equivalent up to and including order $g^3$. 
2-loop resummed entropy

from 2PI-skeleton expansion (Luttinger-Ward 1960
Cornwall, Jackiw, Tomboulis, 1974)

\[ \log \mathcal{Z}[D] = -\beta \Omega[D] \]
\[ = -\frac{1}{2} \text{Tr} \log (D_0^{-1} + \Pi[D]) + \frac{1}{2} \text{Tr} \Pi[D] \cdot D + \overline{\Phi}[D] \]
\[ \overline{\Phi}[D] = \frac{1}{12} \Theta + \frac{1}{8} \Theta_0 + \frac{1}{48} \Theta + \ldots \]
\[ D = \left( D_0^{-1} + \Pi[D] \right)^{-1} \]
\[ \frac{1}{2} \Pi = \frac{\delta \Omega[D]}{\delta D} \]

self-consistent 2-loop approx. : \[ \overline{\Phi} = \frac{1}{12} \Theta + \frac{1}{8} \Theta_0 \]

\[ \rightarrow \Pi = \frac{1}{2} \Theta - \overline{\Phi}[D] \]

Matsubara sum evaluated \[ \rightarrow \text{RTF (Minkowski)} \]

\[ \mathcal{Z} / V = \frac{1}{(2\pi)^4} \int \text{d}^4k \ n(k_0) \text{Im} \{ \log D_0^{-1} + \Pi D \} - \frac{T \overline{\Phi}}{V} \]

\[ n(k_0) = \frac{1}{\text{e}^{\beta k_0} - 1} \]

entropy density \[ S = -\frac{\partial \mathcal{Z} / V}{\partial T} \]

stationarity \[ \Rightarrow \frac{\partial \Omega}{\partial T} \] drops out

only \( n \)'s to be differentiated

(after Matsubara sum in \( \overline{\Phi} \) also)
dramatic simplification to:

\[ S = - \left( \frac{\partial^2 k \theta_n}{\partial \mathbf{k} \partial \mathbf{n}} \right) \ln \log D^{-1} + \left( \frac{\partial^2 k \theta_n}{\partial \mathbf{k} \partial \mathbf{n}} \right) \ln \mathbf{TT} \cdot \Re D + S' \]

\[ S' = \left( \frac{\theta_n}{\phi} \right)^{2 \text{-loop}} \left( \frac{\partial^2 k \theta_n}{\partial \mathbf{k} \partial \mathbf{n}} \right) \Re \mathbf{TT} \cdot \ln D = 0 \quad (3 \text{-loop}) \]

(Riedel 1968 in Fermi-liquid theory,
Vandersheyden + Baym 1999 in QED,
Blazot (lanca + A.R. 1999: QCD)

+ bears out:

dynamical quasi-particle description
with (residual) interactions of 3-loop order only

+ manifestly UV finite \( \frac{\theta_n}{\partial \mathbf{T}} \rightarrow 0 \) for \( k_0 \rightarrow \pm \infty \)

multiplicative renormalization of \( \mathbf{TT} \) and \( D \) drop on

approximate self-consistency through

\[ \mathbf{TT} \rightarrow \mathbf{TT}^{\text{HTL}} \]

\[ \rightarrow \mathbf{TT}^{\text{HTL}} \& \delta \mathbf{TT} \]

within HTL pert. th.

removes problem of \( \mathbf{e} \)-derivable approach that

\[ \begin{align*}
- & \text{ in gauge theories:} \\
& \quad \text{gauge dependent } \sim g^4 \sim 3 \text{-loop} \\
- & \quad \text{incomplete } (\mathbf{g})
\end{align*} \]
1st approximation to

HTL-resummed entropy in QCD:

\[ S_{1L} = S_{1L}^{QP} + S_{1L}^{LD} \]

\[ S_{1L}^{QP} = -2 \int d^3k \, \partial_T \left[ T \log (e^{\beta \omega_T} - 1) \right] \]
\[ - \int d^3k \, \partial_T \left[ T \log \frac{e^{\beta \omega_L} - 1}{e^{\beta k} - 1} \right] \]

\[ S_{1L}^{LD} = -2 \int d^4k \, \Theta(k^2 - \omega^2) \frac{\partial n}{\partial T} \partial_T \left( \Im T_T \cdot \Re \left( \frac{1}{\omega^2 - k^2 + \Pi_T} \right) \right) \]
\[ + 2 \int d^4k \, \Theta(k^2 - \omega^2) \frac{\partial n}{\partial T} \Im T_T \cdot \Re \left( \frac{1}{\omega^2 - k^2 - \Pi_T} \right) \]
\[ - \int d^4k \, \Theta(k^2 - \omega^2) \frac{\partial n}{\partial T} \partial_T \left( \Im T_L \cdot \Re \left( \frac{1}{k^2 + \Pi_L} \right) \right) \]
\[ + \int d^4k \, \Theta(k^2 - \omega^2) \frac{\partial n}{\partial T} \Im T_L \cdot \Re \left( \frac{1}{k^2 + \Pi_L} \right) \]

numerical evaluation with

\[ m_{\omega,0}^2 = \frac{N_c g^2 T^2}{6} \]

\( g(T) \) through 2-loop RGE with \( T_c = 1.14 \Lambda_{MS} \) (pure glue)

\[ \Pi_L(\omega,k) = 2 m_{\omega}^2 \left[ 1 - \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} \right] \]

\[ \Pi_T(\omega,k) = m_{\omega}^2 + \frac{\omega^2 - k^2}{2k^2} \Pi_L(\omega,k) \]
Completion of plasmon effect $\sim g^3$ requires:

$$\Delta m^2_{\omega} = \text{Re} \left. STT_{\omega} \right|_{\omega = k}$$

$k$ hard $\sim T$

NB: $\frac{3}{4}$ of (soft) plasmon effect reshuffled into hard part of spectral fct.

calculable by standard HTL pert. th.

$$\text{for numerical estimates of HTL & NLO effects:}$$

- approximate mono-local $\Delta m^2_{\omega}(k)$ by averaged value $\overline{\Delta m^2_{\omega}} / m^2_{\omega} = -\frac{\sqrt{3N}}{T} g$

- include only for hard momenta $k \geq \sqrt{2\pi T} \cdot m_D$

($\Delta m_{\omega}$ or $\Delta m$ behave differently and do not contribute to $\sim g^3$ effects)

- include such that $m_\omega$ monotousous in $g$

(Padé or approx. self-cons. gap equation)
LO: HTL-resummed entropy for pure-gluon QCD

leading order with ren. scale μ ∈ [πT, 4πT]

NLO estimate by adjusting \( m_0 \) for

hard momenta \( k ≥ \sqrt{2πTm_0} × (0.5...2) \) (Pade)

\[ \frac{S}{S_{\text{free}}} \]

\[ T/T_c \]

lattice result (Boyd et al., 1996
Okamoto et al., 1998)

⇒ good quantitative agreement

already for \( T > 2...3T_c \)
Dynamics of the chiral phase transition at finite chemical potential
Talk by Fred Cooper at Brookhaven, July 2000

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Here we want to study the dynamics of what happens when we start at high temperatures and traverse the 3 possible QCD phase transitions - first and second order chiral transition and first order superconducting transition.

- Obtain a Model with phase structure of QCD
- Determine Phase Structure as a function of $\mu$ and $T$
- Use model to study dynamics of QCD-like phase transitions
- Look for signatures for RHIC
- HERE WE IGNORE THE SUPERCONDUCTING PHASE

MODEL:

The COMPLETE MODEL with chiral and superconducting phase:

$$\mathcal{L} = \sum_i \bar{\psi}^{(i)} \gamma_\mu \psi^{(i)} + \frac{1}{2} g^2 [\bar{\psi}^{(i)} \psi^{(i)}] [\bar{\psi}^{(j)} \psi^{(j)}] + 2g^2 (\bar{\psi}^{(i)} \gamma_5 \psi^{(j)})(\bar{\psi}^{(i)} \gamma_5 \psi^{(j)}) - \mu \bar{\psi}^{(i)} \psi^{(i)}.$$ (1)

The flavor indices, summed on from 1 to $N$, have been explicitly indicated. The first term is the Gross-Neveu interaction, whereas the second such term, which differs in the arrangement of its flavor indices, induces the pairing force to leading order in $1/N$. In the final term, $\mu$ is the chemical potential. The FULL phase structure is in the Fig 1.

NEXT we restrict ourselves to the Chiral Sector $G=0$. The Lagrangian is

$$\mathcal{L} = -i \bar{\psi}_i \gamma_\mu \partial_\mu \psi^i - \frac{1}{2} g^2 (\bar{\psi}_i \psi^i)^2,$$ (2)

which is invariant under the discrete chiral group: $\Psi_i \to \gamma_5 \Psi_i$. In leading order in large $N$ the effective action is

$$S_{\text{eff}} = \int d^2x \left[ -i \bar{\psi}_i \partial_\mu \psi^i - i \sigma \bar{\psi}_i \psi^i - \frac{\sigma^2}{2\rho^2} \right] + \text{tr} \ln S^{-1}[\sigma],$$ (3)

where $S^{-1}(x, y)[\sigma] = [\gamma^\mu \partial_\mu + \sigma] \delta(x - y)$.

The equilibrium properties of the GN model at finite temperature and chemical potential have been known for a long time - L. Jacobs, Phys. Rev. D 10 (1974) 3956. U. Wolff, Phys. Lett. B 157, 303 (1985) - and is summarized by Fig. 2.
The phase structure is determined from the renormalized effective potential
\[ V_{\text{eff}}(\sigma^2, T, \mu) = \frac{\sigma^2}{4\pi} \ln \frac{\sigma^2}{m_f^2} - 1 \]
\[ -\frac{2}{T} \int_0^\infty \frac{dk}{2\pi} \ln (1 + e^{-\beta(E-\mu)}) + \ln (1 + e^{-\beta(E+\mu)}) \]  

(4)

Here \( m_f \) is the physical mass of the fermion in the vacuum sector.

The critical temperature at small chemical potential is given by
\[ T_c = \frac{m_f}{\pi} \gamma [1 - \frac{7\mu^2 \zeta(3)}{4\gamma m_f^3}] \]

The tricritical point occurs at
\[ \frac{\mu_c}{m_f} = 0.608, \quad \frac{T_c}{m_f} = 0.318. \]

We have chosen to renormalize the effective potential so its value at \( T = 0 \) in the false vacuum \( \sigma = 0 \) is zero. In the true vacuum \( \sigma = m_f \) the energy density has the value
\[ \epsilon/m_f^2 = -\frac{1}{4\pi}. \]

**Expansion of the plasma through a phase transition**

Following a heavy ion collision, the ensuing plasma expands and cools traversing the chiral phase transition. In hydrodynamic simulations of these collisions, a reasonable approximation is to treat the expansion as a 1+1 dimensional boost invariant expansion (Bjorken, Landau, Cooper et al) along the beam (z) axis. In this approximation, the fluid velocity scale as

\[ v = z/t \]

In terms of the variables fluid rapidity

\[ \eta = \frac{1}{2} \ln \left( \frac{t + z}{t - z} \right), \quad \tau = (t^2 - z^2)^{1/2} \]

physical variables such as \( \sigma, \epsilon, p \) become independent of \( \eta \).

Although the effective mass \( \sigma \) and \( \epsilon \) is a function solely of \( \tau \), two point correlation functions depend on fluid rapidity \( \eta \) differences as well.

We shall use the metric convention \((-+)-\). In our approximation, the dynamics are described by the Dirac equation with self-consistently determined mass term. Rescaling the fermion field,

\[ \psi(x) = \frac{1}{\sqrt{\tau}} \Phi(x), \]

and introducing conformal time \( u \) via

\[ \tau = m_f e^u \]

we obtain

\[ \left[ \gamma^0 \partial_u + \gamma^3 \partial_t + \tilde{\sigma}(u) \right] \Phi(x) = 0, \quad (5) \]

where \( \tilde{\sigma}(u) = \sigma_1 = \frac{z}{m_f} e^u \). Further letting \( g^2 = \lambda/2N \) we have the gap equation

\[ \sigma = -i \frac{\lambda}{2N} \langle [\Psi_i^\dagger, \gamma^0 \Psi_i] \rangle = -i \frac{\lambda}{2} \langle [\psi_i^\dagger, \gamma^0 \psi_i] \rangle, \quad (6) \]

and we have assumed here that we have \( N \) identical \( \Psi_i = \psi \).

These equations are to be solved subject to initial conditions at \( \tau = \tau_0 \). It is sufficient to describe the initial state of the charged fermion field by the initial particle and anti particle number densities, which we will assume to be Fermi-Dirac distributions described by \( \mu_0 \) and \( T_0 \).

Expanding the fermion fields \( \Phi \) in terms of Fourier modes at fixed conformal time \( u \),

\[ \Phi(x) = \int \frac{dk}{2\pi^2} \left[ b(k) \phi_+^u(k) e^{ik\eta} + d((-k) \phi_-^u(k) e^{-ik\eta}) \right], \quad (7) \]

the \( \phi^u_\pm \) then obey

\[ \left[ \gamma^0 \frac{d}{du} + i \gamma^3 k_\eta + \tilde{\sigma}(u) \right] \phi^u_\pm (u) = 0. \quad (8) \]

The superscript \( \pm \) refers to positive- or negative-energy solutions. Introducing
\[ \phi^\pm_k(u) = \left[-\gamma^0 \frac{d}{du} - i \gamma^3 k_\eta + \hat{\sigma}(u) \right] f^\pm_k(\tau) \chi^\pm, \]

where the momentum independent spinors \( \chi^\pm \) are chosen to be the orthonormal \( \pm 1 \) eigenstates of \( i \gamma^0 \), we obtain

\[ \left( -\frac{d^2}{du^2} - \hat{\omega}_k^2 \pm i \frac{d\hat{\sigma}}{du} \right) f^\pm_k(u) = 0, \]

where \( \hat{\omega}_k^2 = k_\eta^2 + \hat{\sigma}^2(u) \). We parametrize the positive-energy solutions \( f^+_k \) in a WKB manner:

\[ f^+_k(u) = N_k \frac{1}{\sqrt{2\hat{\omega}_k}} \exp\left\{ \int_0^u \left( -i \hat{\Omega}_k(u') - \frac{\hat{\sigma}(u')}{2\hat{\Omega}_k(u')} \right) du' \right\}. \]

\( \hat{\Omega}_k \) obeys the real equation

\[ \ddot{\Omega}_k + \frac{\dot{\Omega}_k}{\hat{\omega}_k} = \frac{\dot{\sigma}(3\dot{\Omega}_k)}{4\hat{\omega}_k} = \hat{\omega}_k^2(u) - \hat{\omega}_k^2. \]

The lowest order WKB has

\[ \hat{\Omega}_k = \hat{\omega}_k, \quad \hat{\sigma} = 0. \]

Using the mode decomposition and the definitions \( \langle \delta^l(k) \delta^q(q) \rangle = 2\pi \delta(k-q) N_+(q) \) and \( \langle d^l(k) d^q(q) \rangle = 2\pi \delta(k-q) N_-(q) \), we obtain for the gap equation

\[ \ddot{\sigma} = \lambda \int \frac{dk_\eta}{2\pi} \left( 1 - N_+(k) - N_-(k) \right) R_k(u), \]

where \( R_k(u) = 1 - 2k_\eta^2 |f^+_k(u)|^2 \). and

\[ \lambda^{-1} = \int \frac{dk_\eta}{2\pi} \frac{1}{\sqrt{k_\eta^2 + m_f^2}} = \int \frac{dk}{2\pi} \frac{1}{\sqrt{k^2 + m_f^2}}. \]

This equation is solved simultaneously with eq. (10).

We choose our initial state to be in local thermal and chemical so that

\[ N_\pm(k, \mu, T) = \left[ e^{\frac{\omega_k(0) \mp \mu}{T}} + 1 \right]^{-1} \]

where \( \omega_k(0) = E - \sqrt{k_\eta^2 + \sigma^2(0)} = \frac{\hat{\omega}_k(0)}{\hbar} \). Since we start our simulation in the unbroken mode, \( \hat{\sigma}(0) = 0 \). We choose the initial \( u_0 = \frac{1}{m_f} \) and measure the proper time in these units. We use adiabatic initial conditions on the mode functions \( f \), i.e. \( f_k(u) = \frac{N_k}{\sqrt{2\hat{\omega}_k}} \), \( f^+_k(0) = -i\hat{\omega}_k f^+_k(u) \) and \( N_\pm^2 = [\omega_k(u) + \sigma(u)]^{-1} \).

We have studied three separate starting points on the phase diagram of Fig. 1 in our numerical simulations. We determined the energy density and the pressure from the expectation value of the energy momentum tensor.

\[ \left\langle \frac{T^\mu_\nu}{N} \right\rangle = \left\langle \frac{i}{2} \bar{\psi}_{\eta} \nabla_{\nu} \psi - \frac{i}{2} \nabla_{\mu} \bar{\psi} \psi - g_{\mu\nu} \frac{\sigma^2}{2\lambda} \right\rangle. \]

In the \( \eta, \tau \) coordinate system \( T^\mu_\nu \) is diagonal which allows us to read off the comoving pressure and energy density. After renormalization we obtain

\[ e(\tau) \tau^2 = \int_0^\Lambda \frac{dk_\eta}{2\pi} \left[ \frac{\sigma^2}{\sqrt{k_\eta^2 + m_f^2}} + 4\Omega_k(\dot{\sigma}^2 - \omega_k^2)|f_k|^2 \right] \]

\[ (N_+ + N_-) \left[ 2\dot{\sigma} + 4\Omega_k(\dot{\sigma}^2 - \omega_k^2)|f_k|^2 + 2(k_\eta - \dot{\sigma}) \right], \]

\[ p\tau^2 = \int_0^\Lambda \frac{dk_\eta}{2\pi} \left( 1 - N_+ - N_- \right) 4\Omega_k(\dot{\sigma}^2 - \omega_k^2)|f_k|^2 \]
In the massless phase, one finds that the exact equation of state is \( p = \epsilon \). To compare our field theory calculation with a local equilibrium hydrodynamical model we assume

\[
T^{\alpha \beta} = pg^{\alpha \beta} + (\epsilon + p)u^\alpha u^\beta
\]  

The conservation law of energy and momentum

\[
T_{\alpha \beta} = 0,
\]

combined with scaling law \( v = z/t \) and \( p = c \) yields

\[
\frac{\epsilon}{\epsilon_0} = \left( \frac{\tau_0}{\tau} \right)^2, \quad \frac{T}{T_0} = \left( \frac{\tau_0}{\tau} \right),
\]

From Eq. 15 and Eq. 16 we can also determine \( p(\mu, T) \) and \( c(\mu, T) \). Assuming \( T/T_0 = \tau_0/\tau \) and \( \mu/\mu_0 = \tau_0/\tau \) we find that the local equilibrium expressions for \( \epsilon \) and \( p \) evolve identically to the numerically determined field theory evolution before the phase transition.

With the same assumptions we find the distributions for \( N^\pm \) plotted against \( k \), are independent of \( \tau \). This also agrees with the exact evolution before the phase transition.

We want to understand how the particle number distributions evolve in time. In relativistic quantum mechanics, particle number is not conserved. However in a mean field approximation one can define an interpolating number operator which at late times becomes the outstate number operator. By fitting the interpolating number densities for both fermions and antifermions to Fermi-Dirac distributions we extract the best value of \( \mu \) and \( T \) for that value of the proper time. To define the interpolating number operator we use a set of orthonormal mode functions \( y_k \) which are the adiabatic approximation to the exact mode functions.

\[
y_k^+ = u_k e^{-i \int \omega_k du}, \quad y_k^- = v_k e^{i \int \omega_k du}
\]

with

\[
u_k = \frac{-i \gamma^\mu k_\mu + \tilde{\sigma}}{\sqrt{2\omega_k(\tilde{\omega}_k + \tilde{\sigma})}} \chi^+, \quad v_{-k} = \frac{i \gamma^\mu k_\mu + \tilde{\sigma}}{\sqrt{2\omega_k(\tilde{\omega}_k + \tilde{\sigma})}} \chi^-
\]

The creation and annihilation operators then become time dependent and the expansion of the quantum field becomes

\[
\Phi(x) = \int \frac{dk}{2\pi} [a(k, u)y_k^+(u) + c^\dagger(k, u)y_k^-(u)] e^{ik\eta}.
\]

This is an alternative expansion to that found in eq. (7), and the two sets of creation and annihilation operators are related by a Bogoliubov transformation

\[
a(k, u) = \alpha_k(u) h(k) + \beta_k^* \dagger(k)
\]

\[
c^\dagger(k, u) = -\beta_k(u) h(k) + \alpha_k^* \dagger(k).
\]

To ensure that at \( u = 0 \) the two number operators match, one chooses adiabatic initial conditions: \( y = \phi \), so that \( \alpha_k(0) = 1, \beta_k(0) = 0 \). The interpolating number operators for "quarks" and "antiquarks"
are defined by
\[ N^+(k, u) = \langle a^+(k, u) a(k, u) \rangle; N^-(k, u) = \langle c^+(k, u) c(k, u) \rangle. \]

With \[ \frac{\dot{\Omega}_k}{2 \Omega_k} \] we have explicitly
\begin{equation}
|\beta_k|^2 = \frac{k^2}{2 \omega_k (\omega_k + \bar{\sigma})} \frac{(\Omega_k - \bar{\omega}_k)^2 + \Delta_k^2}{[\Omega_k^2 + (\omega_k + \bar{\sigma})^2 + 2\Omega_k \bar{\sigma} + \Delta_k^2]},
\end{equation}
\begin{equation}
N^\pm(k, u) - N^\pm(k) + [1 - N^+(k) - N^-(k)] |\beta_k(u)|^2.
\end{equation}

We have solved the simultaneous equations Eq. 10 and Eq. 12 numerically. Comparing \( N^\pm(k, u) \) with an equilibrium parameterization we have determined \( T(k, u) \) and \( \mu(k, u) \) as a function of \( k \) as in Aarts. When these quantities are independent of \( k = kT \) this defines a time evolving temperature and chemical potential. We found that \( T \) and \( \mu \) are independent of \( k \) except at high momentum before the chiral phase transition.

From Fig. 5 we see that for both the 1st and 2nd order transitions, \( \sigma(\tau) \) shows a sharp transition during evolution from the unbroken mode to the broken symmetry mode. Before the phase transition the temperature falls consistent with the equation of state \( p - \epsilon \). For the 2nd order transition, the chemical potential follows the temperature and falls as \( \frac{1}{\tau} \). After the phase transition, there is now a mass scale \( m_f \) which leads to oscillations of \( \sigma \). For the 1st order transition the chemical potential falls faster than \( \frac{1}{\tau} \). If instead we chose initial conditions so that the trajectory passed through the tricritical point, one finds a fall off for \( \mu \) intermediate between the two above cases.

The order of the transition has a more noticeable effect on the spectrum of particles and antiparticles. As we have discussed, if the system evolves in local thermal equilibrium with \( \sigma = 0 \), then when \( N^\pm(k_{\text{sta}}, u) \) is plotted vs. \( k = kT \) it should be independent of \( u \). Thus any change in this spectra is an indication of the system going out of equilibrium. We expect and find that because of the latent heat released during a first order transition that the distortion of the spectra is greatest in that case. (see Fig. 6). Going through a second order phase transition has a smaller effect in distorting the Fermi-Dirac distribution whereas passing through the tricritical regime one finds results (not shown here) just intermediate between these two cases.

In local equilibrium with \( \sigma = 0, \epsilon = p \propto \tau^{-2} \), simulations shown in Fig. 6 agree with this before the phase transition occurs.
After the phase transition we find that the energy density oscillates around the true broken symmetry values discussed earlier, namely $\epsilon_0 = -1/4\pi$. These oscillations would be damped if we went beyond mean field theory and include hard scatterings between the fermions.

1. "PION" CORRELATION FUNCTION

We can define an effective (neutral) pion field via

$$\pi(x) \equiv c \bar{\psi}(x) i \gamma^5 \psi(x)$$

(21)

where $c$ is a constant. Using our mode expansion we find that

$$\langle \bar{\psi} i \gamma^5 \psi \rangle = \frac{2}{r^2} \langle \langle \psi(x) | r^3 \phi(x) \rangle \rangle$$

$$= \int dk_\parallel k_\parallel [2 - N_+(k) - N_-(k)] \frac{d}{du} |f_k|^2$$

(22)

Because the integrand in eq. 22 is odd, the expectation value is zero (otherwise there would be spontaneous breakdown of parity).

For the equal time correlation function in lowest order in large-$N$, we obtain the usual Fermion self energy loop, with the interpolating number density for fermions and anti-fermions participating in the loop evolving with the proper time. Apart from an overall constant one can write the connected correlation function in the form:

$$D(\eta - \eta'; \tau) = \langle \pi(\eta, \tau) \pi(\eta', \tau) \rangle_c = \frac{1}{r^4} \text{Tr}[\gamma^3 S(\eta - \eta'; \tau) \gamma^3 S(\eta' - \eta; \tau)]$$

(23)
where at equal times, the propagator is just

\[ S(\eta, \eta'; \tau)_{\alpha\beta} = \langle [\phi_\alpha(\eta, \tau), \phi^\dagger_\beta(\eta', \tau)] \rangle. \]  

(24)

Here \( \alpha, \beta \) take on the values \{1, 2\} and are the spinor indices. Using the mode expansion we find that the equal time propagator can be written as

\[ S(\eta, \eta'; \tau)_{\alpha\beta} = \int \frac{dk \eta dq e^{i(k-\eta)q}}{2\pi} \left[ \left( \frac{1}{1 - 2N_+(k)} \right) \phi^\dagger_{\kappa\alpha}(\tau) \phi^\dagger_{\kappa\beta}(\tau) \right. \\
\left. + \left( 2N_-(k) - 1 \right) \phi_{\kappa\alpha}(\tau) \phi_{\kappa\beta}(\tau) \right]. \]  

(25)

Evaluating the trace we find that we obtain

\[ r^4 D(\eta - \eta'; \tau) = \int \frac{dk \eta dq}{2\pi} e^{i(k-\eta)q} D(k, q; \tau) \]  

(26)

where

\[ D(k, q) = \left[ \left( 1 - 2N_+(k) \right) \left[ 1 - 2N_+(q) \right] + \left[ 1 - 2N_-(k) \right] \left[ 1 - 2N_-(q) \right] \right] F_1(k, q) \]
\[ + \left[ \left( 1 - 2N_+(k) \right) \left[ 1 - 2N_-(q) \right] + \left[ 1 - 2N_+(q) \right] \left[ 1 - 2N_-(k) \right] \right] F_2(k, q) \]  

and

\[ F_1(k, q; \tau) = \left| f_k \right|^2 \left| f_q \right|^2 \left\{ (k\Delta_q + q\Delta_k)^2 + \left[ k(\vec{\Delta}_q + \vec{\sigma}) - q(\vec{\Delta}_k + \vec{\sigma}) \right]^2 \right\}, \]

(28)

and

\[ F_2(k, q; \tau) = \left| f_k \right|^2 \left| f_q \right|^2 \left\{ (kq - \Delta_k\Delta_q + (\vec{\Delta}_k + \vec{\sigma})(\vec{\Delta}_q + \vec{\sigma}))^2 \right. \\
\left. + \left[ \Delta_k(\vec{\Delta}_q + \vec{\sigma}) + \Delta_q(\vec{\Delta}_k + \vec{\sigma}) \right]^2 \right\}. \]

(29)
Hard thermal loops on the lattice

D.Bödeker, in collaboration with G.D. Moore and K. Rummukainen

We have investigated an out-of-equilibrium problem in a hot non-Abelian gauge theory. The rate for baryon number dissipation $\gamma$ in the electroweak theory is much larger than the thermalization of the other degrees of freedom (except, of course, hydrodynamic modes). Therefore, the system is in a quasi-equilibrium state when observed on time scales of order $\gamma^{-1}$. Due to the chiral anomaly the change of baryon number is proportional to the change of the Chern-Simons number of the gauge fields. A fluctuation-dissipation theorem relates $\gamma$ to the diffusion rate of the Chern-Simons number $\Gamma_{\text{CS}}$.

The rate $\Gamma_{\text{CS}}$ is obtained from an unequal time correlation function in thermal equilibrium. It is non-perturbative since it is mainly determined by soft gauge fields with momenta of order of the magnetic scale $g^2 T$. Therefore it has to be evaluated on a lattice. It is not possible to perform lattice simulation of a quantum fields theory in real time. However, the non-perturbative gauge fields have long wavelengths and therefore behave classically. Thus one can use an effective classical theory to determine $\Gamma_{\text{CS}}$.

To obtain such an effective theory the hard $k \sim T$ modes, which are not classical, must be integrated out. But even then one has to deal with the problem that the resulting effective theory does not have a continuum limit. One can also integrate out the $k \sim g T$ modes. In this way one obtains an effective theory which does have a continuum limit and which has recently been used to compute $\Gamma_{\text{CS}}$. The drawback is that it is only valid at leading and next-to-leading order in $\log(1/g)$.

Here we have used an effective theory, the hard thermal loop effective theory, which keeps the $k \sim g T$ physics in the game. It contains the classical gauge fields and, in addition, the fields $W(x, \vec{v})$ representing the color charge due to hard particles with velocity $\vec{v}$ ($v^2 = 1$). We have expanded $W(x, \vec{v})$ in spherical harmonics $W_{lm}(x)$ with respect to $\vec{v}$ and we kept only components with $l \leq l_{\max}$. This gives a field theory in $(3+1)$ dimensions which was put on a lattice. We have measured $\Gamma_{\text{CS}}$ for various values of the lattice spacing $a$, of $l_{\max}$ and the Debye mass $m_D$. While analytic estimates indicate that one has to use rather large values of $l_{\max}$ in order to reach the $l_{\max} \to \infty$ limit, we found to our surprise that the $l_{\max}$ dependence is very weak for even $l_{\max}$. Already the result for $l_{\max} = 2$ is basically identical to the $l_{\max} \to \infty$ limit.

The $a$-dependence could be reduced by adding an $a$-dependent counter-term to $m_D^2$. Our results agree remarkably well with two independent calculations, one which uses particle degrees of freedom to represent the hard modes, and one which uses only gauge field degrees of freedom. Our result is about a factor 4-5 larger than the leading log result, while the agreement appears to be better when the next-to-leading log corrections are included in the latter.
Hard thermal loops on the lattice
hep-ph/9907545, PR D61 056003, 2000

in collaboration with G. Moore, K. Rummukainen

* non-equilibrium: baryon number dissipation

* non-perturbative "gluo"-dynamics at high T (magnetic scale physics)

* effective classical theories

* HTL on the lattice

* results
integrate out hard modes \((1\text{-loop})\)

\[ \rightarrow \text{hard thermal loop effective theory} \]

Breiten, Pisarski; Freinkel, Taylor; Taylor, Wong

physical picture: classical fields + classical particles

\[ \bar{u}^2 = 1 \]
\[ \nu r = (\lambda, \bar{\nu}) \]

\[ D_\mu F^{\mu\nu} = m_3^2 \int_0^\infty \nu^r W(x, \bar{\nu}) \]

\[ \nu r D_\mu W = \bar{\omega} \cdot \vec{E} \]

Blaizot, Iancu; Aoki-
Langevin equation solves problems 18.2

- captures all relevant physics of high momentum \((k \gg g^2 T)\) modes
- has a finite continuum limit

Hot sphaleron rate:

\[
\Gamma_{cs} = \pi e \frac{g^2 T}{m_D^2} \log \left( \frac{m_D}{g^2 T} \right) \alpha^5 T^4
\]

\(\alpha = 10.8 \pm 0.4\)  G.D. Moore
price: only valid at leading log
and next-to-leading log order

(Arnold, 1998)

here: keep HngT physics in the game

idea: make mD large at fixed a

\[ \Pi_w \text{ will dominate over } \Pi_{\text{latt}}. \]

G.D. Moore

should work since scaling of Tcs with mD known:

\[ \Pi_{\text{cs}} \sim \left( \frac{gT}{m_D} \right)^2 \ g^{10} T^4 \]

"improvement": add counterterm to mD

Such that

\[ \lim_{k_0 \to 0} \frac{1}{k_0} \left< \Pi_w^{\text{tr.}} + \Pi_{\text{latt.}}^{\text{tr.}} \right> \hat{k} = \Pi_{\text{physical HTc}}^{\text{tr.}} \]
Lattice regularization

D.B., McLerran, Smilga; Hu, Mueller; Moore

Particles

\[ \text{link: } U_i \in SU(N) \]

\[ U_i = \text{e}^{iga_i} \]

Here:

\[ W(x, \vec{u}) \rightarrow \sum_{\ell = 0}^{\ell_{\text{max}}} \sum_{l \leq i \leq e} W_{\text{em}}(\ell) \cdot W_{\text{em}}(x) \]
Chern–Simons diffusion

\[ \tau_{CS} = \alpha e' \frac{g^2 T^2}{m_D^2} \alpha^5 T^4 \]
Summary

* non-perturbative out-of-equilibrium problem in hot QCD

* lattice implementation of Hard Thermal Loop Effective Theory

* measured the hot sphaleron rate result for MSM:

\[ T_{\text{sph}} = (25.4 \pm 2) \alpha^5 T^4 \]

* \( \alpha, \) \( \text{max, method dependence weak} \)
YANG-MILLS in (q+1), MAGNETIC MASS, COVARIANCE, etc.  

W.J. Y, WITH KARABALI & KIM  

WHY (q+1) DIMENSIONS?  

1. INTERESTING IN ITS OWN RIGHT.  

2. MAGNETIC MASS: MASS GAP OF YM2+1  

= MAGNETIC MASS OF YM3+1 AT T≠0.  

HAMILTONIAN ANALYSIS  

→ SOME EXACT RESULTS.  

→ MASS GAP, VACUUM WAVEFUNCTION, STRING TENSION.  

→ (q+1)-COVARIANT APPROACH FOR USE IN (RESUMME PERTURBATION THEORY?  

RECAPITULATION OF RESULTS (AGE ≥ 1 YEAR)  

1. GAUGE-INvariant VARIABLES  

2. GAUGE-INvariant MEASURE, INNER PRODUCT.  

3. H IN THESE VARIABLES  

4. PROPAGATOR MASS  

5. $\Psi_0[A], STRING TENSION$ {COMPARE WITH LATTICE  

RECENT RESULTS hep-th/0007184  

"UNIVERSALITY OF $\sigma$"  

H, MASS GAP, IMPROVED" PERTURBATION THEORY  

COVARIANT MASS TERM, GAP EQUATION.  

HAMILTONIAN ANALYSIS  

$A_0 = 0$ GAUGE. $A_i^0 = g^{-1}A_i^0g + g^{-1}D_i^0g$.  

WAVEFUNCTIONS ARE GAUGE-INvariant.  

1. INNER PRODUCT  

$\langle 1|2 \rangle = \int \delta \mu(c) \psi_1^*(A) \psi_2(A)$  

→ gauge-invariant measure  

$[dA_0 dA]$  

$Vol\Omega$  

2. HAMILTONIAN OPERATOR  

$H = T + V$  

coupling constant  

$T = \frac{e^2}{\alpha} \int E^2 = -\frac{e^2}{\alpha} \int \frac{\delta^2}{\delta A_i^0}$  

$V = \frac{1}{2e^2} \int B^2$  

$B = D_i A_j - D_j A_i [A_i A_j]$
\section*{Variables}

\[ \frac{1}{2} (A_1 \pm i A_2) = A_z, \quad A_z \quad \text{or} \quad A, \bar{A} \]

\[ A = - i \varepsilon M M^{-1} \quad \bar{A} = M^{-1} \bar{\varepsilon} M \]

\[ M = \text{complex matrix} \quad \det M = 1 \]

\[ M \rightarrow M_3 = e^M \]

\[ \delta A = - \varepsilon (8 \varepsilon M^3) + [\varepsilon M M^2, \, 8 \varepsilon M^2] \]

\[ \delta \bar{A} = \bar{\varepsilon} (M^{-1} \varepsilon M) \]

\[ [dA d\bar{A}] = \det (D \bar{D}) \quad \delta \mu (8 \varepsilon M^3, M^{-1} \varepsilon M) \]

\[ \delta \mu (H) \quad \phi (\mu) \quad \text{(Haar measure)} \]

\[ \text{for SU(3)} \quad \text{function of } H = \varepsilon M \]

\[ \text{exact} \quad \delta \mu (e) = \delta \mu (H) \exp \left[ 2 \pi i \alpha S(H) \right] \]

\[ H^{-1} \delta H = \delta \varepsilon^a R_{ab}(H) t_b \quad H = e^{t_a \varepsilon^a} \]

\[ d\mu (H) = [d\mu](\det R) \]

\[ \alpha_3 f_{abc} = f_{aun} f_{bmn} \]

\[ S(H) = \frac{1}{2} \int \nu (H, 5 H^{-1}) - \frac{i}{12 \pi} \int \text{Tr}(H^3 d H)^3 \]

\[ \langle \psi | H \psi \rangle = \int d \mu (H) e^{2 \alpha_3 S(H)} \]

\[ \text{A "collective" field} \]

\[ H = M M \quad \text{gauge-invariant} \]

\[ W(\alpha) = \text{Tr} \quad P e^{-2 \phi \alpha} \]

\[ J = \alpha_3 \frac{\partial}{\partial H} H^{-1} \quad \text{"current"} \]

\[ \text{write } H \text{ in terms of } J \]

\[ V = \frac{1}{2 e^2} \int \sqrt{2} \quad = \frac{2 \pi^2}{e^2 c^2} \int \frac{1}{5} J^a 5 J^a \]

\[ \text{find } T \text{ by change of variables on } \Psi (J). \]
\[ H = T + V \]

\[ T \psi = \frac{-e^2}{\hbar} \int \frac{\psi_s}{S\alpha_S S\alpha_S} \psi \]

\[ = \frac{-e^2}{\hbar} \int \left[ \frac{\partial \psi}{\partial S\alpha_S} \frac{S_j}{S\alpha_S} \frac{S^2 \psi}{S\alpha_S} + \frac{S_j \psi}{S\alpha_S} \frac{S^2 \psi}{S\alpha_S} \right] \]

\[ = \int \omega_s \psi + \frac{\Omega_{ab}(u,v) S^2 \psi}{S_j S_j} \]

\[ \omega_s = \frac{-e^2}{\hbar} \int \frac{S_j \omega_s}{S\alpha_S S\alpha_S} \]

\[ = \left( \frac{e^2 c_a}{2 \pi} \right) \mathcal{M}_{\alpha\alpha}(\alpha) \operatorname{Tr}[\mathcal{D}^{-1}(\alpha, \alpha)] \]

\[ \mathcal{M}(\alpha) \rightarrow \frac{e^2 c_a}{2 \pi} \]

\[ T = \mathcal{M} \left[ \int S_j S_j + \Omega_{ab}(u,v) \frac{S_j}{S_j S_j} \right] \]

\[ \Omega_{ab}(u,v) = \frac{c_a}{2 \pi} \frac{\delta_{ab}}{(u-v)^2} - \frac{c_a}{\pi^2} \frac{\delta_{ab}}{(u-v)^2} + G(u) \]

(Recheck \( \int \frac{S_j}{S_j} \) by self-adjointness)

---

**BACK TO A's**

\[ S' \rightarrow \mathcal{S}^M + \mu^2 F - \Delta F \]

\[ \mu^2 = \Delta = m^2 = \left( \frac{e^2 c_a}{2 \pi} \right)^2 \]

tree level in "improved" perturbation theory.

→ Resummation procedure
by adding and subtracting \( F \).

**COVARIANTIZATION**

\[ \theta_x \rightarrow \eta \cdot \theta_x \quad \theta_x \rightarrow \eta \cdot \theta_x \]

\[ \eta = (1, \iota, 0), \quad \bar{\eta} = (1, -\iota, 0) \]

→ GENERAL \( \eta, \bar{\eta} \)

\[ \eta^2 = 0, \bar{\eta}^2 = 0 \quad \eta \cdot \bar{\eta} = 2 \]

\[ F(A, \bar{A}) \rightarrow \int F(\eta \cdot A, \bar{\eta} \cdot A) \frac{d\eta}{4\pi} \]

**MINKOWSKI VERSION**

\[ \eta = (- \cos \theta \cos \chi, - \sin \theta \cos \chi, - \cos \theta \sin \chi + \iota \cos \chi) \quad \text{sm} \theta \]

MINKOWSKI VERSION USING TRACES WITH NXN MATRICES, \( N \rightarrow \infty \).
VACUUM Wavefunction

Ignore $V$ for the moment.

Vacuum: $\psi_0 = 1 \quad T \psi_0 = 0$

Normalizable: $\int \psi_0^* \psi_0 \, d\mu(\sigma) < \infty$

Include $V$ perturbatively: $\psi = e^P$

$$P = -\frac{\pi}{\hbar^2 c \alpha} \int \left[ \frac{\hbar^2}{\alpha^2} \int \frac{\hbar^2}{\alpha^2} \int \right]$$

$$= -\frac{2}{\epsilon^2} \left[ \frac{\pi^2}{\alpha^4} \int \left( \frac{1}{m + \sqrt{m^2 - \epsilon^2}} \right) \, dJ_x \right]$$

$$\pm \text{ fake } \int f(k, y, z) \, J_x(k) \, J_y(k) \, J_z(k) + G(k, y, z)$$

$$f(k, y, z) = \int e^{i \epsilon y} \, S(k + q, y, z) \left( \frac{\pi}{2 \alpha c} \right)^3 \frac{(\hbar^2 m^2 - m)(\hbar^2 m^2 - \omega)}{\sqrt{m^2 + \hbar^2 m^2} \sqrt{m^2 + \hbar^2 m^2}}$$

$$\psi \approx e^{\frac{-1}{2\epsilon^2} \int B \frac{1}{\sqrt{\omega^2 - \epsilon^2}} B}$$

$$(\text{perturbative})$$

$k \gg \alpha$

$$W(\sigma) = \text{Tr} \left( P e^{-\frac{\sigma}{4} A} \right)$$

$$\langle W_f(\sigma) \rangle = \int \psi_0^* \psi_0 \, W(\sigma) \, d\mu(\sigma)$$

$$= \text{const} \cdot e^{-\frac{\sigma}{4} A(\sigma)}$$

$$\sqrt{\sigma} = e^{\sqrt{\frac{N^2-1}{8\pi}}}$$

$$\frac{\sqrt{\sigma}}{\epsilon^2} = 0.345 \quad 0.564 \quad 0.772 \quad 0.977$$

$$\text{SU(2)} \quad \text{SU(3)} \quad \text{SU(4)} \quad \text{SU(6)}$$

0.335 \quad 0.553 \quad 0.758 \quad 0.966

Difference $\leq 3\%$

Lattice (Tepfer et al)

Analogy to chiral perturbation theory
MASS GAP, "RESUMMED" PERTURBATION THEORY

\[ T = \tau \left( \frac{g \Delta}{3} + \int_{\Omega} \frac{g \Delta}{3} \frac{g \Delta}{3} \right) \quad m = \frac{e^2 cA}{2\pi} \]

\[ \int J^a = \int J^a \quad (E X A C T) \]

\[ \psi = e^{-cA \sin \Phi} \]

\[ \langle \Phi | \Phi \rangle = \int \phi \mu (H) \phi^* \phi \]

\[ H \rightarrow e^{-cA \sin \Phi} \]

\[ H = e^{-cA \sin \Phi} \phi \sim \sqrt{\frac{\tau}{3}} \phi \]

\[ H = \frac{\tau}{2} \left( -\frac{\Delta^2}{4} + \phi^* (-\nabla^2 \phi) + \frac{A^2}{2} \phi^* \phi + \ldots \right) \]

"UNIVERSALITY" OF \( \sigma \)

CONTRIBUTION FROM \( J^3 \)-TERMS:

\[ e^2 \rightarrow e^2 \int \frac{f(\mu)}{\mu^2} \quad f(\infty) = 1 \]

\[ e^2 \phi^2 = e_{\Phi}^2 \]

\[ \left( \frac{\sqrt{\tau}}{e^2} \right)_{\text{LARGE LOOPS}}^{\text{LARGE LOOPS}} \sim \sqrt{\frac{N^2-1}{8\pi}} \quad \text{(c.f. Thomson scattering)} \]

MASS GAP \( \sim m = \frac{e^2 cA}{2\pi} \) \( \text{(NOT } e^2) \)

POINT OF DISCREPANCY?

REDO KARSCH ET AL \& TEUPER WITH COMPARABLE DEFINITION OF \( e^2 \)

\[ \frac{m}{e^2} = \begin{cases} 
0.32 & \text{SU(3)} \\
0.35, 0.46, 0.31 - 0.40 & \text{LATTICE} 
\end{cases} \]
MORE EXACT FORM OF MASS TERM

\[ \text{MASS TERM} = m^2 F \]

\[ F \approx - \frac{1}{e^2} \int Tr (\bar{\Theta} H \bar{H}^{-1}) \]

REQUIRED & DESIRABLE PROPERTIES

1. EXPRESSIBLE IN TERMS OF H (NOT JUST \( \Phi \))
2. SHOULD AGREE WITH F ABOVE
3. SHOULD HAVE "HOLONOMIC" INVARIANCE

\[ A = - \bar{\Theta}_a M^{-1} M, \quad M \bar{\nabla}z \rightarrow \text{SAME A} \]

H = M^T M \quad \bar{\nabla}z H \bar{\nabla}z, \quad H \rightarrow \text{SAME PHYSICS.}

MINIMAL GENERALIZATION OF F

\[ F_{\text{min}} = - \frac{2\pi}{e^2} \frac{S(H)}{\omega_{2\pi}} \]

ANOTHER

\[ F = \frac{\pi^2}{2e^2} \int G \delta J^a \quad H_{a6} \quad G \bar{\Theta} \bar{J}^6 \]

\[ \Rightarrow F_{\text{min}} + \text{TERM CUBIC IN A, GAUGE-INVARIANT.} \]
Solving the problem of initial conditions in heavy ion collisions

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October 27, 2000

The initial conditions for particle production at central rapidities in heavy ion collisions are determined in a classical effective field theory of QCD. The space-time evolution of the produced glue is determined by solving Yang-Mills equations for two singular light cone sources. Boost invariance is assumed—thereby allowing for a dimensional reduction of the theory to a 2+1-dimensional theory. The energy and number distributions of gluons at late times are determined and shown to depend simply on the saturation scale $Q_s$ of gluons in the nuclear wavefunction. With initial conditions motivated by the lattice simulations, a Landau equation for parton transport is solved to study the equilibration of the system achieved by 2 to 2 processes. The equilibration time, the initial temperature and the chemical potential are determined as a function of $Q_s$. 
• What do parton distributions look like at high energies?

  Minnesota Mobsters*

• How are partons produced in high energy nuclear collisions?

  Alex Krasnitz.

• Do the partons produced equilibrate to form a gluon-quark plasma?

  Jeff Bjoraker.

E. Iancu

* A. Ayala, I. Ibacua-Melian, Y. Kouchegar, A. Kovner, A. Leonidov,
  L. McLerran, H. Weigert.
**The Classical Field Of A Nucleus.**

\[ A^+ = 0 \]

\[ A^- = 0 \]
\[ A_\perp = 0 \]

\[ A^- = 0 \]
\[ A_\perp = \frac{i}{g} U(x_\perp) \nabla_\perp U^+(x_\perp) \]

\[ x^\perp = 0 \]

Partons "live" on 2-D sheet (well, almost...)

**Ground State Properties**

Analogous to those of Ising spins in random "quenched" magnetic fields

- Parisi-Sourlas

**Smeearing in \( x^\perp \) required in practice**
\[ D^{\mu} F_{\mu \nu} = J^\nu \]

\[ J_\nu^a = f_1^a \delta(x^-) \delta^{\nu+} + f_2^a \delta(x^+) \delta^{\nu-} \]

\[ A^i = \Theta(x^+) \Theta(x^-) \alpha_1^i + \Theta(-x^+) \Theta(x^-) \alpha_2^i \]
\[ + \Theta(x^+) \Theta(-x^-) \alpha_3^i(x) \]

\[ A^\pm = \pm x^\pm \Theta(x^-) \Theta(x^+) \alpha \]

- **Initial Conditions by Matching Equations of Motion at \( \tau = 0 \).**

\[ \alpha_1^i \bigg|_{\tau = 0} = \alpha_1^i + \alpha_2^i \]

\[ \alpha \bigg|_{\tau = 0} = \frac{i}{2} \left( \alpha_1^i, \alpha_2^i \right) \]
Figure 1: $\varepsilon_T/(g^2 \mu)^2$ as a function of $g^2 \mu \tau$ for $g^2 \mu L = 5.66$ (diamonds), 35.36 (pluses) and 296.98 (squares). Both axes are in dimensionless units. Note that $\varepsilon_T = 0$ at $\tau = 0$ for all $g^2 \mu L$. The lines are exponential fits $\alpha + \beta e^{-\tau}$ including all points beyond the peak.

"FORMATION TIME" $T_{Bj} = 1/\gamma$

RHIC: $T_{Bj} \sim 1/0.62 \text{ GeV}^{-1}$

LHC: $T_{Bj} \sim 1/1.4 \text{ GeV}^{-1}$
FROM THE INITIAL GLUON DISTRIBUTION TOWARDS EQUILIBRATION.
At Equilibrium

\[ Q_s \ (GeV) \]

- \( t_{eq} \) (fm)
- \( T_{eq} \) (MeV)
- \( \mu \) (MeV)

\[ 0 \alpha = 0.3 \]
\[ 0 \alpha = 0.2 \]
Reorganizing Finite Temperature Field Theory

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The heavy-ion collision experiments at RHIC and LHC will allow us to study the properties of high-temperature phase of QCD for the first time. There are many methods that can be used to calculate the properties of the quark-gluon plasma. On the non-perturbative end, there are now reliable lattice results for equilibrium (Euclidean) properties. In the other extreme, the weak-coupling limit, standard perturbation theory techniques can be applied to both static and dynamical quantities. The pressure of a gas of quarks and gluons, for instance, is known up to $O(g^5)$.

Unfortunately, in order for the series to converge, the temperature has to be on the order of $10^5$ GeV. At temperatures expected in RHIC and LHC heavy-ion collisions (0.2-1 GeV) the weak-coupling expansion seems to be of little use.

Here I will concentrate on scalar $\phi^4$ since the poor convergence of the weak-coupling expansion also shows up there. In Part II, I will discuss the extension of the method to gauge theories. The weak-coupling expansion of the pressure is known up to order $g^5$. In Fig. 1, we show the successive perturbative approximations to $P/P_{\text{ideal}}$ as a function of $g(2\pi T)$. Each partial sum is shown as an error band obtained by varying the renormalization scale $\mu$ from $\pi T$ to $4\pi T$. The shaded bands can be considered as a lower bound on the theoretical error. The difference between successive approximations should also be considered which leads us to conclude that the error grows quickly for $g > 1.5$.

Clearly perturbation theory is not converging in this regime. This is not surprising since the "soft" scale ($q \sim gT$) and the "hard" scale ($p \sim T$) are of the same order. In this regime we have to more carefully take into account the interaction of the soft and hard degrees of freedom. One possible way to do this is to change the expansion point for the loop expansion. Instead of expanding around a massless ground state we expand around a gas of massive quasiparticles which incorporate the "soft" scale physics. In the following section I will discuss a technique called screened perturbation theory that can be used to do this in a systematic manner.

I. FORMALISM

Within screened perturbation theory the scalar lagrangian density is written as

$$L_{\text{SPT}} = -\mathcal{E}_0 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} (m^2 - m_0^2) \phi^2 - \frac{1}{24} g^2 \phi^4 + \Delta L + \Delta L_{\text{SPT}} , \tag{1}$$

where $\mathcal{E}_0$ is a vacuum energy density parameter and we have added and subtracted mass terms. If we set $\mathcal{E}_0 = 0$ and $m_0^2 = m^2$, we recover the original lagrangian. Screened perturbation theory is defined by taking $m^2$ to be of order $g^0$ and $m_0^2$ to be of order $g^2$, expanding systematically in powers of $g^2$, and setting $m_0^2 = m^2$ at the end of the calculation. This defines a reorganization of perturbation theory with

$$L_{\text{free}} = -\mathcal{E}_0 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 , \tag{2}$$

$$L_{\text{int}} = -\frac{1}{24} g^2 \phi^4 + \frac{1}{2} m_0^2 \phi^2 + \Delta L + \Delta L_{\text{SPT}} . \tag{3}$$

At each order in screened perturbation theory, the effects of the $m^2$ term in (2) are included to all orders. However, when we set $m_0^2 = m^2$, the dependence on $m$ is systematically subtracted out at higher orders in perturbation theory by the $m_0^2$ term in (3). At nonzero temperature, screened perturbation theory does not generate any infrared
divergences, because the mass parameter $m^2$ in the free part of the lagrangian provides an infrared cutoff. The resulting perturbative expansion is therefore a power series in $g^2$ and $m_1^2 = m^2$ whose coefficients depend on the mass parameter $m$.

In order to complete the calculation we must specify the way in which this mass determined. In the data shown here I will identify $m$ with the tadpole mass. For a more complete discussion of the various mass prescriptions and results obtained with each see Ref. [4].

![Graph](image)

**FIG. 1.** One-, two-, and three-loop results for the pressure of $\lambda \phi^4$ theory as a function of $g(2\pi T)$ for two different values of the renormalization scale $\mu$: (a) $\pi T < \mu < 4\pi T$ (b) $1/2m < \mu < 2m$.

In Figure 1, the one-, two-, and three-loop renormalized pressures are shown as a function of $g(2\pi T)$. The bands in Figure 1a and 1b were obtained varying the renormalization scale by a factor of two about the values of $2\pi T$ and $m$ respectively. In both cases, the convergence of the successive approximations is much better than the conventional perturbative calculation with the two- and three-loop results lying within the minimal error bars of the conventional $O(g^2)$ calculation. [4] The convergence for $\mu \propto m$ seems to be better than with $\mu \propto T$, but at this point it is impossible to conclusively settle this issue. A reasonable estimate would take the appropriate scale to be somewhere between these two scales.

In order to make contact with RHIC and LHC we need to extend these methods to QCD. For gauge theories you can't simply add and subtract a momentum independent mass because this would violate gauge invariance. One possibility is to use instead the non-local functions related to the gluon self-energy and vertex functions obtained in the high-temperature limit of QCD. [5]

**ACKNOWLEDGEMENTS**

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Failure of perturbative QCD

- The weak coupling expansion of the QCD free energy, $\mathcal{F}$, has been calculated to order $\alpha_s^{5/2}$.\(^1,2\)

- The successive terms contributing to $\mathcal{F}$ can strictly only form a decreasing series if $\alpha_s \lesssim 1/20$ which corresponds to $T \sim 10^5$ GeV.

![Graph](Image)

Figure 1. Perturbative QCD Free Energy vs Temperature. ($\pi T \leq \mu_4 \leq 4\pi T$)

Lattice results from G. Boyd et al, 95/96.
Screened perturbation theory

- Within screened perturbation theory, a mass, which can be treated as a variational parameter, is added to the Lagrangian and the loop expansion is recomputed.\textsuperscript{7,8}

\[
\mathcal{L}_{\text{SPT}} = -\mathcal{E}_0 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} (m^2 - m_1^2) \phi^2 \\
- \frac{1}{24} g^2 \phi^4 + \Delta \mathcal{L} + \Delta \mathcal{L}_{\text{SPT}}
\]  

We can split this into free and interaction parts

\[
\mathcal{L}_{\text{free}} = -\mathcal{E}_0 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2
\]  

\[
\mathcal{L}_{\text{int}} = - \frac{1}{24} g^2 \phi^4 + \frac{1}{2} m_1^2 \phi^2 + \Delta \mathcal{L} + \Delta \mathcal{L}_{\text{SPT}}
\]

The additional counterterms that are required have the form

\[
\Delta \mathcal{L}_{\text{SPT}} = -\Delta \mathcal{E}_0 - \frac{1}{2} (\Delta m^2 - \Delta m_1^2) \phi^2.
\]

with

\[
\Delta \mathcal{E}_0 = Z_E (m^2 - m_1^2)^2 \\
\Delta m^2 = (Z_\phi Z_m - 1) m^2 \\
\Delta m_1^2 = (Z_\phi Z_m - 1) m_1^2
\]  

- The previous work performed a two-loop calculation for $N = 1$, and a three-loop calculation only in the large-$N$ limit. We have recently finished the full $N = 1$ three-loop calculation of the thermodynamic functions.\textsuperscript{9}

\textsuperscript{7} F. Karsch, A. Patkós, and P. Petreczky, 97.  \textsuperscript{8} S. Chiku and T. Hatsuda, 98.  \textsuperscript{9} ABS, hep-ph/0002048,
Figure 2. SPT results for the pressure, screening mass, and entropy.
Hard-thermal-loop perturbation theory

Hard-thermal-loop (HTL) perturbation theory is a reorganization of the perturbative series for QCD.\(^1\)

\[
\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}} + \Delta \mathcal{L}_{\text{HTL}}(g, m_g^2 - \bar{m}_g^2) \tag{6}
\]

The HTL improvement term is

\[
\mathcal{L}_{\text{HTL}} = -\frac{3}{2}(m_g^2 - \bar{m}_g^2)T \left( G_{\mu \alpha} \left\langle \frac{n^{\alpha}n^{\beta}}{(n \cdot D)^2} \right\rangle G_{\mu \beta} \right) \tag{7}
\]

Transverse propagator

\[
\omega^2 - k^2 \rightarrow \omega^2 - k^2 - \Pi_T(\omega, k)
\]

\[
\Pi_T(\omega, k) = \frac{3}{2} m_g^2 \omega^2 \left[ 1 - \frac{\omega^2 - k^2}{2\omega k} \log \frac{\omega + k}{\omega - k} \right] \tag{8}
\]

Longitudinal propagator

\[
k^2 \rightarrow k^2 - \Pi_L(\omega, k)
\]

\[
\Pi_L(\omega, k) = 3m_g^2 \left[ \frac{\omega}{2k} \log \frac{\omega + k}{\omega - k} - 1 \right] \tag{9}
\]

Weak coupling limit (High T)

\[
\lim_{\alpha_s \rightarrow 0} m_g^2 = \frac{4\pi}{3} \alpha_s T^2 \tag{10}
\]
Conclusions and Outlook

- We have completed the three-loop calculation of the free energy of a massive scalar field. We were also able to calculate the three-loop free energy (pressure), two-loop screening mass, and three-loop entropy for massless scalar fields using screened perturbation theory.

- For QCD, we have completed the first step in systematically including some of the effects of gluonic and fermionic quasi-particles, screening, and Landau damping in the free energy of a quark-gluon plasma.

- All divergences can be dealt with in a systematic way and quantities are properly renormalized.

- The htlQCD results agree with the lattice results considering that order $\alpha_s$ and higher order corrections will be generated at the two-loop level.

- We are in the process of computing the htlQCD two-loop contribution.

- Once the thermodynamics have been described, this method can be used to calculate real-time processes within a systematic framework.

- Some immediate phenomenological applications include:
  1. heavy quark production in QGP
  2. photon/dilepton emission from QGP
We study some aspects of the vacuum energy of pure Yang-Mills and QCD at $\theta \sim \pi$. We show that there are domain walls, separating phases in which CP is spontaneously broken, compute or estimate their tension and exhibit the phase diagram of QCD with two light flavors as function of the quark mass difference and the angle $\theta$.

\textbf{Why } \theta \sim \pi \ ?

In Nature, \( \theta \sim 0 \) if CP is conserved by Strong Interactions. A priori, \( \theta = \pi \) is also CP conserving, because

\[ \pi \xrightarrow{CP} -\pi = \pi \mod 2\pi \]

However, CP can be spontaneously broken! This is Dashen’s phenomenon.

This is forbidden at \( \theta = 0 \) by the Vafa-Witten theorem.
Puzzles at large Nc

Consider pure Yang-Mills with large number of colors, Nc. The vacuum energy as function of θ takes the form

\[ E[θ] = N_c^2 F(θ/N_c) \]

This is because there are \( O(N_c^2) \) degrees of freedom, and of the 't Hooft rescaling of the coupling,

\[ \tau = \frac{θ}{2\pi} + i \frac{4\pi}{g^2} \rightarrow \tilde{\tau} = \frac{τ}{N_c} \]

with \( \tilde{\tau} \) fixed as \( N_c \to \infty \).

This seems incompatible with the fact that \( θ \to θ + 2\pi \) should describes the same physics.

To leading order in Nc, Witten's solution is

\[ E[θ] = \frac{χ}{2} \min_k (θ + k2\pi)^2 \]

where \( χ \) is the topological susceptibility of pure Yang-Mills.
Simple construction of Witten’s solution

Consider QCD with one light flavor. At large $N_c$ and low energy, the effective lagrangian is

$$V(\eta, \theta) = -m\Sigma \cos \frac{\eta}{f_\pi} + \frac{\chi}{2} (\eta + \theta)^2$$

where $m$ is the quark mass and $\Sigma$ is the quark condensate.

For fixed $\theta$ and if $m\Sigma \gg \chi$, there are many local minima at $\langle \eta/f_\pi \rangle \approx k2\pi$. Integrating out the $\eta$ field gives

$$V(\theta) \approx \frac{\chi}{2} \min(\theta + \langle \eta/f_\pi \rangle)^2$$

which reproduces Witten’s solution.

At $\theta = \pi$, there are two degenerate vacua, $\langle \eta \rangle \leftrightarrow -\langle \eta \rangle$, and CP is spontaneously broken. There is a Domain Wall, with tension $\sigma \sim N_c$. For $\theta \neq 0$, there is a unique groundstate. The other vacua become stable in the limit $N_c \to \infty$. The false vacuum decay rate is

$$\Gamma \sim e^{-N_c^4}$$

Shifman; Gabadadze; M.T.
QCD with two light flavors

At $\theta \sim \pi$ and for three light flavors, CP is spontaneously broken if the quark mass satisfy

$$\frac{m_u m_d}{m_s} > |m_d - m_u|$$

Naively, as $m_s \to \infty$, CP is never spontaneously broken if there are only two flavors !? But we expect a first order phase transition at $\theta \sim \pi$. A second order transition would imply that the meson excitations get massless, but chiral symmetry is always broken for $m_q \neq 0$?

Smilga

If one keeps track of the singlet mode, there is no puzzle. The expression valid for two light flavors is

$$\left| \frac{m_u - m_d}{m_u + m_d} \right| < \frac{m_\pi^2}{m_\eta^2}$$

M.T.
This can be summarized by the following phase structure diagram for QCD with two light flavors:

Where $z = m_d/m_u$. For large or small mass differences, the line of first order transitions at $\theta = \pi$ become second order at two endpoints.
Decrease of the Non-perturbative cutoff in QGP

E. Shuryak

→ The cutoff in vacuum \( p_{\text{cutoff}} \approx 1 \text{ GeV} \) (on gluons)

Where does it come from?

→ Kinetics of QGP formation (overview)
  - "hot glue scenario" → \( q, g \) lag behind and do not equilibrate chemically
  - gluon equilibration: \( gg \to gg, gg \to ggg, gg \to ng \ldots \)
  - self-screening idea or LPM?
    \( \to \) at \( T_c = 2(1.3)T_c \) (RHIC) \( p_{\text{cutoff}} \approx 1 \text{ GeV} \) still

→ What happens next, as \( T \to T_c \)?
  The cutoff decreases
  - Pert. argument \( M \sim gT \)
  - lattice data

→ Do we have experimental evidences?
  - dijectons \( \text{CERES} + \text{NA50} \)
  - photons \( \text{WA98} \)

→ Digression about soft photons/LPM suppression.

(Just observed by TAPS/SLI)
Brief summary
on parton kinetics

"Parents" (Partons before collisions)

\[ x \gg x_0 = 0.02, q \gg q \]
\[ N_g \sim 800, N_{qg} \sim 1 \]
in any beam

\[ N_{\text{dau}} \sim 350 \]

Very far from chemical equilibrium, even if kinetic one is (approx.) OK
\[ \frac{dN}{dp_T} \sim 5 \cdot e^{-p_T/T} \]
\[ p_T \sim 1 \text{GeV} \]

"Daughters" (Partons after 1st collision)

If ticked down, grows to \( N_{\text{max}} \sim 1000 \) (= parents)
\[ \frac{dN}{dy} \leq 100 \text{ (still very very dilute)} \]

\[ T \sim \frac{1}{p_T} \leq 0.1 \text{ fem} \]

Very far from chemical equilibrium, even if kinetic one is (approx.) OK
\[ \frac{dN}{dp_T} \sim 5 \cdot e^{-p_T/T} \]
\[ p_T \sim 1 \text{ GeV} \]

"Grand-daughters"

Quarks are never able to catch-up
"hot glue scenario"

\\[ E_S \cdot g \]

Described either by \( g^* \rightarrow gg \) (Geiger)
or on-shell processes \( gg \rightarrow gg \)

\[ g \sim 1/6 \text{ at } T \sim 2 \text{ fm} \]

Biro et al. 93
Prakash + Leibbrandt
ES + Li Xiong 99
Langer + et al.

Further processes \( gg \rightarrow ng \)

\[ g \sim 1 \text{ at } T \sim 1.5 \text{ fm} \]

ES + Li Xiong 95 and 95

\[ T \rightarrow \infty \] observed prios give

\[ \frac{dN_{\text{sys}}}{dp_T} (1.1) \cdot \left( \frac{2}{e^2 + ...} \right) = 1000 \text{ (at reduced energy)} \]

Note: Very far from \# of parents!!!

Another 1.18 expect toward 2\cdot100 \text{ GeV} \cdot \text{A}
The point I want to make about P_cutoff:

- Generally speaking...
  - P_{cutoff} in vacuum \( \sim 1 \text{ GeV} \) on gluons
  - \( \Gamma \) in non-equilibrium
  - P_{cutoff} in "color glass" \( \sim g T^2 \) (\( T/\sqrt{\Lambda} \) for q's)
  - Properly generalized if non-equilibrium...
    \( \sim g \mu^2 \) \( \rightarrow \) Mekareev-Venugopala

- However: at RHIC all available estimates for all three give \( P_{cutoff} \approx 1\text{ GeV} \) at \( T \approx 0 \). (as a coincidence)
  - at LHC we should get larger than in-vacuum cutoff.
  - So \( \rightarrow \) HIJING works without modifications with vacuum cutoffs...

- "Parents" \( \rightarrow \) "Daughters" with \( p_T \sim 2 \text{ GeV} \) or so \( \rightarrow \) \( T \sim (p_T^2) \sim 600 \text{ MeV} \)

- Kinetics at later times depends on \( P_{cutoff} \) for \( T \) going down, toward \( T_c \sim 160 \text{ MeV} \).
  - \( P_{cutoff} \) must go down, by a significant factor, \( \sim 3 \)
  - Cross sections \( \gamma \) by an order of magnitude...
Arguments

- Perturbative
  
  \[ W_q = \frac{gT}{\sqrt{s}} \]
  
  at \( T = 200 \text{ MeV} \)
  
  \( g \approx 1.2 \)
  
  \( \approx 100 \text{ MeV} \)
  
- Non-pert.: \( \tilde{T} \) pairing, \( \tilde{W}_q \) deconfinement (which is at least too small to be true?)

- Lattice 1: provides an estimate of \( m^2 \): \( O(m^2) \) corrections to \( p, e \)

  \( \Rightarrow \) Fixed cutoff \( 1 \text{ GeV} \) is clearly unacceptable \( \Rightarrow \) \( 3T = 600 \text{ MeV} \)

  is the peak energy

  \( \Rightarrow \) \( m_q \approx \frac{1}{3} \text{ GeV} \) as estimate

  compared to pert., \( g \) grows to \( \approx 2 \).

- Lattice 2: direct \( m_{\text{gluon}} \)

  electric mass ...

  \( \Rightarrow \) \( \frac{m_{\text{electric}}}{2} \) indeed

  (although below \( T = 170 \text{ MeV} \), \( m_{\text{gluon}} \) masses should go to \( \infty \)).

- Phenomenology:

  dileptons:

  \( - e^+ e^- \) in vacuum

  \( \Rightarrow \) pert. th.

  \( \Rightarrow e^+ e^- \) from hot matter

  seem to produce continuum till \( E_{\text{cutoff}} \leq 0.5 \text{ GeV} \).

  \( \Rightarrow \) from hot matter \( \text{(WA98)} \)

  \( \Rightarrow \) all \( q \) can be explained by "partonic rates"
Conclusions

- $P_{\text{cut off}} \neq P_{\text{cut off initially}} \neq P_{\text{cut off}}$
  - vacuum
  - $P_{\text{cut off}} \approx 1 \text{ GeV}$, instantaneous
  - $P_{\text{cut off}} \approx g^T$, plasma scale
  - $P_{\text{cut off}} \approx g^S$, saturation scale

But at RHIC all $\approx 1\text{ GeV}$

- As QGP cools down toward $T_c \approx 160\text{ MeV}$,
  $P_{\text{cut off}}$ goes down as well.
  (There exist gluons with $p$ below $1\text{ GeV}$!)

- Dilepton/photons data from hot matter (at $T \approx T_c$, not QGP but "mixed phase" at best) show that.

- New window for "direct $\bar{\phi}$" $p_L \approx 30\text{ MeV}$
  First evidences for LPM effect $\rightarrow$ TAPS(651)

- $gg \rightarrow ggg$ (processes $gg \rightarrow ng, n > 3$)
  are the main "entropy production" tool
  $\Rightarrow$ deserve further studies
  $\Rightarrow$ can equilibrate glue in $\sim 1.5\text{ fm/c}$

- We would like to see "QGP push"
  in transverse flow $\rightarrow V_0$ and especially, $V_2$!
Brief Closing Remarks

4 Generic Topics

\begin{align*}
\text{Vacuum} & \quad \begin{cases} \text{In} & 2 \\ \text{Out} & 6 \end{cases} \\
T \neq 0 & \quad \begin{cases} \text{In} & 4 \\ \text{Out} & 11 \end{cases} \\
\mu \neq 0 & \\
\text{None of above} & \quad \text{Out}
\end{align*}

What are the outstanding problems?

What are the physical problems and why do we care?

What are the techniques?
Generic Techniques

Coupling big

Lattice gauge theory

Exactly solvable models

Brave attempts:

Instantons
Monopole condensates
Variational vacua
Schwinger-Dyson Egare

Physics Issues

Chiral symmetry not
Free quarks not
Phase transitions at \( t \neq 0, \mu \neq 0 \)
Chiral symmetry not
\[ U(1) \text{ (\(q'\) not Goldstone boson)} \]
\[ \partial_\mu J^\mu \sim FF^\dagger \]
** ** instantons

\[ SU(2) \]

instantons interact other modes: make act as Goldstone

** **

lattice hard as \(M_g > 0\)

but good semi-quant evidence

Confinement

No quarks:

hbar \[ \times \times \times \]

Exactly soluble

** **

Models

With quarks:

hbar \[ \times \times \]
Finite $T$ & $\mu$ when coupling big

\[ T \rightarrow T_c \]
\[ T < T_c \]

No quarks

$T \rightarrow T_c$ is good

$T < T_c$ bad

$e^{-mg_{1/2}/T} \ll 1$

With quarks:

168
\[ \mu = 0 \]
\[ m_0 = m_d = 0 \]

\[ \frac{g}{\beta} \]

Lattice: \( \times \times \)

No first order deconfinement favored

\[ \mu \neq 0 \]

No lattice m.c.

\[ g_{388.6eV} \]

\[ E/\eta - m \approx 10^6 \text{ MeV} \]
Both nuclear matter \( \rightarrow \) nuclei

Hot nuclear matter: liquid gas \( PT \)
Cold nuclear matter: superfluid, conduct.

\(? \quad ? \quad ? \quad ?\)

Nuclear matter computations
only reliable for \( T \ll 10^3 \text{ MeV} \)

\[ \frac{N - NN_m}{NN_m} \ll 1 \]

Very little known at \( T + \mu \neq 0, \rho \ll \text{ big} \)
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<thead>
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list of participants 172
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</table>
# Agenda

**EQUILIBRIUM & NON-EQUILIBRIUM ASPECTS OF HOT, DENSE QCD**

*July 17th – July 30th, 2000*

A RIKEN BNL Research Center & BNL Nuclear Theory Workshop

**All lecturers are allotted one hour plus half hour for discussions.**

### Monday 17 July

#### Morning

**Small Seminar Room, Physics, Bldg. 510 1st floor**

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker(s)</th>
<th>Topic</th>
</tr>
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<tbody>
<tr>
<td>10:00</td>
<td>Registration &amp; Welcome Reception</td>
<td><em>(Coffee &amp; Breakfast Treats will be provided)</em></td>
</tr>
<tr>
<td>11:00</td>
<td>Laurence Yaffe (University of Washington)</td>
<td>Non-Perturbative Dynamics of Hot Non-Abelian Gauge Fields</td>
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<td></td>
<td>Discussion</td>
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<tr>
<td>12:30</td>
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#### Afternoon

**Small Seminar Room, Physics Bldg. 510, 1st floor**

| Time   | Speaker(s)                                      | Topic                                                               |
|--------|-------------------------------------------------|                                                                    |
| 14:15  | Eric Braaten (Ohio State University)            | Thermodynamics of Hot QCD                                          |
|        | Discussion                                      |                                                                      |
| 16:30  | Pawel Danielewicz (Michigan State University)   | Low-Momentum Correlation from Real-Time Theory                     |
|        | Discussion                                      |                                                                      |

### Tuesday 18 July

#### Morning

**Room 2-160, Physics Bldg. 510, 2nd floor**

| Time   | Speaker(s)                                      | Topic                                                               |
|--------|-------------------------------------------------|                                                                    |
| 11:00  | Krishna Rajagopal (MIT)                         | Traversing the QCD Phase Transition: Quenching, Slowing or Bubbling Out of Equilibrium |
|        | Discussion                                      |                                                                      |
| 12:30  | LUNCH                                          |                                                                      |

#### Afternoon

**Small Seminar Room, Physics Bldg. 510, 1st floor**

| Time   | Speaker(s)                                      | Topic                                                                |
|--------|-------------------------------------------------|                                                                     |
| 14:15  | Randy Kobes (University of Winnipeg)            | Calculating Viscosity                                               |
|        | Discussion                                      |                                                                      |
| 16:30  | Peter Arnold (University of Virginia)           | Hot Scalar Theories in Large N: Bose-Einstein Condensation         |
|        | Discussion                                      |                                                                      |
**Wednesday 19 July**

**Morning**  *Berkner Hall Auditorium (Cafeteria Building)*

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>11:00 - 11:30</td>
<td>Coffee</td>
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<tr>
<td>11:30 - 12:30</td>
<td>SPECIAL Announcement “First Physics Results from PHOBOS@RHIC” Wit Busza, PHOBOS Collaboration</td>
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<tr>
<td>12:30 -</td>
<td>LUNCH</td>
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**Afternoon**  *Small Seminar Room, Physics Bldg. 510, 1st floor*

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker/Title</th>
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<tbody>
<tr>
<td>14:15 - 15:45</td>
<td>H. Arthur Weldon <em>(W. Virginia University)</em>  <em>A Space Time View of Thermal Field Theory</em> Discussion</td>
</tr>
<tr>
<td>16:30 - 18:00</td>
<td>Rudolf Baier <em>(University of Bielefeld)</em>  <em>Resonant Decay of Parity Odd Bubbles in Hot Hadronic Matter</em> Discussion</td>
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**Thursday 20 July**

**Morning**  *Room 2-160, Physics Bldg. 510, 2nd floor*

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<th>Time</th>
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<tbody>
<tr>
<td>11:00 - 12:30</td>
<td>Berndt Mueller <em>(Duke University)</em>  <em>The Quantum Limit of Classical Field Dynamics</em> Discussion</td>
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<td>12:30 -</td>
<td>LUNCH</td>
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**Afternoon**  *Room 2-160, Physics Bldg. 510, 2nd floor*

<table>
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<th>Time</th>
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<tbody>
<tr>
<td>14:15 - 15:45</td>
<td>Teiji Kunihiro <em>(Kyoto University)</em>  <em>The Renormalization Group Method Applied to Kinetic Equations</em> Discussion</td>
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**Thursday 20 July (cont)**

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<th>Time</th>
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<tr>
<td>16:30 – 18:00**</td>
<td>Keijo Kajantie (University of Helsinki)</td>
<td>Thermodynamics of Hot QCD; the Effective Theory Approach</td>
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<td></td>
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<td><strong>Discussion</strong></td>
</tr>
<tr>
<td>18:00 – 21:00</td>
<td></td>
<td><strong>WORKSHOP BBQ at Brookhaven Center, Bldg 30 - Patio</strong></td>
</tr>
</tbody>
</table>

**Friday 21 July**

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>Small Seminar Room, Physics Bldg. 510, 1st floor</td>
<td>Hector de Vega (Paris VI) Non-Equilibrium Quantum Plasmas, Quantum Kinetics and the Dynamical Renormalization Group</td>
</tr>
<tr>
<td>11:00 – 12:30</td>
<td></td>
<td>Discussion</td>
</tr>
<tr>
<td>12:30 -</td>
<td></td>
<td>LUNCII</td>
</tr>
<tr>
<td>Afternoon</td>
<td>Small Seminar Room, Physics Bldg. 510, 1st floor</td>
<td>Ariel Zhitnitsky (British Columbia University) Early Universe at the QCD Scale</td>
</tr>
<tr>
<td>14:15 – 15:45</td>
<td></td>
<td>Discussion</td>
</tr>
</tbody>
</table>
### Monday 24 July

**Morning**  
Small Seminar Room, Physics Bldg. 510, 1st floor  

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00</td>
<td>Misha Stephanov</td>
<td>QCD at Finite Density of Isospin (Illinois University &amp; RIKEN BNL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
<tr>
<td>12:30</td>
<td></td>
<td>LUNCH</td>
</tr>
</tbody>
</table>

**Afternoon**  
Small Seminar Room, Physics Bldg. 510, 1st floor  

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:15</td>
<td>Dirk Rischke</td>
<td>Properties of Gluons in Color Superconductors (RIKEN BNL Theory)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
<tr>
<td>16:30</td>
<td>Eduardo Fraga</td>
<td>Quark Stars from Perturbative QCD (BNL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
</tbody>
</table>

### Tuesday 25 July

**Morning**  
Room 2-160, Physics Bldg. 510, 2nd floor  

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00</td>
<td>Emil Mottola</td>
<td>The Electrical Conductivity of a QED Plasma (LANL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
<tr>
<td>12:30</td>
<td></td>
<td>LUNCH</td>
</tr>
</tbody>
</table>

**Afternoon**  
Small Seminar Room, Physics Bldg. 510, 1st floor  

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:15</td>
<td>Anton Rebhan</td>
<td>Hard-thermal/dense-loop Thermodynamics of the Quark-gluon Plasma (University of Vienna)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
<tr>
<td>16:30</td>
<td>Fred Cooper</td>
<td>Simulations of the Chiral Phase Transition on Both Sides of the Tricritical Point (LANL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discussion</td>
</tr>
</tbody>
</table>
Wednesday 26 July

<table>
<thead>
<tr>
<th>Morning</th>
<th>Small Seminar Room, Physics Bldg. 510, 1st floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00 – 12:30</td>
<td>Dietrich Bodeker (Heidelberg University) Discussion</td>
</tr>
<tr>
<td>Hard Thermal Loops on the Lattice</td>
<td></td>
</tr>
<tr>
<td>12:30 -</td>
<td>LUNCH</td>
</tr>
</tbody>
</table>

Afternoon | Small Seminar Room, Physics Bldg. 510, 1st floor |
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>15:30 – 17:00</td>
<td>V.P. Nair (City College of New York) Discussion</td>
</tr>
<tr>
<td>Covariance &amp; Magnetic Mass</td>
<td></td>
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</tbody>
</table>

Thursday 27 July

<table>
<thead>
<tr>
<th>Morning</th>
<th>Room 2-160, Physics Bldg. 510, 2nd floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00 – 12:30</td>
<td>Raju Venugopalan (BNL Theory) Discussion</td>
</tr>
<tr>
<td>Solving the Problem of Initial Conditions in HI Collisions: from the Nuclear Wave Function to a Gluon Plasma</td>
<td></td>
</tr>
<tr>
<td>12:30 -</td>
<td>LUNCH</td>
</tr>
</tbody>
</table>

Afternoon | Room 2-160, Physics Bldg. 510, 2nd floor |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>14:15 – 15:45</td>
<td>Michael Strickland (Ohio State University) Discussion</td>
</tr>
<tr>
<td>HTL Perturbation Theory</td>
<td></td>
</tr>
<tr>
<td>16:30 – 18:00**</td>
<td>Michel Tytgat (CERN / BNL) Discussion</td>
</tr>
<tr>
<td>QCD at Theta ~ Pi</td>
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</tr>
</tbody>
</table>

**16:30 talk and discussion to be held in the Small Seminar Room, Physics Bldg 510, 1st floor**
<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:00 - 12:30</td>
<td>Ed Shuryak (Stony Brook University) <em>Decrease of the Non-perturbative Cutoff in QCP at RHIC</em></td>
</tr>
<tr>
<td>12:30</td>
<td>Lunch</td>
</tr>
<tr>
<td>14:15 - 15:45</td>
<td>Larry McLerran (BNL Theory) <em>CLOSING LECTURE</em></td>
</tr>
</tbody>
</table>
For information please contact:

Ms. Tammy Heinz or Ms. Pamela Esposito
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Upton, NY 11973-5000, USA
Phone: (631)344-5864 (631) 344-3097
Fax: (631)344-2562 (631) 344-4067
E-Mail: theinz@bnl.gov pesposit@bnl.gov
Homepage: http://quark.phy.bnl.gov/www/riken.html
RIKEN BNL RESEARCH CENTER

EQUILIBRIUM & NON-EQUILIBRIUM ASPECTS OF HOT, DENSE QCD

JULY 17–30, 2000

Speakers:

P. Arnold  R. Baier  D. Bödker  E. Braaten
F. Cooper  P. Danielewicz  H. de Vega  E. Fraga
K. Kajantie  R. Kobes  T. Kunihiro  L. McLerran
E. Mottola  B. Müller  V.P. Nair  K. Rajagopal
A. Rebhan  D. Rischke  E. Shuryak  M. Stephanov
M. Strickland  M. Tytgat  R. Venugopalan  H.A. Weldon
L. Yaffe  A. Zhitnitsky