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by Counter-propagating Laser Beams

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Parametric excitations of fast plasma waves by

counter-propagating laser beams

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Abstract

Short and long wavelength plasma waves can become strongly coupled in the presence of two counter-propagating laser pump pulses detuned by twice the cold plasma frequency $\omega_p$. What makes this four-wave interaction important is that the growth rate of the plasma waves occurs much faster than in the more obvious co-propagating geometry.

An important nonlinear process in plasma physics is the beatwave excitation of the electron plasma wave using high-frequency lasers, with applications including plasma heating and current drive [1,2], studying and controlling the ionosphere [3], and accelerating charged particles [4,5]. In this Letter we demonstrate that a fast plasma wave with phase velocity close to the speed of light can be generated by crossing two counter-propagating laser beams, which are detuned by $|\Delta \omega| \approx 2\omega_p$, where $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ is the plasma frequency, $-e$ and $m$ are the electron charge and mass, and $n_0$ is the plasma density. The counter-propagating geometry departs from the geometry employed in the traditional plasma beatwave accelerator (PBWA) approach to generating fast plasma waves for particle acceleration, which utilizes co-propagating laser pulses [6] detuned by $\Delta \omega = \omega_p$.

That a plasma wave can be driven unstable by the $2\omega_p$ beatwave was originally proposed by Rosenbluth and Liu (RL) [7], who calculate a growth rate of a fast plasma wave $\gamma_{RL} \approx \omega_p a_0 a_1/2$ (co-propagating lasers). Note that this decay is high-order, with growth rate going as pump amplitude squared. Thus, for pump waves of sub-relativistic intensity, i.e. $a_0, a_1 \ll 1$, this decay instability is too slow to be of great practical interest.
What we propose here is that a counter-propagating pump geometry results in a growth rate also second order in the pump amplitude, but strongly enhanced by the factor $2\omega_p^2/\omega_0^2$. We consider the four-wave interaction, in which the four participating waves are: the counter-propagating lasers, and two plasma waves, one (slow) with about twice the laser wavenumber, and one (fast) with the small wavenumber ($\omega_p/c$). For the similar reason why Raman backscattering is much faster than Raman forward scattering, here the counter-propagating geometry enhances the growth rate, but now in the much different context of a four-wave interaction in which there is decay to a fast plasma wave capable of particle acceleration.

To proceed, consider then the interaction of two counter-propagating laser beams (labeled by 0 and 1), with the corresponding normalized vector potentials given by $\vec{a}_{0,1} = a_{0,1}(\vec{e}_z \exp (i\theta_{0,1}) + c. c.)$, where $\vec{e}_z(\pm) = (\vec{e}_x \pm i\vec{e}_y) / 2$, $\theta_0 = k_0 z - \omega_0 t$, and $\theta_1 = k_1 z + \omega_1 t$. We assume that the duration of the forward-moving laser pulse is short (several plasma periods) and the duration of the backward-moving pulse is twice the length of the plasma. Tenuous plasma $\omega_p \ll \omega_0$ is assumed, ensuring that the lasers propagate almost as in vacuum: $v_{g0} \approx c$ and $|\vec{k}_0 - \vec{k}_1| \approx 2k_0$. The 4-wave instability we consider involves a short-wavelength (slow) and a long-wavelength (fast) plasma wave. The wavenumber of the slow wave is $k_s = 2k_0 - k_p$. The wavenumber of the fast wave $k_p$ is determined by the group velocity of the short pulse $v_{g,p}$: $k_p = \omega_p/v_{g,p} \approx \omega_p/c$.

Just as for the co-propagating geometry, the time-averaged ponderomotive force $\vec{F} = -mc^2 \nabla (\vec{a}_0 \cdot \vec{a}_1) \approx 2k_0 mc^2 \sin (2k_0 z - \Delta \omega)$ due to the pump lasers drives the plasma waves:

$$\ddot{\xi} + \omega_p^2 \xi = ik_0 e^2 a_0 a_1 e^{i(\Delta \omega t - 2k_0 z)} + c. c.,$$

where $\xi \equiv z - z_0$ is the Lagrangian electron displacement. For the co-propagating geometry, RL [7] used a single-wave ansatz for the plasma electron displacement $\xi = A(t) \sin [k z - \omega_p t + \phi(t)]$. The single-wave ansatz used by RL, however, is not sufficiently general for the case of counter-propagating lasers. Consider instead then the two-wave ansatz:

$$\xi = A_f \sin [k_f z_0 - \omega_p t + \phi_f] + A_s \sin [k_s z_0 - \omega_p t + \phi_s],$$

(2)
where $A_f (\phi_f)$ and $A_s (\phi_s)$ are the amplitudes (phases) of the fast and slow plasma waves. For simplicity, in the analytic calculation we assume monochromatic laser waves. Short-pulse effects are numerically treated later in the paper. Substituting $z = z_0 + \xi$, where $\xi$ is given by Eq. (2), into the RHS of Eq. (1) yields

\[
\frac{\partial^2 \xi}{\partial t^2} + \omega_p^2 \xi = i k_0 e^2 a_0 a_1 \sum_{k,l} (-1)^{k+l} J_k(2k_0 A_f) J_l(2k_0 A_s) e^{i[k \omega_0 - \omega_p t + \phi_f]} e^{i[l \omega_0 - \omega_p t + \phi_s]} e^{i[\Delta \omega t - 2k_0 \xi]} + \epsilon, \quad \epsilon,
\]

(3)

where $J_{k,l}$ are Bessel functions, and $\Delta \omega = \omega_0 - \omega_1 = 2 \omega_p + \delta \omega$. In writing the RHS of Eq. (3), we used the identity $e^{i \alpha \sin \phi} = \sum_k J_k(\alpha) e^{i k \phi}$. A set of purely time-dependent equations can now be obtained by separating the $z_0$ dependent terms on both sides of Eq. (3). Thus, substituting Eq. (2) into LHS of Eq. (3) and matching the corresponding harmonics of $k_p z_0$ and $k_s z_0$ on both sides of the equation, we can write for the $(k = 0, l = 1)$ and $(k = 1, l = 0)$ terms the following:

\[
\frac{\partial \phi}{\partial t} = \delta \omega - \frac{\Omega_B^2}{4} \omega_p G(A_f, A_s) \sin \phi
\]

(4)

\[
\frac{\partial (k_0 A_f)}{\partial (\omega_p t)} = \frac{\Omega_B^2}{4} J_0(2k_0 A_f) J_1(2k_0 A_s) \cos \phi
\]

(5)

\[
\frac{\partial (k_0 A_s)}{\partial (\omega_p t)} = \frac{\Omega_B^2}{4} J_1(2k_0 A_f) J_0(2k_0 A_s) \cos \phi,
\]

(6)

where $\phi = \phi_s + \phi_f + \pi/2 + \delta \omega t$, $\Omega_B^2 = 4 a_0 a_1 \omega_0^2 / \omega_p^2$ is the square of the electron bounce frequency in the optical lattice created by the interference of the counter-propagating lasers, and

\[
G(A_f, A_s) = \frac{J_0(2k_0 A_f) J_1(2k_0 A_s)}{k_0 A_f} + \frac{J_1(2k_0 A_f) J_0(2k_0 A_s)}{k_0 A_s}.
\]

Higher order Bessel terms are of the same order in the plasma wave amplitudes $2k_0 A_{f,s}$, and are assumed small.

At resonance $(\delta \omega = 0)$, the relative phase $\phi$ locks at $\phi = 0$. For small amplitudes of the plasma waves, $2k_0 A_{f,s} \ll 1$, Eqs. (5,6) can then be linearized, predicting the simultaneous exponential amplification of the fast and slow waves with the growth rate $\Omega_i = \omega_0^2 a_1 a_0 / \omega_p$. In
a nutshell, this is the principal result of this work: fast plasma wave capable of accelerating relativistic particles can be produced with a high temporal growth rate \( \Omega_i \). This growth rate is much higher than that predicted by RL for the co-propagating lasers: \( \Omega_i/\gamma_{RL} \approx 2\omega_0^2/\omega_p^2 \). The fast wave \( A_f \) grows so rapidly because it is parametrically coupled to the slow wave \( A_s \). The coupling mechanism is the ponderomotive force due to the counter-propagating optical mixing of the laser beams.

An important practical issue is the sensitivity of the instability to the deviation from the exact two-plasmon resonance \( \delta \omega \). For the finite frequency detuning from resonance \( \delta \omega \neq 0 \), there is an intensity threshold: phase-locking takes place only if \( \Omega_B^2/2 > \delta \omega/\omega_p \). Here, again, the counter-propagating geometry offers an advantage over the co-propagating case: the intensity threshold is given by

\[
\sqrt{I_0I_1}[W/cm^2] = 1.4 \times 10^{-3}(\delta \omega/\omega_p)n_0[cm^{-3}] \tag{7}
\]

For example, if the laser wavelengths are \( \lambda_0 = 0.8 \mu m \) and \( \lambda_1 = 1.0 \mu m \), and plasma density is \( n_0 = 10^{19} \text{ cm}^{-3} \) (corresponding to \( \omega_0 - \omega_p = 2.5\omega_p \)), the geometric mean of the laser intensities should exceed the threshold value of \( 8.0 \times 10^{15} \text{ W/cm}^2 \). Since this threshold is not too high, the instability is quite robust to the plasma inhomogeneity and detuning errors.

Equations (4,5,6) can be simplified by noting that, from the last two equations, \( J_0(2k_0A_f)/J_0(2k_0A_s) = \text{const.} \). If both waves start out negligibly small, the constant is equal to unity, and one can assume that \( A_f = A_s \) at all times. This assumption is only meaningful when the instability significantly amplifies both \( A_s \) and \( A_f \), so that the small absolute difference of the initial amplitudes is unimportant. The equations for the phase and the normalized amplitude \( u = 2k_0A_s = 2k_0A_f \) become

\[
\dot{\phi} = (\delta \omega/\omega_p) - \Omega_B^2 J_0(u)J_1(u)/u \sin \phi \tag{8}
\]

\[
\dot{u} = \frac{\Omega_B^2}{2}J_0(u)J_1(u) \cos \phi, \tag{9}
\]

where the dot indicates a derivative with respect to \( \omega_p t \). The conserved invariant of Eqs. (8,9) is \( \mathcal{H} = \Omega_B^2 u^2 \sin \phi - 2(\delta \omega/\omega_p) F(u) \), where
\[ F(u) = \int_{0}^{u} \frac{x^2}{J_0(x)J_1(x)} \, dx. \]

Note is that \( F(u) \) diverges for \( u \to \mu_0 \), where \( \mu_0 = 2.405 \) is the first zero of \( J_0 \).

For the excitation which starts out infinitesimally small \( \mathcal{H} \approx 0 \), and the sin \( \phi \) can be expressed in terms of the amplitude \( u \). The expression for the cos \( \phi \) is then substituted into Eq. (9):

\[ \dot{u} = \frac{J_0(u)J_1(u)}{2} \left[ \Omega_B^4 - \frac{4F^2(u)(\dot{\delta\omega})^2}{u^4\omega_p^2} \right]^{1/2}. \]  

Equation (10) gives the trajectory of the wave amplitude as a function of time. The plus (minus) sign corresponds to the increasing (decaying) portions of the trajectory. For a finite detuning \( \delta\omega \), the “motion” of \( u \) is periodic between its initial starting value \( u_0 \) and the maximum value \( u_{\text{max}} \).

For a perfect laser detuning \( \delta\omega = 0 \), the mode amplitude has a stable attractor at \( u = \mu_0 \). Since \( \mu_0 > 1 \), Eq. (3) no longer holds because of the breaking of the slow wave [8]. For \( 0 < (\delta\omega/\omega_p) < \Omega_B^2/2 \) the amplitude \( u \) oscillates periodically between its initially small value \( u_0 \) and \( u_{\text{max}} < \mu_0 \) which is found by solving the equation \( F(u_{\text{max}})/u_{\text{max}} = \Omega_B^2\omega_p/2\delta\omega \). This equation has no solutions for \((\delta\omega/\omega_p) < \Omega_B^2/2\), i.e. there is no instability. Defining \( \delta\omega \) according to \( \Omega_B^2/2 = (1 + \epsilon)(\delta\omega/\omega_p) \), we plotted in Fig. 1 the temporal evolution of \( u \) for a fixed \( \Omega_B = 1 \) and three different detunings corresponding to \( \epsilon = 0.2, 0.1, 0.05 \).

Analytic progress can be made in the limit of \( u < 1 \), which is, in any case, the applicability limit of Eq. (3). Then \( F(u) = u^2 + 3/16u^4 + \ldots \), and the maximum amplitude can be evaluated as \( u_{\text{max}} = 4\sqrt{\epsilon}/3 \). The oscillation period is given by \((\delta\omega)T = 8\sqrt{2/3}\ln[2u_{\text{max}}/u_0]/u_{\text{max}} \), where \( u_0 \ll u_{\text{max}} \) is the initial mode amplitude. Figure 1 confirms that the smaller is the peak amplitude of the wave, the longer is the oscillation period.

The physics of the amplitude oscillation can be understood as follows. Initially, \( u \) is very small, and since the ratio \( F(u)/u^2 \) is approximately a constant, the relative phase is locked at a constant \( \phi = \sin^{-1} 2(\delta\omega/\omega_p)/\Omega_B^2 \). As \( u \) undergoes an exponential growth, the phase “unlocks” and drifts towards \( \phi = \pi/2 \), at which time the amplitude peaks at \( u = u_{\text{max}} \) and
starts dropping. After the amplitude drops to its initial value $u_0$, the phase locks again, and 
the process repeats.

While the fastest instability corresponds to $\Delta \omega = \omega_s + \omega_f$, it is instructive to understand 
qualitatively how two plasma oscillations $\omega_s$ and $\omega_f$ (fast and slow) can become strongly 
coupled by a beatwave which has a frequency $\Delta \omega \neq \omega_s + \omega_f$. It turns out that both $\omega_f$ 
and $\omega_s$ are shifted from $\omega_p$ in the presence of the lasers. The simplified description of the 
instability, expressed by Eqs. (4-6), predicts that the frequency shifts are proportional to 
$(a_0a_1)^2$. Indeed, consider the small-intensity regime $(\delta \omega/\omega_p) \gg \Omega_B^2/2$. Then using $\phi \approx (\delta \omega)t$ 
and expanding Bessel functions to the lowest order in $A_{s,f}$, it can be shown from that both 
$\phi_s$ and $\phi_f$ acquire a time-averaged drift $\dot{\phi}_{s,f} = -\delta \Omega_{s,f}$, where $\delta \Omega_s = \delta \Omega_f = \Omega_B^4/(2 \delta \omega/\omega_p)$. 
Therefore, in the presence of the nonresonant beatwave the frequencies of both modes are 
shifted in the direction of $(\delta \omega)$. A rough estimate of the instability threshold can be obtained 
by requiring that $\delta \Omega_s + \delta \Omega_f = (\delta \omega/\omega_p)$. This results in $\Omega_B^2 = 4(\delta \omega/\omega_p)$, overestimating the 
earlier obtained expression for the intensity threshold by a factor 2. As shown below, there is 
an additional mechanism of shifting the frequency of the slow plasma wave via backscattering 
the short laser pulse. This frequency shifting can significantly modify the threshold intensity.

Since multiple plasma and laser waves are involved, Eqs. (4-6) describe the instability 
only approximately. Some of the missed effects are: (i) plasma perturbation driven at 
frequency $\Delta \omega$; (ii) modification of $a_1$ by the backscattering of $a_0$ off this driven density 
perturbation; (iii) the renormalization of the slow wave frequency due to its interaction with 
the short laser pulse. Therefore, we supplement the above calculation by a more rigorous 
two-scale particle simulation, which takes advantage of the scale separation between the 
short period of the slow plasma wave and a much longer period of the fast wave. We also 
assume for simplicity that the forward propagating laser pulse $a_0$ is much shorter than $a_1$.

The small-scale dynamics of the plasma electrons is characterized by their location (or 
phase) $\theta_j = \theta_0 + \theta_1 \approx 2k_0 z_j$ inside the optical lattice produced by the interference of the two 
lasers. Equations of motion for the $j$'s electron in a reference frame moving with the short 
pulse are described in Refs. [9,10]:
\[
\ddot{\theta}_j + \Omega_B^2 \sin (\theta_j - \Delta_0 \zeta) = - \sum_{i=1}^{\infty} \dot{n}_i e^{i \theta_j} - \bar{\varepsilon}_z + c. c.,
\]  
(11)

where a dot denotes a derivative with respect to \(\zeta = \omega_p (t - z/c)\), \(\dot{n}_i = i \langle e^{-i \theta_j / l} \rangle_{\lambda_0 / 2}\) is the \(l\)-th harmonic of the small-scale electron plasma wave averaged over one lattice period, and \(\Delta_0 = \Delta \omega_0 / \omega_p\). The global electric field \(\bar{\varepsilon}_z = 2 \omega_0 e E_z / m c \omega_p^2\) is generated owing to the average momentum deposition from the lasers into the plasma [11]. In normalized units, equations for \(\bar{\varepsilon}_z\) and \(a_1\) can be written as

\[
\frac{\partial \bar{\varepsilon}_z}{\partial \zeta} = \langle \ddot{\theta}_j \rangle_{\lambda_0 / 2}, \quad \frac{\partial a_1}{\partial \zeta} = - \frac{i \omega_p a_0^2}{4 \omega_0} \langle e^{-i \theta_j} \rangle_{\lambda_0 / 2}
\]  
(12)

Equations (11,12), supplemented by the initial conditions at \(\zeta = -\infty\), are numerically solved using macro-particles. As an initial condition, we assume that at \(\zeta = -\infty\) plasma is uniform \((\dot{n}_i = 0 \text{ for all } l)\) and stationary \((\dot{\theta}_j = 0 \text{ for all } j)\), and that a small initial fast plasma wave is present \((\bar{\varepsilon}_z = \bar{\varepsilon}_0)\). The presence of a much larger plasma wave inside the short pulse (taken here in the form \(a_0 = 0.5 \bar{a}_0 [\tanh (-\zeta / \tau_L) + 1]\)) indicates an instability.

The fast electric field \(E_z\) obtained by integrating Eqs. (11,12) is shown in Fig. 2 for two sets of laser field amplitudes \(a_0 \text{ and } a_1\). Simulation parameters are \(\omega_0 / \omega_p = 10\), \(\omega_0 - \omega_1 = 2.5 \omega_p\), and \(\bar{\varepsilon}_0 = 10^{-3}\). In Fig. 2(a) \(a_0 = a_1 = 0.06\) were assumed fixed. Evolving \(a_1\) according to the second Eq. (12) did not result in any significant change of \(E_z\). We also simulated the case of the fixed \(a_0 = 0.19 \text{ and } a_1 = 0.015\), which did not show any instability since in this case \(\Omega_B^2\) is smaller than in Fig. 2(a). However, when \(a_1\) was self-consistently evolved, a large electric field was excited, as shown in Fig. 2(b). This result is a manifestation of the physics which was not included in the above two-wave analysis which predicted that the threshold for the instability is determined by the frequency detuning \(\delta \omega\) and \(\Omega_B^2 = 4 a_0 a_1 \omega_0^2 / \omega_p^2\), which only depends on the product of the laser amplitudes, not on the individual amplitudes.

As was explained earlier, the instability threshold arises because the finite \(\Omega_B^2\) is needed to shift the frequencies of the fast and slow plasma waves to compensate for the frequency detuning \(\delta \omega\). However, there may be other mechanisms of frequency shifting unaccounted for by the two-wave treatment. In particular, it follows from Eq. (12) that a slow wave with
amplitude $\hat{n}_1 \sim e^{-i\zeta}$ excites a backward wave $\delta a_1 = \omega_p a_0^* \hat{n} / 4\omega_0 (\Delta_0 - 1)$, which then forms a beatwave with $a_0$ and acts back on the plasma electrons. Substituting $\delta a_1$ into Eq. (11), obtain an additional frequency shift of the slow plasma wave $\delta \Omega_s^z = \omega_p^2 |a_0|^2 / 4\omega_0 (\Delta \omega - \omega_p)$. For the simulation parameters of Fig. 2(b), this additional frequency shift, independent of $a_1$, effectively reduces the $\delta \omega = 0.5\omega_p$ frequency mismatch. Hence, $\Omega_B^2$ required to bridge the remaining gap is reduced as well. For the simulation parameters of Fig. 2(a) this reduction was negligible because of the smallness of $a_0^2$.

The relatively modest intensity threshold, given by Eq. (7), can be further lowered by employing a chirped laser pulse. Frequency chirp $\delta \omega(\zeta)$ also provides the benefit of suppressing the Raman backscattering of the more intense short pulse which can evolve from noise [12]. In Fig. 3 we plotted the amplitudes of the fast and slow plasma waves, $c_z$ and $\langle \cos \theta_j \rangle$, for a linearly-chirped Gaussian pulse. Assuming that $\lambda_l = 1\mu$m, the central frequency of the laser $\omega_0 = \omega_1 + 2.35\omega_p$ corresponds to $\lambda_0 = 810$ nm, and the plasma frequency $\omega_p / \omega_1 = 0.1$ corresponds to $n_0 = 10^{19}$ cm$^{-3}$, the pulse profile is as follows: $a_0 = 0.15 \exp \left[ -\zeta^2 / 2\tau_L^2 \right]$ with $\tau_L = 25$ (160 fs FWHM) and $d\delta \omega / d\zeta = -9.5 \times 10^{-3} \omega_p$ (3% bandwidth). The initial fast plasma wave $\hat{c}_0 = 10^{-3}$ and $a_1 = 0.0165$ have been assumed. In this example an accelerating plasma field of up to 9 GeV/m is generated.

In conclusion, we showed that large-amplitude fast plasma waves might be very effectively excited by two counter-propagating laser pulses detuned by approximately two plasma frequencies. In this arrangement, a slow plasma wave is incidentally excited, which is very effective in coupling the laser energy to the very useful for particle acceleration fast plasma wave.

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REFERENCES


FIGURES

FIG. 1. Fast and slow wave amplitude \( u = 2k_0A_J \) as functions of time for three detunings:

\[ \Omega_B^2/2 = (1 + \epsilon)(\delta \omega/\omega_p). \]  
Initial excitation \( a_0 = 10^{-3}, \Omega_B^2 = 1 \)

FIG. 2. Solid line: fast electric field \( \tilde{e}_z \), dashed line: normalized intensity of short pulse \( a_{0n}^2 \),
(a) \( a_0 = a_1 = 0.06 \), fixed \( a_1 \); (b) \( a_0 = 0.19, a_1(\zeta = 0) = 0.015 \), and \( a_1 \) is solved for from Eq. (12)

FIG. 3. Solid line: fast electric field \( \tilde{e}_z \), long-dashed line: normalized intensity of short pulse \( a_{0n}^2 \), dashed line: density bunching of the slow plasma wave \( \Re(\hat{n}_1) = \langle \cos \theta_j \rangle \). Rapidly-varying part part of \( \hat{n}_1 \) is the driven plasma response inside the laser pulse.
\( n = \varepsilon \) for different values of \( \varepsilon \): 
- \( \varepsilon = 0.2 \)
- \( \varepsilon = 0.1 \)
- \( \varepsilon = 0.05 \)
Normalized Intensity

\[ k_p (ct - z) \]

\[ \text{Normalized Intensity} \]

\[ e_z, \quad \text{Re}(n_1) \]

\[ \omega p / \omega m c \]

\[ 2 \pi \omega / \omega m c \]

\[ a_{0n} \]

\[ \text{Re}(n_1) \]

\[ \text{Normalized Intensity} \]

\[ k_p (ct - z) \]
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