



# Stochastic Resonance in a Periodically Modulated Dissipative Nuclear Dynamics

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December 31, 2000

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## Abstract

A fission decay of highly excited periodically driven compound nuclei is considered in the framework of Langevin approach. We have used residual-time distribution (RTD) as the tool for studying of dynamic features in a presence of periodic perturbation. The structure of RTD essentially depends on the relation between Kramers decay rate and the frequency  $\omega$  of the periodic perturbation. In particular, intensity of the first peak in RTD has a sharp maximum at certain nuclear temperature depending on  $\omega$ . This maximum should be considered as first-hand manifestation of stochastic resonance in nuclear dynamics.

## 1 Introduction

The atomic nucleus since its discovery has been constantly used for verifying of new physical ideas such as tunneling [1], superfluids [2], superconductivity [3], supersymmetry [4], dynamical chaos [5]. Thus it seems unnatural that one of most recent and intriguing discoveries in nonlinear physics—stochastic resonance (SR) (see [6] for a recent review) up till have not found response of the nuclear community. This is particularly odd because there is no doubt that the theory of the collective nuclear motion pretending on a consistent description of nuclear dynamics must be essentially nonlinear theory. The aim of the present work is to demonstrate the principle possibility of observation of SR in nuclear dynamics. As a concrete example we consider a process of induced nuclear fission in the presence of weak periodic perturbation.

SR was introduced nearly 20 years ago to explain the periodicity of the Earth's ice ages [7, 8] and has found its numerous applications into such diverse fields like physics, chemistry and biology (see [6]).

The mechanism of SR can be explained in terms of the motion of a particle in a symmetric double-well potential subjected to noise and time periodic forcing. The noise causes incoherent transitions between the two wells with a well-known Kramers rate [9]  $\tau_k$ . If we apply a weak periodic forcing noise-induced hopping between the potential wells can become synchronized with periodic signal. This statistical synchronization takes place at the condition

$$\tau_k^{-1} = \pi/\omega \quad (1)$$

where  $\omega$  is a frequency of periodic forcing. Two prominent features of SR arise from synchronization condition (1):

(i) signal-to-noise ratio does not decrease with increasing noise amplitude (as it happens in linear system), but attains a maximum at a certain noise strength (optimal noise amplitude can be found from (1) as  $r_k$  is simply connected with it);

(ii) the residence-time distribution (RTD) demonstrates a series of peaks, centered at odd multiples of the half driving period  $T_n = 2(n - \frac{1}{2})\frac{\pi}{\omega}$  with exponentially decreasing amplitude. Notice that if a single escape from a local potential well is the event of interest then RTD reveals the dynamics of the system more transparently than the signal-to-noise ratio. These signatures of SR are not confined to the special models, but occur in general bi- and monostable systems and for different types of noise.

## 2 Langevin Equation

Kramers [9] was the first to consider nuclear fission as a process of overcoming the potential barrier by the Brownian particle. A slow fission degree of freedom (with large collective mass) is considered as Brownian particle, and fast nucleon degrees of freedom — as a heat bath. Adequacy of such description is based on the assumption that the time of equilibrium achievement in the system of nucleon degrees of freedom is much less than the characteristic time scale of collective motion. The most general way of description of dissipative nuclear dynamics is Fokker-Planck equation [10]. However for demonstration of qualitative effects it is convenient to use Langevin equation [11] that is equivalent to Fokker-Planck equation but is more transparent. As it has been shown the description based on Langevin equation adequately represents nuclear dissipative phenomena such as heavy-ion reactions and fission decay [12, 13, 14] and possesses a number of advantages over Fokker-Planck description.

Because we only intend to qualitatively demonstrate SR in nucleus let us consider the simplest type of Langevin equation — one-dimensional problem with inertial  $M$  and friction  $\gamma$  parameters independent on coordinates. Fission coordinate  $R$  is considered as a coordinate of Brownian particle. The rest degrees of freedom play a role of heat bath being modeled by random force  $\xi(t)$ .

The particle motion is described by Langevin equation for canonically conjugate variables  $\{P, R\}$

$$\begin{aligned} \frac{dR}{dt} &= \frac{P}{M} \\ \frac{dP}{dt} &= -\gamma P - \frac{dV}{dR} + \xi(t) \\ \beta &= \gamma/M \end{aligned} \quad (2)$$

$\xi(t)$  is stochastic force possessing statistical properties of white noise:

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'), \quad D = \gamma T \quad (3)$$

The nuclear temperature  $T(\text{MeV}) = \sqrt{E^*/a}$  where  $E^*$  is an excitation energy and the level density parameter  $a = A/10$  ( $A$  being a mass number). The (deformation) potential  $V$  is given as [12]

$$V(R) = \begin{cases} 37.46(R-1)^2 \text{ (MeV)} & \text{for } 0 < R < 1.27 \\ 8.0 - 18.73(R-1.8)^2 \text{ (MeV)} & \text{for } R > 1.27 \end{cases} \quad (4)$$

(these are parameters of  $^{205}\text{At}$  nucleus [12]).

Plausible sources of periodic perturbation are considered below.

The discretized form of the Langevin equation is given by [13, 14]

$$\begin{aligned} R_{n+1} &= R_n + \tau P_n/M \\ P_{n+1} &= P_n(1 - \beta\tau) - \left[ \left( \frac{dV(R)}{dR} \right)_n - A \cos \omega t_n \right] \tau + \sqrt{\frac{2\beta MT\tau}{N}} \eta(t_n) \end{aligned} \quad (5)$$

Here  $t_n = n\tau$  and  $\eta(t_n)$  is a normalized Gaussian-distributed random variable which satisfies

$$\langle \eta(t) \rangle = 0, \quad \langle \eta(t_n) \eta(t_{n'}) \rangle = N \delta_{nn'} \quad (6)$$

Efficiency of numerical algorithm (5) was checked for the following cases:

(i)  $V = 0$ ,  $A = 0$ , where numerical and analytical results for  $\langle P^2 \rangle$  and  $\langle R^2 \rangle$  can be compared [12];

(ii)  $V \neq 0$ ,  $A = 0$ , where numerical and analytical values for Kramers decay rate  $r_k$  can be compared. According to [9]

$$r_k = \frac{\omega_{min}}{2\pi} \left[ \sqrt{\beta^{*2} + 1} - \beta^* \right] \exp(-\Delta V/T), \quad \beta^* = \frac{\beta}{2\omega_{max}} \quad (7)$$

Here  $\omega_{min}$  and  $\omega_{max}$  are the angular frequencies of the potential (4) at the potential minimum and at the top of barrier respectively,  $\Delta V$  is the height of the potential barrier. Numerical values of Kramers decay rates  $r_k^i$  for the time bin  $i$  is calculated by sampling the number of fission events  $(N_f)_i$  in the  $i^{th}$  time bin width  $\Delta t$  normalized to the number of events  $N_{total} - \sum_{j < i} (N_f)_j$

which have not fissioned

$$r_k^i = \frac{1}{N_{total} - \sum (N_f)_j} \frac{(N_f)_i}{\Delta t} \quad (8)$$

Comparison of (7) with asymptotic value of (8) was used for determination of the time interval  $\tau$ , which provides saturation for numerical integration (5). On the other hand, the interval  $\tau$  should be chosen larger than the correlation time of the random process  $\xi(t)$ . Results of numerical calculations are plotted on Fig.1 according to (8) under different number of time steps per unit nuclear time  $\hbar/MeV$ . One can see that even 20 steps per nuclear time provides a sufficient saturation.

### 3 Stochastic Resonance in Nuclear Fission

Now let us proceed to the description of expected effect — manifestation of SR in nuclear fission. In the absence of periodic forcing, RTD  $N(t)$  has the exponential form (see [6])  $N(t) \exp(-r_k t)$ . In the presence of the periodic forcing, one observes a series of peaks, centered at odd multiples of the half driving period  $T_\omega = 2\pi/\omega$ . The heights of these peaks decrease exponentially with their order number. These peaks are simply explained [15]. The best time for the particle to escape potential well is when the potential barrier assumes a minimum. A phase of periodic perturbation may be chosen in such a special way that the potential barrier  $V(R) - AR \cos(\omega t + \phi)$  assumes its first minimum at  $t = 1/2 T_\omega$ . Thus  $t = 1/2 T_\omega$  is a preferred residence time interval. Following "good opportunity" to escape occurs in a full period, when potential barrier achieves its minimum again. The second peak in the RTD is therefore located at  $3/2 T_\omega$ . The location of the other peaks is evident. The peak heights decay exponentially because the

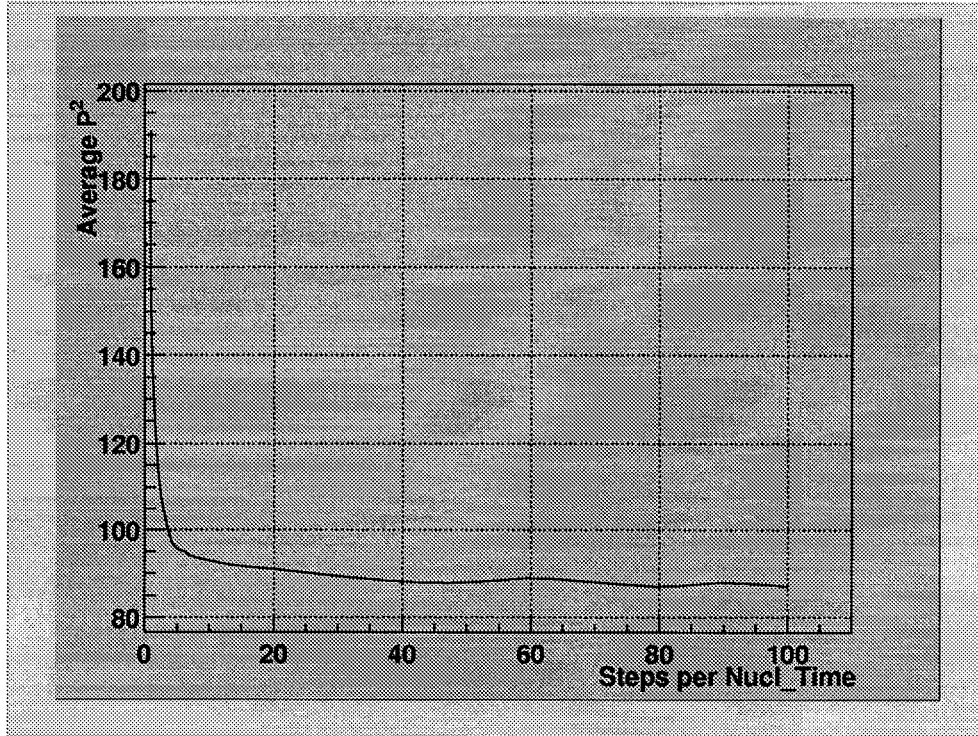


Figure 1: The results of numerical calculations for  $r_k$  under different number of time steps per nuclear time  $\hbar/MeV$

probabilities of the particle to jump over a potential barrier are statistically independent. As is shown for symmetric double-well potential [16], the strength  $P_1$  of the first peak at  $1/2 T_\omega$  (the area under peak) is a measure of the synchronization between the periodic forcing and the switching between the wells. So, if the mean residence time (MRT) of the particle in one potential well is much larger than the period of the driving, the particle is not likely to jump over the first time the relevant potential barrier assumes its minimum. The RTD exhibits in such a case a larger number of peaks where  $P_1$  is small. If the MRT is much shorter than the period of the driving RTD has already decayed practically to zero before the time  $1/2 T_\omega$  is reached and the weight  $P_1$  is again small. Optimal synchronization, i.e., maximum  $P_1$ , is reached when the MRT matches half driving period, i.e., condition (1). This resonance condition can be achieved by varying the noise intensity  $D$  (or  $\omega$ ).

We will show that the same correlation between periodic forcing and escape time takes place for a decay of excited states (fission) with a single potential minimum as well. For RTD constructing (and following  $P_1$  calculation) we use the numerical solutions of Langevin equation (5). Let us study evolution of  $P_1$  within the temperature interval  $1 MeV \leq T \leq 6 MeV$ . Corresponding Kramers rates  $r_k$  and resonance frequency satisfying (1) are represented in Table 1. Let us fix a frequency of periodic perturbation  $\omega = 0.0267 MeV/\hbar$  ( $T_\omega/2 = 117\hbar/MeV$ ) — a resonance frequency at  $T = 3 MeV$  (see Table 1). On account of the exponential decay of peaks heights in RTD ( $H_n \sim \exp(-r_k T_n)$ ,  $T_n = 2(n1/2)\pi/\omega$ ), one must observe a series of resonance peaks at  $T < 3 MeV$ . On the other hand, at  $T > 3 MeV$  (and for the same frequency of periodic perturbation) vast majority of nuclei would decay in a while shorter than  $T_1 \sim T_\omega/2$ . Due to this a sharp maximum of first peak intensity should be observed in the

Temperature (MeV)	Kramers decay rate (MeV/ $\hbar$ )	Resonant frequency(MeV/ $\hbar$ )
1	$4.13 \cdot 10^{-5}$	$1.3 \cdot 10^{-4}$
2	0.0023	0.007
3	0.0085	0.027
4	0.0166	0.050
5	0.0248	0.074
6	0.0324	0.097

Table 1:

vicinity of  $T \sim 3\text{MeV}$ , that is to be interpreted as a manifestation of SR.

The results of numerical procedure for RTD are presented on Fig.2. Pictures correspond to values of Kramers decay rate ( $T = 1 - 6 \text{ MeV}$ ) under fixed parameters of periodic perturbation ( $A = 1, \omega = 0.0267$ ). In accordance with expected behavior in the first case (at low  $r_k$ ) one can distinctly see three peaks located near  $t = T/2 (\sim 117.7), 3/2T (\sim 353), 5/2T (\sim 588)$ , and in the second case almost all RTD is concentrated near  $t = 0$  (with width less than  $T/2$ ). Connected with these variations of 1st peak intensity (that represents the measure of the synchronization between the periodic forcing and the nuclear temperature and consequently measure of SR) are depicted on Fig.3 for two frequencies of periodic perturbation. Maxima of intensities  $P_1$  coincide with chosen frequencies of periodic perturbation (see 1).

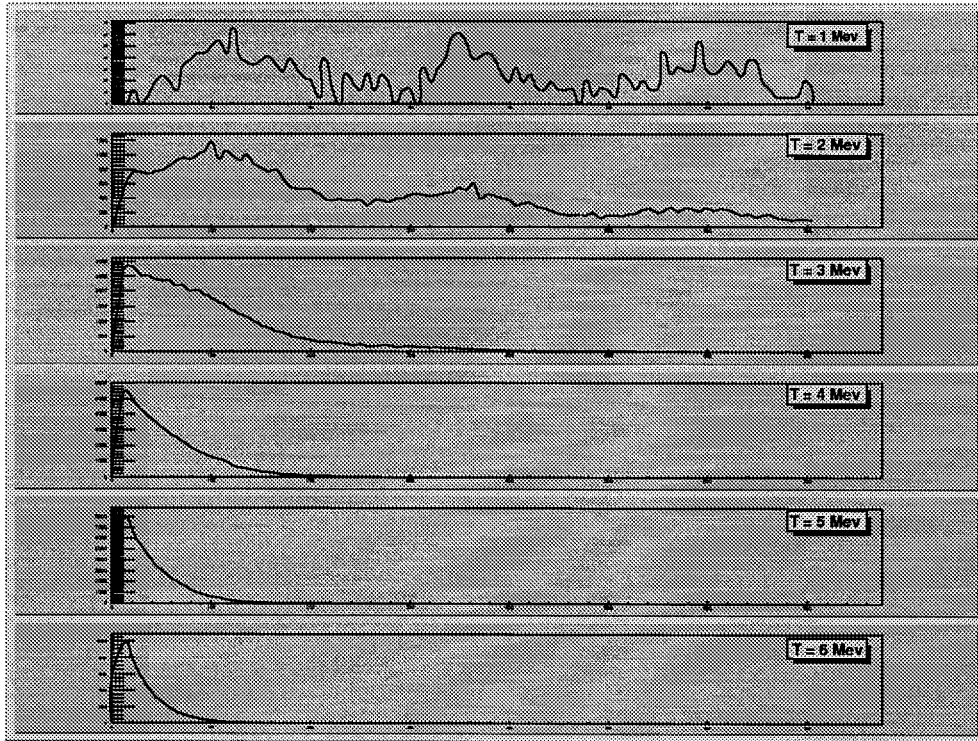


Figure 2: RTD for  $T = 1 - 6 \text{ MeV}$

In conclusion, let us briefly consider the possible sources of periodic perturbation. The first possibility is the fissile nucleus as a component of double nuclear system formed, for example, in heavy-ion collisions [17]. In this case, deformational potential will experience

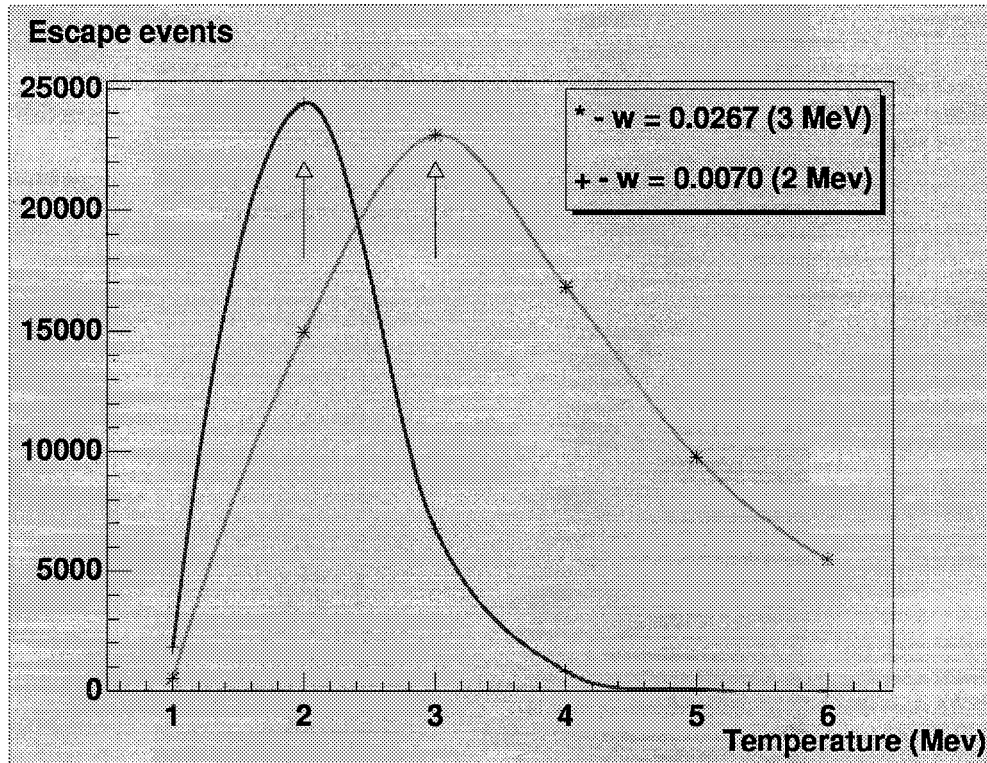


Figure 3: Dependence of  $P_1$  on  $T$  for two different  $\omega$ .

periodic perturbation similar to tide-waves on the Earth caused by the Moon rotation. In the case of asymmetric fission the source of periodic perturbation may be alternating electric field. The problem of choice of periodic perturbation would be discussed separately.

We would like to thank A.Yu. Korchin for valuable discussions.

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