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Title: Quantum Mechanics Without Statistical Postulates

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QUANTUM MECHANICS WITHOUT STATISTICAL POSTULATES

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Abstract The Bohmian formulation of quantum mechanics describes the measurement process in an intuitive way without a reduction postulate. Due to the chaotic motion of the hidden classical particle all statistical features of quantum mechanics during a sequence of repeated measurements can be derived in the framework of a deterministic single system theory.

1. INTRODUCTION

In the usual formulation of quantum mechanics the principal impossibility to predict the outcome of a quantum measurement and the associated probability distribution function for the possible results are usually introduced ad hoc together with the reduction of the wavefunction. In order to avoid the difficulties connected with this assumption, a deterministic formulation of quantum mechanics has been suggested by David Bohm [Bohm], where the dynamics of a nonlocal hidden variable x is determined by the wavefunction $\Psi(x, t) = R(x, t)e^{\frac{i}{\hbar}S(x, t)}$ via the potential $V_{\text{qu}}(x, t) := -\frac{\hbar^2}{2m} \frac{\partial^2 R(x, t)}{\partial x^2} + V(x, t)$. Originally an additional assumption concerning the distribution $\mathcal{P}(x, t)$ of the particles in a fictive ensemble $\{x_i\}$ had to be made, in order to recover all statistical features of the experiment [Bohm, Dürr]. In the following it will be sketched briefly, how this statistical assumption can be *derived* from the chaotic dynamics of the Bohmian particle, which shows that quantum mechanics can be un-

derstood completely on the basis of a nonstatistical formulation. Details can be found elsewhere [Geiger].

2. CHAOS IN THE MEASUREMENT

A series of von Neumann measurements with a consecutive reparation of the initial state $|\Psi|^2(x, t_i) := |\Psi|^2(x, t_0)$ is considered in the Bohmian formulation. This corresponds to the series of real physical experiments, which are necessary to verify quantum statistical properties in a *single* system.

This sequence of position measurements and reparation is illustrated schematically in the figure for the case of two possible outcomes $x \in [0, 1[$ or $x \in [1, 2[$:

- (a) At the initial time $t = t_0$ an arbitrary wavefunction $\Psi(x, y, t_0)$ is to be measured by a detector with a (highdimensional) coordinate $y = 0$.
- (b) During the von Neumann interaction $H_{WW} = \lambda i \hat{x} \partial / \partial y$ between the system coordinate x (with accuracy $\Delta x = 1$) and detector y there is a separation of the wavefunction $\Psi(x, y, t) = \sum_n c_n \Psi_n(x) \Phi_0(y - \lambda x t)$ in two parts along the detector coordinate y . The coordinate y of the particle after the measurement is stored (in another macroscopic coordinate z) and reflects the original position of the particle x in the intervals $[0, 1[$, $[1, 2[$.
- (c) After the measurement the wavepacket without the particle (here $[0, 1[$) does not influence the particle dynamics anymore (because of the locality of $V_{qu}(x, t)$) and can therefore be neglected.
- (d) For a new measurement with the same initial state (a) the remaining wavepacket has to flow from one to both intervals into the original form $|\Psi(x, y, t_0)|$.

Chaotic phenomena within the Bohmian quantum mechanics have been demonstrated repeatedly [Schwengelbeck, Dürr, Dwedney]. In the following it is pointed out that in this interpretation also the measurement is a deterministic process, whose chaotic properties can be studied. As the dimension of both the system x and the detector y is at least one, the Poincaré-Bendixson-Theorem, which excludes chaos in autonomous systems of dimension $n \leq 2$, is not applicable. Therefore and due to the high nonlinearity of the many particle system (including the detector) it is conceivable that the dynamics of the hidden variable (x, y) during the measurement process is in general chaotic and ergodic in particular.

One of the main consequences is the loss of predictability of the next measurement in a series of measurements. Although the position of the particle evolves deterministically from the initial value $x(0)$, the intrinsic inaccuracy Δx of any measurement together with the mixing property

of the dynamics prevents from the complete knowledge of the system for the future.

It will be shown next that the distribution $P(x)$ of measurement results in a series of measurements with reparation of $\Psi(x, t_0)$ at times t_i is given by $P(x) = |\Psi(x, t_0)|^2$, as it is demanded by experiment.

This argument also solves the problem of "quantum equilibrium" [Dürr] in an ensemble $\{x_i\}$ of independent systems with the same wavefunction $\Psi(x, t)$, but different positions $x_i(t)$ of the particles, which demands $\mathcal{P}(x, t) = |\Psi(x, t)|^2$ for their distribution function $\mathcal{P}(x, t)$.

Due to ergodicity of the trajectory $x(t)$ the distribution $P(x)$, which is obtained along one trajectory $x(t)$ at different times t_i , can also be expressed by the probability distribution $\mathcal{P}(x, t_0)$ of an appropriate fictive ensemble of particles $\{x_i\}$ at a fixed time t_0 :

$$P(x) = \mathcal{P}(x, t_0). \quad (1.1)$$

Formally this identity can be concluded from the coincidence of the time average $\langle x^k \rangle_t := \frac{1}{M} \sum_i x^k(t_i)$ and the ensemble average $\langle x^k \rangle_e := \frac{1}{V} \int x^k dx$ in ergodic systems for all natural k . Note that here the ensemble $\{x_i\}$ is not introduced by an additional statistical assumption, but follows from the proven ergodicity of the dynamical system under investigation. A short calculation using the continuity equations for $\mathcal{P}(x, t)$ and $|\Psi(x, t)|^2$ shows $\frac{d}{dt} f(x(t), t) = 0$ for any trajectory $x(t)$, $f(x, t)$ being

defined by $\mathcal{P}(x, t) = f(x, t)|\Psi(x, t)|^2$. As the trajectory is ergodic, it is a dense subset of phase space and $f(x, t)$ is constant in the whole accessible area. Because of the normalisation $\int \mathcal{P}(x, t)dx = \int |\Psi(x, t)|^2 dx = 1$ it follows that $f(x, t) = 1$ and

$$\mathcal{P}(x, t) = |\Psi(x, t)|^2 \quad . \quad (1.2)$$

Finally we can conclude that the distribution $P(x)$ of the real experimental results during a sequence of measurements in a single system is given by

$$P(x) \stackrel{\text{equ.1.1}}{=} \mathcal{P}(x, t_0) \stackrel{\text{equ.1.2}}{=} |\Psi(x, t_0)|^2 = |\Psi(x, t_i)|^2 \quad , \quad (1.3)$$

where the last step follows from the consecutive reparation of the system at times t_i in the initial state. This means that the density $P(x)$ of particle positions in a sequence of measurements is identical to $|\Psi(x, t)|^2$ of the wavefunction of the single systems which is prepared before each measurement.

3. CONCLUSION

Bohmian quantum mechanics gives the usual statistical predictions of quantum mechanics without any statistical assumptions within a deterministic, single system theory. The uncertainty in the result of a quantum mechanical measurement and the probability distribution $P(x) = |\Psi(x, t)|^2$ in a sequence of measurements follows from the interplay of the chaotic motion of the hidden variable $x(t)$ and the finite accuracy Δx of any real measurement.

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