# DETERMINANTS OF NODULATION COMPETITIVENESS IN *RHIZOBIUM ETLI*

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### 1. Kondo insulators

Most Kondo insulators are van Vleck paramagnets at low temperatures, e.g. CeNiSn, $Ce_3Bi_4Pt_3$, $CeFe_4P_{12}$, $SmB_6$, $YbB_{12}$, and FeSi. Exceptions are TmSe and UNiSn for which antiferromagnetic long-range order was found and $UFe_4P_{12}$ orders ferromagnetically at low $T$. UNiSn, SmS, and TmTe undergo a metal-insulator transition as a function of temperature and pressure, respectively, and a Bose-Einstein condensation of excitons has been reported in $TmSe_{1-x}Te_x$ and $Sm_{1-x}La_xS$. The properties of these narrow gap semiconductors (hybridization gap at the Fermi level of the order of the Kondo temperature) strongly depend on defects, e.g. nonmagnetic impurities like Kondo holes, ligand defects or dopants, in the compound.

There are several experimental studies on impure Kondo insulators. Our prediction of a $c^{1/2}$-dependence [13,26] in the low-$T$ specific heat has been verified for La-substituted ($c$ is the concentration) $Ce_3Bi_4Pt_3$. The valence instability and electrical properties of Yb and La substituted $SmB_6$ have been studied long ago. The gap of CeNiSn was found to close with 15% La substitution and is also smeared with increasing $x$ in $CeNi_{1-x}Pt_xSn$. Recently, a metal-insulator transition was discovered in FeSi with the chemical substitution of Al at the Si site.

A Kondo hole is an isolated *nonmagnetic* impurity in a heavy fermion lattice (e.g. a La (Th) atom replacing a Ce (U) ion) introduced with a large local potential that prevents the occupation of the f level. The impurity gives rise to a $\delta$-function-like boundstate in the density of states inside the hybridization gap [5]. In our calculation we mostly limited ourselves to the electron-hole symmetric situation, a local hybridization and a nearest-neighbor tight-binding dispersion for the conduction band on a simple cubic lattice. The correlations among the f electrons were introduced (a) within the local approximation (via a selfenergy for the f electrons in the spirit of the $d \to \infty$ limit) and (b) within the mean-field Kotliar-Ruckenstein slave-boson approach (related to the Gutzwiller approximation). Both approaches yield the same results showing the robustness to the approximations used to incorporate the correlations.

Our main results are the following:

- We have studied the local density of f-states in the neighborhood of the Kondo hole [5]. The Kondo hole boundstate pins the Fermi level (charge neutral substitution) and has the magnetic properties of a free spin 1/2, i.e. it is polarized by a magnetic field, follows a Curie susceptibility, a magnetic field induces a Schottky anomaly in the specific heat and it should be observable by electron paramagnetic resonance (assuming that inhomogeneous broadening is sufficiently small) [5,40]. A larger concentration of Kondo holes should be observable by infrared spectroscopy [40].
- Other cases of Kondo-hole-like defects we studied are (i) doping, e.g. Th impurities substituting Ce atoms introduce an additional conduction electron, and (ii) ligand defects, which locally change the hybridization. In case (i) the Fermi level is not pinned by the impurity (or impurity band) and hence the impurity has nonmagnetic properties at low $T$. In case (ii) a pair of boundstates may form in the gap close to the band-edges. These results have been presented at the 38th MMM Conference as an invited paper [40].

- For several Kondo holes, multiple scattering of the electrons and the interference between the different scattering sites give rise to interactions among the impurities [42,49]. There are as many boundstates in the gap as Kondo holes in the system. An expression for the Green's function solving the scattering problem has been obtained exactly with the local approximation for the correlations.
- A pair of impurities has been discussed in [42]. If the Kondo holes are nearest neighbors they form a bonding (filled with two electrons) and an (empty) antibonding state. The groundstate is a singlet and the energy difference between the two boundstates is the exchange energy. If the Kondo holes are not on neighboring sites the boundstates are degenerate; they pin the Fermi level and have magnetic properties.
- For an arbitrary cluster of $N_i$ Kondo holes the energies of the $N_i$ boundstates depend on the spatial distribution of the Kondo holes [49]. The distribution of states is symmetric with respect to the center of the gap (electron-hole symmetry). If $N_i \to \infty$ there is an accumulation of boundstates with energy close to zero (Fermi level), which eventually leads to an *impurity band.* These states determine the low temperature thermodynamics and transport properties.
- For boundstates at the Fermi energy the spectral weight is nonzero only on sites neighboring one of the Kondo holes. Hence, wavefunctions of boundstates at the Fermi energy corresponding to a cluster of Kondo holes can be classified into connected and disconnected (see Fig. 1). The connectivity of the boundstates increases with increasing $N_i$ until eventually a percolating boundstate is obtained. The percolation threshold corresponds to the $T = 0$ *insulator-metal transition* in the dirty Kondo insulator.

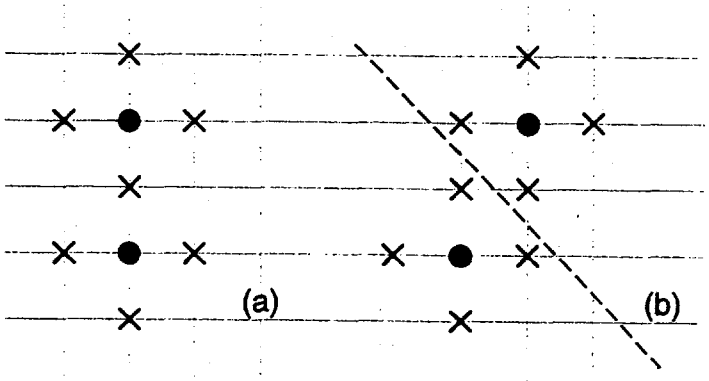


Fig. 1: Examples of (a) connected and (b) disconnected Kondo hole boundstates on a square lattice with nearest neighbor hopping. The dots represent the substituted atoms and the crosses sites with nonzero spectral weight. The localized electrons cannot cross the dashed line.

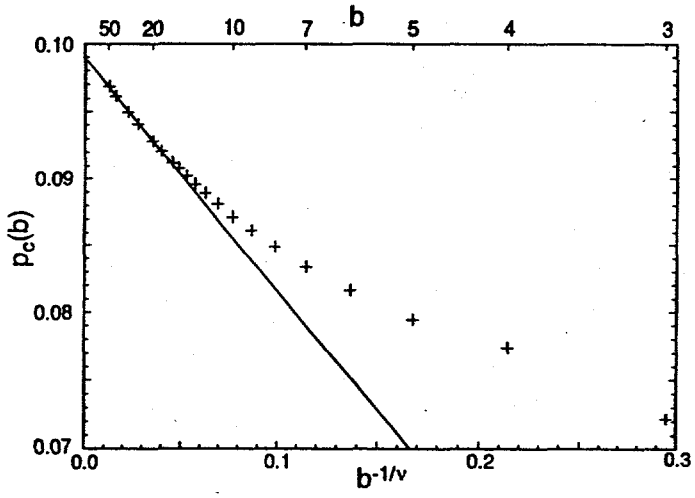


Fig. 2: Small cell renormalization for the site percolation of of Kondo holes with first, second and fourth nearest neighbor bonds on a sc lattice. $b^3$ sites are replaced by a single top and bottom faces of the supersite, which is occupied if the cell are connected. $p_c(b)$ is the nontrivial fixed point of the renormalization and $\nu = 0.9$ [56].

- For a simple cubic lattice with nearest neighbor hopping and electron-hole symmetry, the connectivity problem can be mapped onto the site percolation of Kondo holes with first, second and fourth nearest neighbor bonds [49,56]. We estimated the critical concentration of Kondo holes (percolation threshold) using (i) the low density mean cluster size expansion and (ii) a small cell renormalization which yields $c_{cr}$ as the scaling fixed point (see Fig. 2). The results of both methods are consistent with $c_{cr} = 0.099$.
- If the tight-binding of the conduction band extends beyond nearest neighbors, the physical extension of the boundstates and the connectivity increase accordingly. This lowers the percolation threshold, so that 10% of Kondo holes is an upper bound for the insulator to become a metal. Electron-hole asymmetry also lowers $c_{cr}$.

The impurity band gradually smears the hybridization gap with increasing concentration of Kondo holes $c$ [13,26,40]. For small $c$ the width and the height of the band are proportional to $c^{1/2}$ and the Fermi level is pinned in the band. The low temperature specific heat is proportional to $T$ with strongly enhanced $\gamma$ coefficient. The $c^{1/2}$-dependence of the $\gamma$ coefficient has been observed for La-substituted $Ce_3Bi_4Pt_3$. The susceptibility follows a Curie-Weiss-like behavior with an antiferromagnetic Weiss-temperature.

**Magnetic instabilities of Kondo insulators :** We considered a stoichiometric Kondo insulator described by the symmetric Anderson lattice without orbital degeneracy and in average two electrons per site within a mean-field approximation formulated in terms of four slave-bosons per site in analogy to Kotliar and Ruckenstein's approach for the Hubbard model [17]. A hybridization gap on the scale of the Kondo temperature opens in the paramagnetic phase, giving rise to semiconducting properties at low temperatures. The paramagnetic solution is stable for sufficiently small $U$, but not stable with respect to a metallic ferromagnetic phase if $U > 1.54V$ (first order transition) and antiferromagnetic long-range order for $U > 0.45V$ (second order transition). In zero-field the energy of the antiferromagnetic phase is always lower than the energy of the ferromagnetic state. Quantum fluctuations and the RKKY interaction are expected to stabilize the paramagnetic and antiferromagnetic phases as compared to the ferromagnetic one. In a strong magnetic field the ferromagnetic solution becomes the stable one.

We further considered a stoichiometric metallic Anderson lattice with orbital degeneracy in the $U \to \infty$ limit within mean-field approximation formulated now in terms of three slave bosons per site [21]. We found that for the orbitally nondegenerate case the paramagnetic solution is unstable towards ferromagnetism if the valence is smaller than a critical one. This instability is suppressed with increasing orbital degeneracy. The orbitally degenerate metallic situation with finite $U$ requires a large number of auxiliary bosons [25]. The combined boson-fermion Hamiltonian is formulated for the general case of arbitrary electronic configurations, but the mean-field approximation has been discussed only for the case in which the occupancy of the $f^n$-configurations for $n > 2$ is excluded. The occupation probabilities were obtained as a function of $U$ and the f-level position. For sufficiently large $U$ the paramagnetic solution becomes unstable towards ferromagnetism. Again, this instability is suppressed with increasing orbital degeneracy. The case of an orbitally degenerate Kondo insulator with electron-hole symmetry was also discussed in mean-field theory.

We studied the **interplay between the Kondo hole impurity band and magnetic long-range order**, extending this way our investigation of the magnetic phase diagram of stoichiometric Kondo insulators [59,61,63]. UNiSn is an antiferromagnetic Heusler compound with intrinsic disorder (Ni and Sn are frequently on the 'wrong' sites). We used the symmetric non-degenerate Anderson lattice with two electrons per site. Correlations were treated within Kotliar and Ruckenstein's mean-field approach formulated in terms of four slave-bosons per site. The main results are the following:

- A hybridization gap on the scale of the Kondo temperature is only weakly renormalized by the impurity band [59]. This phase is only stable for small $U$.
- With increasing $U$ there is a second order transition to an insulating antiferromagnetic phase. As a function of Kondo hole concentration there is a re-entrant paramagnetic-antiferromagnetic-paramagnetic phase boundary [63].
- Due to the finite density of states of the impurity band there is a Stoner instability from the paramagnetic to a ferromagnetic state [61,63]. The critical $U$ is, however, too large to be physically relevant.
- A metallic ferromagnetic phase is stable in a sufficiently large magnetic field (first order transition).

Most of the above results are reviewed in a chapter of the book *Current Problems in Condensed Matter*

published by Plenum Press [83].

## 2. The n-channel Kondo problem

The n-channel Kondo problem consists of an impurity of spin $S$ interacting via exchange with conduction electrons in $n$ orbital channels. For a given spin $S$ the model has two parameters, namely the Kondo temperature $T_K$ and the number of channels $n$. Three qualitatively different situations occur as a function of $n$ [7,81]. (i) For $n = 2S$ the spin of conduction electron channels *compensates* the impurity spin into a singlet, giving rise to Fermi-liquid behavior at low $T$. This situation is realized for Fe and Cr impurities in simple metals like Cu and Ag [81]. (ii) If $S > n/2$ the impurity spin is only partially screened and said to be *undercompensated.* The spin-density of the conduction electrons is not large enough to form a singlet, leaving an asymptotically free spin of $(S - n/2)$ [7,81]. Examples for this case are Tm or Tb [52] impurities embedded in a metal. (iii) If $S < n/2$ there are more channels than required to compensate the impurity spin (*overcompensated* impurity). This gives rise to critical behavior (non-Fermi-liquid) as the temperature and the external magnetic field tend to zero [7,81]. Applications are the quadrupolar Kondo effect and electron assisted tunneling of an atom in a double-well potential [1]. A convincing experimental realization is the differential resistance of metal point contacts containing structural disorder.

We extensively reviewed the above results in [81]. We also presented a brief summary of applications of the n-channel Kondo problem as an invited paper at the International Conference on Strongly Correlated Electron System in Amsterdam [43] and published a brief review of multichannel properties as a chapter in the book *New Trends in Magnetic Materials and their Applications* [38]. The multichannel Kondo problem is also closely related to magnetic impurities in antiferromagnetic Heisenberg chains [3,4] (see Sect. 8.).

The following two issues, part of P. D. Sacramento's Ph.D. thesis (Temple University), were completed and published:

- The thermodynamics of the n-channel Kondo model for general $n$ and impurity spin $S$ in a magnetic field [7] was obtained by solving the thermodynamic Bethe ansatz equations. Results were obtained both analytically and numerically. Several numerical procedures had to be employed to obtain satisfactory results for the entire parameter range (temperature, magnetic field, $n$ and $S$). The main difficulty we had to confront was that the problem requires a substantial increase in the numerical precision as compared to similar efforts for the Kondo and Coqblin-Schrieffer models, in particular when properties vary on a logarithmic scale or diverge resembling critical behavior. This study involves all three cases: (i) the perfectly compensated ($n = 2S$), the (ii) undercompensated and (iii) the overcompensated impurity. Physical applications have been discussed in [6].
- We considered the electron assisted tunneling of an atom in a double-well potential (parametrized by a two-level system (TLS)) [1,6]. The scattering of the electrons gives rise to logarithmic infrared singularities. We derived and analyzed the renormalization group equations for an arbitrary number of relevant orbital channels. Close to the strong coupling fixed point the model is equivalent to the overcompensated $n$-channel Kondo problem with $S = 1/2$ (the TLS) [1] and an effective number of channels that depends on the initial conditions of the renormalization procedure. The Zeeman splitting in the Kondo problem represents the asymmetry of the well. The diverging susceptibility (to an asymmetry in the well) as $H \to 0$ and $T \to 0$ indicates that the symmetric TLS is unstable to a local lattice deformation via coupling to phonons [1,6], which disappears above a critical temperature $T_c$. The groundstate equilibrium situation of the TLS corresponds to $H \neq 0$ and a Fermi liquid picture applies (the atom is not localized in one of the minima). The specific heat $\gamma$-values become giant as $H \to 0$. For small asymmetry the specific heat shows a double peak structure which is particularly pronounced for $n = 2$. There is a very close relationship between this model and the quadrupolar Kondo effect. Possible realizations of multichannel Kondo behavior are (see [6,81]) the effect of the motion of Ge on the resistivity of $Pb_{1-x}Ge_xTe$, the $log(T)$-dependence in heavily doped conducting polymers and the differential resistance of metal point contacts with structural disorder.

The most interesting case is the overcompensated low-$T$ fixed point, believed to be related to the non-Fermi-liquid behavior observed in several heavy-fermion compounds and alloys and to high $T_c$ compounds in the underdoped regime. Of crucial importance is the stability of the non-Fermi-liquid fixed point to symmetry breaking perturbations. It was found to be unstable to **(a)** a magnetic field (Zeeman splitting) [1,6,81], and

**(b)** channel-symmetry breaking in the exchange coupling, i.e. orbital channels with different $J$. **(c)** On the other hand, an exchange anisotropy, i.e. $J_\parallel \neq J_\perp$, is an irrelevant perturbation for $S = 1/2$ and $n = 2$.

We studied the stability of all the fixed points with respect to **(d) a crystalline field splitting** of the orbital channels [48,54], **(e) interactions among the host electrons** on a lattice [70], and **(f)** calculated the finite size corrections to the overcompensated fixed point [53,60]. The calculations were performed within the framework of Bethe's Ansatz and the quantum inverse scattering method. Our main results are the following:

**(d)** We considered crystalline fields splitting the channel manifold into two multiplets, of $n^*$ (lower energy) and $(n-n^*)$ (higher energy) orbitals, respectively [48,54]. The exchange coupling is isotropic and the same for all channels. The crystalline field populates the channels differently and hence quenches the orbital singlet. The groundstate Bethe ansatz equations (two coupled bands corresponding to spin-composites of length $n$ and $n^*$) have been derived and solved as a function of $n$, $n^*$, $S$, and the (magnetic and crystalline) fields. The strings of length $n^*$ modify the overcompensated fixed point, so that new non-Fermi-liquid behavior is obtained [48].

- If $S = n/2$ or $S = n^*/2$ (new) the conduction electrons compensate the impurity spin into a singlet.
- If $S > n/2$ the impurity remains undercompensated to an effective spin $S - n/2$.
- In all other cases the impurity spin is overcompensated yielding critical behavior. The critical exponents depend on the quenching of the orbits, i.e. on $n^*$. This changes the universality class of the model.
- In the physically most interesting case, $S = 1/2$, $n = 2$ and $n^* = 1$, the overcompensated fixed point is totally quenched by the crystalline field. This was presented as an invited paper at the International Conference on Strongly Correlated Electron Systems in Goa, India [54]. Applications: The crystalline field corresponds to an external magnetic field in the electron assisted tunneling of an atom in a double well potential and to a splitting of the $\Gamma_8$ conduction state quartet in the quadrupolar Kondo effect. Another possible application is a quantum dot coupled to a metallic ring with two bands (the crystalline field splitting corresponds to the potential difference between the bands).

**(e)** We have studied a magnetic impurity embedded into a multichannel correlated host. The correlated lattice is the one-dimensional SU(3) invariant supersymmetric t-J model and the impurity has intermediate valence with two magnetic configurations (see Sect. 5). The solution was obtained via the quantum inverse scattering method. The main result is that [70]

- the correlations in the host only affect the charge sector driving the impurity into the mixed valence regime, but leaving the spin sector unchanged. Hence, interactions in the host are irrelevant (in the sense of a renormalization group) to the low-$T$ fixed points.

**(f)** We also investigated the finite size corrections to the groundstate energy of the multichannel Kondo impurity ($S = 1/2$, $n = 2$) as a function of the magnetic and crystalline fields [53].

- If either field is non-zero the conformal towers are of the Luttinger liquid type, while in the absence of fields the parafermion sector of the two-level SU(2) Wess-Zumino-Witten model contributes giving rise to the non-Fermi-liquid properties (see Sect. 4). We discussed some transport properties [60].

In summary, the exact solution of the multichannel model allows a detailed study of the stability of the non-Fermi-liquid fixed point to the various perturbations. This can be used as a testing ground for more complex (and non-integrable) models.

## 3. One-dimensional conductors

One-dimensional conductors are a long-standing topic of research with applications to quasi-1D organic conductors, inorganic CDW materials, polymers, mesoscopic systems, and semiconductor heterostructures, e.g. quantum wires. Numerous recent studies of electrons in 1D were motivated by the discovery of high $T_c$. Quantum fluctuations are fundamental in 1D, since even arbitrarily weak interactions transform the Fermi liquid into a Luttinger liquid. Hence, interacting electrons in 1D are always highly correlated. The residue of the Fermi-liquid quasi-particle pole vanishes and hence the discontinuity of the momentum distribution at the Fermi points disappears. The quasi-particle pole is replaced by incoherent structures, which involve non-universal power-law singularities, consequence of the conformal invariance. In addition, for electron systems the charge and the spin separate, i.e., they propagate independently with different group velocities. A detailed account of the properties of electrons in 1D can be found in my recent review article [82].

Some electron systems in 1D are integrable and an exact solution can be constructed by repeatedly applying Bethe's Ansatz. Several properties are accessible from the Bethe Ansatz solution: (a) Groundstate properties such as the energy, the susceptibility, the magnetization, the chemical potential as a function of the number of electrons, the magnetic field and interaction parameters, (b) the classification of all states and the excitation spectrum from the groundstate, (c) the thermodynamics, in particular the specific heat, and (d) the finite size corrections to the groundstate energy, i.e. the conformal towers, which determine transport properties and the critical exponents for the asymptotic of correlation functions. Our results on finite size effects are summarized in Sect. 4.

We derived exact results for the following integrable models.

- *Universal properties* of one-dimensional systems with SU($N$) symmetry [8,9]. The zero-temperature susceptibility to a field breaking the SU($N$)-invariance of an integrable one-dimensional system has logarithmic singularities as the field tends to zero (the third derivative of the energy with respect to the field diverges). The logarithms arise from the interference of the two "Fermi points" of the spin wave spectrum. The spin wave velocity is shown to be inversely proportional to the zero-field susceptibility [8,9]. These properties hold for a large variety of models, e.g., the multicomponent Fermi gas with $\delta$-function interaction [8], the SU($N$)-invariant Heisenberg chain of arbitrary spin [9] including the Babujian-Takhtajan model, the $N$-component supersymmetric t-J model in one dimension, the Gross-Neveu model, and the SU($N$) generalization of the Hubbard chain.

- *Metal-insulator* transitions are an unusual feature in 1D systems. An exception is the SU($N$) generalization of Lieb and Wu's Bethe ansatz solution of the standard Hubbard chain to $N$ spin degrees of freedom, which is integrable by construction and has several unusual properties at low temperatures [10]. (i) In the absence of fields and in the continuuum limit the effective interaction between charges corresponds to a potential of the form $[Sinh(ax)]^{-2}$, where $x$ is the distance between the particles involved and $a$ is an inverse length scale. (ii) In the limit $N \rightarrow \infty$ and in the continuum limit the model can be mapped onto a Bose gas interacting via a $\delta$-function potential. (iii) For $N > 2$ the model shows a Mott metal-insulator transition at $T = 0$ at a finite critical $U_c$ for exactly one electron per site. $U_c$ depends on $N$ ($U_c = 0$ for $N = 2$). For $U < U_c$ the system is metallic, while for $U > U_c$ it is insulating. (iv) A qualitative change in the charge rapidity distribution is found at $U_c$. (v) For one electron per site the Fermi velocity is finite for $U < U_c$, diverges as $U \rightarrow U_c$ and vanishes for $U > U_c$. Such a metal-insulator transition is also predicted by mean-field theories, which should hold for sufficiently large $N$, independently of the dimension.

- We studied a 1D Hubbard model with two bands of equal nearest-neighbor hopping, interband splitting $\Delta$ and a Hubbard-like repulsion $U$ [45,46]. The Mott transition disappears as a function of $\Delta$ [46]. The spin and the two bands constitute four internal degrees of freedom, so that the Bethe ansatz solution is the SU(4) generalization of Lieb and Wu's equations for the Hubbard chain. $U_c$ decreases monotonically with $\Delta$, vanishing if the excited electron band is empty, and $U_c = 2.981$ when the bands are degenerate. We derived groundstate properties, the spectrum of elementary excitations, the low-$T$ specific heat and the magnetic susceptibility, in both the metallic and insulating phases, as a function of the band splitting for exactly one electron per site [45]. There are four branches of excitations: (i) charge, (ii) interband and (iii) two branches of spinwave excitations. The group velocity of the charges is finite in the metallic phase, diverges as the metal-insulator transition is approached from the metallic side and vanishes for the insulator.

- We considered the one-dimensional gas of fermions with parabolic dispersion and $N$ spin-components (or spin $S$, $N = 2S + 1$) interacting via a $\delta$-function potential [29,32]. We classified all the states of the model by using the string hypothesis and derived the thermodynamic Bethe ansatz equations for the model with (i) repulsive and (ii) attractive interaction for arbitrary band-filling in terms of the dressed energy potentials [29]. Several limiting cases were discussed, e.g., $S = 1/2$, the groundstate, vanishing interaction strength, the strong coupling limit, high temperatures, and the large $N$ limit. In [32] we presented an extensive discussion of the groundstate properties (e.g. the energy, the chemical potential, and the magnetic susceptibility as a function of the band-filling and the interaction strength) and the spectrum of elemental excitations for both the repulsive and the attractive interaction cases. These

results were obtained by numerically solving the Bethe ansatz equations. Luttinger liquid properties are obtained for a repulsive interaction, while in the attractive case the electrons form boundstates of up to $N$ fermions of different spin-components (generalized Cooper pairs). In the attractive case all spin excitation branches are gapped.

- The *degenerate supersymmetric t-J model* consists of electrons with $N$ spin components on a lattice with nearest neighbor hopping, spin exchange $J$ and excluded multiple occupation of the lattice sites [16,20,22,50]. The model is integrable at the supersymmetric point, $t = J$, for arbitrary band filling. At the supersymmetric point the charge degree of freedom at acts similarly to the spinons, giving rise to a graded superalgebra. The model is a generalization of the standard t-J model to arbitrary spin. By analizing the discrete Bethe ansatz equations and we classified all the solutions according to the string hypothesis and derived the thermodynamic Bethe ansatz equations [16]. These equations were then solved in limiting cases, e.g., $S = 1/2$, the groundstate, the high temperature limit and, when the charge fluctuations are suppressed, the SU($N$)-invariant Heisenberg chain. We also studied [20,22] the groundstate properties and the spectrum of elemental excitations for arbitrary band-filling and several degeneracies $N$.
- We calculated the low-$T$ specific heat coefficient of the 1D supersymmetric t-J model with $N$ spin components [50]. Using the Sommerfeld expansion we extracted the $\gamma$-coefficient from the thermodynamic Bethe ansatz equations and expressed $\gamma$ in terms of the group velocities of the charges and spinwaves for arbitrary $N$, band-filling, and splitting of the levels (magnetic and crystalline fields). The results agree with conformal field theory and contain the following special cases: (i) For $N = 2$ the traditional supersymmetric t-J model, (ii) for one electron per site the SU($N$)-Heisenberg chain, and (iii) for $N = 4$ the two-band supersymmetric t-J model with crystalline field splitting [18].
- We studied an integrable *two-band model* consisting of two parabolic bands of electrons of equal mass (split by a crystalline field) with a local $\delta$-function exchange interaction [14]. The interaction is *attractive or repulsive* depending on whether the pair of electrons is in a spin-singlet or triplet state. This leads to the factorization of the scattering matrix into a product of a spin scattering matrix and an orbital one (for the band-indices). We classified all the states and derived the thermodynamic Bethe ansatz equations [33]. The attractive interaction leads to Cooper-pair-like boundstates (existing at every $T$), which do not condense nor have long-range order. A threshold magnetic field is required to overcome the binding energy and hence at $T = 0$ there is no response to fields smaller than the critical one, $H_c$. We obtained groundstate properties, the low-$T$ specific heat, the magnetization and the spectrum of elementary excitations as a function the interaction strength, the magnetic field, and the splitting between the bands [33]. For a magnetic field only slightly larger than $H_c$ the magnetization is proportional to $(H - H_c)^{1/2}$ [2]. This model was then extended to an arbitrary number of bands [37].
- We considered a *two-band model* of electrons on a chain with nearest neighbor hopping $t$ constrained by the excluded multiple occupancy of the sites, an interband splitting $\Delta$, and intra- and interband exchange $J$ between nearest neighbor sites. This model is integrable at the supersymmetric point [18,24] and reduces to the SU(4)-invariant supersymmetric t-J model with crystalline field splitting [16,20,22]. We obtained the exact solution via Bethe's ansatz and solved the groundstate Bethe ansatz equations for arbitrary band-filling. The groundstate properties and the four branches of elementary excitations, namely, charge, crystalline field and two spin-wave excitations, are discussed as a function of the crystal field splitting. As a function of $\Delta$ the model interpolates between the fourfold and the standard twofold degenerate t-J models [18]. The two bands could represent the $3d_{z^2}$ and $3d_{x^2-y^2}$ orbitals of Cu in high $T_c$ materials.
- We considered a soluble 1D model for the formation of *excitons and electron-hole droplets* [34,35,39], which consists of two parabolic bands of opposite mass ($m = \pm 1/2$) separated by a gap, $2\Delta$. The bands contain spinless fermions and represent the conduction and valence bands of a semiconductor. The holes in the valence band and the particles in the conduction band are locally attracted via a $\delta$-function potential. This model is mapped onto the two-component Fermi gas (the two components label the bands, the chemical potential corresponds to a magnetic field, and $\Delta$ to the chemical potential) and Gaudin and Yang's exact solution is used to study the formation of exciton bands. In the groundstate the particles and holes are either paired in exciton boundstates or unpaired. We also discussed metastable

states, the excitation spectrum and the thermodynamics [34,35]. At finite $T$ many-particle boundstates are populated, which at low $T$ are strongly delocalized and interpreted as electron-hole droplets.
We also considered the corresponding lattice model, which is mapped onto the Hubbard model with attractive $U$ interaction [39]. The continuum model requires an ad-hoc cutoff for the valence band; in contrast the lattice provides a natural cutoff. The Hubbard model yields an additional class of excitations, which however has no impact at low-$T$. Our results for a direct gap semiconductor are straightforwardly extended to an indirect gap semiconductor.

Most of the above results are part of the extensive review article on "Exact Results for Highly Correlated Electron Systems in One-Dimension" that recently appeared published in the International Journal of Modern Physics [82]. They were placed into the context of many other rigorous results known to present for interacting electrons in 1D.

### 4. Finite size scaling, Aharonov-Bohm-Casher interference phenomena and quantum dots

The finite size of a 1D system manifests itself in several ways: (i) The finite length $L$ of a ring gives rise to quantum topological effects, i.e. to persistent currents with oscillation patterns of the Aharonov-Bohm (AB) and Aharonov-Casher (AC) type, (ii) the asymptotic dependence at long times and distances of correlation functions is given by the finite size corrections to the groundstate energy and conformal field theory, and (iii) impurities may qualitatively change the properties [41]. For integrable systems the finite size corrections to the groundstate energy can be extracted from the Bethe ansatz solution and are given in terms of a series of quantum numbers, e.g. the change in the number of occupied states, the back scattering quantum number, one quantum number describing the particle-hole excitations about each of the Fermi points, and the Aharonov-Bohm-Casher phase-shifts. Each band (e.g. charges and spinons) with a Fermi surface (Luttinger liquid) has its own group velocity and contributes to the finite size corrections. The coupling between the bands due to the interaction is given by the matrix of generalized dressed charges. The finite size excitation spectrum constitutes the conformal towers.

The groundstate energy is expanded in powers of $1/L$. The extensive term represents the energy density in the thermodynamic limit, impurity scattering gives rise to terms independent of $L$, and the terms proportional to $1/L$ are the mesoscopic contributions yielding the conformal towers. Persistent currents have been observed experimentally in small metal and semiconductor rings in the geometry of the AB effect. Persistent charge (AB) and spin (AC) currents arise from the phase shifts picked-up by the wavefunction due to a magnetic flux threading the ring and a radially directed electric field, respectively. The quantization of the fluxes leads to periodic oscillations of the currents.

Our main results are the following:

- In [47] we studied the **period halving** of AB and AC oscillations due to the interference of Fermi surfaces of two Dirac seas. The calculation was performed for the AC pattern of an antiferromagnetic spin-1/2 Heisenberg ring with a concentration $x$ of impurity spins $S$ (see [41,44]). The persistent spin current reveals the two periods of oscillation associated with the two Fermi seas, corresponding to spinons and spin composites represented by strings of rapidities of length $2S$. The energy is piecewise parabolic, with the parity depending on the back-scattering quantum numbers, and shows microscopic discontinuities of the energy of the order of the Heisenberg uncertainty caused by the interaction between the two Fermi surfaces. Period halving and the energy jumps are a novel feature that appear quite generally in systems with more than one Fermi surface. For instance, similar results (for AC) are obtained for the Hubbard chain with attractive $U$ in a magnetic field large enough to close the spin-gap and the t-J model in non-zero magnetic field (AB interference). Period halving was experimentally observed, but it is not clear if it is caused by this mechanism.
- Experiments are always performed at finite $T$. It is therefore of interest to investigate the **effect of the temperature on the AB interference pattern**. For systems with one Fermi surface the groundstate persistent current has the shape of a saw-tooth. The temperature smears the Fermi surface and reduces the amplitudes of oscillation. The amplitude of higher harmonics decreases faster with $T$ than the fundamental one, changing the saw-tooth to a more sinusoidal form. The controlling parameter is $LT/zv$, where $v$ is the group velocity, and $z$ the dressed generalized charge [57]. We also derived the

temperature effects for two interfering Dirac seas and discussed the results in the context of the t-J and Hubbard models [69]. Impurity scattering introduces a 'Dingle temperature', explaining why experimentally always a sinusoidal pattern is observed.

- The finite size spectrum of the **multichannel Kondo problem** is of great interest because of the non-Fermi-liquid-like low-$T$ fixed point. We derived the conformal tower structure [53] of the finite size corrections to the groundstate energy for a spin-1/2 impurity and electrons in two orbital channels, including the effects of magnetic and crystalline fields [48,54]. The charge (one band) and spin (two bands) sectors always decouple. We considered two partial waves (forward and backward movers) and distinguished three cases. (i) In a crystalline field the two spin rapidity bands are Luttinger liquid-like. (ii) Without crystalline field splitting but in a magnetic field only one band of spin rapidities (the two-strings) contributes to the finite size corrections. (iii) In the absence of crystalline and magnetic fields, the parafermion sector of the two-level SU(2) Wess-Zumino-Witten model contributes giving rise to the non-Fermi-liquid properties. We used these results to study the AB and AC interference patterns. While the AB effect has no unusual features, the AC pattern shows up to three periods of oscillation and mesoscopic discontinuities in the energy as discussed above [53,64]. The $T = 0$ magnetoresistivity [53,60] is a continuous function of the crystalline and magnetic fields and depends only on the impurity magnetization (implicit function of the crystal field). The non-Fermi-liquid behavior re-appears in the low-$T$ dependence of the resistivity.
- We studied the persistent current oscillations of electrons in a mesoscopic **metallic ring coupled to a quantum dot** within the framework of the exact Bethe ansatz solution of the Anderson impurity model [55,58]. Electrons localized in the dot experience a large Coulomb repulsion, in analogy to the 3d or 4f electrons in a magnetic impurity, and the conduction electrons moving along the ring can tunnel in and out of the dot (hybridization). The dot can either be connected to the ring as a side branch or be embedded into the ring. For the embedded dot we used a simple model to discuss the Coulomb blockade resonances as a function of the voltage $V$ between the ring and the dot. For the side-branched dot the resistivity as a function of $V$ follows steps which occur in the "intermediate valent" region of the dot. The persistent current oscillations (periods, period halving, parity, and edge states in the dot) as a function of the magnetic flux were studied using the conformal towers for the Anderson impurity, and the t-J and Hubbard models. We discussed our results in the context of recent experiments.

### 5. Magnetic impurities in correlated electron lattices

Magnetic impurities in normal, noninteracting, metals are reasonably well understood. In actual materials, however, the conduction electrons interact among each other. This is especially the case in mixed-valent and heavy fermion systems, where states of ligand ions with d or p character are not uncommon. It is then important to understand the effect the correlations in the host have on the screening mechanism of the impurity spin. There are two reasons why we chose a one-dimensional lattice. (1) The interactions manifest

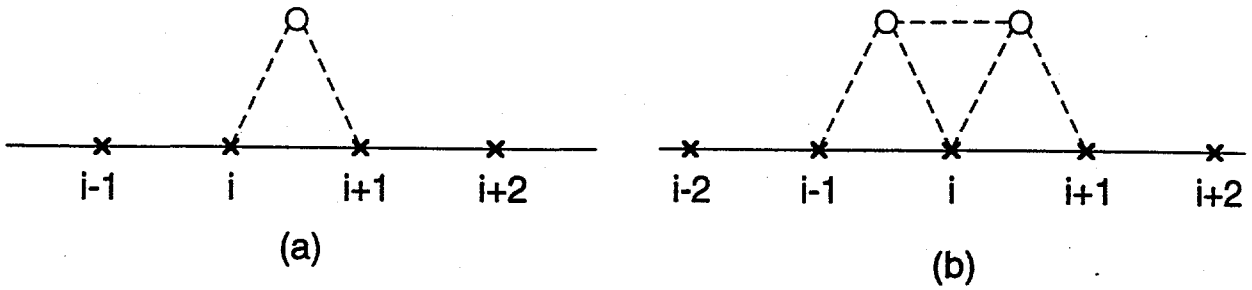


Fig. 3: Schematic representation of impurity spins interacting with itinerant electrons. The crosses indicate lattice sites of the host, the circles impurities and the lines denote interactions. (a) An isolated impurity only interacts with electrons on nearest neighbor lattice sites. (b) A pair of nearest neighbor impurities interacts with electrons on nearest neighbor lattice sites and among each other.

themselves most dramatically in 1D, where they transform the free electron gas into a Luttinger liquid. (2) We can construct integrable lattice systems using the quantum inverse scattering method and discuss the properties by solving the Bethe Ansatz equations.

The starting point in the quantum inverse scattering method is the scattering matrices (i) between the interacting conduction states, and (ii) the conduction electrons with the impurity. The scattering matrices have to be carefully chosen to satisfy the triangular Yang-Baxter relation (necessary and sufficient condition for the integrability). The monodromy (transfer) matrix can then be constructed as a function of a spectral parameter. On the one hand, the monodromy matrix yields the Bethe Ansatz equations diagonalizing the system, while on the other hand, the Hamiltonian (and higher conserved quantities) can be derived. The latter is rather tedious for lattice systems, but we succeeded to obtain the general expression for some of the cases discussed below. The impurity Hamiltonian is more accessible (and has been explicitly constructed) in the continuum limit, where the conduction states are still correlated (Luttinger liquid), but the interaction with the impurity has the form of a $\delta$-function. The solution of the Bethe Ansatz equations yields the properties of the impurity and the host.

We have studied the following combinations of impurity and host:

- A mixed valent impurity with two magnetic configurations of spin $S$ and $(S+1/2)$, respectively, embedded in the supersymmetric t-J model [62].
- A Kondo exchange impurity of spin $S$ in the supersymmetric t-J model [65].
- A mixed valent impurity with two magnetic configurations of spin $S$ and $(S-1/2)$, respectively, in the SU(3) invariant t-J model (BBB graded, i.e. spinons and holons play identical roles) [66].
- A magnetic impurity embedded into the Hubbard chain with repulsive $U$ [67].
- A magnetic impurity embedded into the Hubbard chain with attractive $U$ [67].
- A multichannel Kondo impurity incorporated into the t-J model with two channels [70].
- A Coqblin-Schrieffer impurity of spin $S$ in the in the SU$(2S+2)$ invariant supersymmetric t-J model ($B^{2S+2}$ graded, i.e. spinons and holons play identical roles) [76].
- An $N$-fold degenerate Anderson impurity in the $U \rightarrow \infty$ limit embedded into the $F^{N-1}B$ graded supersymmetric t-J model [77].

The overall picture emerging from the above investigation is

- Correlations in the host strongly couple to the charge sector without affecting the spin sector. The screening of the impurity spin is then unchanged with respect to the free electrons (same fixed points).
- The correlations drive the impurity away from integer valence into the mixed valence region with the concomitant increase of the Kondo temperature. A fraction of itinerant electron is localized at the impurity site. In the special case a Kondo exchange impurity of spin $S$ in the supersymmetric t-J model, not an electron but a hole is localized [65].
- An impurity embedded into a correlated lattice interacts with itinerant states of both partial waves, i.e. with states of even and of odd parity with respect to the impurity site. This contrasts the situation of a noninteracting host, where the interaction is only with even parity (s-wave) conduction states.
- In the case of gapped spin excitations (e.g. attractive Hubbard model) the spin compensation of the impurity is suppressed, but due to the integrability no boundstate is formed in the gap (lack of backward scattering). The impurity does not respond to fields smaller than the critical field (binding energy of "Cooper-pair"-like singlets).
- The multichannel Kondo fixed points remain unchanged, but the impurity acquires intermediate valence.

By construction the impurity resides on a link of the chain and interacts with both neighboring lattice sites (see Fig. 3 (a)). Hence, the impurity couples to itinerant electrons in both, even and odd parity states with respect to the impurity site. So far we succeeded to obtain the Hamiltonian in the continuum limit (Luttinger liquid) for almost all the cases and the general structure of the *lattice impurity Hamiltonian* in some of the cases. The explicit construction of the lattice impurity Hamiltonian (as a conserved current of the monodromy matrix) is a quite complicated task.

The above results for isolated magnetic impurities in a correlated electron lattice have been extended to finite concentrations of magnetic ions, i.e. to a *one-dimensional Kondo lattice or Kondo alloy.* The motivation for this calculation is the following. In many heavy fermion systems the thermodynamic properties

of alloys do not significantly differ from those of the stoichiometric compound. For instance, in $Ce_{1-x}La_xPb_3$ the thermodynamic properties scale with the Ce concentration over almost the entire range of $x$. Hence, the thermodynamics does not primarily depend on the coherence of the narrow heavy electron band nor on the distribution of the Ce ions on the Ce-La sublattice. There is a large (quasi-)degeneracy of states as a function of the distribution of Ce ions. On the other hand, the RKKY interaction leads to antiferromagnetic long-range order for $CePb_3$ and should play a role for all concentrations $x$. A successful approach should then treat the Kondo screening and the antiferromagnetic correlations between ions on equal footing and yield a quasi-locality structure (independence on the distribution of magnetic ions). Since the lattice includes forward and backward moving electrons, the magnetic moments interact with each other via (a 1D variant of) the RKKY interaction, giving rise to a large frustration of spins (in the sense of RVB the antiferromagnetic bonds cannot all be statisfied simultaneously). The magnetic moments are located on links of the chain, interacting with itinerant electrons on nearest neighbor lattice sites (see Fig. 3(b)). The starting point are again the scattering matrices and the Bethe Ansatz is then constructed via the quantum inverse scattering method. Since the scattering matrices satisfy the Yang-Baxter triangular relation, the model automatically has the locality structure built in, i.e. the physical properties are independent on the relative positions of the magnetic ions.

Three integrable one-dimensional lattice models displaying properties of heavy fermion alloys are constructed via the inverse quantum scattering method from the scattering matrices (a) between correlated itinerant electrons and (b) of electrons with the impurity. This includes Kondo screening, interaction among impurities and correlations between itinerant electrons all treated (for the first time) on equal footing. The concentration of heavy fermions and the conduction electron density can be varied continuously.

- Mixed valent impurities with two magnetic configurations of spin $S$ and $(S+1/2)$, respectively, embedded in the supersymmetric t-J model (FFB graded superalgebra) [70].
- Mixed valent impurities with two magnetic configurations of spin $S$ and $(S-1/2)$, respectively, in the SU(3) invariant t-J model (BBB graded superalgebra) [76].
- Magnetic impurities embedded into the Hubbard chain with attractive $U$ [77].

Some common aspects of these soluble models are:

- The number of itinerant electrons and the concentration of magnetic moments can be varied continuously over the entire range. The isolated impurity, the heavy fermion compound and the Heisenberg chain are contained as the limiting end-points.
- For magnetic moments of spin-1/2 the density of states of spinon rapidities acquires a second peak, which is interpreted as the Kondo impurity band. The spectral weight of this band strongly depends on the concentration of magnetic moments. As a function of magnetic field this band is depleted first (narrow band), giving rise to a steep increase of the magnetization. Two critical fields are obtained, corresponding to the van Hove singularity of the impurity band and the total spin polarization of the itinerant electrons.
- For magnetic moments of spin larger than 1/2 string states are important in the ground state. They are irrelevant only in the isolated impurity limit, where the spin is undercompensated. For a finite concentration the Fermi sea of strings is responsible for the spin-compensation into a singlet groundstate, i.e. the magnetic moments are antiferromagnetically correlated similarly as for a Heisenberg chain.
- A distribution of Kondo temperatures flattens the impurity bands causing a steep increase of the magnetization. This situation is studied in the context of the metamagnetic transition and non-Fermi-liquid behavior.
- The Hubbard model with attractive $U$ has a gapped spin excitation spectrum. The gap is reduced with increasing concentration of impurities and is closed at a critical concentration. There are no boundstates formed in the gap as a consequence of the integrability. For concentrations larger than the critical a novel itinerant ferromagnetic phase is obtained at $T = 0$, arising from the unpaired electrons. A fractions of the electrons is unpaired due to pair-breaking/weakening of the 'Cooper' pairs by the magnetic moments.

In summary, although only one-dimensional and hence without direct application to experiment, this is the first approach for the Kondo lattice (alloy) that treats the Kondo effect and RKKY simultaneously and on equal footing. The system to be studied contains many of the properties found in heavy fermions. Antiferromagnetic correlations lead to frustrations among the spins. This introduces a large degeneracy of

states (narrow band in the density of states), which is broken up by a magnetic field. Experimentally, in some heavy fermion alloys the frustration leads to a quantum critical point at $T = 0$ with non-Fermi-liquid properties (suppressed in our case by the 1D phase space).

## 6. Mixed valence in manganites

The discovery of a thousand-fold change in the resistivity of $La_{0.67}Ca_{0.33}MnO_3$ films as a function of the magnetic field, an effect now known as the Colossal Magneto-Resistance, renewed the interest in the compounds of the (La,Ca)$MnO_3$ family. The undoped end-compounds, $LaMnO_3$ and $CaMnO_3$, are insulating antiferromagnets with trivalent and tetravalent Mn ions, respectively. Substituting a fraction $x$ of divalent ions (e.g. Ca or Sr) for $La^{3+}$ introduces tetravalent Mn, and the materials become ferromagnetic and metallic at low $T$ for $0.2 < x < 0.5$, while long-range order of charges was observed by electron diffraction for $x \geq 0.5$. Recent studies on $^{18}O$ substitution strongly suggests that the coupling to the lattice, either in the form of polarons or the Jahn-Teller effect, plays an important role. This coupling to the lattice manifests itself in changes of the Mn-O bond lengths and the Mn-O-Mn bond angles, i.e. in local quadrupolar distortions in an otherwise nearly cubic environment.

The Mn ions are in a mixed valent state of two magnetic configurations, $Mn^{4+}$ and $Mn^{3+}$. $Mn^{4+}$ has three $t_{2g}$ electrons in the d shell and $Mn^{3+}$ has in addition a localized $e_g$ electron. All d electrons are ferromagnetically correlated (first Hund rule) forming spins $S = 3/2$ and $S' = 2$, respectively. The intermediate valence character of the Mn ions arises from the $e_g$ electron, which may be localized at the Mn ion or become itinerant either via hopping or hybridization. At low $T$ two types of mixed valence states have to be distinguished: (i) The $e_g$-electron hops (via the O p-orbitals) from one Mn ion to a nearest neighbor, so that the Mn ions are an inhomogeneous dynamic mixture of $Mn^{3+}$ and $Mn^{4+}$ (realized e.g. in $Sm_3S_4$ and $Eu_3S_4$). (ii) A linear superposition of the two atomic configurations via hybridization (intermediate valence, realized in heavy fermions). Depending on doping, the manganites interpolate between these two situations. The $e_g$ orbitals couple to the lattice in the form of polarons or the Jahn-Teller effect, giving rise to changes in the Mn-O bond-lengths (local quadrupolar distortions in an otherwise cubic environment). Lifting the degeneracy of the two $e_g$ levels corresponds to a local change in the Mn-O bond lengths.

We investigated the partial aspect of a single Mn ion embedded into a crystal represented by a band of conduction states. This has the advantage that the powerfull methods known for impurity problems can be used to treat all the details of the local aspects of a Mn ion. The results are expected to be complementary to those for a lattice of Mn ions without the local details. In this sense, as for the physics of heavy fermions (note: the manganites are similar to low carrier heavy fermion compounds), the single impurity problem could be an important step toward the understanding of the full system. Of special interest is the quadrupolar susceptibility, which is related to the lifting of the degeneracy of the $e_g$ orbitals, i.e. to the dynamical Jahn-Teller effect. We considered *three impurity models* that consist of the coherent superposition of the two electronic configurations via hybridization and contain all the correlations to satisfy the first Hund's rule.

**(1)** An adaptation of Tsvelik and Wiegmann's multichannel generalization of the Anderson impurity model, consisting of conduction states hybridized with localized d electrons in five orbitals. A strong exchange interaction among the d electrons favors aligned spins maximizing the total spin of the d-shell (first Hund rule). A cubic crystalline field quenches the orbital moment splitting the orbitals into $t_{2g}$ and $e_g$. A tetragonal crystalline field lifts the degeneracy of the $e_g$ orbitals. The model is integrable and we succeeded in calculating the quadrupolar response in zero magnetic field and for the spin-polarized case as a function of cubic crystalline field [71].

**(2)** While model (1) involves the moment formation (Anderson model) of the $t_{2g}$ multiplet, in the second model their spin is incorporated directly within the framework of a mixed valence impurity with two magnetic configurations of spins $S = 3/2$ and $S' = 2$. Only the two channels for the $e_g$ orbitals need to be considered. This model is integrable by construction and the groundstate quadrupolar response in zero magnetic field and for the totally spin-polarized impurity have been calculated [72].

Both of these models have as advantage the integrability, so that an analytic solution is available, and as disadvantage that the number of d electrons varies between 3 and 5. Their applicability to Mn is then restricted to large cubic crystal fields. The results for both models are essentially the same, indicating that the problem of moment formation does not play any significant role in this case.

**(3)** The third model corresponds to model (2) but with an infinite Coulomb repulsion that prevents the two $e_g$ levels being occupied simultaneously [73], which is integrable only in the spin-polarized limit, where the $e_g$ levels are screened into a quadrupolar singlet by the quadrupolar Kondo effect. We studied this model in the integer-valence and the mixed-valence regimes, using several techniques.

- The $Mn^{4+}$ limit is obtained by means of a Schrieffer-Wolff transformation. The exchange Hamiltonian is diagonal in the orbital and the coupling is ferromagnetic. The renormalization of the model leads to a weak coupling fixed point, so that the impurity is asymptotically free [73].
- In the integer-valent $Mn^{3+}$ limit a Schrieffer-Wolff transformation yields a Hamiltonian that exchanges simultaneously the spin and orbital indices. The renormalization of this limit leads to a strong coupling fixed point and to simultaneous spin and quadrupolar Kondo effects. The spin is partially screened (undercompensated) from $S' = 2$ to $S = 3/2$, while a quadrupolar singlet is formed. The energy scale is of the form of the Kondo temperature, but depends on the spin [73]. We constructed the corresponding Yosida ground state wave-function.

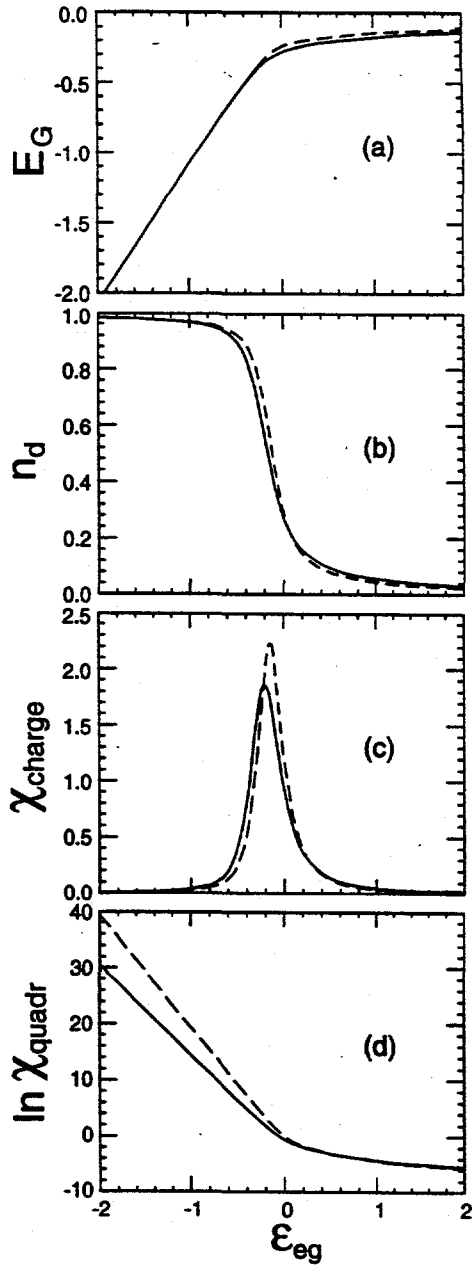


Fig. 4: (a) Ground state energy, (b) number of localized $e_g$ electrons, (c) charge and (d) (logarithm of) quadrupolar susceptibilities as a function of the energy of the $e_g$ electron, $\epsilon_{e_g}$ for a Mn imurity obtained within the non-crossing diagram approximation. The dashed curves correspond to the ferromagnetic lattice, while the solid curves represent the zero-magnetic field situation.

- In the mixed-valence regime we used (i) the saddle-point slave-boson formulation, (ii) the non-crossing diagram approximation, and (iii) the Bethe *Ansatz* (for the spin-polarized impurity only) [78]. We calculated the ground state energy, the valence, and the charge and quadrupolar susceptibilities as a function of $\epsilon_{e_g}$, the energy of the $Mn^{3+}$ configuration relative to the Fermi level, for zero-magnetic field and the spin-polarized limit. The results of the three methods agree remarkably well. The peaks of the charge and quadrupolar susceptibilities are larger in the spin-polarized case, indicating that the ferromagnetic lattice is more favorable for charge order and lattice distortions.

More definite answers to these questions will be obtained from the two-impurity model, which is currently under investigation. Here, the hopping of the $e_g$ electron between the two sites gvives rise to the double exchange mechanism. This ferromagnetic exchange competes with the antiferromagnetic superexchange between the spins.

## 7. Heavy fermion and Kondo systems

Characteristic of heavy fermions is an unusually large electron mass for states close to the Fermi surface which arises through the hybridization and high correlations via the competing action of the Kondo screening and RKKY-interactions. The competition of these mechanisms may also be the origin of the antiferromagnetic phase with low $T_N$ and faint ordered magnetic moment observed in some compounds ($UPt_3$, $CeAl_3$, $CeRu_2Si_2$), and the metamagnetic transition as a function of magnetic field. $T_N$ can be tuned by alloying other components and as $T_N \to 0$, some systems show non-Fermi-liquid-like properties.

The exact solution of the Coqblin-Schrieffer and Anderson impurity models via Bethe's ansatz in combination with $1/N$-expansion results (for instance the NCA for impurities and the Anderson lattice), gave excellent agreement between theory and experiment for numerous Ce and Yb systems, as well as for transition metal ions [81]. This analysis was particularly successful for light heavy-fermion compounds (large Kondo temperature), where crystalline fields and the RKKY interaction do not play a dominant role, e.g. (Ce,La)$Pb_3$, $CeSn_3$, YbCuAl and $YbCu_2Si_2$. For heavier heavy-fermions the RKKY interaction between sites plays an important role and gives rise to non-universal phenomena.

♣ We continued our efforts to obtain a detailed *quantitative comparison between theory and experiment* for Kondo systems. The theoretical basis for this comparison is the exact Bethe-ansatz solution of the degenerate Anderson impurity model. We considered the systems $YbPd_2Si_2$ [12] and $YbCu_4Ag$ [19].

- The available data for the specific heat, magnetic susceptibility, magnetization, valence, NMR Knight shift and relaxation rate, and the quadrupolar moment of $YbPd_2Si_2$ have been examined within the framework of the single-ion Anderson model. For $YbPd_2Si_2$ substantial crystalline field splittings make a quantitative comparison with existing exact solutions of the Anderson model difficult. Inconsistencies with a previously proposed interpretation that a nearly degenerate ground-quadruplet determines the low-temperature thermodynamics are pointed out [12]. It is concluded that at least three of the four Kramers doublets participate in the low $T$ properties. These three doublets should have an overall splitting of the order of $T_K$, i.e. about 100K. We also propose a simple resonant level model [12] to parametrize the NMR relaxation rate at low $T$. Finally, we suggest hydrostatic pressure experiments to study the nature of the faint ordered phase at very low temperatures (which could be surface superconductivity of segregated $Pd_2Si$).
- For $YbAgCu_4$ the crystalline field splittings are smaller than (but of the order of) $T_K$ and do not obscure a quantitative comparison [19] with existing exact solutions of the Anderson model. The available data for the specific heat, magnetic susceptibility, magnetization and inelastic neutron scattering have been examined within the framework of the single-ion Anderson model (see Fig. 5) and a simple resonant level model to parametrize the dynamic susceptibility [19]. Our results are reasonably consistent with previous interpretations in terms of the Coqblin-Schrieffer model.

♣ Dilute $\mathbf{Tb_xTh_{1-x}}$ alloys (small $x$) have been thoroughly studied in the light of a possible valence instability and Kondo effect in $Tb^{3+}$ and $Tb^{4+}$ ions. In collaboration with J. G. Sereni we reviewed the experimental data for the specific heat, susceptibility, magnetization, magnetoresistivity and the change of the superconducting properties. In analogy to Tm ($Tm^{2+}$, $Tm^{3+}$) and Pr ($Pr^{3+}$, $Pr^{4+}$), Tb involves two electronic configurations each with a magnetic Hund's rule ground-multiplet. We used results for a model of valence admixtures of two magnetic configurations to give a qualitative explanation of the

data within a scheme that combines Kondo screening with crystalline fields [52]. Besides Tm impurities in a metal, Tb in Th is only the second known example of *undercompensated* Kondo effect.

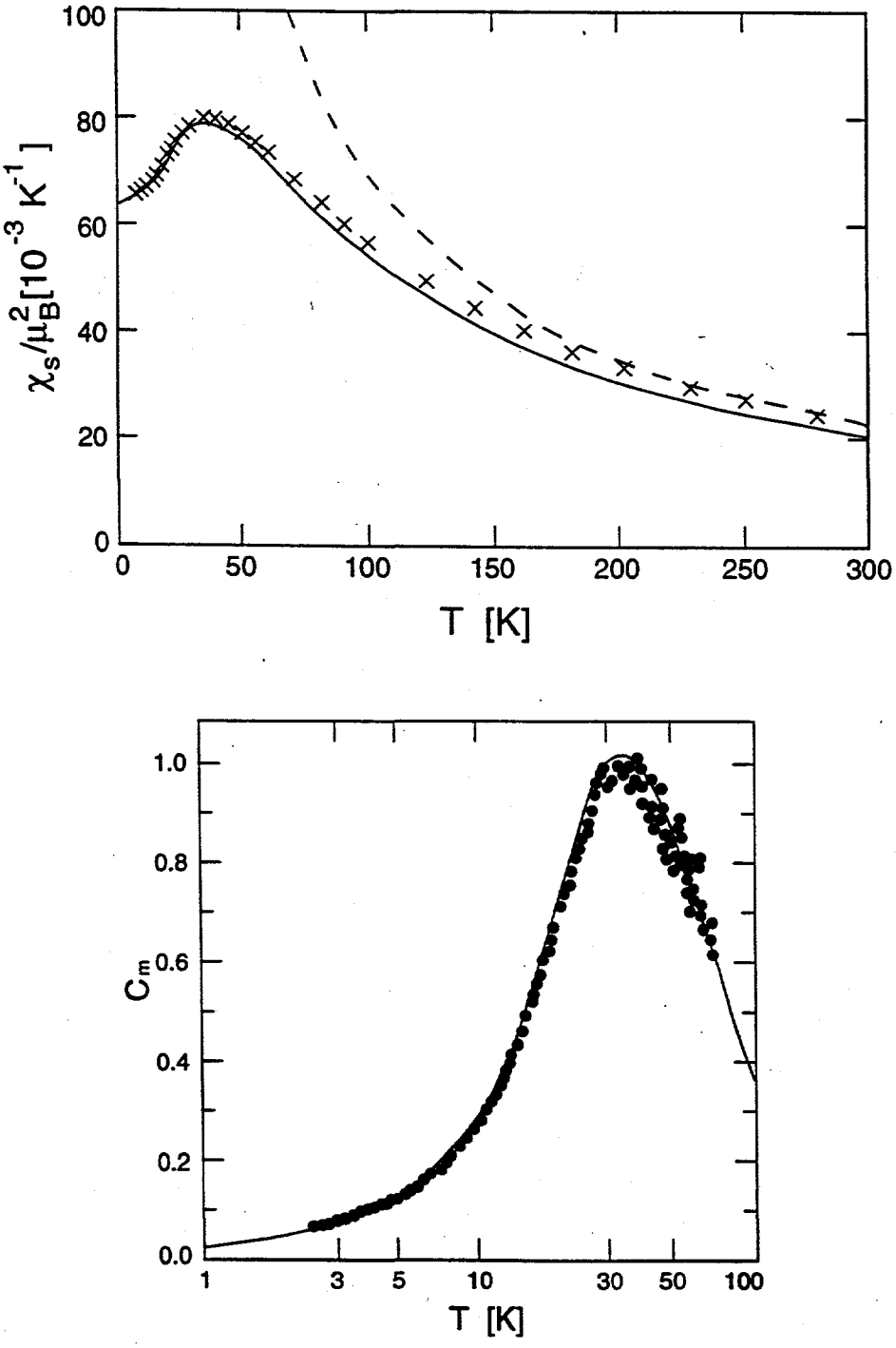


Fig. 5: Experimental magnetic susceptibility and specific heat (difference of $YbCu_4Ag$ and $LuCu_4Ag$) per Yb ion as compared to an Anderson impurity with $N = 8$, $\Gamma = 632K$, $(\epsilon - \mu)/\Gamma = -10.8$ and Landé factor $g = 8/7$. The dashed curve represents the Curie law.

For heavier heavy fermions (small Kondo temperature) the interaction between the different sites via RKKY acquires a crucial importance and cannot be neglected. The interplay between local and intersite correlations is expected to lead to non-universal behavior (antiferromagnetism, strongly enhanced paramagnetism, superconductivity, non-Fermi-liquid, and in some cases ferromagnetism or quadrupolar order). It is necessary to invoke at least two competing energy scales, one of them being the Kondo temperature. The two-impurity problem is the simplest model with two competing energy scales, namely the Kondo temperature and the RKKY interaction between the two spins. The two-impurity Kondo problem has been studied by numerous methods, in particular by the numerical renormalization group and conformal field theory. For strong ferromagnetic RKKY coupling between the impurities, their spins lock into a triplet state, which is compensated in analogy to the two-channel Kondo problem. For strong antiferromagnetic RKKY coupling, on the other hand, the spins of the two impurities compensate each other. These two fixed points are joined by a line of fixed points, that yields the non-universal properties. An unstable fixed point with non-Fermi-liquid properties was found by Jones *et al.* in the presence of electron-hole symmetry, which separates the attraction regimes of the two stable ones. Much of our efforts were directed towards an understanding of this unstable fixed point.

- ♣ The single impurity Kondo problem can be mapped onto a resonant level of spinless fermions with an attractive interaction between the localized and extended states (Falicov-Kimball interaction). We consider two such impurities at sites $\vec{R}_1$ and $\vec{R}_2$ interacting with each other via a hopping matrix element $t$ and an interaction $G$ between the localized fermions [11]. The interactions $t$ and $G$ resemble the RKKY-interaction between the impurities. The physics of the model is most conveniently discussed in terms of even and odd parity states with respect to the point $(\vec{R}_1 + \vec{R}_2)/2$. We obtain the $k$-space renormalization group equations for the model, which are integrated and discussed in terms of Ward-cancellations. Finally, approximate expressions for the static and dynamical susceptibilities for the response to a homogeneous and staggered field are obtained. No dramatic anomalies are found, probably as a consequence of the broken spin-rotational invariance of the model [23].
- ♣ We further considered the two-impurity Kondo problem within an equation of motion approach [28]. In a basis in which the electron states are given by their parity with respect to the midpoint between impurities, we considered the scattering of electrons by the spin-one complexes formed by the impurities, making use of the fact that the RKKY interaction is diagonal in this basis and acts in much the same way as a crystal field acts in the degenerate Kondo problem. By expanding the equations of motion to leading logarithmic order (Nagaoka approximation) we show that the resulting Kondo temperature is reduced with increasing antiferromagnetic RKKY interaction, and for one of the electron parity channels crosses the antiferromagnetic RKKY singlet. We interpret this crossing as reflecting the divergence of Fermi liquid properties found in numerical renormalization group studies along the lines of a phenomenological model introduced previously by us.

## 8. Magnetism in low-dimensional systems

We have studied several topics related to antiferromagnetically correlated Heisenberg spins in one and two dimensions.

- ♠ We considered an *isolated ferromagnetic bond* embedded into a two-dimensional Heisenberg antiferromagnet. Our motivation is that doping or impurities may introduce isolated ferromagnetic bonds, frustrating plaquettes of the square lattice, and be the origin of the spin-glass phase in $La_{2-x}Sr_xCuO_4$.
  - We first investigated the anisotropic spin-1/2 antiferromagnet on a square lattice within the linearized spinwave approximation (LSWA) [15]. Two competing interactions affect the local ordered magnetic moment $\langle S_{iz} \rangle$ and the correlation $\langle S_{ix} S_{jx} \rangle$: the longitudinal terms, $J_z$, tend to enhance the sublattice magnetization, while the transverse terms, $J_{xy}$, represent the quantum fluctuations that suppress the long-range order. We analyzed the interplay between these two effects as a function of $K_z$ and $K_{xy}$, the coupling constants of the impurity link. The LSWA breaks down for sufficiently large $K_z$ and $K_{xy}$ as a consequence of the frustration of the two plaquettes adjacent to the ferromagnetic bond. The work has been extended by Haas [31] to a missing link for an arbitrary spin of the anisotropic lattice.
  - Although the LSWA provides a reasonable description of the pure quantum antiferromagnet at

$T = 0$ (the groundstate has broken symmetry, the Néel state, and quantum fluctuations reduce the sublattice magnetization), the approximation breaks down due to frustration effects if a ferromagnetic bond is incorporated. The two spins joined by the ferromagnetic bond tend to form a triplet state with spin projection perpendicular to the Néel staggered magnetization. We devised a simple perturbative approach [36], in which the two-site cluster consisting of the spins linked by the ferromagnetic bond of coupling $K$ is embedded into the antiferromagnet treated within the LSWA. The resolvent for the local states of the cluster is calculated in three successive levels of approximation. We obtained the groundstate wavefunction, the magnetization at the impurity link and the transverse correlations across the impurity link as a function of $K$. This approximation scheme prevents the formation of a spurious infrared singularity, which lead to the breakdown of the LSWA [15].

- *Impurity-induced critical behavior* in antiferromagnetic Heisenberg chains can be obtained in close analogy to the $n$-channel Kondo problem [3,4,6,81]. We considered an integrable SU(2)-invariant model consisting of the Heisenberg chain of arbitrary spin $S$ (Takhtajan-Babujian model) interacting with an impurity of spin $S'$. The impurity is assumed to be located on a link of the chain and interacts only with both nearest neighbor sites. Our starting point is a set of commuting transfer matrices, whose local weights satisfy the triangular Yang-Baxter relations (the condition of integrability). The diagonalization of the transfer matrices yields the Bethe ansatz equations and the Hamiltonian of the model. The thermodynamics of the system has been studied. As in the $n$-channel Kondo problem three situations have to be distinguished [3,81]: (i) If $S' = S$ the impurity just corresponds to one more site in the chain. (ii) If $S' > S$ the impurity spin is only partially compensated at $T = 0$, leaving an effective spin $(S' - S)$ (undercompensated impurity). (iii) If $S' < S$ the entropy has an essential singularity at $T = H = 0$, giving rise to critical behavior as $H$ and $T$ tend both to zero. As a consequence a two-peak structure arises in the specific heat in a small but finite field and giant $\gamma$-values are obtained. These properties are in close analogy to those of the $n$-channel Kondo problem [6].

- We studied an integrable antiferromagnetic spin-1/2 Heisenberg chain containing a *finite concentration* $x$ of impurity spins $S$ [41,44]. $x$ adds one more parameter to the phase diagram without spoiling the integrability. We investigated the disappearance of the multicritical fixed point as a function of $x$ and the magnetic field, and found that the limits $x \to 0$, $H \to 0$ and $T \to 0$ cannot be interchanged [41,44]. A second spin rapidity band (in addition to the spinons) corresponding to string states of length $2S$ acquires finite spectral weight (proportional to $x$) [41]. This band plays a role only if $x \neq 0$ yielding a singlet groundstate, but not for a finite number of impurities. As a function of field the van Hove singularities of the empty bands define two critical fields $H_{c1}(x)$ and $H_{c2}(x)$, at which the susceptibility and the specific heat $\gamma$ coefficient diverge [44]. Both track the combined density of states of spinons and $2S$-strings. The non-Fermi-liquid behavior as $x \to 0$ arises from nonanalyticities as $H_{c2}$ vanishes. In [47] we obtained the finite size corrections to the ground state energy, the conformal towers of elementary excitations (see also section 4) and the Aharonov-Casher interference pattern.

- *Unbinding transition in a many-string system* [27]. We considered a system of $n$ thermally fluctuating strings on a square lattice. The strings are nonintersecting and subject to the solid-on-solid restriction, meaning that each string crosses a horizontal line across the system only once. The strings are under tension, i.e., changes in the coordinate $x$ cost energy, and they exert short-range forces on each other. Systems similar to this are of interest in connection with the wetting transition, the commensurate-incommensurate transition, the unbinding transition in membranes, and the statistics of "drunken walkers". Our work is focussed on the binding-unbinding transition that takes place in the presence of short-range attractive forces between strings as the temperature or the interaction strength is varied.

  We provided an exact solution for such a model [27]. For arbitrary finite $n > 2$ there is a second order binding-unbinding transition with the same critical exponents as for $n = 2$. In the limit $n \to \infty$ the transition becomes first order, in analogy with the transition from the Luttinger-liquid phase to the ferromagnetic phase in the anisotropic Heisenberg model in one-dimension. In the Luttinger-liquid phase the correlations of the strings decay as power laws with non-universal exponents.

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## REVIEW ARTICLES

[81] P. Schlottmann and P. D. Sacramento, Multichannel Kondo problem and some applications, Advances in Physics **42**, 641 (1993).

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[83] P. Schlottmann, Impurity states in Kondo insulators, in *Current Problems in Condensed Matter*, edited by J.-L. Morán-López, Plenum Press, (New York, 1997), p. 53.

## OTHER ACTIVITIES

[1] Short Course at the Naval Coastal Research Center, Panama City, June 1991; P. Schlottmann and P. Wise, lecturers. Superconductivity and applications.

[2] Member of the Local Organizing and Program Committees of the Conference on PHYSICAL PHENOMENA at HIGH MAGNETIC FIELDS, Tallahassee, May 1991.

[3] Member of the Program Committee of the International Conference on Physical Phenomena in Highly Correlated Systems, San Diego, 1993.

## INVITED TALKS

[4] Invited participant at the workshop on Novel Superconductors: Concepts, Models and Methods; Institute for Scientific Interchange Foundation, Turin, Italy, November 1991.

[5] Invited speaker at the II. Latin-American Workshop on Magnetism, Magnetic Materials and their Applications, Guanajuato, Mexico, 1993.

[6] Invited paper at the 38th Annual Conference on Magnetism and Magnetic Materials, Minneapolis, November 1993.

[7] International Conference on Strongly Correlated Electron Systems, Amsterdam, August 1994.

[8] International Summer School on Magnetism, Kharkov, Ukraine, September 1994 (Lecturer).

[9] International Conference on Strongly Correlated Electron Systems, Goa, India, September 1995.

[10] Workshop on Perturbative Methods for Strongly Correlated Electrons, Brasilia, Brazil, June 1996.

[11] XXI International Conference on Low-Temperature Physics, Prague, Czech Republic, August 1996.

[12] International Workshop on the Current Problems in Condensed Matter: Theory and Experiments, Morelos, Mexico, January 1997.

I was also invited to the Workshop on Exactly Solvable Models of Strongly Correlated Electrons held in July 1994 at the Aspen Center. Unfortunately I could not attend due to health reasons.

## ABSTRACTS SUBMITTED TO CONFERENCES

*March 1991 Meeting of the American Physical Society, Cincinnati*

[1] Low temperature properties of a two level system interacting with conduction electrons, P.D. Sacramento and P. Schlottmann.

[2] Influence of potential scattering on the diffusion of a heavy charged particle in a metal, P. Schlottmann and J. W. Rasul.

[3] A simple theory of the Kondo hole, R. Sollie and P. Schlottmann.

[4] Isolated ferromagnetic bonds in the two dimensional spin 1/2 Heisenberg antiferromagnet, K. Lee and P. Schlottmann.

[5] Thermal conductivity of impurity doped high $T_c$ superconductors, S. T. Ting, P. Pernambuco-Wise, J. E. Crow and P. Schlottmann.

*The 5th Joint MMM-Intermag Conference, Pittsburgh, June, 1991*

[6] Local density of states in the vicinity of a Kondo hole, R. Sollie and P. Schlottmann.

[7] Low temperature properties of a two level system interacting with conduction electrons, P.D. Sacramento and P. Schlottmann.

[8] Impurity induced critical behavior in antiferromagnetic Heisenberg chains, P. Schlottmann.

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[9] Some universal properties of one-dimensional systems with SU($N$) symmetry, P. Schlottmann.

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[10] Some applications of the $n$-channel Kondo problem, P. Schlottmann.

*March 1992 Meeting of the American Physical Society, Indianapolis*

[11] Two interacting magnetic impurities in a metal: Renormalization group treatment of a simple model, P. Schlottmann and J. W. Rasul.

*International Conference on Strongly Correlated Electron Systems, Sendai, Japan, 1992*

[12] Impurity bands in Kondo insulators, P. Schlottmann.

[13] Groundstate, excitation spectrum and thermodynamics of the degenerate supersymmetric t-J model in one-dimension, P. Schlottmann.

[14] Two interacting magnetic impurities in a metal: Renormalization group treatment of simple models, J. W. Rasul and P. Schlottmann.

*37th Annual Conference on Magnetism and Magnetic Materials, Houston, 1992*

[15] Magnetic instabilities in the Kondo lattice, V. Dorin and P. Schlottmann.

[16] The moderate heavy-fermion compounds $YbPd_2Si_2$ and $YbCu_4Ag$ at low temperatures, P. Schlottmann.

[17] Groundstate, excitation spectrum and thermodynamics of the degenerate supersymmetric t-J model with crystalline fields in one dimension, P. Schlottmann.

[18] Magnetic properties of an isolated missing link in the anisotropic two-dimensional Heisenberg antiferromagnet, S. Haas.

*XX International Conference on Low Temperature Physics, Eugene, August 1993*

[19] Exact solution of a simple exciton model in one-dimension, P. Schlottmann.

*II Latin-American Workshop on Magnetism, Magnetic Materials and their Applications, Guanajuato, Mexico, August 1993*

[20] The multichannel Kondo problem, P. Schlottmann, (invited paper).

[21] The one-dimensional Hubbard model with attractive $U$ as a soluble model for exciton bands in a semiconductor, P. Schlottmann.

*38th Annual Conference on Magnetism and Magnetic Materials, Minneapolis, November 1993*

[22] Effects of doping in impure Kondo insulators, P. Schlottmann (invited paper).

[23] Groundstate properties and excitations of an integrable one-dimensional two-band model with $\delta$-function interaction, P. Schlottmann.

[24] Perturbative approximation scheme for isolated impurity bonds in the two-dimensional spin-1/2 Heisenberg antiferromagnet, P. Schlottmann.

*International Conference on Strongly Correlated Electron Systems, Amsterdam, August 1994*

[25] Interacting Kondo holes in a Kondo insulator, P. Schlottmann.

[26] Some applications of the multichannel Kondo problem, P. Schlottmann and P. D. Sacramento (invited paper).

*International Conference on Magnetism, Warsaw, August 1994*

[27] Metal-insulator transition in an one-dimensional Hubbard-like model with degeneracy and crystalline field splittings, P. Schlottmann.

[28] Susceptibility and specific heat of the Heisenberg chain with finite concentration of impurities, P. Schlottmann.

*International Summer School on Magnetism, Kharkov, Ukraine, September 1994*

[29] Exact solution of two-band models for highly correlated electrons in one dimension, P. Schlottmann (lecturer).

*International Conference on Strongly Correlated Electron Systems, Goa, India, September 1995*

[30] Impure Kondo insulators, P. Schlottmann.

[31] Quenching of overcompensated Kondo impurities via channel asymmetry, P. Schlottmann and K. Lee (invited paper).

*40th Annual Conference on Magnetism and Magnetic Materials, Philadelphia, November 1995*

[32] Aharonov-Casher effect in a spin chain with two Fermi surfaces and Aharonov-Bohm oscillations at finite temperatures, A. A. Zvyagin and P. Schlottmann.

[33] Metal-insulator transition in dirty Kondo insulators, P. Schlottmann.

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[36] Impurity states in Kondo insulators, P. Schlottmann (invited paper).

*XXI International Conference on Low Temperature Physics, Prague, Czech Republic, August 1996*

[37] Impurity states in Kondo insulators, P. Schlottmann (selected contributed paper).

[38] Persistent current oscillations in a metallic ring embedding a quantum dot, A. A. Zvyagin, T. V. Bandos and P. Schlottmann.

*International Conference on Strongly Correlated Electron Systems, Zurich, Switzerland, August 1996*

[39] Two-channel Kondo impurity: Some transport properties, P. Schlottmann and A. A. Zvyagin.

[40] Impurity states in Kondo insulators, P. Schlottmann.

*41th Annual Conference on Magnetism and Magnetic Materials, Atlanta, November 1996*

[41] Some transport properties of a two-channel Kondo impurity, P. Schlottmann and A. A. Zvyagin.

*Workshop on Current Problems in Condensed Matter: Theory and Experiments, Cocoyoc, Morelos, Mexico, January 1997*

[42] Impurity states in Kondo insulators, P. Schlottmann (invited paper).