String Theory and Non-Commutative Geometry

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I Final Report on DOE Grant DE-FG02-94ER-25228

During the period of the three-year DOE grant DE-FG02-94ER-25228, granted for the period August 1, 1997 to July 31, 2000, and extended until July 31, 2001, the investigator made significant progress and achieved most of the proposed goals. He completed certain older work, discovered a number of fundamental new theoretical phenomena, developed theories about them, and we began a project to integrate *twist quantum fields* into constructive quantum field theory, and to determine the effects of twisting on breaking supersymmetry. Two postdoctoral fellows — filling a single postdoctoral position — have overlapped the period of this grant. Olivier Grandjean and David Ellwood collaborated on this work with Arthur Jaffe. The work also included completion of a collaboration with Olivier Grandjean and student Jon Tyson, after Grandjean left Harvard.

- Jaffe wrote a lengthy paper introducing non-commutative harmonic analysis and looking at it from the point of view of non-commutative geometry [3].
- Jaffe discovered a new representation for invariants obtained by pairing a *JLO cocycle* with a square-root of the identity [4].
- Jaffe gave the *holonomy expansion* as a regularization technique in quantum theory [6], and as a tool to compute the invariants discussed above.
- Jaffe discovered *twist positivity* [5], a condition that leads to a positive partition function for a density matrix twisted by a one-parameter unitary group. This structure arises naturally in quantum theory and leads to a corresponding functional integral.
- Jaffe extended the notion of twist positivity to a general class of symmetries of Lagrangians of free fields [2].
- Jaffe defined appropriate twist fields and established twist positivity for them [1]. Using these fields, he wrote down the restrictions on twists and on interactions in order to attempt to define $N = 2$ supersymmetric non-linear field theories on a cylinder [1].

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- Jaffe used the twist fields to give a mathematical complete computation of the elliptic genus in quantum field theory [7,8].
- Jaffe began a study to implement twist fields in constructive quantum field theory [10], and in particular we are investigating using twist fields as an infrared cutoff in quantum field theory.

In addition, Olivier Grandjean and David Ellwood have pursued other projects, leading to the references [11, 12, 13], and a new set of projects was begun in the summer of 2000 with student William Ritter [14].

II References

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